Extended Higgs sectors and the alignment limit

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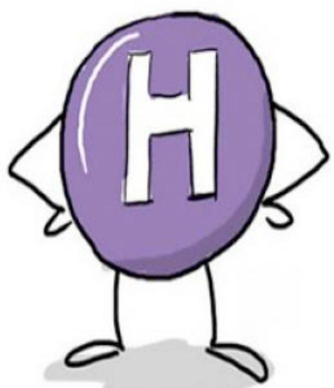

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Outline

- Evidence for a Standard Model (SM)-like Higgs boson
- Motivations for extended extended Higgs sector
- The alignment limit—approaching the SM Higgs boson
- Adding a Higgs singlet to the SM
- The two-Higgs doublet model (2HDM)
- Adding a Higgs singlet to the 2HDM
- Beyond Higgs singlets and doublets

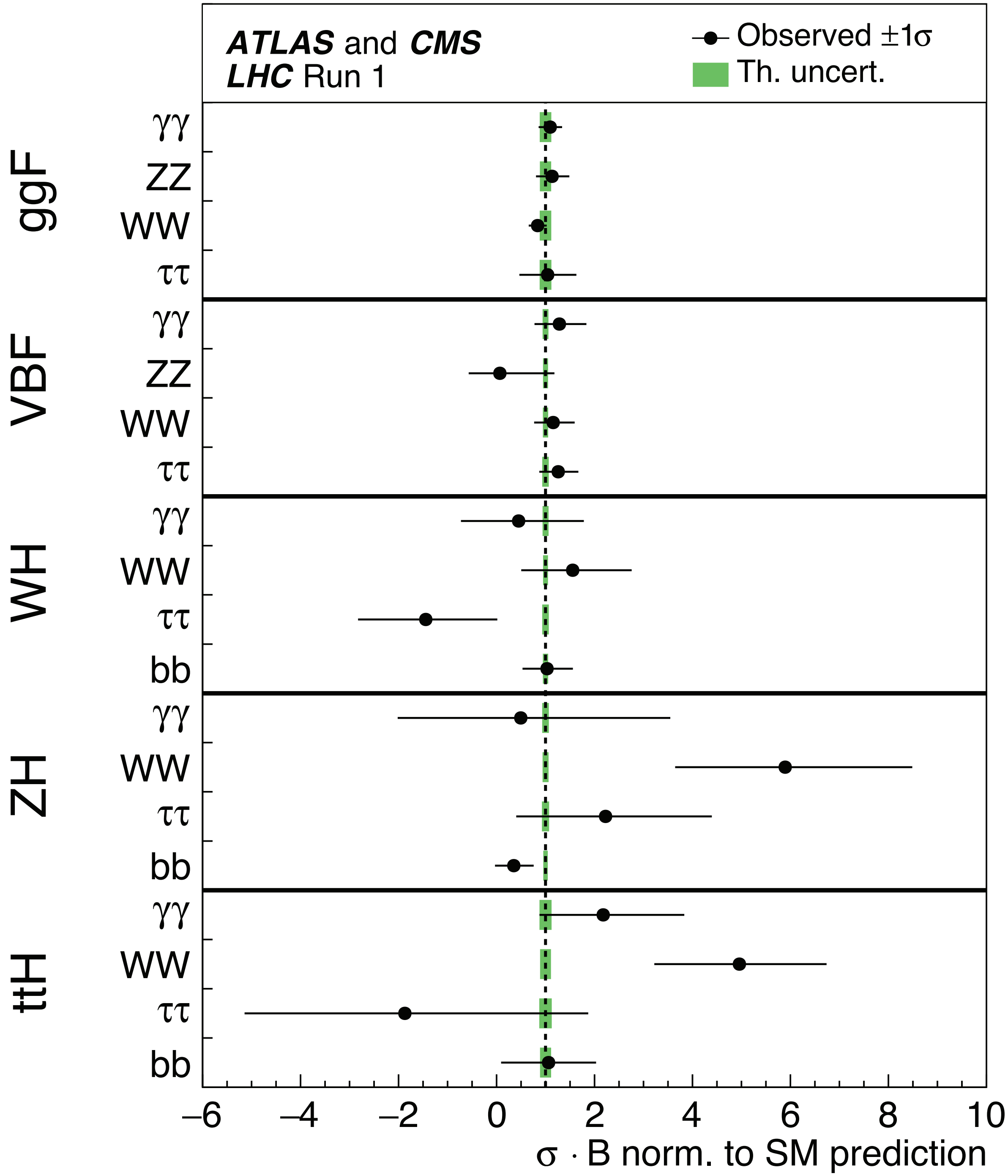
Evidence for a SM-like Higgs boson

In the summary slide of his talk on Monday, Eliot Lipeles summarized the results of the ATLAS Higgs experiment as follows:

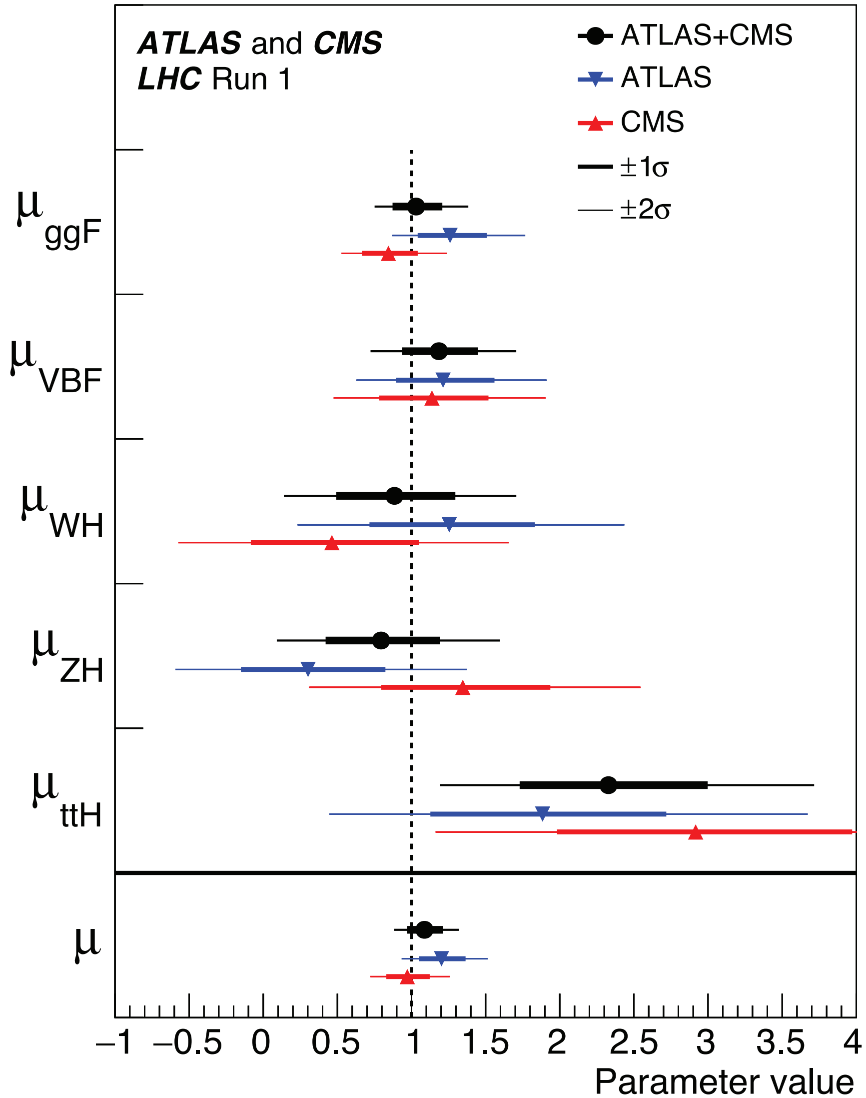
Summary

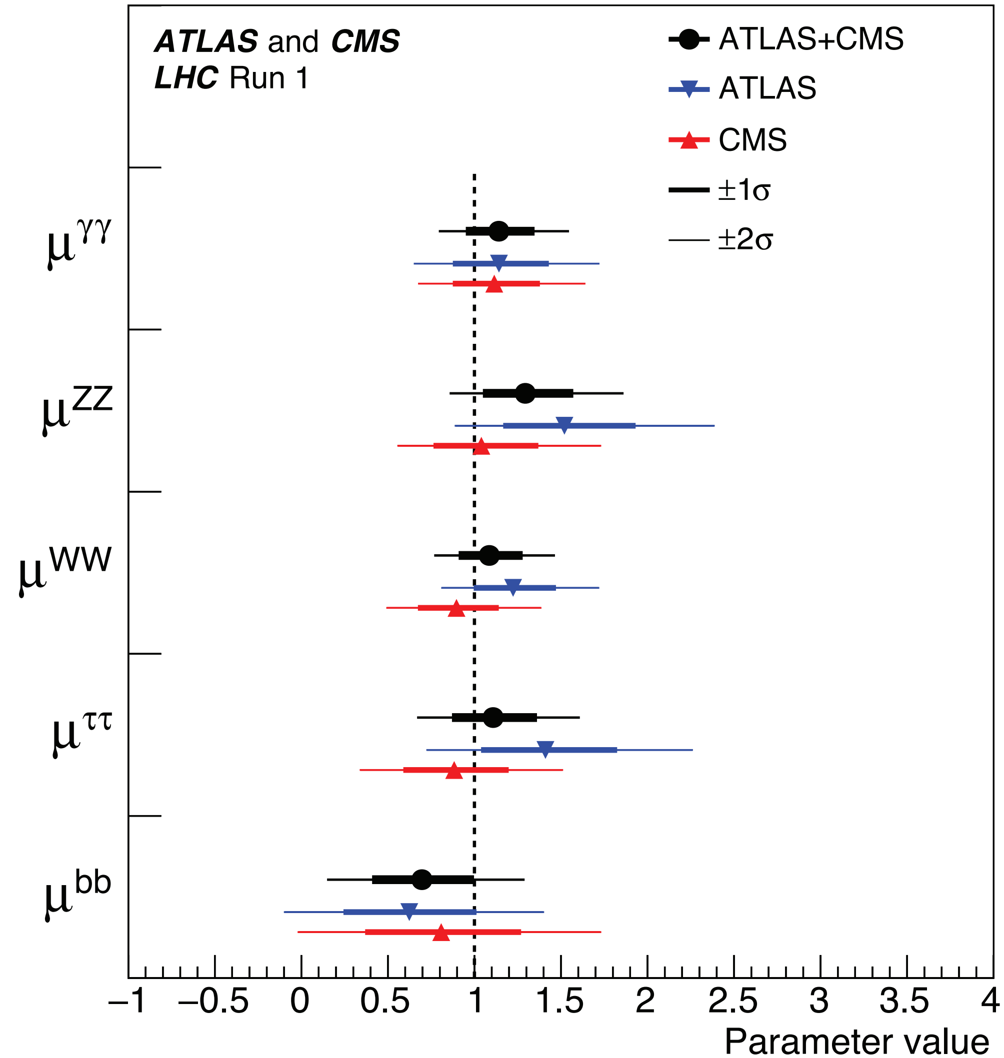
Four years ago today, a particle discovery was announced by ATLAS and CMS

We now know this particle strongly resembles the SM Higgs boson



ATLAS and CMS
LHC Run 1
Observed ±1σ
Th. uncert.
ggF
VBF
WH
ZH
ttH
γγ
ZZ
WW
ττ
bb
−6
−4
−2
0
2
4
6
8
10
σ · B norm. to SM prediction





Assumes that Higgs decays are according to the SM

Assumes that Higgs production modes are according to the SM

Motivations for an extended Higgs sector

- The fermion sector of the SM is not of minimal form ("Who ordered that?"). So, why should the scalar sector be minimal?

- Adding new scalar states can alleviate the metastability of the vacuum, allowing the Higgs-sector-extended SM to be valid all the way up to the Planck scale.

- Extended Higgs sectors can provide a dark matter candidate.

- Extended Higgs sectors can provide new sources of CP violation (which may be useful in baryogenesis).

- Models of BSM physics often require additional scalar Higgs states. The MSSM is a famous example of this.

For an arbitrary Higgs sector, the tree-level ρ-parameter is given by

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \quad \Longleftrightarrow \quad (2T+1)^2 - 3Y^2 = 1\,,$$

independently of the Higgs vacuum expectation values (vevs), where T and Y specify the weak-isospin and the hypercharge of the Higgs multiplet.* The simplest solutions are Higgs singlets $(T,Y)=(0,0)$ and hypercharge-one complex Higgs doublets $(T,Y)=(\frac{1}{2},1)$. These Higgs representations will be our main focus.

More generally, one can achieve $\rho=1$ by conspiracy if

$$\sum_{T,Y}\left[4T(T+1)-3Y^2\right]|V_{T,Y}|^2 c_{T,Y} = 0\,,$$

where $V_{T,Y}\equiv\langle\Phi(T,Y)\rangle$ and $c_{T,Y}=1$ for complex Higgs representations and $c_{T,Y}=\frac{1}{2}$ for real $Y=0$ Higgs representations.

*Y is normalized such that the electric charge of the scalar field is $Q=T_3+Y/2$.

The alignment limit—approaching the SM Higgs boson

Consider an extended Higgs sector with at n hypercharge-one Higgs doublets Φ_i and m additional singlet Higgs fields ϕ_i.

After minimizing the scalar potential, we assume that only the neutral Higgs fields acquire vevs (in order to preserve U(1)$_{\rm EM}$),

$$\langle\Phi_i^0\rangle = v_i/\sqrt{2}\,, \qquad\qquad \langle\phi_j^0\rangle = x_j\,.$$

Note that $v^2 \equiv \sum_i |v_i|^2 = 4m_W^2/g^2 = (246~\text{GeV})^2$.

We define new linear combinations of the hypercharge-one doublet Higgs fields (the so-called *Higgs basis*). In particular,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} = \frac{1}{v}\sum_i v_i^*\Phi_i\,, \qquad\qquad \langle H_1^0\rangle = v/\sqrt{2}\,,$$

and $H_2, H_3, \ldots, H_n$ are the other linear combinations such that $\langle H_i^0\rangle = 0$.

That is H_1^0 is aligned with the direction of the Higgs vev in field space. Thus, if $\sqrt{2}\,\mathrm{Re}(H_1^0) - v$ is a mass-eigenstate, then the tree-level couplings of this scalar to itself, to gauge bosons and to fermions are precisely those of the SM Higgs boson. This is the exact alignment limit.

In general, $\sqrt{2}\,\mathrm{Re}(H_1^0) - v$ is not a mass-eigenstate due to mixing with other neutral scalars. In this case, the observed Higgs boson is SM-like if either

- the mixing of $\sqrt{2}\,\mathrm{Re}(H_1^0) - v$ with other neutral scalars is suppressed,

  and/or

- the diagonal squared masses of the other scalar fields are all large compared to the mass of the observed Higgs boson (the so-called *decoupling limit*).

Although the alignment limit is most naturally achieved in the decoupling regime, it is possible to have a SM-like Higgs boson without decoupling. In the latter case, the masses of the additional scalar states could lie below ~ 500 GeV and be accessible to LHC searches.

Extending the SM Higgs sector with a singlet scalar

The simplest example of an extended Higgs sector adds a real scalar field S. The most general renormalizable scalar potential (subject to a $\mathbb{Z}_2$ symmetry to eliminate linear and cubic terms) is

$$\mathcal{V} = -m^2\Phi^\dagger\Phi - \mu^2 S^2 + \tfrac{1}{2}\lambda_1(\Phi^\dagger\Phi)^2 + \tfrac{1}{2}\lambda_2 S^4 + \lambda_3(\Phi^\dagger\Phi)S^2\,.$$

After minimizing the scalar potential, $\langle\Phi^0\rangle = v/\sqrt{2}$ and $\langle S\rangle = x/\sqrt{2}$. The squared-mass matrix of the neutral Higgs bosons is

$$\mathcal{M}^2 = \begin{pmatrix} \lambda_1 v^2 & \lambda_3 vx \\ \lambda_3 vx & \lambda_2 x^2 \end{pmatrix}\,.$$

The corresponding mass eigenstates are h and H with $m_h \leq m_H$. An approximate alignment limit can be realized in two different ways.

- $x \gg v$. This is the *decoupling limit*, where h is SM-like and $m_H \gg m_h$.
- $|\lambda_3|x \ll v$. Then h is SM-like if $\lambda_1 v^2 < \lambda_2 x^2$. Otherwise, H is SM-like.

The Higgs mass eigenstates are explicitly defined via

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \sqrt{2}\,\mathrm{Re}\,\Phi^0 - v \\ \sqrt{2}\,S - x \end{pmatrix},$$

where

$$\lambda_1 v^2 = m_h^2 \cos^2\alpha + m_H^2 \sin^2\alpha\,,$$

$$\lambda_2 x^2 = m_h^2 \sin^2\alpha + m_H^2 \cos^2\alpha\,,$$

$$\lambda_3 x v = (m_H^2 - m_h^2)\sin\alpha\cos\alpha\,.$$

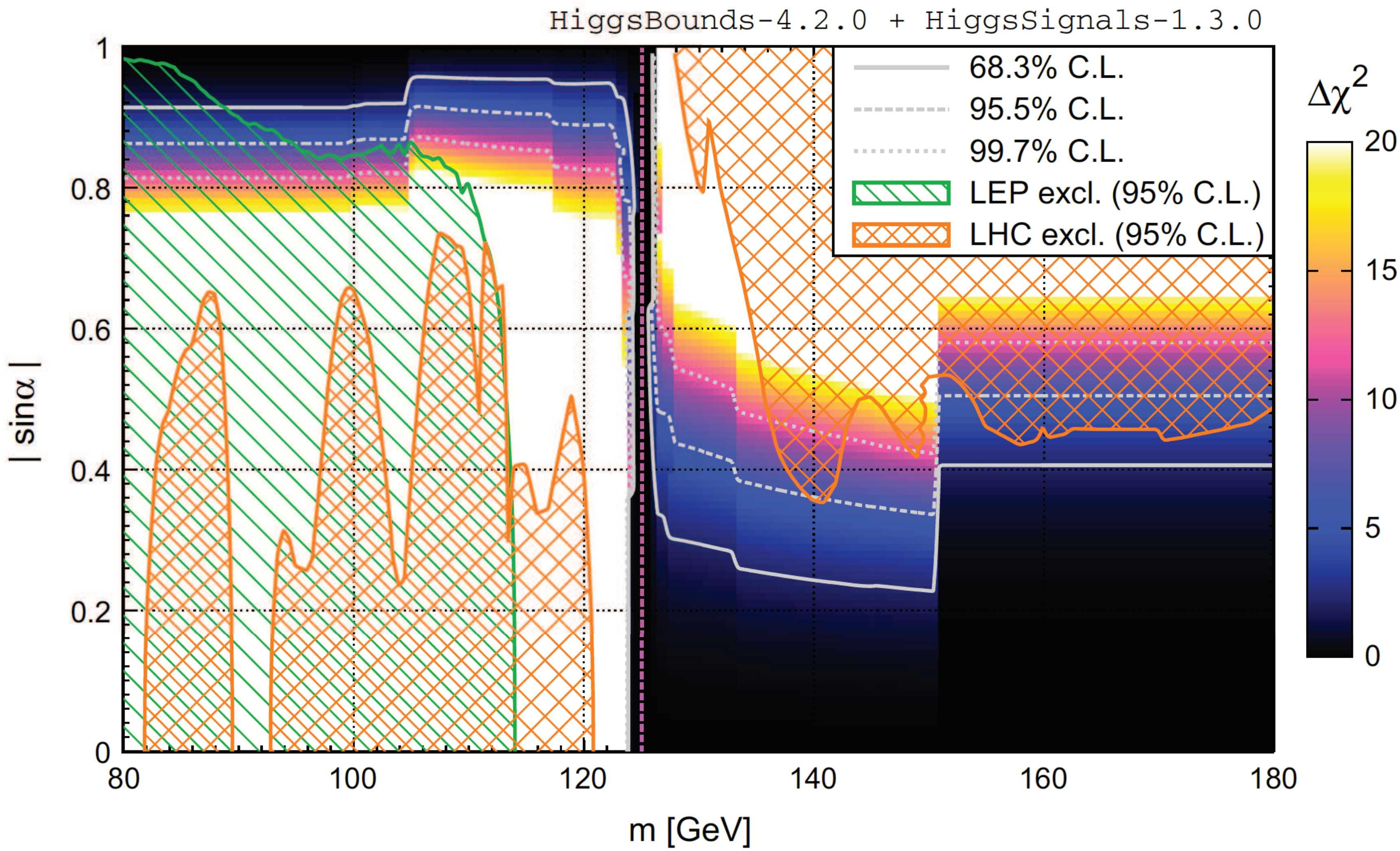
The SM-like Higgs must be approximately $\sqrt{2}\,\mathrm{Re}\,\Phi^0 - v$.

If h is SM-like, then $m_h^2 \simeq \lambda_1 v^2$ and

$$|\sin\alpha| = \frac{|\lambda_3| v x}{\sqrt{(m_H^2 - m_h^2)(m_H^2 - \lambda_1 v^2)}} \simeq \frac{|\lambda_3| v x}{m_H^2 - m_h^2} \ll 1\,,$$

If H is SM-like, then $m_H^2 \simeq \lambda_1 v^2$ and

$$|\cos\alpha| = \frac{|\lambda_3| v x}{\sqrt{(m_H^2 - m_h^2)(\lambda_1 v^2 - m_h^2)}} \simeq \frac{|\lambda_3| v x}{m_H^2 - m_h^2} \ll 1\,.$$



Taken from T. Robens and T. Stefaniak, Eur. Phys. J. **C75**, 104 (2015).

Theoretical structure of the 2HDM

Consider the most general renormalizable 2HDM potential,

$$
\begin{aligned}
\mathcal{V} = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 \\
& + \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\
& + \left\{ \tfrac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)\right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\} .
\end{aligned}
$$

After minimizing the scalar potential, assume that $\langle \Phi_i^0 \rangle = v_i$ (for $i = 1, 2$).

Define the Higgs basis fields,

$$
H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} \equiv \frac{v_1^* \Phi_1 + v_2^* \Phi_2}{v} , \qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \equiv \frac{-v_2 \Phi_1 + v_1 \Phi_2}{v} ,
$$

such that $\langle H_1^0 \rangle = v/\sqrt{2}$ and $\langle H_2^0 \rangle = 0$. The Higgs basis is uniquely defined up to an overall rephasing, $H_2 \to e^{i\chi} H_2$.

In the Higgs basis, the scalar potential is given by:

$$\begin{aligned}\mathcal{V} = {} & Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2} Z_1 (H_1^\dagger H_1)^2 \\ & + \tfrac{1}{2} Z_2 (H_2^\dagger H_2)^2 + Z_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1) \\ & + \left\{ \tfrac{1}{2} Z_5 (H_1^\dagger H_2)^2 + \left[Z_6 (H_1^\dagger H_1) + Z_7 (H_2^\dagger H_2) \right] H_1^\dagger H_2 + \text{h.c.} \right\} ,\end{aligned}$$

where Y_1, Y_2 and $Z_1, \ldots, Z_4$ are real and uniquely defined, whereas Y_3, Z_5, Z_6 and Z_7 are complex and transform under the rephasing of H_2,

$$[Y_3, Z_6, Z_7] \to e^{-i\chi} [Y_3, Z_6, Z_7] \quad \text{and} \quad Z_5 \to e^{-2i\chi} Z_5 \, .$$

Physical observables must be independent of χ.

After minimizing the scalar potential, $Y_1 = -\frac{1}{2} Z_1 v^2$ and $Y_3 = -\frac{1}{2} Z_6 v^2$.

Remark: Generically, the Z_i are $\mathcal{O}(1)$ parameters.

Type I and II Higgs-quark Yukawa couplings in the 2HDM

In the Φ_1–Φ_2 basis, the 2HDM Higgs-quark Yukawa Lagrangian is:

$$-\mathscr{L}_{\rm Y} = \overline{U}_L \Phi_i^{0\,*} h_i^U U_R - \overline{D}_L K^\dagger \Phi_i^- h_i^U U_R + \overline{U}_L K \Phi_i^+ h_i^{D\,\dagger} D_R + \overline{D}_L \Phi_i^0 h_i^{D\,\dagger} D_R + {\rm h.c.}\,,$$

where K is the CKM mixing matrix, and there is an implicit sum over i. The $h^{U,D}$ are 3×3 Yukawa coupling matrices.

In order to naturally eliminate tree-level Higgs-mediated FCNC, we shall impose a discrete symmetry to restrict the structure of $\mathscr{L}_{\rm Y}$.

Under the discrete symmetry, $\Phi_1 \to +\Phi_1$ and $\Phi_2 \to -\Phi_2$, which restricts the form of the scalar potential by setting $m_{12}^2 = \lambda_6 = \lambda_7 = 0$.Two different choices for how the discrete symmetry acts on the fermions then yield:

- Type-I Yukawa couplings: $h_1^U = h_1^D = 0$,
- Type-II Yukawa couplings: $h_1^U = h_2^D = 0$.

If the discrete symmetry is unbroken, then the scalar potential and vacuum are automatically CP-conserving (and all scalar potential parameters and the Higgs vevs can be chosen real).

Actually, it is sufficient for the discrete symmetry to be broken softly by taking $m_{12}^2 \neq 0$. In this case, an additional source of CP-violation will be present if $\mathrm{Im}(\lambda_5^*[m_{12}^2]^2) \neq 0$. Nevertheless, Higgs-mediated FCNC effects remain suppressed.

Note that the parameter

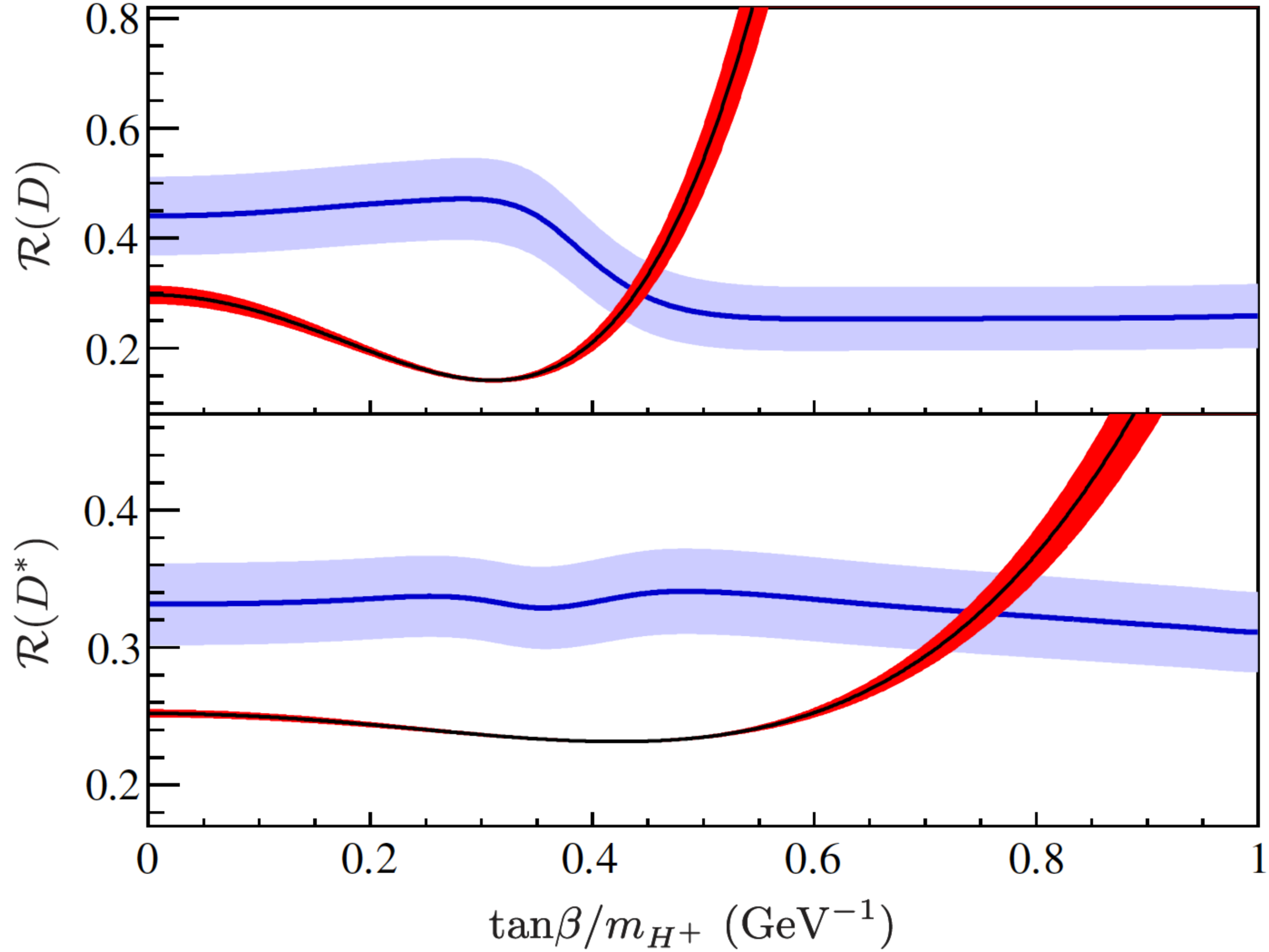
$$\tan\beta \equiv \frac{v_2}{v_1},$$

is now meaningful since it refers to vacuum expectation values with respect to the basis of scalar fields where the discrete symmetry has been imposed.

Beware of rare B decays

The decays $B^{\pm} \to \tau^{\pm}\nu_{\tau}$ and $B \to D^{(*)}\tau^{-}\overline{\nu}_{\tau}$ are noteworthy, since these processes possess *tree-level* charged Higgs exchange contributions that can compete with the dominant W-exchange in models with extended Higgs sectors.

The BaBar Collaboration measured values of the rates for $\overline{B} \to D\tau^{-}\overline{\nu}_{\tau}$ and $\overline{B} \to D^{*}\tau^{-}\overline{\nu}_{\tau}$ that showed a combined 3.4σ discrepancy from the SM predictions, which were also not compatible with the 2HDM Type-I and II Higgs Yukawa couplings. Subsequent measurements of the Belle and LHCb Collaborations are consistent with the BaBar measurements, although with larger error bars.



Taken from J.P. Lees et al. [BaBar Collaboration], Phys. Rev. **D88**, 072012 (2013).

The alignment limit in the CP-conserving 2HDM

We take $m_{12}^2 \neq 0$ and impose a Type-I or II structure of the Higgs–quark interactions. For simplicity, we assume CP-conservation, in which case all scalar potential parameters of the Higgs basis can be chosen real.

The CP-odd Higgs boson is $A = \sqrt{2}\,\mathrm{Im}\,H_2^0$ with $m_A^2 = Y_2 + \frac{1}{2}(Z_3 + Z_4 - Z_5)v^2$. After eliminating Y_2 in favor of m_A^2, the CP-even Higgs squared-mass matrix with respect to the Higgs basis states, $\{\sqrt{2}\,\mathrm{Re}\,H_1^0 - v\,,\ \sqrt{2}\,\mathrm{Re}\,H_2^0\}$ is given by,

$$\mathcal{M}_H^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix} .$$

The CP-even Higgs bosons are h and H with $m_h \leq m_H$. An approximate alignment limit can be realized in two different ways.

1. $m_A^2 \gg (Z_1 - Z_5)v^2$. This is the *decoupling limit*, where h is SM-like and $m_A \sim m_H \sim m_{H^\pm} \gg m_h$.
2. $|Z_6| \ll 1$. h is SM-like if $m_A^2 + (Z_5 - Z_1)v^2 > 0$. Otherwise, H is SM-like.

In particular, the CP-even mass eigenstates are:

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_{\beta-\alpha} & -s_{\beta-\alpha} \\ s_{\beta-\alpha} & c_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} \sqrt{2}\,\mathrm{Re}\,H_1^0 - v \\ \sqrt{2}\,\mathrm{Re}\,H_2^0 \end{pmatrix},$$

where $c_{\beta-\alpha} \equiv \cos(\beta-\alpha)$ and $s_{\beta-\alpha} \equiv \sin(\beta-\alpha)$ are defined in terms of the mixing angle α that diagonalizes the CP-even Higgs squared-mass matrix when expressed in the original basis of scalar fields, $\{\sqrt{2}\,\mathrm{Re}\,\Phi_1^0 - v_1\,,\ \sqrt{2}\,\mathrm{Re}\,\Phi_2^0 - v_2\}$.

Since the SM-like Higgs must be approximately $\sqrt{2}\,\mathrm{Re}\,H_1^0 - v$, it follows that

- h is SM-like if $|c_{\beta-\alpha}| \ll 1\,$,
- H is SM-like if $|s_{\beta-\alpha}| \ll 1$.

The case of a SM-like H necessarily corresponds to alignment without decoupling.

Remark: Although the tree-level couplings of $\sqrt{2}\,\mathrm{Re}\,H_1^0 - v$ coincide with those of the SM Higgs boson, the one-loop couplings can differ due to the exchange of non-minimal Higgs states (if not too heavy). For example, the $H^\pm$ loop contributes to the decays of the SM-like Higgs boson to $\gamma\gamma$ and γZ.

The alignment limit in equations

The CP-even Higgs squared-mass matrix yields,

$$
\begin{aligned}
Z_1 v^2 &= m_h^2 s_{\beta-\alpha}^2 + m_H^2 c_{\beta-\alpha}^2 \,, \\
Z_6 v^2 &= (m_h^2 - m_H^2) s_{\beta-\alpha} c_{\beta-\alpha} \,, \\
Z_5 v^2 &= m_H^2 s_{\beta-\alpha}^2 + m_h^2 c_{\beta-\alpha}^2 - m_A^2 \,.
\end{aligned}
$$

If h is SM-like, then $m_h^2 \simeq Z_1 v^2$ and

$$
|c_{\beta-\alpha}| = \frac{|Z_6| v^2}{\sqrt{(m_H^2 - m_h^2)(m_H^2 - Z_1 v^2)}} \simeq \frac{|Z_6| v^2}{m_H^2 - m_h^2} \ll 1 \,,
$$

If H is SM-like, then $m_H^2 \simeq Z_1 v^2$ and

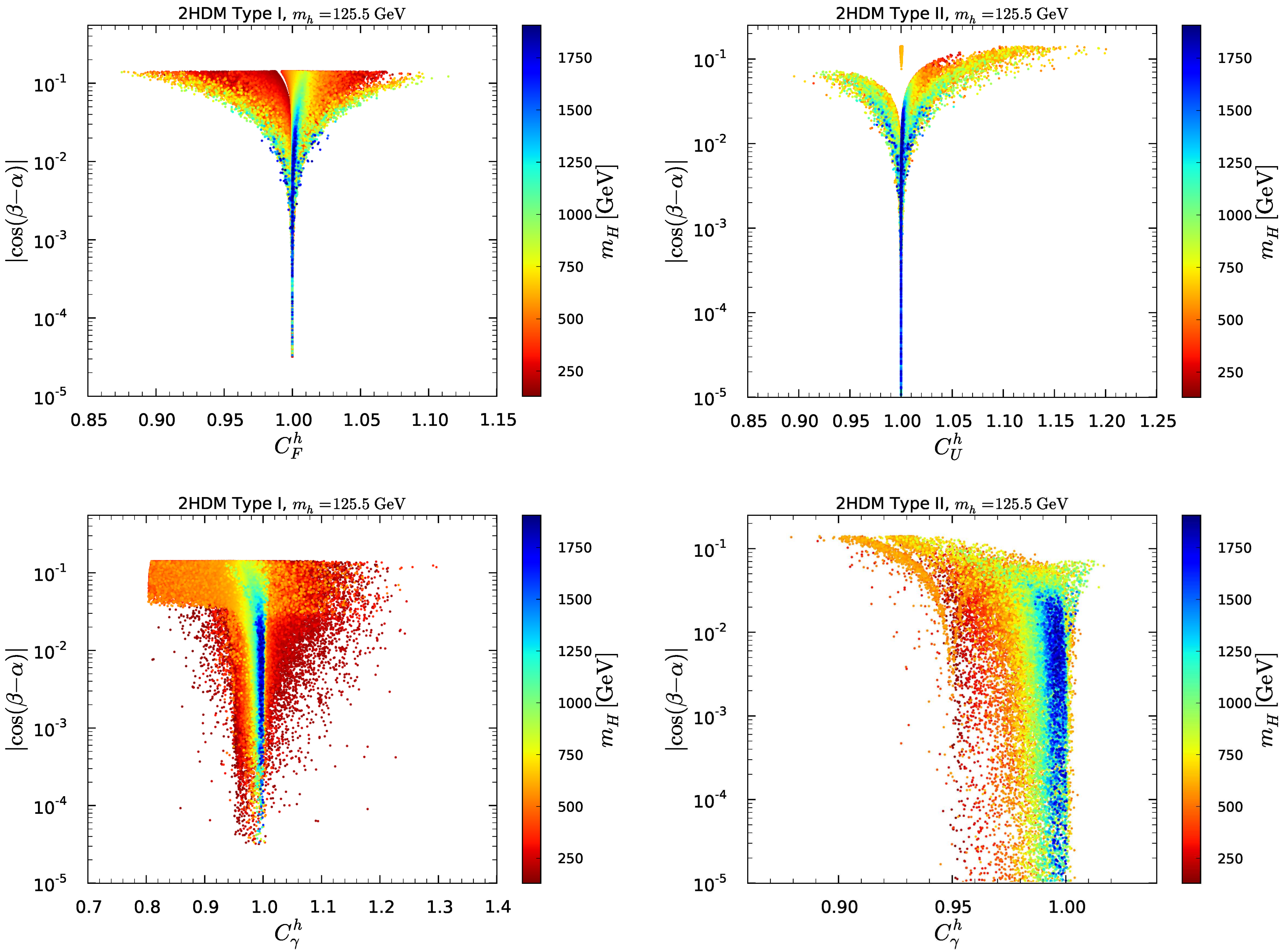
$$
|s_{\beta-\alpha}| = \frac{|Z_6| v^2}{\sqrt{(m_H^2 - m_h^2)(Z_1 v^2 - m_h^2)}} \simeq \frac{|Z_6| v^2}{m_H^2 - m_h^2} \ll 1 \,.
$$

Higgs interaction	2HDM coupling	approach to alignment limit
hVV	$s_{\beta-\alpha}$	$1-\frac{1}{2}c^2_{\beta-\alpha}$
hhh	*	$1+2(Z_6/Z_1)c_{\beta-\alpha}$
hH^+H^-	*	$\frac{1}{3}\left[(Z_3/Z_1)+(Z_7/Z_1)c_{\beta-\alpha}\right]$
$hhhh$	*	$1+3(Z_6/Z_1)c_{\beta-\alpha}$
$h\overline{D}D$	$s_{\beta-\alpha}\mathbb{1}+c_{\beta-\alpha}\rho^D_R$	$\mathbb{1}+c_{\beta-\alpha}\rho^D_R$
$h\overline{U}U$	$s_{\beta-\alpha}\mathbb{1}+c_{\beta-\alpha}\rho^U_R$	$\mathbb{1}+c_{\beta-\alpha}\rho^U_R$

Type I and II 2HDM couplings of the SM-like Higgs boson h normalized to those of the SM Higgs boson, in the alignment limit. The hH^+H^- coupling given above is normalized to the SM hhh coupling. The scalar Higgs potential is taken to be CP-conserving. For the fermion couplings, D is a column vector of three down-type fermion fields (either down-type quarks or charged leptons) and U is a column vector of three up-type quark fields. In the third column, the first non-trivial correction to alignment is exhibited. Finally, complete expressions for the entries marked with a * can be found in H.E. Haber and D. O'Neil, Phys. Rev. D **74**, 015018 (2006) [Erratum: ibid. D **74** (2006) 059905].

$$\text{Type I}: \qquad \rho^D_R=\rho^U_R=\mathbb{1}\cot\beta\,,$$

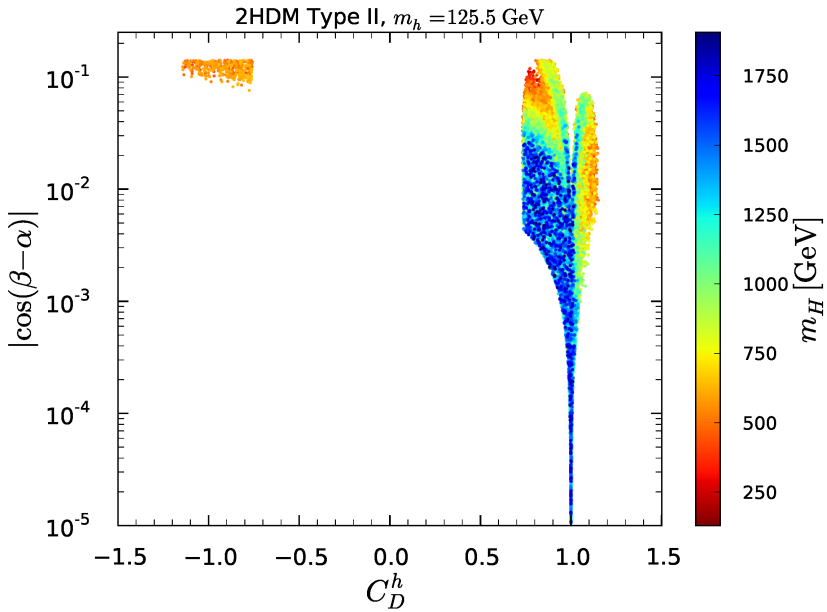
$$\text{Type II}: \qquad \rho^D_R=-\mathbb{1}\tan\beta\,, \qquad \rho^U_R=\mathbb{1}\cot\beta\,.$$

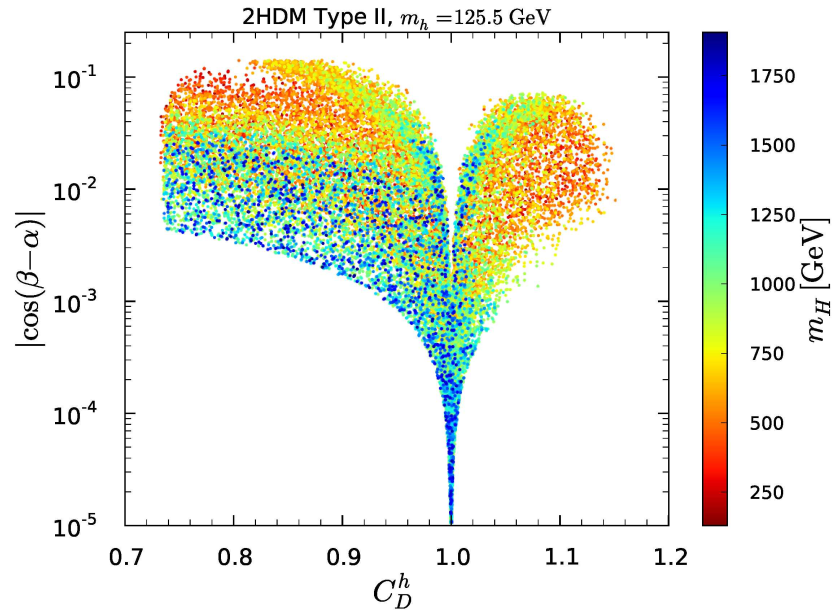

2HDM Type I, $m_h = 125.5$ GeV
2HDM Type II, $m_h = 125.5$ GeV
2HDM Type I, $m_h = 125.5$ GeV
2HDM Type II, $m_h = 125.5$ GeV
$|\cos(\beta-\alpha)|$
C_F^h
C_U^h
C_γ^h
m_H [GeV]
250
500
750
1000
1250
1500
1750
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20
1.25
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4

In Type-II models, the $h\overline{D}D$ coupling relative to the SM is

$$s_{\beta-\alpha} - \tan\beta\, c_{\beta-\alpha}\,.$$

Thus, for $|c_{\beta-\alpha}| \ll 1$ and $\tan\beta\, c_{\beta-\alpha} \simeq 2$, the $h\overline{D}D$ coupling flips sign.†

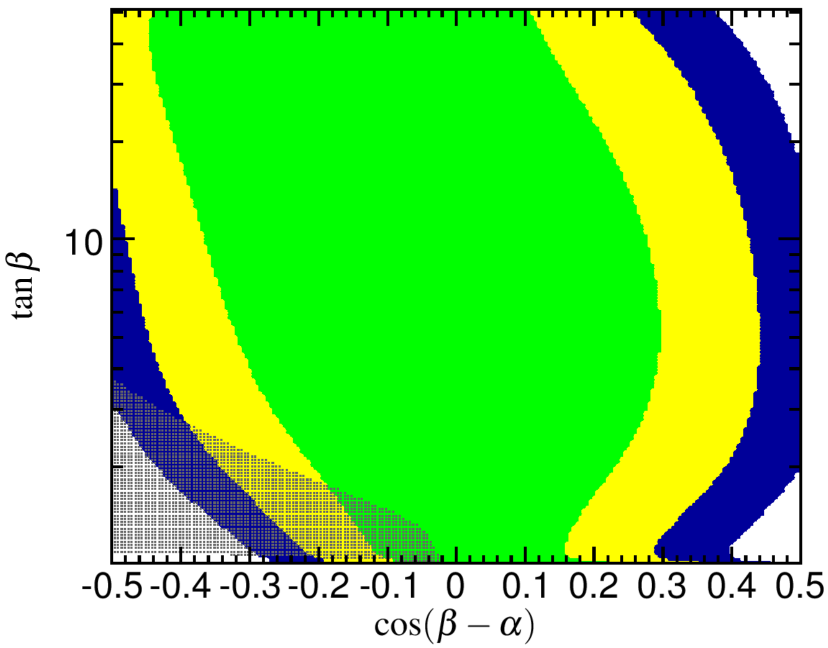


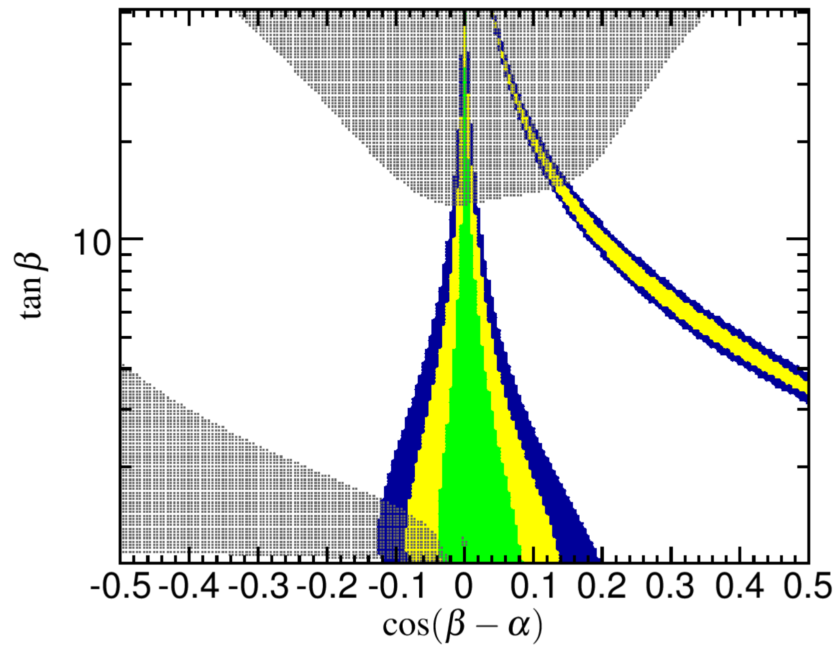


The approach to alignment is "delayed" at large $\tan\beta$.

†Many more figures of this type can be found in J. Bernon, J.F. Gunion, H.E. Haber, Y. Jiang and S. Kraml, Phys. Rev. D **92**, 075004 (2015).

Constraints on Type-I and II 2HDMs from Higgs data

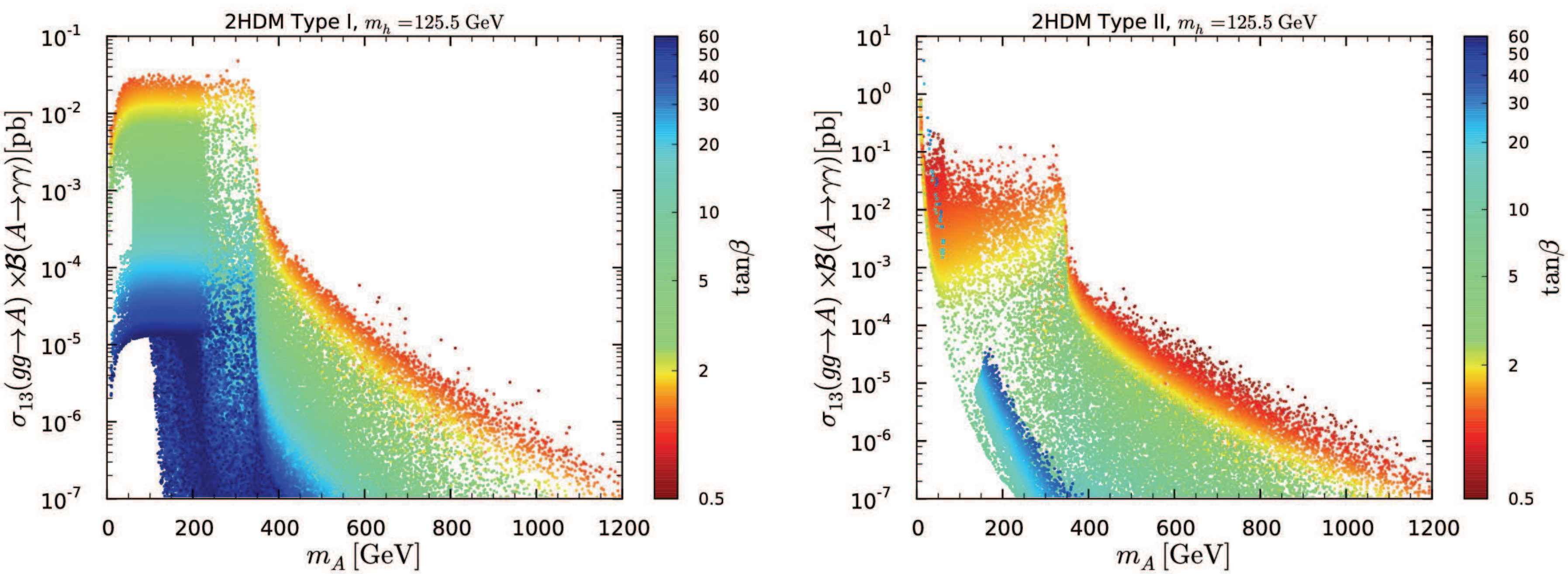




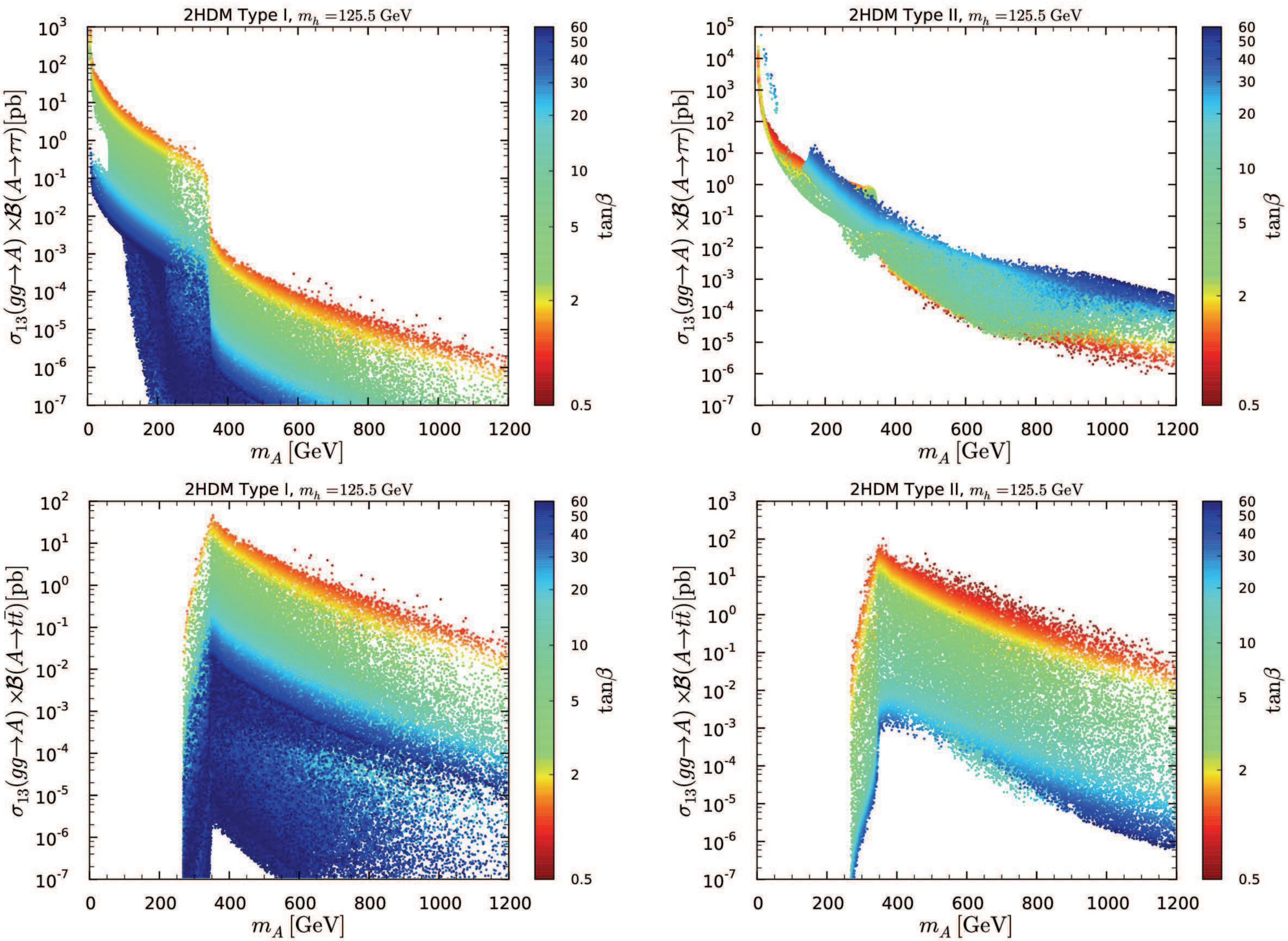
Direct constraints from LHC Higgs searches for Type-I (left) and Type-II (right) 2HDM with $m_H = 300$ GeV with $m_h = 125$ GeV, $Z_4 = Z_5 = -2$ and $Z_7 = 0$. Colors indicate compatibility with the observed Higgs signal at $1\,\sigma$ (green), $2\,\sigma$ (yellow) and $3\,\sigma$ (blue). Exclusion bounds at 95% C.L. from the non-observation of the additional Higgs states overlaid in gray. From H.E. Haber and O. Stål, Eur. Phys. J. C **75**, 491 (2015) [Erratum: ibid., **76**, 312 (2016)].

Projections for future LHC running

Since present data suggests a SM-like Higgs boson, one should take this into account in devising searches for the heavier Higgs states of the 2HDM. For example, sample results are shown below for the search for A in gluon-gluon fusion, scanned the Type-I and II 2HDM parameter spaces, assuming that $|\cos(\beta-\alpha)| \leq 0.14$.[‡]



[‡]See J. Bernon, J.F. Gunion, H.E. Haber, Y. Jiang and S. Kraml, Phys. Rev. D **92**, 075004 (2015).



Cross sections times branching ratio in Type I (left panels) and in Type II (right panels) for $gg \to A \to Y$ at the 13 TeV LHC as functions of m_A for $Y = \gamma\gamma$ (previous page panels), $Y = \tau\tau$ (upper panels) and $Y = t\bar{t}$ (lower panels) with $\tan\beta$ color code.

The alignment limit in the general 2HDM

In the general 2HDM, the scalar potential is generically CP-violating. In this case, the neutral Higgs mass-eigenstates are linear combinations of $\{\sqrt{2}\,\mathrm{Re}\,H_1^0 - v\,,\,\mathrm{Re}\,H_2^0\,,\,\mathrm{Im}\,H_2^0\}$, which are determined by diagonalizing the 3×3 real symmetric squared-mass matrix

$$\mathcal{M}^2 = v^2 \begin{pmatrix} Z_1 & \mathrm{Re}(Z_6) & -\mathrm{Im}(Z_6) \\ \mathrm{Re}(Z_6) & \frac{1}{2}Z_{345} + Y_2/v^2 & -\frac{1}{2}\mathrm{Im}(Z_5) \\ -\mathrm{Im}(Z_6) & -\frac{1}{2}\mathrm{Im}(Z_5) & \frac{1}{2}Z_{345} - \mathrm{Re}(Z_5) + Y_2/v^2 \end{pmatrix},$$

where $Z_{345} \equiv Z_3 + Z_4 + \mathrm{Re}(Z_5)$. The diagonalizing matrix is a 3×3 real orthogonal matrix that depends on three angles: θ_{12}, θ_{13} and θ_{23}, such that θ_{12} and θ_{13} are invariant whereas $\theta_{23} \to \theta_{23} - \chi$ under the rephasing of H_2.[§]

The alignment limit again corresponds to two cases:

1. $Y_2 \gg v^2$, corresponding to the decoupling limit.
2. $|Z_6| \ll 1$, corresponding to alignment with or without decoupling.

[§]See H.E. Haber and D. O'Neil, Phys. Rev. **D74**, 015018 (2006) [Erratum: ibid., **D74**, 059905 (2006)].

The alignment limit of the general 2HDM in equations

To obtain the conditions in which h_1 is the SM-like Higgs boson, noting that:

$$\frac{g_{h_1VV}}{g_{h_{\rm SM}VV}} = c_{12}c_{13}\,, \qquad \text{where } V = W \text{ or } Z\,,$$

where $h_{\rm SM}$ is the SM Higgs boson, we demand that

$$s_{12}\,,\; s_{13} \ll 1\,.$$

Here, $s_{12} \equiv \sin\theta_{12}$, $c_{12} \equiv \cos\theta_{12}$, etc. We denote the masses of the neutral Higgs mass eigenstates by m_1, m_2 and m_3. It follows that:

$$Z_1 v^2 = m_1^2 c_{12}^2 c_{13}^2 + m_2^2 s_{12}^2 c_{13}^2 + m_3^2 s_{13}^2\,,$$

$$\mathrm{Re}(Z_6\, e^{-i\theta_{23}})\, v^2 = c_{13}s_{12}c_{12}(m_2^2 - m_1^2)\,,$$

$$\mathrm{Im}(Z_6\, e^{-i\theta_{23}})\, v^2 = s_{13}c_{13}(c_{12}^2 m_1^2 + s_{12}^2 m_2^2 - m_3^2)\,,$$

$$\mathrm{Re}(Z_5\, e^{-2i\theta_{23}})\, v^2 = m_1^2(s_{12}^2 - c_{12}^2 s_{13}^2) + m_2^2(c_{12}^2 - s_{12}^2 s_{13}^2) - m_3^2 c_{13}^2\,,$$

$$\mathrm{Im}(Z_5\, e^{-2i\theta_L 23})\, v^2 = 2s_{12}c_{12}s_{13}(m_2^2 - m_1^2)\,.$$

Assuming no mass degeneracies in the neutral scalar sector, it then follows that in the alignment limit,

$$\boxed{\begin{aligned} s_{12} &\equiv \sin\theta_{12} \simeq \frac{\mathrm{Re}(Z_6 e^{-i\theta_{23}})v^2}{m_2^2 - m_1^2} \ll 1\,, \\ s_{13} &\equiv \sin\theta_{13} \simeq -\frac{\mathrm{Im}(Z_6 e^{-i\theta_{23}})v^2}{m_3^2 - m_1^2} \ll 1\,, \end{aligned}}$$

One additional small quantity characterizes the alignment limit,

$$\mathrm{Im}(Z_5 e^{-2i\theta_{23}}) \simeq \frac{(m_2^2 - m_1^2)s_{12}s_{13}}{v^2} \simeq -\frac{2\,\mathrm{Im}(Z_6^2 e^{-2i\theta_{23}})v^2}{m_3^2 - m_1^2} \ll 1\,.$$

Finally, the following mass relations in the alignment limit are noteworthy,

$$\begin{aligned} m_1^2 &\simeq Z_1 v^2\,, \\ m_2^2 - m_3^2 &\simeq \mathrm{Re}(Z_5 e^{-2i\theta_{23}})v^2\,. \end{aligned}$$

The alignment limit of the Higgs sector of the MSSM

The MSSM values of Z_1 and Z_6 (including the leading one-loop corrections):

$$Z_1 v^2 = m_Z^2 c_{2\beta}^2 + \frac{3v^2 s_\beta^4 h_t^4}{8\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right],$$

$$Z_6 v^2 = -s_{2\beta}\left\{m_Z^2 c_{2\beta} - \frac{3v^2 s_\beta^2 h_t^4}{16\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t(X_t+Y_t)}{2M_S^2} - \frac{X_t^3 Y_t}{12M_S^4}\right]\right\}.$$

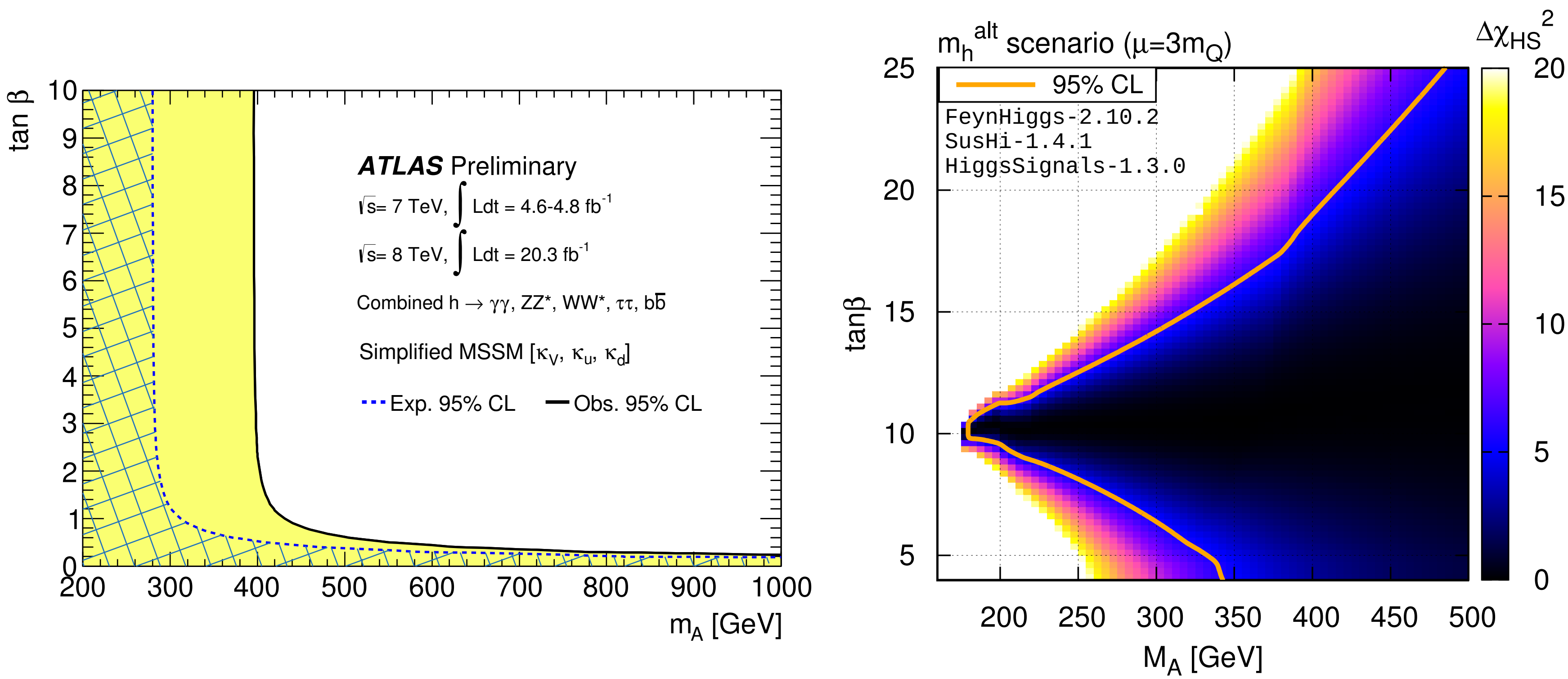
where $M_S^2 \equiv m_{\tilde{t}_1} m_{\tilde{t}_2}$, $X_t \equiv A_t - \mu\cot\beta$ and $Y_t = A_t + \mu\tan\beta$.

Note that $m_h^2 \leq Z_1 v^2$ is consistent with $m_h \simeq 125$ GeV for suitable choices for m_S and X_t. Exact alignment (i.e., $Z_6 = 0$) can now be achieved due to an accidental cancellation between tree-level and loop contributions,¶

$$m_Z^2 c_{2\beta} = \frac{3v^2 s_\beta^2 h_t^4}{16\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t(X_t+Y_t)}{2M_S^2} - \frac{X_t^3 Y_t}{12M_S^4}\right].$$

That is, $Z_6 \simeq 0$ is possible for a particular choice of $\tan\beta$.

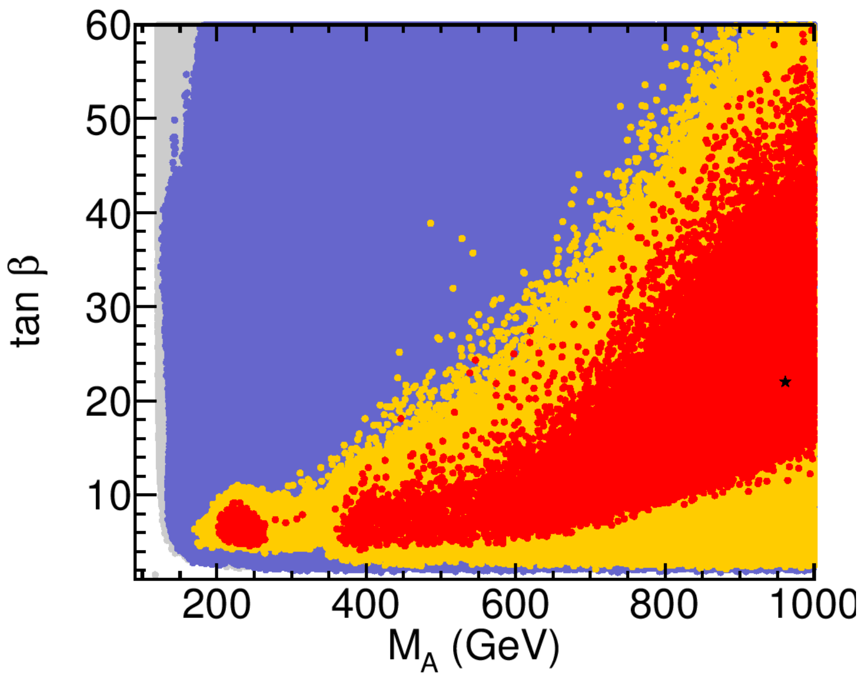
¶See M. Carena, H.E. Haber, I. Low, N.R. Shah and C.E.M. Wagner, Phys. Rev. D **91**, 035003 (2015).

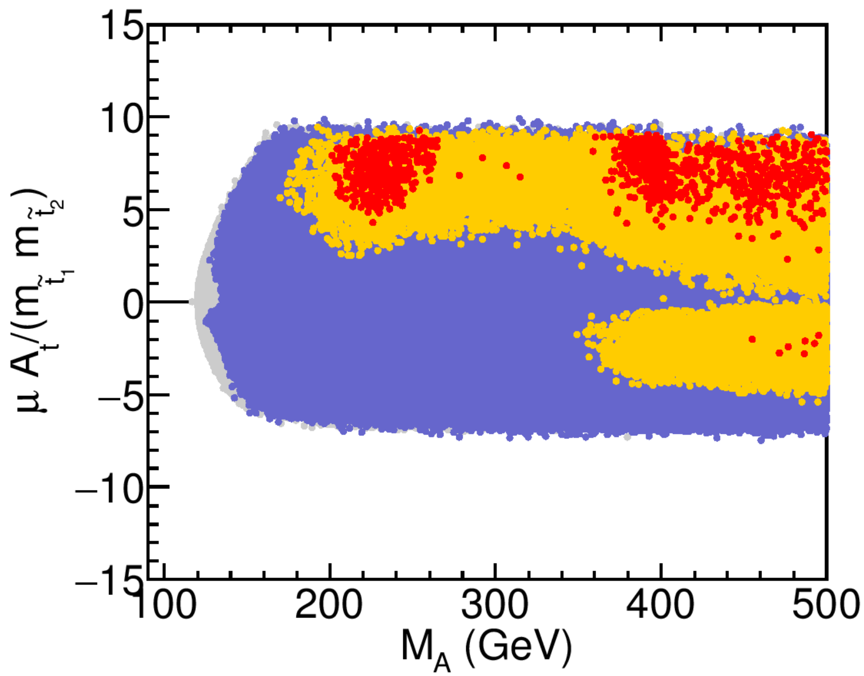


Left panel: Regions of the $(m_A, \tan\beta)$ plane excluded in a simplified MSSM model via fits to the measured rates of the production and decays of the Higgs boson h. Taken from ATLAS-CONF-2014-010.

Right panel: Likelihood distribution, $\Delta\chi^2_{\rm HS}$ obtained from testing the signal rates of the Higgs boson h against a combination of Higgs rate measurements from the Tevatron and LHC experiments, obtained with `HiggsSignals`, in the alignment benchmark scenario. From P. Bechtle, S. Heinemeyer, O. Stål, T. Stefaniak and G. Weiglein, EPJC **75**, 421 (2015).

Likelihood analysis: allowed regions in the $\tan\beta$–m_A plane





Preferred parameter regions in the $(M_A,\ \tan\beta)$ plane (left) and $(M_A,\ \mu A_t/M_S^2)$ plane (right), where $M_S^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$, in a pMSSM-8 scan. Points that do not pass the direct constraints from Higgs searches from `HiggsBounds` and from LHC SUSY particle searches from `CheckMATE` are shown in gray. Applying a global likelihood analysis to the points that pass the direct constraints, the color code employed is red for $\Delta\chi_h^2 < 2.3$, yellow for $\Delta\chi_h^2 < 5.99$ and blue otherwise. The best fit point is indicated by a black star. (Taken from P. Bechtle, H.E. Haber, S. Heinemeyer, O. Stål, T. Stefaniak, G. Weiglein and L. Zeune, in preparation.)

A symmetry origin for alignment without decoupling

For simplicity, we examine the CP-conserving 2HDM, for which one can rephase the Higgs basis field H_2 such that Z_5, Z_6 and Z_7 are real. Given a scalar potential in the Φ_1–Φ_2 basis, one can derive

$$Z_6 = -\tfrac{1}{2}\big[\lambda_1 c^2_\beta - \lambda_2 s^2_\beta - \lambda_{345} c_{2\beta}\big] s_{2\beta} + \lambda_6 c_\beta c_{3\beta} + \lambda_7 s_\beta s_{3\beta}\,,$$
$$Z_7 = -\tfrac{1}{2}\big[\lambda_1 s^2_\beta - \lambda_2 c^2_\beta + \lambda_{345} c_{2\beta}\big] s_{2\beta} + \lambda_6 s_\beta s_{3\beta} + \lambda_7 c_\beta c_{3\beta}\,.$$

If the alignment condition $Z_6 = 0$ holds independently of $\tan\beta$, then it follows that[‖]

$$\lambda_1 = \lambda_2 = \lambda_{345}\,, \qquad \lambda_6 = \lambda_7 = 0\,.$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$. The above *natural alignment condition* can be achieved by imposing a particular Higgs flavor or generalized CP symmetry.

Note that the natural alignment condition also sets $Z_7 = 0$. Indeed, if the natural alignment condition holds in one basis, then it holds in any basis.

[‖] See P.S. Bhupal Dev and A. Pilaftsis, JHEP **1412**, 024 (2014) 024 [Erratum: ibid. **1511**, 147 (2015)].

The natural alignment condition can be relaxed. It is sufficient to impose a discrete $\mathbb{Z}_2$ symmetry where the Higgs basis field H_1 is unchanged but $H_2 \to -H_2$. It then follows that

$$Y_3 = Z_6 = Z_7 = 0\,.$$

Note that the minimum condition $Y_3 = -\frac{1}{2}Z_6 v^2$ requires that $Y_3 = 0$ if $Z_6 = 0$, so this $\mathbb{Z}_2$ symmetry *cannot* be softly broken.

No conditions are imposed on $Z_1, \ldots, Z_5$. The natural alignment condition is a special case where $Z_1 = Z_2 = Z_{345}$.

Having imposed the above $\mathbb{Z}_2$ symmetry in the bosonic sector of the theory, we can extend it to the Yukawa interactions. If we demand that all fermions are even under the $\mathbb{Z}_2$ symmetry, then the H_1 couplings to fermions are those of the SM Higgs boson and the Yukawa couplings of H_2 to the fermions are absent. This is the inert doublet model (IDM).

Further details on the IDM

By imposing the discrete $\mathbb{Z}_2$ symmetry, the scalar potential is CP-conserving. The SM Higgs state is $h = \sqrt{2}\,\mathrm{Re}\; H_1^0 - v$. The inert doublet is

$$H_2 = \begin{pmatrix} H^+ \\ (H + iA)/\sqrt{2} \end{pmatrix},$$

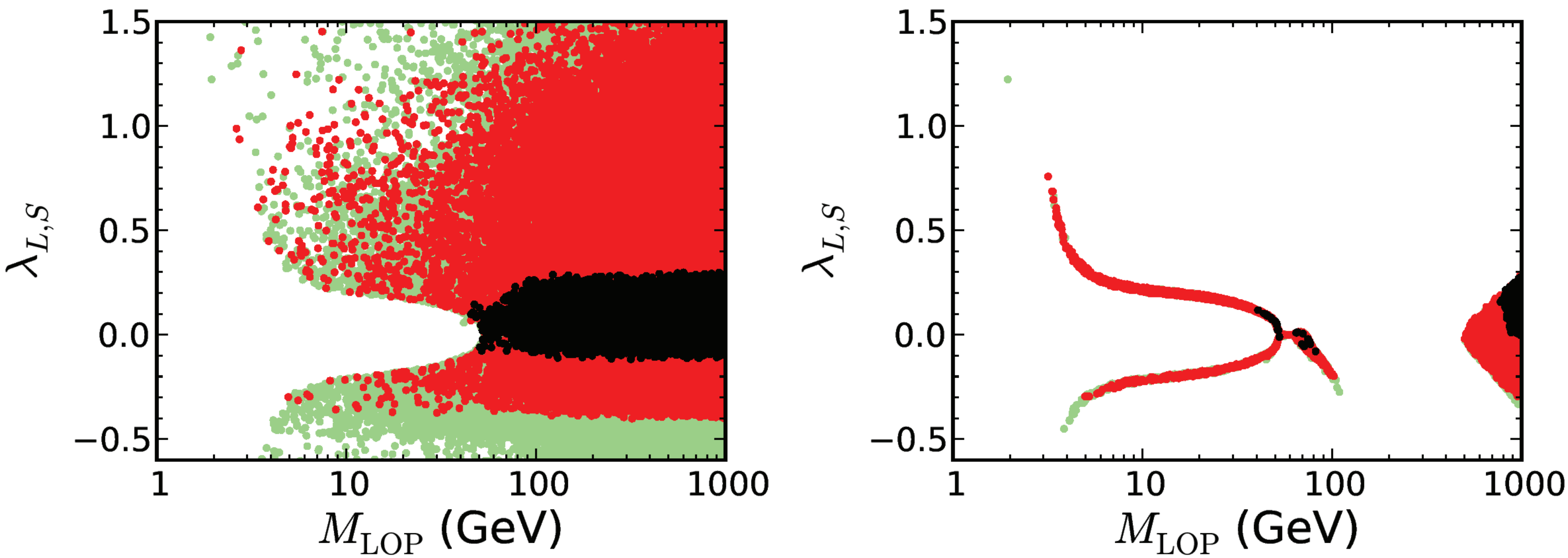
where the mass eigenstates consist of two neutral scalars, H, A and a charged Higgs pair. The physical Higgs masses are

$$m_h^2 = Z_1 v^2\,, \qquad m_{H^\pm}^2 = Y_2 + \tfrac{1}{2} Z_3 v^2\,,$$
$$m_{H,A}^2 = m_{H^\pm}^2 + \tfrac{1}{2}(Z_4 \pm |Z_5|) v^2\,.$$

H and A have opposite CP-quantum numbers, but there is no interaction that can determine separate CP quantum number for these states. The lighter of these two states will henceforth be denoted as H_L.

The lightest $\mathbb{Z}_2$–odd particle (LOP) is stable. If $Z_4 < |Z_5|$ (in which case H_L is lighter than $H^\pm$), then the LOP is a neutral scalar.

The LOP is a candidate for dark matter. Including the exclusion limits from the current dark matter direct detection experiments, a cosmologically relevant LOP is ruled out by Goudelis, Herrmann and Stål for all LOP masses below 500 GeV except for a narrow window around $\frac{1}{2}m_h$.



The viable IDM parameter space projected on the $(M_{\rm LOP}\,,\,\lambda_{L,S})$ plane imposing only the upper limit (left) and the upper and lower limits (right) of the WMAP range, $0.1018 \leq M_{\rm LOP}h^2 \leq 0.1234$. The green points correspond to all valid points in the scan, while the red and black regions show the points which remain valid when the model satisfies stability and perturbativity up to a scale $\Lambda = 10^4$ GeV and the GUT scale $\Lambda = 10^{16}$ GeV, respectively. Above, $\lambda_{L,S} \equiv \frac{1}{2}(Z_3 + Z_4 \mp |Z_5|)$; when multiplied by v the latter corresponds to the hH_LH_L coupling. Taken from A. Goudelis, B. Herrmann and O. Stål, JHEP **1309** (2013) 106.

Adding a Higgs singlet to the 2HDM

Consider a Higgs sector that consists of two hypercharge-one complex doublet and a complex neutral singlet S. We can define the doublet fields of the Higgs basis, H_1 and H_2 as before. The relevant scalar potential is more complicated than that of the 2HDM. Here we focus on the terms that are relevant for the scalar squared-mass matrices.

$$\begin{aligned}
\mathcal{V} \ni \ldots &+ \tfrac{1}{2}Z_1(H_1^\dagger H_1)^2 + \ldots + \left[\tfrac{1}{2}Z_5(H_1^\dagger H_2)^2 + Z_6(H_1^\dagger H_1)H_1^\dagger H_2 + \text{h.c.}\right] + \ldots \\
&+ S^\dagger S\left[Z_{s1}H_1^\dagger H_1 + \ldots + (Z_{s3}H_1^\dagger H_2 + \text{h.c.}) + Z_{s4}S^\dagger S\right] \\
&+ \left\{Z_{s5}H_1^\dagger H_1 S^2 + \ldots + Z_{s7}H_1^\dagger H_2 S^2 + Z_{s8}H_2^\dagger H_1 S^2 + Z_{s9}S^\dagger S\, S^2 + Z_{s10}S^4 + \text{h.c.}\right\} \\
&+ \left[C_1 H_1^\dagger H_1 S + \ldots + C_3 H_1^\dagger H_2 S + C_4 H_2^\dagger H_1 S + C_5(S^\dagger S)S + C_6 S^3 + \text{h.c.}\right].
\end{aligned}$$

For simplicity, we shall assume that the scalar potential is CP-invariant. We then write the squared-mass matrix of the CP-even Higgs bosons with respect to the basis $\{\sqrt{2}\,\text{Re}\ H_1^0 - v\,,\ \sqrt{2}\,\text{Re}\ H_2^0\,,\ \sqrt{2}\,(\text{Re}\ S - v_s)\}$.

The squared-mass matrix for the CP-even scalars is a real symmetric matrix,

$$\mathcal{M}_S^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 & \sqrt{2}\, v\big[C_1 + (Z_{s1} + 2Z_{s5})v_s\big] \\ & \overline{M}_A^2 + Z_5 v^2 & \dfrac{v}{\sqrt{2}}\Big[C_3 + C_4 + 2(Z_{s3} + Z_{s7} + Z_{s8})v_s\Big] \\ & & -C_1 \dfrac{v^2}{2v_s} + 3(C_5 + C_6)v_s + 4(Z_{s4} + 2Z_{s9} + 2Z_{s10})v_s^2 \end{pmatrix},$$

where $\overline{M}_A^2$ is the 11 element of the CP-odd squared-mass matrix with respect to the basis $\{\sqrt{2}\,\mathrm{Im}\ H_2^0,\ \sqrt{2}\,\mathrm{Im}\ S\}$.

Exact alignment occurs when $(\mathcal{M}_S^2)_{12} = (\mathcal{M}_S^2)_{13} = 0$. That is,

$$Z_6 = 0\,, \qquad\qquad C_1 + (Z_{s1} + 2Z_{s5})v_s = 0\,.$$

The decoupling limit corresponds to $\overline{M}_A \gg v$ and $v_s \gg v$ and yields approximate alignment.

Approximate alignment can also be achieved with a combination of a subset of the above conditions. For example, $C_1 + (Z_{s1} + 2Z_{s5})v_s \simeq 0$ and $\overline{M}_A \gg v$ [with $Z_6 \sim \mathcal{O}(1)$] yields approximate alignment.

The alignment limit of the Higgs sector of the NMSSM

In the NMSSM, including the leading one-loop radiative corrections,

$$Z_1 v^2 = (m_Z^2 - \tfrac{1}{2}\lambda^2 v^2)c_{2\beta}^2 + \tfrac{1}{2}\lambda^2 v^2 + \frac{3v^2 s_\beta^4 h_t^4}{8\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right],$$

$$Z_6 v^2 = -s_{2\beta}\left\{(m_Z^2 - \tfrac{1}{2}\lambda^2 v^2)c_{2\beta} - \frac{3v^2 s_\beta^2 h_t^4}{16\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t(X_t+Y_t)}{2M_S^2} - \frac{X_t^3 Y_t}{12M_S^4}\right]\right\}.$$

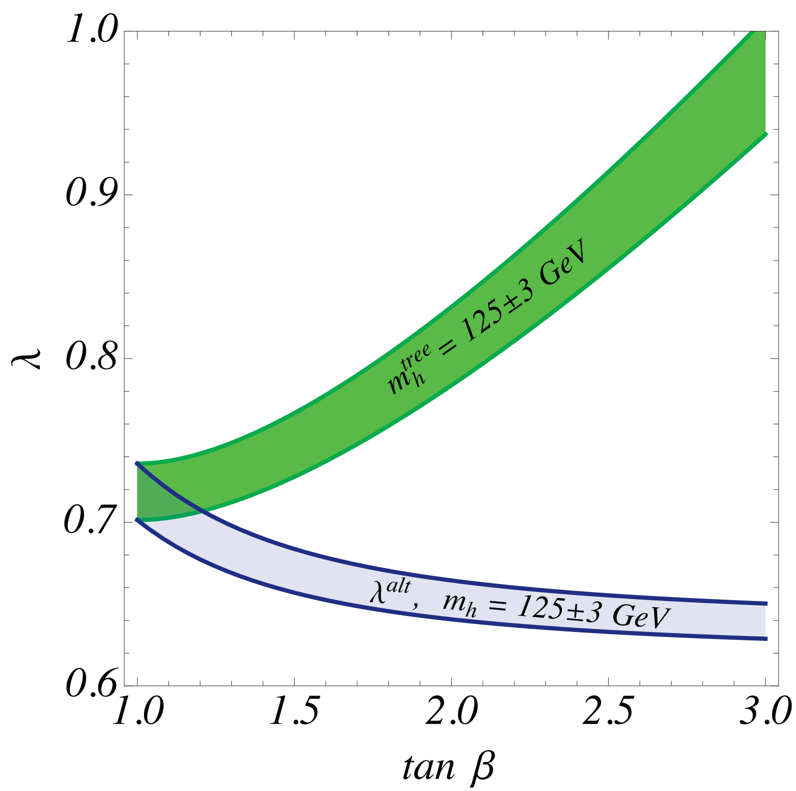
The exact alignment limit requires that $Z_6 = 0$ and $C_1 + (Z_{s1} + 2Z_{s5})v_s = 0$. In the NMSSM, the latter condition yields

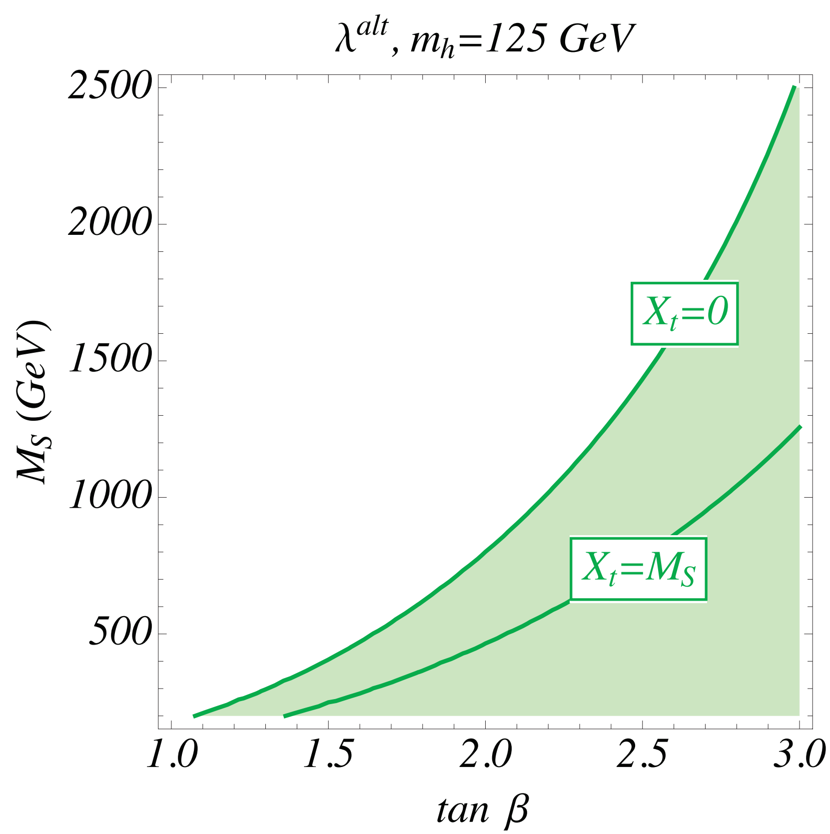
$$\frac{\overline{M}_A^2 s_{2\beta}^2}{4\mu^2} + \frac{\kappa s_{2\beta}}{2\lambda} = 1\,,$$

where $\overline{M}_A^2 \equiv 2\mu(A_\lambda + \kappa v_s)/s_{2\beta}$ and $\mu \equiv \lambda v_s$. Note that κ governs the self-coupling of the singlet scalar field.

In contrast to the MSSM, in the NMSSM one can set $Z_6 = 0$ and obtain $m_h = 125$ GeV, with only small contributions from the one-loop radiative corrections. This leads to a preferred choice of NMSSM parameters,**

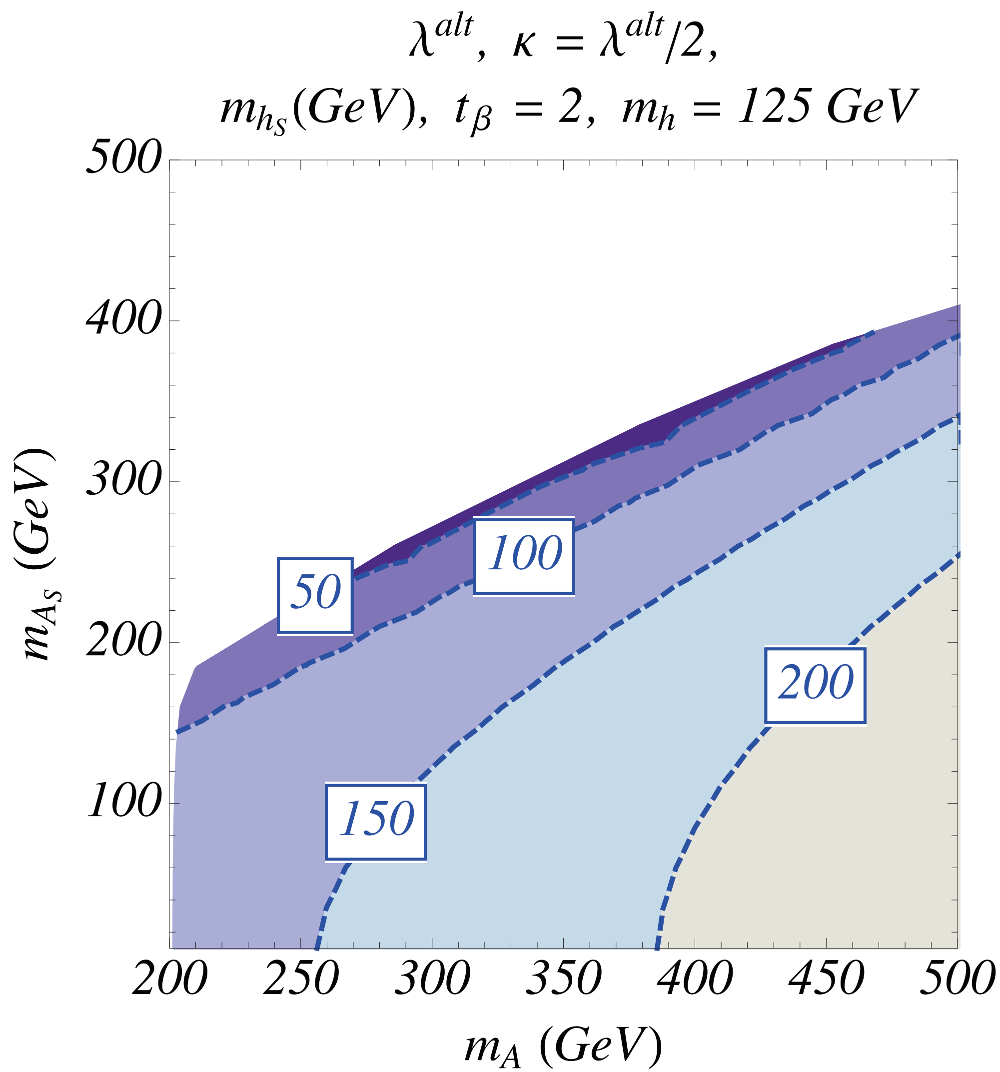
$$\lambda \sim 0.65\,, \qquad\qquad \tan\beta \sim 2\,.$$





See M. Carena, H.E. Haber, I. Low, N.R. Shah and C.E.M. Wagner, Phys. Rev. D **93, 035013 (2016).

The second alignment limit condition leads to further correlations of the NMSSM parameter space.



Near the alignment limit, we have $m_A \simeq m_H \simeq \overline{M}_A$.

Beyond singlets and doublets

If one considers a scalar sector with triplet Higgs fields, then one must include addition Higgs multiplets in such a way that $\rho \simeq 1$.

Georgi and Machacek constructed an amusing model in which $\rho = 1$ at tree-level due to a well chosen scalar potential that respects custodial symmetry. The model contains a complex $Y = 1$ doublet, a complex $Y = 2$ triplet and a real $Y = 0$ singlet. Without going into details, there is a doublet vev, v_ϕ, and a common triplet vev, v_χ, with $v^2 \equiv v_\phi^2 + 8v_\chi^2 = (246~\mathrm{GeV})^2$.

The physical scalars make up custodial SU(2) multiplets: a 5-plet of states ($H_5^{\pm\pm}$, $H_5^\pm$ and H_5^0) with common mass m_5, a triplet ($H_3^\pm$, H_3^0) with common mass m_3, and custodial singlets that mix with squared-mass matrix

$$\mathcal{M}^2 = \begin{pmatrix} Z_{11} v_\phi^2 & v_\phi v_\chi (Z_{12} - 2\sqrt{3}\, m_3^2/v^2) \\ v_\phi v_\chi (Z_{12} - 2\sqrt{3}\, m_3^2/v^2) & \frac{3}{2} m_3^2 - \frac{1}{2} m_5^2 + v_\chi^2 (Z_{22} - 12 m_3^2/v^2) \end{pmatrix},$$

where the Z_{ij} depend on dimensionless quartic couplings.

The custodial singlet CP-even Higgs bosons are h and H with $m_h \leq m_H$. An approximate alignment limit can be realized in two different ways.

1. In the *decoupling limit*, h is SM-like and $m_H \simeq m_3 \simeq m_5 \gg m_h$.[††]
2. $v_\chi \ll v$. Then h is SM-like if $Z_{11}v^2 < \frac{3}{2}m_3^2 - \frac{1}{2}m_5^2$. Otherwise, H is SM-like.

Remark: Implications of a modified unitarity sum rule

In the Georgi-Machacek model, the existence of doubly-charged Higgs bosons implies that

$$\sum_i g^2_{h_i W^+ W^-} = g^2 m_W^2 + \sum_k |g_{\phi_k^{++} W^- W^-}|^2 \,,$$

where the sum is taken over all CP-even Higgs bosons of the model. The presence on the last term on the right hand side above means that individual $h_i VV$ couplings can exceed the corresponding coupling of the SM.

[††]For details, see K. Hartling, K. Kumar, and H.E. Logan, Phys. Rev. **D90**, 015007 (2014).

It is convenient to write $c_H \equiv \cos\theta_H = v_\phi/(v_\phi^2 + 8v_\chi^2)^{1/2}$, and $s_H \equiv \sin\theta_H$. Then, the following couplings are noteworthy:

$$\begin{array}{llll} H_1^0 W^+W^- : & g c_H m_W \,, & H_1^{\prime\,0} W^+W^- : & \sqrt{8/3}\, g m_W s_H \,, \\ H_5^0 W^+W^- : & \sqrt{1/3}\, g m_W s_H \,, & H_5^{++} W^-W^- : & \sqrt{2}\, g m_W s_H \,, \end{array}$$

where H_1^0 and $H_1^{\prime\,0}$ are the custodial singlet interaction eigenstates. Note that $H_1^{\prime\,0}$ and H_5^0, H_5^{++} have no coupling to fermions, whereas

$$H_1^0 f\bar{f} : \quad \frac{g m_q}{2 m_W c_H} \,.$$

In the absence of H_1^0–$H_1^{\prime\,0}$ mixing, $c_H = 1$ corresponds to the alignment limit. But consider the strange case of $s_H = \sqrt{3/8}$. In this case, the $H_1^{\prime\,0}$ coupling to W^+W^- matches that of the SM. Nevertheless, this does not saturate the HWW sum rule! Moreover, it is possible that the $H_1^{\prime\,0}W^+W^-$ coupling is *larger* than $g m_W$, without violating the HWW sum rule. Including H_1^0–$H_1^{\prime\,0}$ mixing allows for even more baroque possibilities not possible in a multi-doublet extension of the SM.

Conclusions

- The Higgs data strongly suggests that the observed Higgs boson is SM-like.
- If the Higgs sector in nature is non-minimal, then it must contain a SM-like Higgs boson.
- In the alignment limit, the mass eigenstate corresponding to the observed Higgs boson points is aligned with direction (in field space) of the doublet vacuum expectation value.
- Departures from the alignment limit encode critical information that will provide important clues for the structure of the non-minimal Higgs sector.