Nambu–Jona-Lasinio Model of Dynamical Supersymmetry Breaking

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(Collaboration with Yan-Min Dai, Gaber Faisel and Otto Kong)

Motivation and Background

Symmetry is the organizing principle in the theory of modern physics.

- spontaneous symmetry breaking as a key feature of many theories
- Viable models with all symmetry breaking and mass generation arising dynamically have had limited success.

Supersymmetry (SUSY) is a symmetry of unique importance and undoubtedly a popular candidate for physics beyond the Standard Model.

- **soft SUSY breaking masses** as the seed for the electroweak symmetry breaking
- The origin of soft supersymmetry breaking masses has been usually depicted intricately in the literature via extra hidden/mediating sectors.

Motivation and Background

A **simple** theory for the generation of the soft masses would be more compelling.

- We present the prototype model with a four-superfield interaction term, which gives rise to supersymmetry breaking and soft masses dynamically.
- Along with the supersymmetry breaking, the presence of the expected Goldstino is verified.

Nambu and Jona-Lasinio (NJL) Model Mechanism

In 1961, Y. Nambu and G. Jona-Lasinio proposed a classic model of dynamical mass generation and symmetry breaking, with the basic Lagrangian density being

Nambu and Jona-Lasinio (1961)

$$\mathcal{L} = -\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + g_0\left[\left(\bar{\psi}\psi\right)^2 - \left(\bar{\psi}\gamma_5\psi\right)^2\right].$$

- strong attractive four-fermion interaction
- arise the bound state of fermion pair which behaves as a scalar composite
- dynamical electroweak symmetry breaking
- Dirac fermion mass

Nambu and Jona-Lasinio (NJL) Model Mechanism

Explicitly, we have

$$\mathcal{L} = i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- + g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_-$$

$$\rightarrow\mathcal{L} - (\phi^* - g\psi_+\psi_-)\left(\phi - g\bar{\psi}_+\bar{\psi}_-\right)$$

$$= i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- - \phi^*\phi + g\left(\phi^*\bar{\psi}_+\bar{\psi}_- + \phi\psi_+\psi_-\right)$$

- equation of motion for $\phi^\dagger$ gives $\phi = g\bar{\psi}_+\bar{\psi}_-$
- no kinetic term for ϕ
- non-zero $\langle\phi\rangle$ gives electroweak symmetry breaking and fermion mass

Supersymmetric Nambu–Jona-Lasinio (SNJL) Model

The basic Lagrangian density for the Supersymmetric NJL Model is

$$\mathcal{L} = \int d^4\theta \ \left[\bar{\Phi}_+\Phi_+ + \bar{\Phi}_-\Phi_- + g^2\bar{\Phi}_+\bar{\Phi}_-\Phi_+\Phi_-\right] .$$

- Interaction term $g^2 \int d^4\theta \bar{\Phi}_{+a}\bar{\Phi}^a_-\Phi^b_+\Phi_{b-}$ is introduced.
- In component fields, *i.e.*, $\Phi = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$, one can see that it contains dimension-six operator $\bar{\psi}_+\bar{\psi}_-\psi_+\psi_-$.

Buchmüller and Love (1982)

Supersymmetric Nambu–Jona-Lasinio (SNJL) Model

We can rewrite the Lagrangian by intrducing two auxiliary chiral superfields and the soft SUSY breaking term.

$$\rightarrow \mathcal{L} = \int d^4\theta \ \left[\left(\bar{\Phi}_+\Phi_+ + \bar{\Phi}_-\Phi_-\right)\left(1 - \tilde{m}^2\theta^2\bar{\theta}^2\right) + \bar{\Phi}_1\Phi_1\right] + \int d^2\theta \left[\mu\Phi_2\left(\Phi_1 + g\Phi_+\Phi_-\right)\right] + \text{h.c.} \ .$$

- The equation of motion for Φ_2 gives $\Phi_1 = -g\Phi_+\Phi_-$, implying $\int d^4\theta \ \bar{\Phi}_1\Phi_1 = \int d^4\theta \ g^2\bar{\Phi}_+\bar{\Phi}_-\Phi_+\Phi_-$.
- Φ_1 is the composite scalar while Φ_2 plays the Higgs superfield.
- The soft SUSY breaking term is necessary to have a non-trivial vacuum.

Buchmüller and Ellwanger (1984)

Problems with NJL and SNJL Model

In the top mode Standard Model:

- Higgs as a top quark condensate
- top quark mass > 200 GeV

Bardeen *et al.* (1990); Marciano (1989,1990); Miransky *et al.* (1989); King and Mannan (1990,1991)

In MSSM of supersymmetric NJL:

- Lighter top quark mass is possible.
- Only very small tanβ is allowed.
- NJL mechanism is used to break electroweak symmetry.

Carena *et al.* (1992)

Inspired by the NJL and SNJL models, we proposed a similar (but with a very different behavior) model, of which the NJL mechanism is to break the supersymmetry.

Prototype Model of Superfields

Consider a four-superfield interaction $g^2 \int d^4\theta \Phi^{\dagger}_{+a} \Phi^{b\dagger}_{-} \Phi^{a}_{+} \Phi_{b-}$:

- alternative color index contraction
- if $\left\langle \Phi^{\dagger}_{+a} \Phi^{a}_{+} \Big|_{\theta^2\bar{\theta}^2} \right\rangle$ or $\left\langle \Phi^{\dagger}_{+a} \Phi^{a}_{+} \Big|_{\theta^2} \right\rangle$ develops, we will obtain soft supersymmetry breaking mass for $\Phi^{b\dagger}_{-} \Phi_{b-}$.

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We start with a Lagrangian containing the dimension six interaction term of chiral superfields,

$$\mathcal{L} = \int d^4\theta \left[\Phi^{\dagger}\Phi + \frac{m_0}{2}\Phi^2\delta^2(\bar{\theta}) + \frac{m_0^*}{2}\Phi^{\dagger\,2}\delta^2(\theta) - \frac{g_0^2}{2}\left(\Phi^{\dagger}\Phi\right)^2\right]$$

Component Field and Effective Theory Picture

We introduce auxiliary real superfield U

$$U(x,\theta,\bar{\theta}) = \frac{C(x)}{\mu} + \sqrt{2}\theta\frac{\chi}{\mu} + \sqrt{2}\bar{\theta}\frac{\bar{\chi}}{\mu} + \theta\theta\frac{N(x)}{\mu} + \bar{\theta}\bar{\theta}\frac{\bar{N}(x)}{\mu}$$
$$+\sqrt{2}\theta\sigma^{\mu}\bar{\theta}v_{\mu}(x) + \sqrt{2}\theta\theta\bar{\theta}\bar{\lambda}(x) + \sqrt{2}\bar{\theta}\bar{\theta}\theta\lambda(x) + \theta\theta\bar{\theta}\bar{\theta}D(x),$$

and add $\int d^4\theta\, \frac{1}{2}\left(\mu U + g_o\Phi^\dagger\Phi\right)^2$ to the original Lagrangian.

$$\mathcal{L} = \int d^4\theta \left[\Phi^\dagger\Phi + \tfrac{1}{2}(\mu U + g_o\Phi^\dagger\Phi)^2 - \tfrac{g_o^2}{2}\left(\Phi^\dagger\Phi\right)^2 + \tfrac{m_o}{2}\Phi^2\delta^2(\bar{\theta}) + \tfrac{m_o^*}{2}\Phi^{\dagger\,2}\delta^2(\theta)\right]$$
$$= \int d^4\theta \left[\Phi^\dagger\Phi + \frac{\mu^2 U^2}{2} + \mu g_o U\Phi^\dagger\Phi + \tfrac{m_o}{2}\Phi^2\delta^2(\bar{\theta}) + \tfrac{m_o^*}{2}\Phi^{\dagger\,2}\delta^2(\theta)\right] .$$

- $U = -\frac{g_o}{\mu}\Phi^\dagger\Phi$
- soft SUSY breaking masses $-\mu g_o \langle U\,|_D\rangle = \tilde{m}_o^2$, $-g_o\langle U\,|_N\rangle = \tilde{\eta}_o$

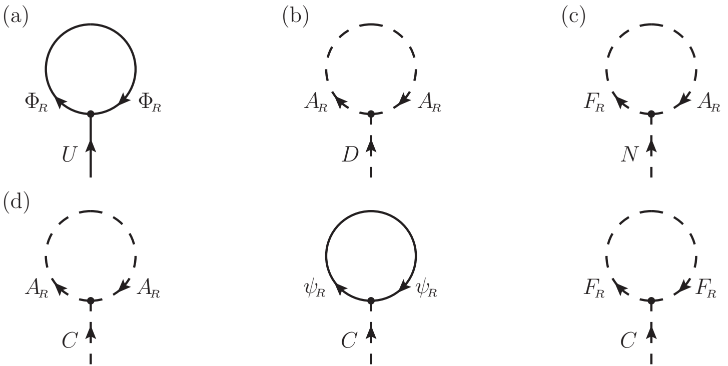
Component Field and Effective Theory Picture

The **effective Lagrangian** in component form is given by

$$
\begin{aligned}
\mathcal{L}_{eff} =& (1 + g_o C) \left[A^* \Box A + i(\partial_\mu \bar{\psi}) \bar{\sigma}^\mu \psi + F^* F \right] \\
&+ \frac{m_o}{2} (2AF - \psi\psi) + \frac{m_o^*}{2} \left(2A^* F^* - \bar{\psi}\bar{\psi} \right) \\
&+ \mu CD - \mu\chi\lambda - \mu\bar{\chi}\bar{\lambda} + NN^* - \frac{\mu^2}{2} v^\nu v_\nu - \mu g_o \psi\lambda A^* - \mu g_o \bar{\psi}\bar{\lambda} A + \mu g_o D A^* A \\
&- i\frac{g_o}{2} \bar{\psi}\bar{\sigma}^\mu \chi \partial_\mu A + i\frac{g_o}{2} (\partial_\mu \bar{\psi}) \bar{\sigma}^\mu \chi A - g_o \chi\psi F^* + g_o N A F^* \\
&+ i\frac{g_o}{2} \bar{\chi}\bar{\sigma}^\mu \psi \partial_\mu A^* - i\frac{g_o}{2} A^* \bar{\chi}\bar{\sigma}^\mu \partial_\mu \psi - g_o \bar{\chi}\bar{\psi} F + g_o N^* A^* F \\
&- \frac{\mu g_o}{\sqrt{2}} \eta^{\mu\nu} v_\mu i A^* \partial_\nu A + \frac{\mu g_o}{\sqrt{2}} \eta^{\mu\nu} v_\mu i (\partial_\nu A^*) A - \frac{\mu g_o}{\sqrt{2}} \eta^{\mu\nu} v_\mu \bar{\psi}\bar{\sigma}_\nu \psi \; .
\end{aligned}
$$

- $\langle C \rangle$ corresponds to the wave function renormalization.

The tadpole diagrams are:



- The minimum conditions for scalar potential $V(C, N, D)$ are corresponding to the so-called **gap equations**.

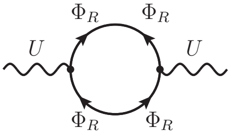
Component Field and Effective Theory Picture

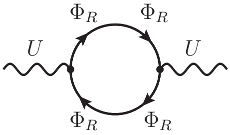
Assuming nonzero VEVs of C, N, D, we have three **gap equations**:

- $\langle C \rangle = -g \int^E \frac{(k^2+|m|^2+\tilde{m}^2+|\tilde{\eta}|^2)}{(k^2+|m|^2+\tilde{m}^2+|\tilde{\eta}|^2)^2-4|m|^2|\tilde{\eta}|^2}$
- $\tilde{\eta} = g^2 \tilde{\eta} \int^E \frac{(k^2-|m|^2+\tilde{m}^2+|\tilde{\eta}|^2)}{(k^2+|m|^2+\tilde{m}^2+|\tilde{\eta}|^2)^2-4|m|^2|\tilde{\eta}|^2}$
- $\tilde{m}^2 = g^2 \int^E \frac{\left[\tilde{m}^2(k^2-|m|^2)+2k^2|\tilde{\eta}|^2\right](k^2+|m|^2+\tilde{m}^2+|\tilde{\eta}|^2)-8k^2|m|^2|\tilde{\eta}|^2}{(k^2+|m|^2)[(k^2+|m|^2+\tilde{m}^2+|\tilde{\eta}|^2)^2-4|m|^2|\tilde{\eta}|^2]}$.

We expect to have a Goldstino with the **supersymmetry breaking**.

- loop corrected two-point function for the composite superfield U
- dynamically generated kinetic term

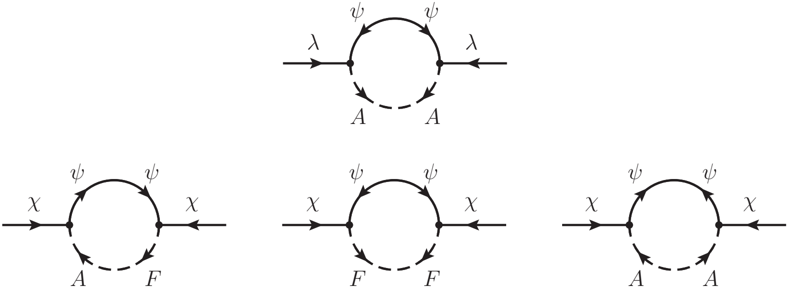




Goldstino in Component Field Theory

There are two fermionic components of U – the χ and λ.

- **tree-level** Dirac mass term $\mu\chi\lambda$
- For $\langle N\rangle = 0$, $\chi\chi$ and $\lambda\lambda$ (Majorana) mass terms are not allowed by $U(1)_R$ symmetry.



There are two diagrams contributing to the $\chi\lambda$ mass.

$$\Omega_{\chi\chi} = -\frac{g^2 \tilde{m}^4}{\tilde{\eta}} |m|^2 I_{3F}(|m|^2, m_{A_-}^2, m_{A_+}^2) + \frac{1}{2\tilde{\eta}} \left(g^2 I_C + \tilde{m}^2 g^2 I_{N'} \right) ,$$

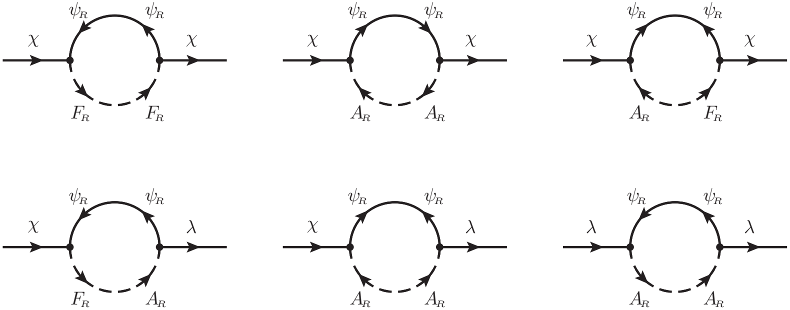
$$\Omega_{\chi\lambda} = 2\mu g^2 \tilde{m}^2 |m|^2 I_{3F}(|m|^2, m_{A_-}^2, m_{A_+}^2) - \mu g^2 I_{N'} ,$$

$$\Omega_{\lambda\lambda} = -\mu^2 g^2 \tilde{\eta} |m|^2 I_{3F}(|m|^2, m_{A_-}^2, m_{A_+}^2) .$$

- The determinant of the fermion mass matrix is zero, indicating the existence of a massless state.

Wave Function Renormalization

On the other hand, the diagrams for the wave function renormalization are



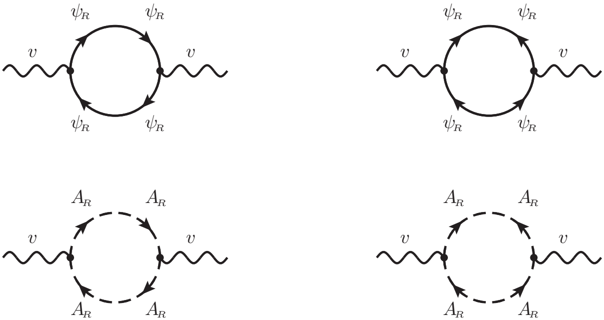
Wave Function Renormalization

The diagrams give non-zero kinetic terms.

- dynamically generated kinetic term
- massless state with a kinetic term $\rightarrow$ we have the Goldstino.

Spin One Vector Boson

The spin one vector boson v^μ is an important characteristic of the model.



- the proper self-energy diagrams
- properly behaved kinetic and mass terms generated

Remarks

We have a dimension six interaction term $\frac{-g^2}{2}\left(\Phi^\dagger\Phi\right)\left(\Phi^\dagger\Phi\right)$ in the superfield model Lagrangian.

- The interaction is like a superspace version of four-scalar interaction.
- With the $\left\langle \Phi^\dagger\Phi|_D \right\rangle$ or $\left\langle \Phi^\dagger\Phi|_N \right\rangle$ coming from the dynamically induced two-superfield condensate, the interaction term can be the source of SUSY breaking.
- It may be possible for the Φ to be played by one of the supersymmetric Standard Model matter superfields.
- $m = 0$ case with pure D-term SUSY breaking also works and is particularly interesting because of no input mass scale.

Conclusion and Discussion

We present a new model to dynamically break supersymmetry, characterized by the generation of **soft mass(es)** and a spin one **composite**.

- Together with models of dynamical electroweak symmetry breaking, it is plausible to have a supersymmetric Standard Model without input mass parameter for which soft SUSY breaking and a subsequent electroweak symmetry breaking both being generated dynamically.
- No other SUSY breaking sector, messenger sector, or hidden sector is needed.
- All mass scales can be generated dynamically.

Thank you for your attention!