Nambu–Jona-Lasinio Model of Dynamical Supersymmetry Breaking

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Motivation and Background

**Symmetry** is the organizing principle in the theory of modern physics.

- spontaneous symmetry breaking as a key feature of many theories
- Viable models with all symmetry breaking and mass generation arising *dynamically* have had limited success.

**Supersymmetry** (SUSY) is a symmetry of unique importance and undoubtedly a popular candidate for physics beyond the Standard Model.

- **soft SUSY breaking masses** as the seed for the electroweak symmetry breaking
- The *origin* of soft supersymmetry breaking masses has been usually depicted intricately in the literature via extra hidden/mediating sectors.
Motivation and Background

A simple theory for the generation of the soft masses would be more compelling.

- We present the prototype model with a four-superfield interaction term, which gives rise to supersymmetry breaking and soft masses dynamically.

- Along with the supersymmetry breaking, the presence of the expected Goldstino is verified.
In 1961, Y. Nambu and G. Jona-Lasinio proposed a classic model of dynamical mass generation and symmetry breaking, with the basic Lagrangian density being

\[ \mathcal{L} = -\bar{\psi} \gamma^\mu \partial_\mu \psi + g_0 \left[ (\bar{\psi} \psi)^2 - (\bar{\psi} \gamma_5 \psi)^2 \right]. \]

- strong attractive four-fermion interaction
- arise the bound state of fermion pair which behaves as a scalar composite
- dynamical electroweak symmetry breaking
- Dirac fermion mass
Explicitly, we have

\[ \mathcal{L} = i\bar{\psi} + \sigma^\mu \partial_\mu \psi + i\bar{\psi} - \sigma^\mu \partial_\mu \psi + g^2 \bar{\psi} \psi \psi + \bar{\psi} \psi - \psi + \psi \rightarrow \mathcal{L} - (\phi^* - g\psi \psi) (\phi - g\bar{\psi} \bar{\psi}) \]

\[ = i\bar{\psi} + \sigma^\mu \partial_\mu \psi + i\bar{\psi} - \sigma^\mu \partial_\mu \psi - \phi^* \phi + g (\phi^* \bar{\psi} \bar{\psi} + \phi \psi \psi) \]

- equation of motion for \( \phi^\dagger \) gives \( \phi = g\bar{\psi} \bar{\psi} \)
- no kinetic term for \( \phi \)
- non-zero \( \langle \phi \rangle \) gives electroweak symmetry breaking and fermion mass
The basic Lagrangian density for the Supersymmetric NJL Model is

\[ \mathcal{L} = \int d^4 \theta \left[ \bar{\Phi}_+ \Phi_+ + \bar{\Phi}_- \Phi_- + g^2 \bar{\Phi}_+ \Phi_- \Phi_+ \Phi_- \right]. \]

- Interaction term \( g^2 \int d^4 \theta \Phi_{+a} \Phi_{-a} \Phi_{+b} \Phi_{-b} \) is introduced.
- In component fields, \( \Phi = A(y) + \sqrt{2} \theta \psi(y) + \theta \theta F(y) \), one can see that it contains dimension-six operator \( \bar{\psi}_+ \bar{\psi}_- \psi_+ \psi_- \).

Buchmüller and Love (1982)
Supersymmetric Nambu–Jona-Lasinio (SNJL) Model

We can rewrite the Lagrangian by introducing two auxiliary chiral superfields and the soft SUSY breaking term.

\[ \mathcal{L} = \int d^4 \theta \left[ (\bar{\Phi} \Phi + \bar{\Phi} \Phi_0) (1 - \tilde{m}^2 \theta^2 \bar{\theta}^2) + \bar{\Phi}_1 \Phi_1 \right] \\
+ \int d^2 \theta [\mu \Phi_2 (\Phi_1 + g \Phi_0 \Phi_0)] + \text{h.c.} \]

The equation of motion for \( \Phi_2 \) gives \( \Phi_1 = -g \Phi_0 \Phi_0 \), implying

\[ \int d^4 \theta \bar{\Phi}_1 \Phi_1 = \int d^4 \theta g^2 \bar{\Phi} \bar{\Phi}_0 \Phi_0 \Phi_0 . \]

\( \Phi_1 \) is the composite scalar while \( \Phi_2 \) plays the Higgs superfield.

The soft SUSY breaking term is necessary to have a non-trivial vacuum.

Buchmüller and Ellwanger (1984)
Problems with NJL and SNJL Model

In the top mode Standard Model:

- Higgs as a top quark condensate
- top quark mass $> 200$ GeV

Bardeen et al. (1990); Marciano (1989,1990); Miransky et al. (1989); King and Mannan (1990,1991)

In MSSM of supersymmetric NJL:

- Lighter top quark mass is possible.
- Only very small $\tan\beta$ is allowed.
- NJL mechanism is used to break electroweak symmetry.

Carena et al. (1992)

Inspired by the NJL and SNJL models, we proposed a similar (but with a very different behavior) model, of which the NJL mechanism is to break the supersymmetry.
Consider a four-superfield interaction $g^2 \int d^4 \theta \Phi_+^a \Phi_-^b \Phi_+^a \Phi_-^b$:

- alternative color index contraction

- if $\Phi_+^a \Phi_+^a | \theta^2 \bar{\theta}^2 \rangle$ or $\Phi_+^a \Phi_+^a | \theta^2 \rangle$ develops, we will obtain soft supersymmetry breaking mass for $\Phi_-^b \Phi_-^b$.
Consider a four-superfield interaction \( g^2 \int d^4 \theta \Phi_{+a}^\dagger \Phi_{-}^b \Phi_{+}^a \Phi_{-}^b \):

- alternative color index contraction
- if \( \langle \Phi_{+a}^\dagger \Phi_{+}^a \rangle_{\theta^2 \bar{\theta}^2} \) or \( \langle \Phi_{+a}^\dagger \Phi_{+}^a \rangle_{\theta^2} \) develops, we will obtain soft supersymmetry breaking mass for \( \Phi_{-}^b \Phi_{-}^b \).

We start with a Lagrangian containing the dimension six interaction term of chiral superfields,

\[
\mathcal{L} = \int d^4 \theta \left[ \Phi^\dagger \Phi + \frac{m_0}{2} \Phi^2 \delta^2(\bar{\theta}) + \frac{m_0^*}{2} \Phi^\dagger \Phi^2 \delta^2(\theta) - \frac{g_0^2}{2} (\Phi^\dagger \Phi)^2 \right]
\]
We introduce **auxiliary real superfield** \( U \)

\[
U(x, \theta, \bar{\theta}) = \frac{C(x)}{\mu} + \sqrt{2} \theta \frac{\chi}{\mu} + \sqrt{2} \bar{\theta} \frac{\bar{\chi}}{\mu} + \theta \theta \frac{\mathcal{N}(x)}{\mu} + \bar{\theta} \bar{\theta} \frac{\bar{\mathcal{N}}(x)}{\mu} \\
+ \sqrt{2} \theta \sigma^\mu \bar{\nu}_\mu(x) + \sqrt{2} \bar{\theta} \bar{\theta} \bar{\chi}(x) + \sqrt{2} \bar{\theta} \theta \lambda(x) + \theta \theta \bar{\theta} \bar{\theta} D(x),
\]

and add \( \int d^4 \theta \frac{1}{2} (\mu U + g_o \Phi^\dagger \Phi)^2 \) to the original Lagrangian.

\[
\mathcal{L} = \int d^4 \theta \left[ \Phi^\dagger \Phi + \frac{1}{2} (\mu U + g_o \Phi^\dagger \Phi)^2 - \frac{g_o^2}{2} (\Phi^\dagger \Phi)^2 + \frac{m_o}{2} \Phi^2 \delta^2(\bar{\theta}) + \frac{m_o^*}{2} \Phi^\dagger 2 \delta^2(\theta) \right]
\]

\[
= \int d^4 \theta \left[ \Phi^\dagger \Phi + \frac{\mu^2 U^2}{2} + \mu g_o U \Phi^\dagger \Phi + \frac{m_o}{2} \Phi^2 \delta^2(\bar{\theta}) + \frac{m_o^*}{2} \Phi^\dagger 2 \delta^2(\theta) \right].
\]

\( \mu U = -\frac{g_o}{\mu} \Phi^\dagger \Phi \)

\( \text{soft SUSY breaking masses} \ - \mu g_o \langle U |_D \rangle = \tilde{m}_o^2, \ -g_o \langle U |_N \rangle = \tilde{\eta}_o \)
The effective Lagrangian in component form is given by

\[
\begin{align*}
\mathcal{L}_{\text{eff}} &= (1 + g_o C) \left[ A^* \Box A + i(\partial_\mu \bar{\psi})\bar{\sigma}^\mu \psi + F^* F \right] \\
&\quad + \frac{m_o}{2} (2AF - \psi \psi) + \frac{m_o^*}{2} (2A^* F^* - \bar{\psi} \bar{\psi}) \\
&\quad + \mu CD - \mu \chi \lambda - \mu \bar{\chi} \bar{\lambda} + NN^* - \frac{\mu^2}{2} \nu^\nu \nu^\nu - \mu g_o \psi \chi A^* - \mu g_o \bar{\psi} \bar{\lambda} A + \mu g_o D A^* A \\
&\quad - i \frac{g_o}{2} \bar{\psi} \bar{\sigma}^\mu \chi \partial_\mu A + i \frac{g_o}{2} (\partial_\mu \bar{\psi})\bar{\sigma}^\mu \chi A - g_o \chi \psi F^* + g_o N A F^* \\
&\quad + i \frac{g_o}{2} \bar{\chi} \bar{\sigma}^\mu \psi \partial_\mu A^* - i \frac{g_o}{2} A^* \bar{\chi} \bar{\sigma}^\mu \partial_\mu \psi - g_o \bar{\chi} \bar{\psi} F + g_o N^* A^* F \\
&\quad - \frac{\mu g_o}{\sqrt{2}} \eta^{\mu \nu} \nu_\mu i A^* \partial_\nu A + \frac{\mu g_o}{\sqrt{2}} \eta^{\mu \nu} \nu_\mu i(\partial_\nu A^*) A - \frac{\mu g_o}{\sqrt{2}} \eta^{\mu \nu} \nu_\mu \bar{\psi} \bar{\sigma}_\nu \psi.
\end{align*}
\]

\( \langle C \rangle \) corresponds to the wave function renormalization.
The tadpole diagrams are:

- The minimum conditions for scalar potential $V(C, N, D)$ are corresponding to the so-called **gap equations**.
Assuming nonzero VEVs of $C$, $N$, $D$, we have three \textit{gap equations}:

- $\langle C \rangle = -g \int^E \frac{(k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)}{(k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - 4|m|^2|\tilde{\eta}|^2}$

- $\tilde{\eta} = g^2 \tilde{\eta} \int^E \frac{(k^2 - |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)}{(k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - 4|m|^2|\tilde{\eta}|^2}$

- $\tilde{m}^2 = g^2 \int^E \frac{\tilde{m}^2 (k^2 - |m|^2) + 2k^2|\tilde{\eta}|^2 (k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2) - 8k^2|m|^2|\tilde{\eta}|^2}{(k^2 + |m|^2)\left((k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - 4|m|^2|\tilde{\eta}|^2\right)}$.

We expect to have a \textit{Goldstino} with the \textit{supersymmetry breaking}.

- loop corrected two-point function for the composite superfield $U$
- dynamically generated kinetic term
There are two fermionic components of $U$ – the $\chi$ and $\lambda$.

- **tree-level** Dirac mass term $\mu \chi \lambda$

- For $\langle N \rangle = 0$, $\chi \chi$ and $\lambda \lambda$ (Majorana) mass terms are not allowed by $U(1)_{R}$ symmetry.
There are two diagrams contributing to the $\chi \lambda$ mass.

\[ \Omega_{\chi\chi} = -\frac{g^2 \tilde{m}^4}{\tilde{\eta}} |m|^2 I_3 F(|m|^2, m_{A_+}^2, m_{A_-}^2) + \frac{1}{2\tilde{\eta}} (g^2 I_N + \tilde{m}^2 g^2 I_{N'}) , \]

\[ \Omega_{\chi\lambda} = 2\mu g^2 \tilde{m}^2 |m|^2 I_3 F(|m|^2, m_{A_+}^2, m_{A_-}^2) - \mu g^2 I_{N'} , \]

\[ \Omega_{\lambda\lambda} = -\mu^2 g^2 \tilde{\eta} |m|^2 I_3 F(|m|^2, m_{A_-}^2, m_{A_+}^2) . \]

- The determinant of the fermion mass matrix is zero, indicating the existence of a massless state.
Wave Function Renormalization

On the other hand, the diagrams for the wave function renormalization are:

\[
\begin{align*}
\chi & \quad \psi_R \quad \psi_R \quad \chi \\
F_R & \quad F_R \\
\chi & \quad \psi_R \quad \psi_R \quad \chi \\
A_R & \quad A_R \\
\chi & \quad \psi_R \quad \psi_R \quad \lambda \\
F_R & \quad A_R \\
\lambda & \quad \psi_R \quad \psi_R \\
A_R & \quad A_R \\
\lambda & \quad \psi_R \quad \psi_R \\
A_R & \quad A_R
\end{align*}
\]
Wave Function Renormalization

The diagrams give non-zero kinetic terms.

- **dynamically** generated kinetic term
- massless state with a kinetic term → we have the **Goldstino**.
The spin one vector boson $v^\mu$ is an important characteristic of the model.

- the proper **self-energy** diagrams
- properly behaved kinetic and mass terms generated
We have a dimension six interaction term $\frac{-g^2}{2} (\Phi^\dagger \Phi) (\Phi^\dagger \Phi)$ in the superfield model Lagrangian.

- The interaction is like a superspace version of four-scalar interaction.
- With the $\langle \Phi^\dagger \Phi \mid D \rangle$ or $\langle \Phi^\dagger \Phi \mid N \rangle$ coming from the dynamically induced two-superfield condensate, the interaction term can be the source of SUSY breaking.
- It may be possible for the $\Phi$ to be played by one of the supersymmetric Standard Model matter superfields.
- $m = 0$ case with pure D-term SUSY breaking also works and is particularly interesting because of no input mass scale.
Conclusion and Discussion

We present a new model to dynamically break supersymmetry, characterized by the generation of soft mass(es) and a spin one composite.

- Together with models of dynamical electroweak symmetry breaking, it is plausible to have a supersymmetric Standard Model without input mass parameter for which soft SUSY breaking and a subsequent electroweak symmetry breaking both being generated dynamically.

- No other SUSY breaking sector, messenger sector, or hidden sector is needed.

- All mass scales can be generated dynamically.
Thank you for your attention!