上州人

上州無智亦無才
剛毅木訥易被欺
唯以正直接萬人
至誠依神期勝利

鑑三

SUSY Inflation? What Else?

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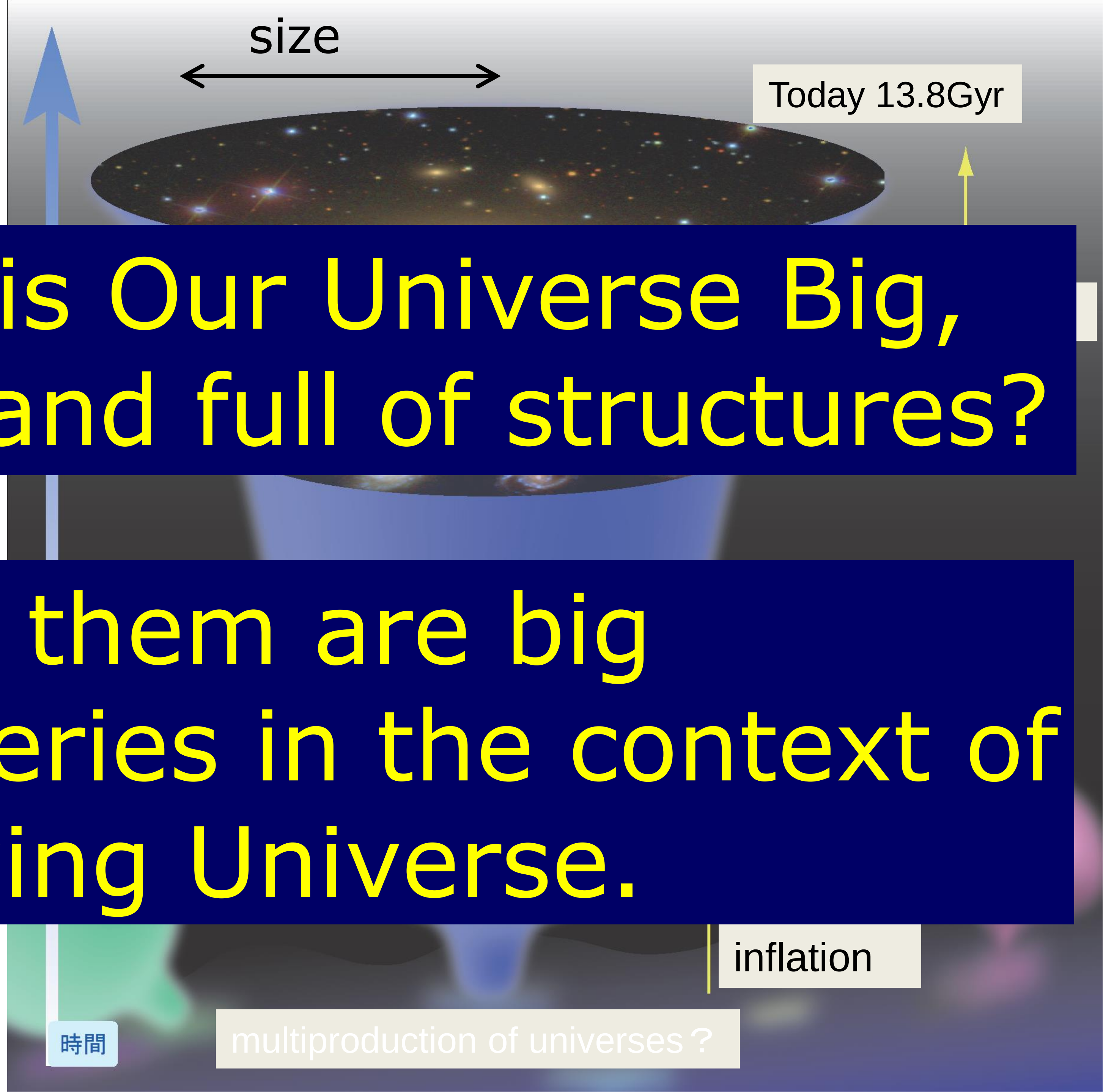
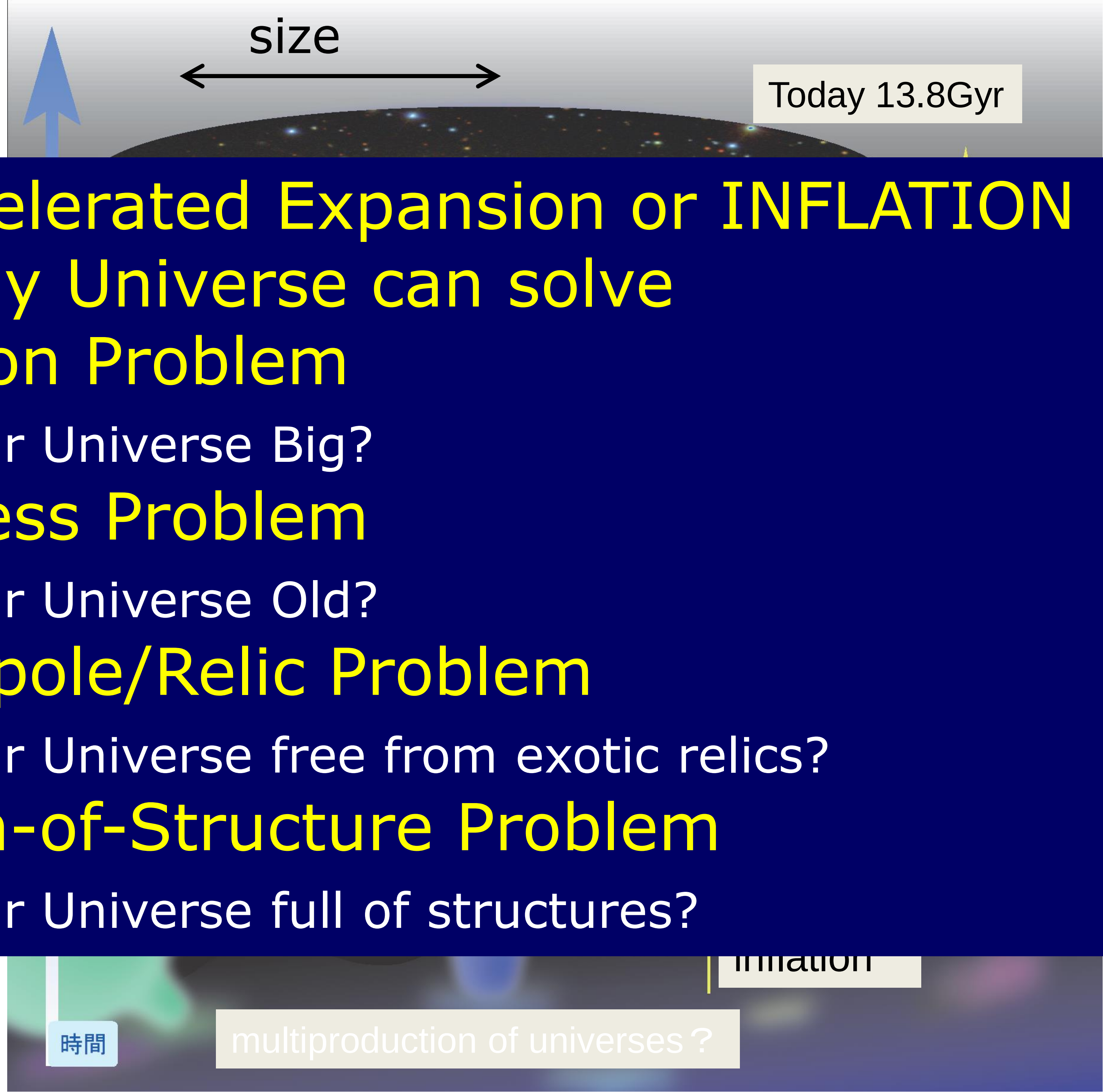


Why is Our Universe Big, Old, and full of structures?

All of them are big mysteries in the context of evolving Universe.



Rapid Accelerated Expansion or INFLATION in the early Universe can solve

The Horizon Problem

Why is our Universe Big?

The Flatness Problem

Why is our Universe Old?

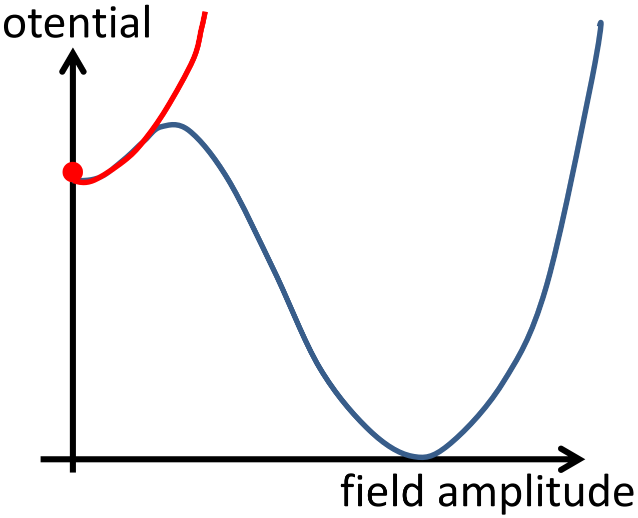
The Monopole/Relic Problem

Why is our Universe free from exotic relics?

The Origin-of-Structure Problem

Why is our Universe full of structures?

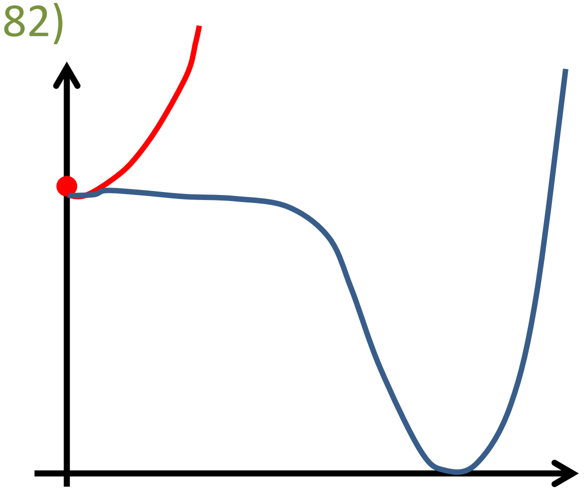
Some historical accounts



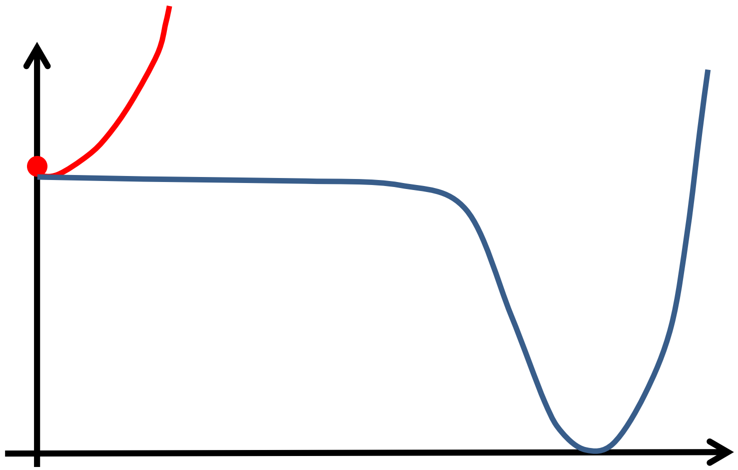
Field theoretic models of inflation
(based on GUT Higgs)

Old inflation: Sato (1981), Guth (1981)
high temperature symmetry restoration
and first-order phase transition
Inflation never ends, the Universe too inhomogeneous

New inflation: Linde (1982), Albrecht & Steinhardt (1982)
high temperature symmetry restoration
Slow-rollover phase transition
Inflation too short,
the Universe too inhomogeneous



Supersymmetric New inflation:
high temperature symmetry restoration
Slow-rollover phase transition
Inflation long enough,
Perturbations at the right amplitude

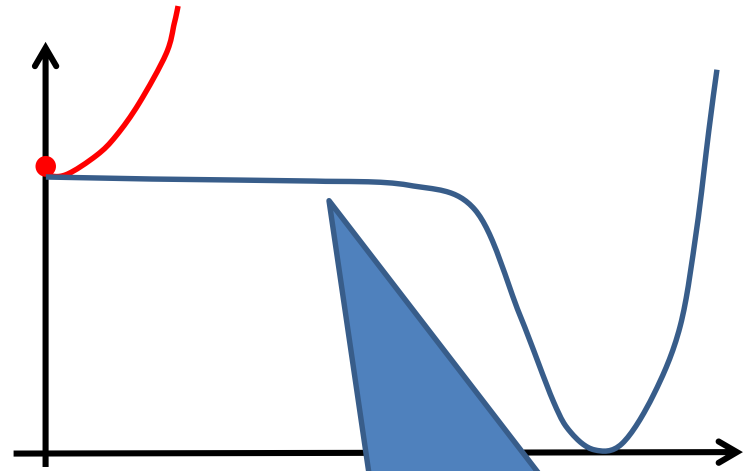


PHYSICS LETTERS Volume 118B, number 4, 5, 6 9 December 1982

COSMOLOGICAL INFLATION CRIES OUT FOR SUPERSYMMETRY

John ELLIS, D.V. NANOPOULOS, Keith A. OLIVE and K. TAMVAKIS
CERN, Geneva, Switzerland

Supersymmetric New inflation:
high temperature symmetry restoration
Slow-rollover phase transition
Inflation long enough,
Perturbations at the right amplitude



SUSY greatly helps to preserve the flatness of the potential against radiative corrections

$$V(\phi) = A\phi^4 (\ln \phi^2/\sigma^2 - \tfrac{1}{2}) + D\phi^2 ,$$

where

$$A = \frac{1}{64\pi^2\sigma^4}\left(\sum_{\mathrm{B}} g_{\mathrm{B}} m_{\mathrm{B}}^4 - \sum_{\mathrm{F}} g_{\mathrm{F}} m_{\mathrm{F}}^4\right)$$

Since SUSY is broken at the mass scale m_S on

$$A = (g_{\mathrm{B(F)}}/32\pi^2\sigma^4)\, m_{\mathrm{S}}^2 \left(\sum_{\mathrm{B}} m_{\mathrm{B}}^2\right)$$

In fact, at finite density/temperature (and/or curved spacetime) SUSY may be broken more severely.

Was the early Universe in a thermal equilibrium state?

Two body reaction rate with a massless gauge particle

$$\Gamma_2 = \langle n\sigma c \rangle \simeq \frac{NT^3}{\pi^2}\frac{\alpha^2}{T^2}$$

must have been larger than

$$H = \left(\frac{1}{3M_P^2} \frac{\pi^2}{30} g_* T^4 \right)^{\frac{1}{2}} .$$

N Number of reaction channel

α Gauge coupling constant

g_* # Relativistic degrees of freedom

$M_P \equiv (8\pi G)^{-\frac{1}{2}}$

Namely, $\Gamma \gg H$ is required to realize a thermal state.

This imposes an upper bound on the radiation temperature,

$$T \ll 10^{15} \left(\frac{\alpha}{0.05}\right)^2 \left(\frac{N}{10}\right) \left(\frac{g_*}{200}\right)^{-1/2} \mathrm{GeV} \equiv T_{\mathrm{eq}}$$

Thermal phase transition at the GUT scale was impossible.

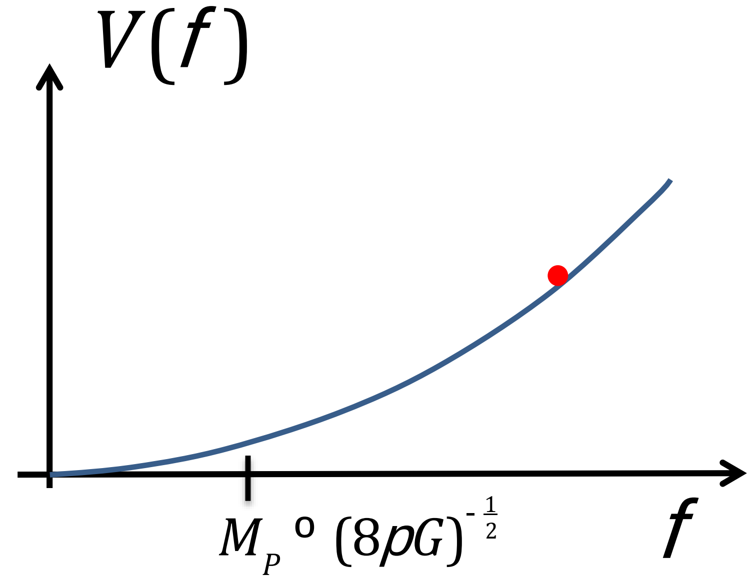
Some nonthermal mechanisms to set up the initial condition for inflation must be invoked.

Chaotic inflation

Linde (1983)

Very early Universe was dominated by large quantum fluctuations, and scalar fields must have taken random values in each coherent domain.
Inflation is naturally realized if f □ M_P.

Sufficiently flat potential is required over super-Planckian field excursion.



Curvature Perturbations

$$\varsigma = \frac{\delta a}{a} = \delta N = H\delta t = H\frac{\delta\phi}{\dot{\phi}} \cong \frac{H^2}{2\pi\dot{\phi}} \cong 10^{-5}$$

Tensor Perturbations

(Quantum gravitational waves)

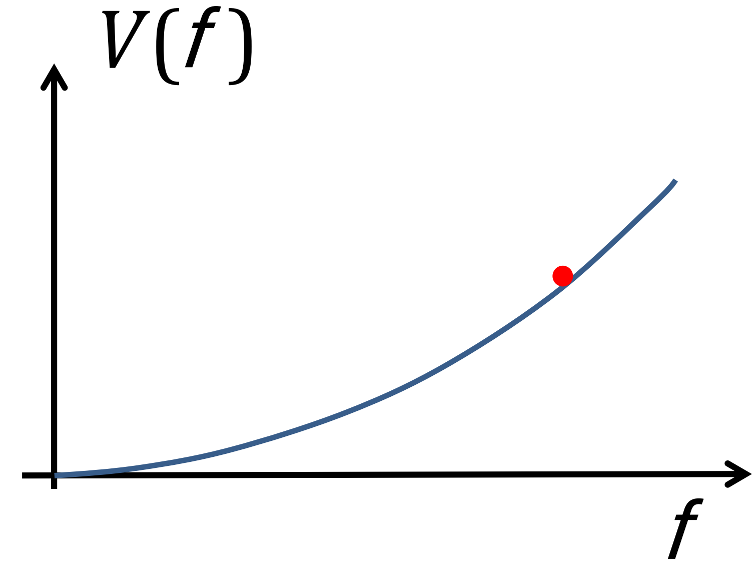
$$\sqrt{\left\langle h_{ij}h^{ij}\right\rangle} = \frac{2^{\frac{3}{2}}}{M_P}\frac{H}{2p}$$

For $V[f] = \frac{1}{2}m^2f^2 + \frac{/}{4}f^4$ we find $m = 10^{13}\mathrm{GeV},\ / < 10^{-13}$

To preserve the smallness of the coupling constant, SUSY is desired.

Chaotic inflation

Linde (1983)



Very early Universe was dominated by large quantum fluctuations, and scalar fields must have taken random values in each coherent domain.
Inflation is naturally realized if $f \square M_P$.

Sufficiently flat potential is required over super-Planckian field excursion.

Difficult to implement in Supergravity

$M_P = 1$ here

$$\mathcal{L} = -K_{ij*}\partial_\mu \Phi_i \partial^\mu \Phi_j^* - V_F$$

$$K_{ij*} = \frac{\partial^2 K}{\partial \Phi_i \partial \Phi_j^*} = \delta_{ij} \Leftarrow K = \sum_i |\Phi_i|^2$$

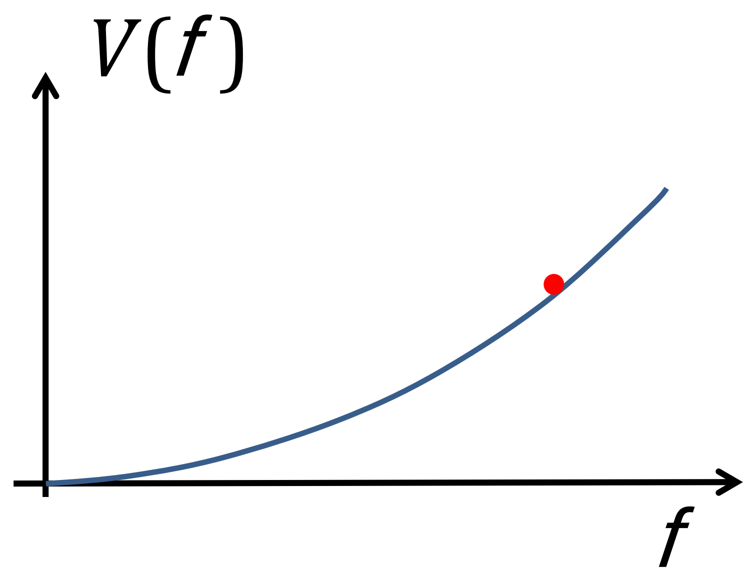
$$V_F = e^K \left[K_{ij*}^{-1} D_{\Phi_i} W D_{\Phi_j^*} W^* - 3|W|^2 \right]$$

$$\boxed{\exp\left(\sum_i |\Phi_i|^2 \right)}$$

$$D_{\Phi_i} W = \frac{\partial W}{\partial \Phi_i} + \frac{\partial K}{\partial \Phi_i} W$$

exponentially steep above M_P

Chaotic inflation



Very early Universe was dominated by large quantum fluctuations, and scalar fields must have taken random values in each coherent domain.
Inflation is naturally realized if $f \gg M_P$.

Sufficiently flat potential is required over super-Planckian field excursion.

This symmetry has been used in a number of contemporary models

Chaotic Inflation in Supergravity

Contrived superpotential in minimial SUGRA — Goncharov and Linde (1984)
Sneutrino inflation in nonminimal SUGRA — Murayama, Suzuki, Yanagida, & JY (1994)

$\boxed{\text{Shift Symmetry} \quad \Phi \to \Phi + iC}$ — Kawasaki, Yamaguchi, & Yanagida (2000)

$$K(\Phi,\Phi^*) = K(\Phi+\Phi^*) = \frac{1}{2}(\Phi+\Phi^*)^2 \Rightarrow \text{canonical kinetic term}$$

$W = mX\Phi$ weakly breaks the shift symmetry

$$\Phi = \frac{1}{\sqrt{2}}(\chi + i\varphi) \longrightarrow V = \frac{1}{2}m^2\varphi^2 + \ldots$$

No exponential increase above M_P along this direction

Tensor-to-scalar ratio

$$H^2 \cong \frac{V(\phi)}{3M_P^2}, \;\; 3H\dot{\phi} \cong -V'(\phi)$$

$$r \equiv \frac{\langle h_{ij}h^{ij}\rangle}{\langle \varsigma^2\rangle} = \frac{8\dot{\phi}^2}{M_P^2 H^2} = 8M_P^2\left(\frac{V'}{V}\right)^2 = 16\varepsilon \qquad \varepsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 = -\frac{\dot{H}}{H^2}$$

The Lyth Bound: Field excursion vs tensor-to-scalar ratio

Lyth (1997)

$$\Delta\phi \gtrsim 5\left(\frac{N}{60}\right)\left(\frac{r}{0.1}\right)^{1/2} M_P$$

Number of e-folds

$$N = \int H\,dt = \int \frac{H}{\dot{\phi}}d\phi \cong \int \frac{3H^2}{V'(\phi)}d\phi = \int \frac{V(\phi)}{M_P^2 V'(\phi)}d\phi = \int \frac{d\phi}{\sqrt{2\varepsilon}\,M_P}$$

Models with canonically normalized inflaton realizing a large tensor-to-scalar fluctuations have super-Planckian field excursion.

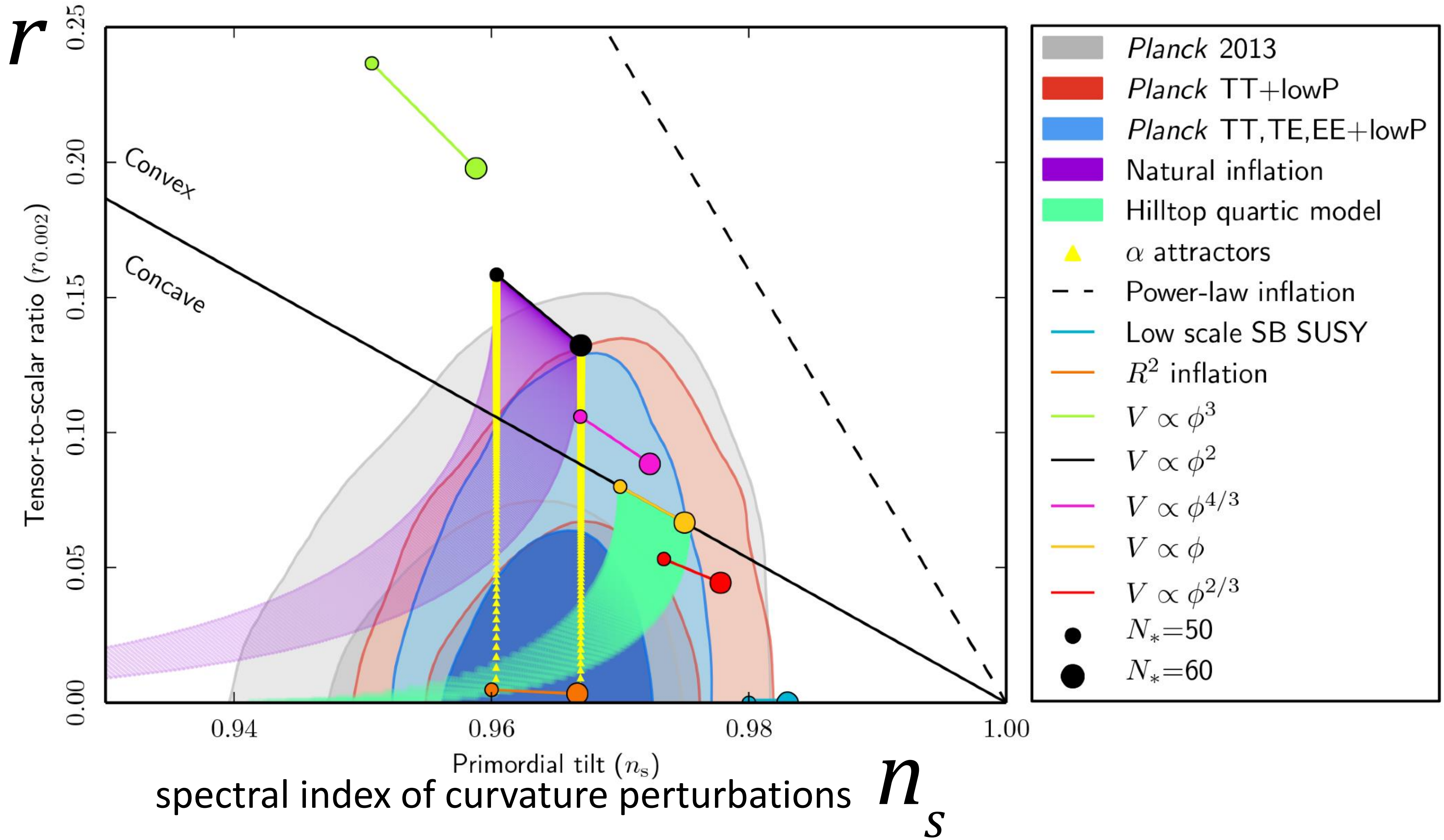
Chaotic inflation is natural, requiring no fine tuning in initial condition.

Chaotic inflation predicts observable tensor-to-scalar ratio.

$$r = 0.20^{+0.07}_{-0.05}$$

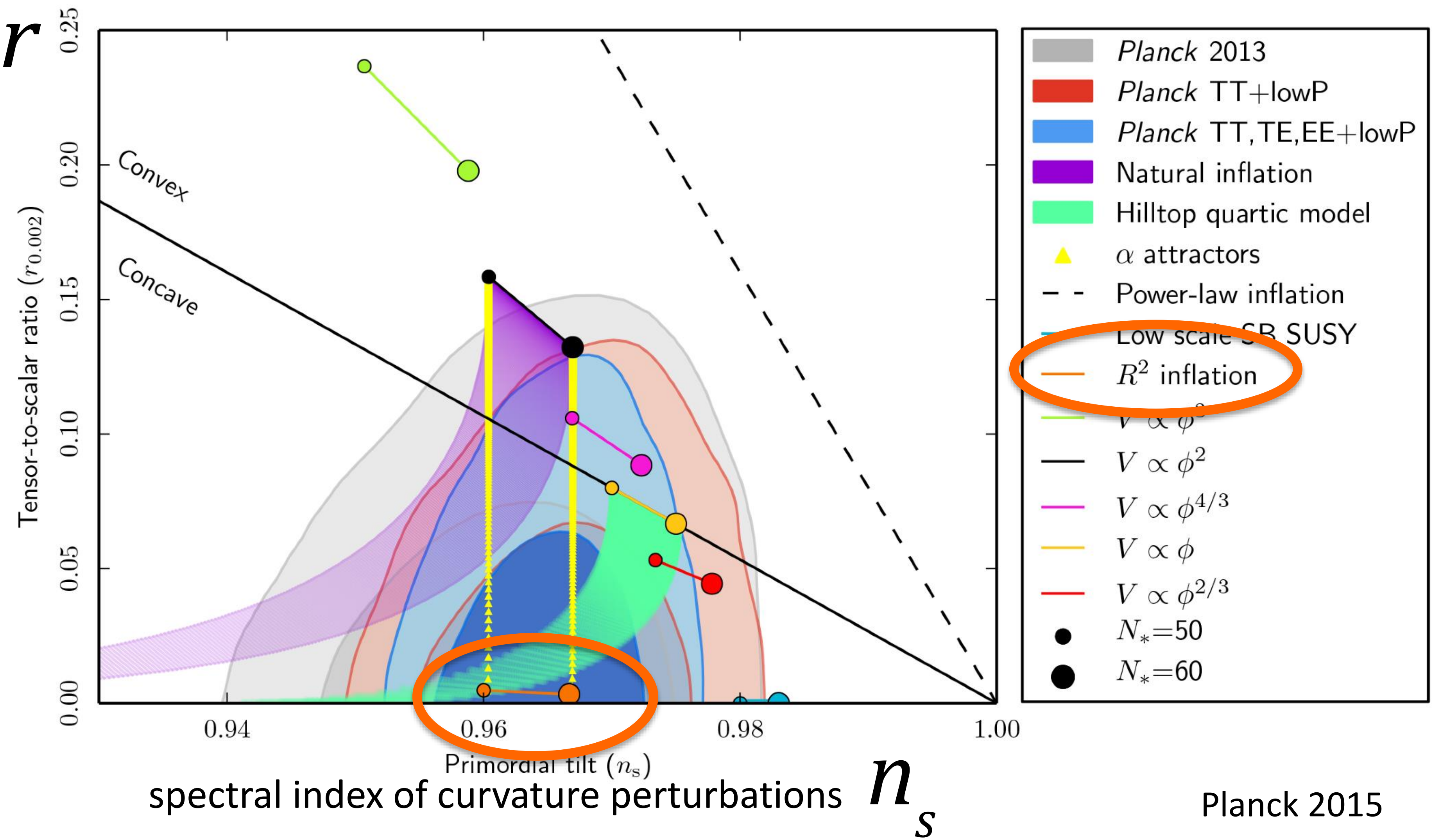
$$r = 0.16^{+0.06}_{-0.05}$$

Nature did not favor (simple polynomial) Chaotic Inflation



Planck 2015

Nature favors R^2 inflation and (original) Higgs inflation



Planck 2015

Nature favors simplistic approach?

The R² inflation (often called Starobinsky model)

Nariai & Tomita (1971)
Starobinsky (1980)

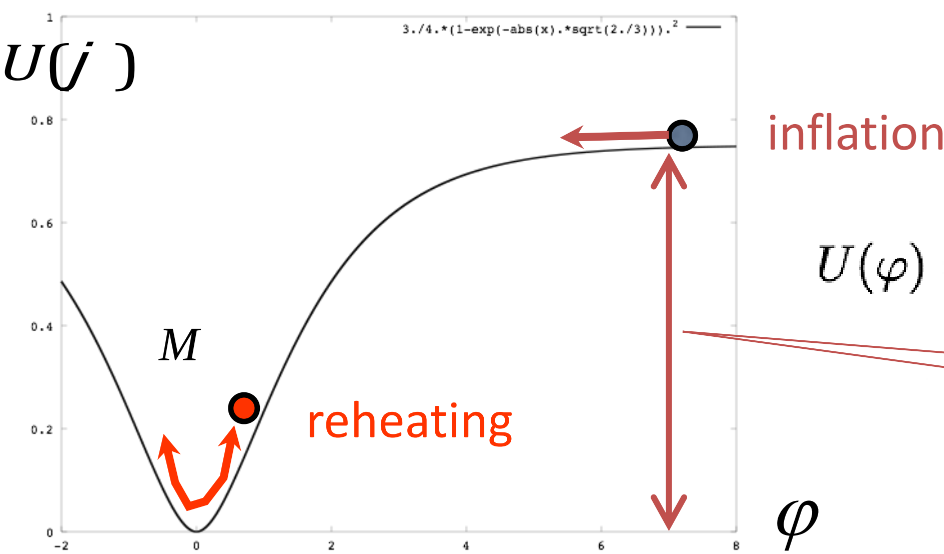
$$S = \frac{M_P^2}{2}\int f(\hat{R})\sqrt{-\hat{g}}\,d^4x \qquad f(R) = \hat{R} + \frac{\hat{R}^2}{6M^2}$$

$\boxed{\text{Conformal Transformation}}$

Jordan frame $\hat{g}_{\mu\nu}$ $\longrightarrow$ Einstein frame $g_{\mu\nu}$

$$g_{\mu\nu} = \hat{g}_{\mu\nu}\Omega^2 \qquad \Omega^2 \equiv \exp\left(\sqrt{\frac{2}{3}}\frac{\varphi}{M_P}\right) \qquad \varphi \equiv \sqrt{\frac{3}{2}}M_P \ln\left|f'(\hat{R})\right|$$

$$S = \int\sqrt{-g}\,d^4x\left[\frac{M_P^2}{2}R - \frac{1}{2}(\partial\varphi)^2 - U(\varphi)\right]$$



$M \approx 3\times10^{13}\,\mathrm{GeV}$ from amplitude of fluctuations

$$U(\varphi) = \frac{3}{4}M^2M_p^2\left(1 - e^{-\sqrt{\frac{2}{3}}\kappa\varphi}\right)^2 \qquad \kappa \propto M_P^{-1}$$

$$= \begin{cases} \frac{3}{4}M^2M_p^2 & \text{for} \quad \varphi \gg \varphi_f \\ \frac{1}{2}M^2\varphi^2 & \text{for} \quad \varphi \ll \varphi_f \end{cases}$$

$$n_s = 1 - \frac{4}{2N+1} = 0.964, \quad r = \frac{48}{(2N+1)^2} = 3.9 \times 10^{-3} \quad \text{for} \quad N = 60$$

Problem with R^2 inflation?

$$f(R) = \hat{R} + \frac{\hat{R}^2}{6M^2} + \alpha \frac{\hat{R}^3}{M^4} + \beta \frac{\hat{R}^4}{M^6} + \ldots$$

Why are higher order terms absent?

Actually, the Lagrangian is

$$\mathcal{L} = \frac{M_P^2}{2} \hat{R} + \boxed{\frac{M_P^2}{12M^2}} \hat{R}^2$$

$\approx 5.3 \times 10^8$

A large dimensionless parameter

The real question may be how the linear (Einstein) term emerged out of an R^2 theory with a large dimensionless coupling (with no Weyl curvature term).

The Original Higgs Inflation

Cervantes-Cota & Dehnen (1995);
Bezrkov & Shaposhnikov (2008)
Barvinsky, Kamenshchik, & Starobinsky (2008)…

$$\mathcal{L} = \frac{M_P^2}{2}\hat{R} - \xi \hat{R} H H^{\dagger} + \mathcal{L}_{SM} \Rightarrow \mathcal{L} = \frac{M_P^2}{2}\hat{R} - \xi \hat{R} \phi^2 - \frac{1}{2}(\hat{\partial}\phi)^2 - \frac{\lambda}{4}\phi^4$$

Conformal Transformation

Jordan frame $\hat{g}_{\mu\nu}$ → Einstein frame $g_{\mu\nu}$

$$g_{\mu\nu} = \hat{g}_{\mu\nu}\Omega^2 \qquad \Omega^2 \equiv \left| 1 - \xi \frac{\phi^2}{M_P^2} \right| \qquad \frac{d\varphi}{d\phi} = \frac{M_P\sqrt{M_P^2 - \xi(1-6\xi)\phi^2}}{M_P^2 - \xi\phi^2}$$

$$\mathcal{L} = \frac{M_P^2}{2}R - \frac{1}{2}(\partial\varphi)^2 - V_H(\varphi) \qquad V_H(\varphi) \simeq \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\kappa\varphi}\right)^2$$

The potential has the same form as conformally transformed R^2 inflation in the field range relevant to inflaton.

$$\boxed{\xi = -4.7 \times 10^4 \sqrt{\lambda}}$$ to match the amplitude of CMB anisotropy.

cf Spokoiny (1984)

Large and negative

Supergravity extension of R^2 inflation and Higgs inflation has also been studied by a number of authors.

SUGRA R^2 inflation (some including R^n corrections)

Ferrara, Grisaru, & van Nieuwenhuizen (1978); Cecotti (1987);
Hindawi, Ovrut, & Waldram (1996); Gates & Ketov (2009);
Ketov & Starobinsky (2012); Ketov (2013); Ketov & Terada (2013);
Kallosh & Linde (2013); Ellis, Nanopoulos, & Olive (2013),
Ferrara, Kallosh & van Proeyen (2013); Watanabe & JY (2013); Pallis (2014);
Giudice & Lee (2014); Kamada & JY (2014); Terada, Watanabe, Yamada & JY (2015)…

SUGRA Higgs inflation

Einhorn & Jones (2010); Ben-Dayan & Einhorn (2010), Lee (2010);
Ferrara, Kallosh, Linde, Marrani, van Proeyen (2010,2011);
Nakayama & Takahashi (2010); Chatterjee & Mazumdar (2011);
Arai, Kawai, & Okada (2011); Pallis & Toumbas (2011);
Choudhury, Chakraborty, & Pal (2014); Terada (2015); Pallis (2016)…

NO longer simple, but we must explore these possibilities
if SUSY is discovered…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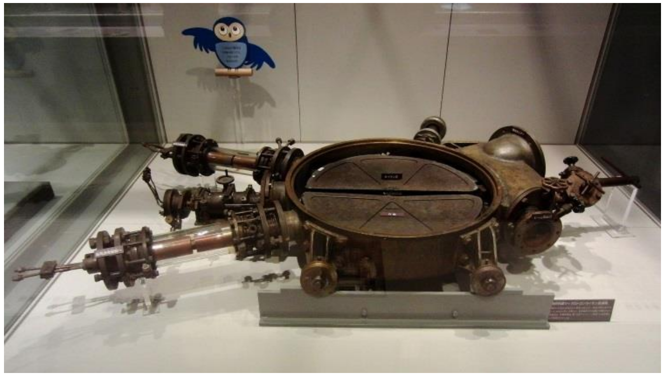
A few remarks on σ

Weather forecast

If the weather forecaster says the probability of rain tonight is 90%, we always take an umbrella and it indeed rains by the time we return home.

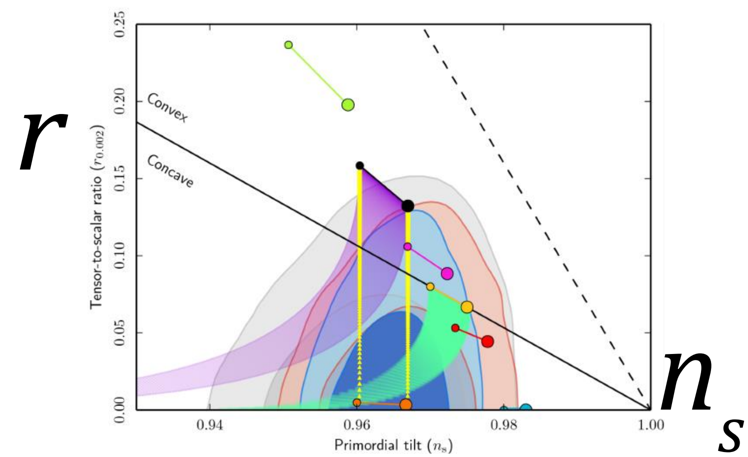
Particle physics experiments

If a 3σ (=99.7%) evidence is reported, first of all, no theorists believe it (but still write many papers), and eventually, it disappears in most cases.



Cosmological observations

We have only 13 years of experience since 1σ and 2σ contours were introduced in papers on cosmology. We are not yet used to "~~statistical~~ real significance" in cosmology and tend to interpret it incorrectly, e.g., focusing on the central region of the contours.



Aspects of SUSY Inflation

Quantum correction under control to sustain a sufficiently flat potential.

Many scalar fields which may serve as an inflaton.

Too many scalar fields, some of which may drive the Universe to a wrong state.

M_P is (one of) the cutoff scale. Shift symmetry helps to achieve field excursion larger than M_P.

For models realizing small enough CMB anisotropy in terms of small coupling constants, SUSY is the most desirable ingredient of the theory.

This is not the case for models realizing small $\frac{dT}{T}$ by a large parameter.

Is there any other way to make a model of inflation where quantum corrections are under control ?

Generalized G-inflation

The most general single scalar action with second order field equations

Kobayashi, Yamaguchi, JY (2011)

$$S = \sum_{i=2}^{5} \int d^4x \sqrt{-g}\mathcal{L}_i$$

$$X = -\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \qquad G_{iX} = \frac{\partial G_i}{\partial X}$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X), \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi, \\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\
\mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right],
\end{aligned}$$

Generalized G-inflation

The most general single scalar action with second order field equations

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$$S=\sum_{i=2}^{5}\int d^4x\,\sqrt{-g}\mathcal{L}_i$$

$$X=-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \qquad G_{iX}=\frac{\partial G_i}{\partial X}$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi,X),\\
\mathcal{L}_3 &= -G_3(\phi,X)\Box\phi,\\
\mathcal{L}_4 &= G_4(\phi,X)R+G_{4X}\left[(\Box\phi)^2-(\nabla_\mu\nabla_\nu\phi)^2\right],\\
\mathcal{L}_5 &= G_5(\phi,X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi-\frac{1}{6}G_{5X}\left[(\Box\phi)^3-3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2+2(\nabla_\mu\nabla_\nu\phi)^3\right],
\end{aligned}$$

Here we consider potential-driven slow-roll Generalized G-inflation

Expand in terms of $X=-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

$$K(\phi,X)=-V(\phi)+\mathcal{K}(\phi)X+\frac{1}{2}h_2(\phi)X^2, \qquad G_i(\phi,X)=g_i(\phi)+h_i(\phi)X$$

Later we take $\mathcal{K}(\phi)=1$ $\quad g_4=\frac{1}{2}M_P^2$ ⇒ Einstein action

Background Field Equations

$$H^2 \simeq \frac{V}{6g_4}, \qquad 3HJ \simeq -V_\phi + 12H^2 g_{4\phi},$$
$$J \simeq \dot{\phi}(\mathcal{K} + Xh_2 + 3H\dot{\phi}h_3 + 6H^2 h_4 + 3H^3\dot{\phi}h_5)$$

Perturbation variables $g_{ij} = a^2 e^{2\zeta}[e^\gamma]_{ij}$

Second-order action for Comoving Curvature Perturbation (Scalar Perturbation)

$$S_{\zeta^2} = \int d^4x a^3 \left[\mathcal{G}_S \dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\partial_i \zeta)^2\right],$$

$$\mathcal{F}_S \simeq \frac{X}{H^2}(\mathcal{K} + h_2 X + 6H^2 h_4) + \frac{4\dot{\phi}X}{H}(h_3 + H^2 h_5),$$
$$\mathcal{G}_S \simeq \frac{X}{H^2}(\mathcal{K} + 3h_2 X + 6H^2 h_4) + \frac{6\dot{\phi}X}{H}(h_3 + H^2 h_5),$$

Second-order action for Tensor Perturbation

$$S_{\gamma^2} = \int d^4x a^3 \left[\mathcal{G}_T \dot{\gamma}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\partial_k \gamma_{ij})^2\right],$$

$$\mathcal{F}_T \simeq 2g_4, \qquad \mathcal{G}_T \simeq 2g_4. \qquad g_4 = \frac{1}{2}M_P^2$$

Tensor-to-Scalar ratio

$$r = \frac{16 c_s \mathcal{F}_S}{c_t \mathcal{F}_T},$$

Scalar Sound speed

$$c_s^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S},$$

Tensor Sound speed

$$c_t^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T} \simeq 1.$$

We find $$\left|\frac{\dot{\phi}}{H}\right| = \sqrt{\frac{g_4 r}{8c_s^2}}\left(\mathcal{K} + h_2 X + 6H^2 h_4 + 4H\dot{\phi}(h_3 + H^2 h_5)\right)^{-1/2}$$

and the Lyth Bound is modified to

$$\Delta\phi \gtrsim \frac{N}{4}(r^{1/2} q_*) \simeq 5\left(\frac{N}{60}\right)\left(\frac{r}{0.1}\right)^{1/2} q_*$$

$$q \equiv \left\{\frac{\overset{= M_P^2}{2g_4}}{\boxed{c_s^2\left|\mathcal{K} + h_2 X + 6H^2 h_4 + 4H\dot{\phi}(h_3 + H^2 h_5)\right|}}\right\}^{1/2}$$

If this factor is much larger than unity, the field excursion during inflation can be smaller than M_P even with observable tensor amplitude.

→ But can this be smaller than the strong coupling scale?

Consider a potential-driven G-inflation as an example

$$\mathcal{L} = \frac{1}{2}M_P^2 R + X + \frac{1}{M^3}X\Box\phi - V(\phi) \quad \left(h_3 = -\frac{1}{M^3}\right) \quad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

At first glance, its strong coupling scale is $E \sim M$,
where perturbative calculation of scattering processes breaks down as the effective coupling exceeds unity.

This is the case in QFT in flat spacetime, but we must estimate it in the inflationary background.

The only scalar degree of freedom during inflation is curvature perturbation ζ , so we should determine the strong coupling scale from its interaction terms during inflation rather than from the form of the original Lagrangian.

cf. Nicolis, Rattazzi (2004)
Bezrukov et al. (2011)

$\left(\varsigma = H\frac{\delta\phi}{\dot{\phi}} \quad \text{in flat gauge}\right)$

Third order action of ζ

Gao, Steer (2011)
De Felice, Tsujikawa (2011)

$$S_{\zeta^3} = \int d^4x a^3 \left[\tilde{\mathcal{C}}_1 M_P^2 \zeta\dot{\zeta}^2 + \frac{1}{a^2}\tilde{\mathcal{C}}_2 M_P^2 \zeta(\partial\zeta)^2 + \tilde{\mathcal{C}}_3 M_P \dot{\zeta}^3 + \mathcal{O}(\epsilon^2) + \boxed{\left.\frac{\delta\mathcal{L}_2}{\delta\zeta}\right|_1 \mathcal{F}_1}\right]$$

$$\tilde{\mathcal{C}}_1 = -\frac{3}{c_s^2}\left(\frac{1}{c_s^2}-1\right)\epsilon_s + \frac{6}{c_s^2}\delta\mathcal{C}_7, \quad \tilde{\mathcal{C}}_2 = -\frac{c_s^2}{3}\tilde{\mathcal{C}}_1,$$

$$\tilde{\mathcal{C}}_3 = \frac{M_P}{c_s^2 H}\left[\left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right)\epsilon_s + 4\delta\mathcal{C}_6 - \frac{4}{c_s^2}\delta\mathcal{C}_7\right].$$

EOM of linear perturbation

$$\left.\frac{\delta\mathcal{L}_2}{\delta\zeta}\right|_1 = -2M_P^2\left[\frac{d}{dt}\left(a^3\frac{\epsilon_s}{c_s^2}\dot{\zeta}\right) - a\epsilon_s\partial^2\zeta\right]$$

$$\begin{aligned}\mathcal{F}_1 \equiv\ & f_1\zeta\dot{\zeta} + f_2\left[(\partial_i\zeta)^2 - \partial^{-2}\partial_i\partial_j(\partial^i\zeta\partial^j\zeta)\right] + f_3\left[\partial_i\zeta\partial^i\partial^{-2}\dot{\zeta} - \partial^{-2}\partial_i\partial_j(\partial^i\zeta\partial^j\partial^{-2}\dot{\zeta})\right]\\ &+ \frac{1}{2M_P^2\epsilon_s}\mathcal{C}_6\dot{\zeta}^2 + \frac{1}{M_P^2\epsilon_s}\mathcal{C}_7\left(\frac{3}{4c_s^2}\dot{\zeta}^2 + \frac{3}{4a^2}(\partial\zeta)^2 - \frac{3H}{c_s^2}\zeta\dot{\zeta}\right)\\ &+ \frac{1}{2M_P^2\epsilon_s}\mathcal{C}_8\left(\zeta\dot{\zeta} + \partial_i\zeta\partial^i\partial^{-2}\dot{\zeta} - \partial^{-2}(\partial_i\dot{\zeta}\partial^i\zeta + \dot{\zeta}\partial^2\zeta)\right) + (\text{higher order in slow-roll}).\end{aligned}$$

$$\mathcal{C}_i \sim \frac{M_P^2}{H^2}\epsilon$$

In terms of canonically Normalized variable $\tilde{\zeta} \simeq \sqrt{\epsilon}M_P\zeta \quad \Rightarrow$

$$\boxed{\Lambda_{\text{strong}} \sim \left(\sqrt{\epsilon}M_P H^2\right)^{1/3}}$$

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}$$

The same result can be obtained by expanding the original action

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2 R}{2} + X + \boxed{\frac{1}{M^3}X\Box\phi} - V(\phi)\right]$$ with respect to $\phi = \phi_0(t) + \boxed{\delta\phi(x,t)}$.

assume this term dominates over the kinetic term

Second-order action for $\delta\phi$

$$S_2 = \int d^4x a^3\left[\left(\frac{1}{2}\boxed{-\frac{3H\dot\phi}{M^3}}\right)\dot{\delta\phi}^2 - \frac{1}{a^2}\left(\frac{1}{2} - \frac{\ddot\phi}{M^3} - \frac{2H\dot\phi}{M^3}\right)(\partial_i\delta\phi)^2 - \frac{1}{2}V''(\phi)\delta\phi^2\right]$$

Then this term dominates the dynamics, and the canonically normalized scalar-field fluctuation reads

$\widetilde{\delta\phi} \simeq \sqrt{3}D^{\frac{1}{2}}\delta\phi$ with $D = H|\dot\phi|/M^3$

Then the cubic interaction of fluctuation reads

$$\mathcal{L}_{\delta\phi^3} \sim \frac{1}{M^3}(\partial\delta\phi)^2\Box\delta\phi \Rightarrow \mathcal{L}_{\delta\phi^3} \sim \frac{1}{D^{\frac{3}{2}}M^3}\left(\partial\widetilde{\delta\phi}\right)^2\Box\widetilde{\delta\phi}$$

Because $D^{\frac{3}{2}}M^3 \simeq \sqrt{\epsilon}M_P H^2$, the strong coupling scale is $(\sqrt{\epsilon}M_P H^2)^{\frac{1}{3}}$, which is larger than the scale of inflation H and M.

Stability of inflation against quantum corrections

First we express the second-order action for field fluctuation of $\phi = \phi_0(t) + \delta\phi(x,t)$

$$S_2 = \int d^4x a^3 \left[\left(\frac{1}{2} - \frac{3H\dot{\phi}}{M^3} \right) \dot{\delta\phi}^2 - \frac{1}{a^2} \left(\frac{1}{2} - \frac{\ddot{\phi}}{M^3} - \frac{2H\dot{\phi}}{M^3} \right) (\partial_i \delta\phi)^2 - \frac{1}{2} V''(\phi) \delta\phi^2 \right]$$

in the form

$$S_2 = \int d^4x \sqrt{-g_{\rm eff}} \left[-\frac{1}{2} g^{\mu\nu}_{\rm eff} \partial_\mu \delta\phi \partial_\nu \delta\phi - \frac{1}{2} \tilde{V}'' \delta\phi^2 \right]$$

We find the effective metric is given by $g^{\rm eff}_{\mu\nu}(\phi_0) = {\rm diag}(A, B, B, B),$

with

$$A = -\left(1 - \frac{2\ddot{\phi}}{M^3} - \frac{4H\dot{\phi}}{M^3} \right)^{\frac{3}{2}} \left(1 - \frac{6H\dot{\phi}}{M^3} \right)^{-\frac{1}{2}},$$

$$B = a^2 \left(1 - \frac{2\ddot{\phi}}{M^3} - \frac{4H\dot{\phi}}{M^3} \right)^{\frac{1}{2}} \left(1 - \frac{6H\dot{\phi}}{M^3} \right)^{\frac{1}{2}},$$

$$\tilde{V}'' = \frac{\sqrt{-g}}{\sqrt{-g_{\rm eff}}} V''.$$

Stability of inflation against quantum corrections

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$$S_2 = \int d^4x a^3 \left[\left(\frac{1}{2} - \frac{3H\dot{\phi}}{M^3} \right) \delta\dot{\phi}^2 - \frac{1}{a^2} \left(\frac{1}{2} - \frac{\ddot{\phi}}{M^3} - \frac{2H\dot{\phi}}{M^3} \right) (\partial_i \delta\phi)^2 - \frac{1}{2} V''(\phi) \delta\phi^2 \right]$$

in the form

$$S_2 = \int d^4x \sqrt{-g_{\rm eff}} \left[-\frac{1}{2} g^{\mu\nu}_{\rm eff} \partial_\mu \delta\phi \partial_\nu \delta\phi - \frac{1}{2} \widetilde{V}'' \delta\phi^2 \right]$$

Calculate the one-loop effective action using the heat kernel method

$$K(\tau) = \exp\left[\tau (\Box_{\rm eff} - \widetilde{V}'') \right]$$

$$\Rightarrow \Gamma = \frac{1}{2} \frac{d}{ds}\bigg|_{s=0} \left(\frac{1}{\Gamma(s)} \int_0^\infty d\tau \tau^{s-1} {\rm Tr} K(\tau) \right)$$

Using the heat kernel method, we find the log-divergent terms of the one-loop effective action are given by

Barvinsky, Vilkovisky (1990)

$$\Gamma \simeq \frac{1}{32\pi^2}\int d^4x\sqrt{g_{\rm eff}}\left[\frac{1}{2}\widetilde{V}''^2-\frac{1}{6}\widetilde{V}''R_{\rm eff}+\frac{1}{120}R_{\rm eff}^2+\frac{1}{60}R^{\rm eff}_{\mu\nu}R^{\mu\nu}_{\rm eff}\right]\ln\frac{\Lambda_c^2}{\mu^2}$$

with

$$R^{\rm eff}_{ij}\simeq\frac{9}{2}a^2H^2\delta_{ij},\ \ R_{\rm eff}\simeq\frac{3\sqrt{6}}{2D}H^2,\ \ \widetilde{V}''\simeq\frac{1}{8\sqrt{6}D^2}V''$$

Compared with the tree-level Lagrangian $\mathcal{L}=\frac{1}{2}M_P^2R+X+\frac{1}{M^3}X\Box\phi-V(\phi)$

$$\sqrt{-g_{\rm eff}}R_{\rm eff}^2\sim\sqrt{-g_{\rm eff}}R^{\rm eff}_{\mu\nu}R^{\mu\nu}_{\rm eff}\sim H^4\ll V$$

$$\sqrt{-g_{\rm eff}}\widetilde{V}''R_{\rm eff}\sim\frac{1}{D}H^2V'',\ \ \sqrt{-g_{\rm eff}}\widetilde{V}''^2\sim\frac{1}{D^2}V''^2$$

These terms are suppressed by powers of $\frac{1}{D}=\left|\frac{M^3}{H\dot{\phi}}\right|\ll 1$ or $\frac{H^2}{M_P^2}\ll 1$

Quantum correction is unimportant during potential driven G-inflation

This is due to the enhancement of the kinetic terms, which makes the theory effectively weakly coupled.

Kunimitsu, Suyama, Watanabe, & JY (2015)

Field excursion in Potential driven G-inflation

$$\mathcal{L} = \frac{1}{2}M_P^2 R + X + \boxed{\frac{1}{M^3}X\Box\phi} - V(\phi) \quad V(\phi) = \begin{cases} \frac{1}{p}m^{4-p}\phi^p & \text{for } p \neq 4 \\ \frac{1}{4}\lambda\phi^4 & \text{for } p = 4 \end{cases}$$

When $\frac{H\dot{\phi}}{M^3} \gg 1$, i.e. $M_P^{\frac{2(p-1)}{p+3}} m^{\frac{2(4-p)}{p+3}} \gg M^{\frac{6}{p+3}}$, dominates the kinetic term

The slow-roll field equation reads $\dot{\phi} = -\sqrt{\frac{M^3 V_\phi}{9H^2}}$

Number of e-folds of inflationary expansion

$$N = \int \frac{H}{\dot{\phi}} d\phi \simeq \frac{1}{M^{\frac{3}{2}} M_P^2} \frac{m^{\frac{4-p}{2}}}{p} \frac{2}{p+3} \phi_N^{\frac{1}{2}(p+3)} - \frac{p}{p+3}$$

$\Downarrow$

starting from the field value $\phi_N = [(p+3)N + p]^{\frac{2}{p+3}} \left(\frac{pM^{\frac{3}{2}} M_P^2}{2m^{\frac{4-p}{2}}} \right)^{\frac{2}{p+3}}$

Amplitude of curvature perturbation

$$\left(\frac{\mathcal{P}_\zeta}{2.2 \times 10^{-9}} \right)^{p+3} = (3.5 \times 10^6)^{p+3} \left(\frac{(p+3)N + p}{2} \right)^{3(p+1)} p^{-6} m^{3(4-p)} M^{3p} M_P^{-12}.$$

For $p=2$

$$\Delta\phi \lesssim \phi_{N=60} = 0.20\left(\frac{M}{10^{10}\mathrm{GeV}}\right)\left(\frac{\mathcal{P}_\zeta}{2.2\times 10^{-9}}\right)^{-\frac{2}{3}}(\sqrt{\epsilon}M_P H^2)^{\frac{1}{3}}$$

The field excursion during last 60 e-folds of inflation is well below the strong coupling scale $(\sqrt{\epsilon}M_P H^2)^{\frac{1}{3}}$ if $M < 10^{10}\,\mathrm{GeV}$.

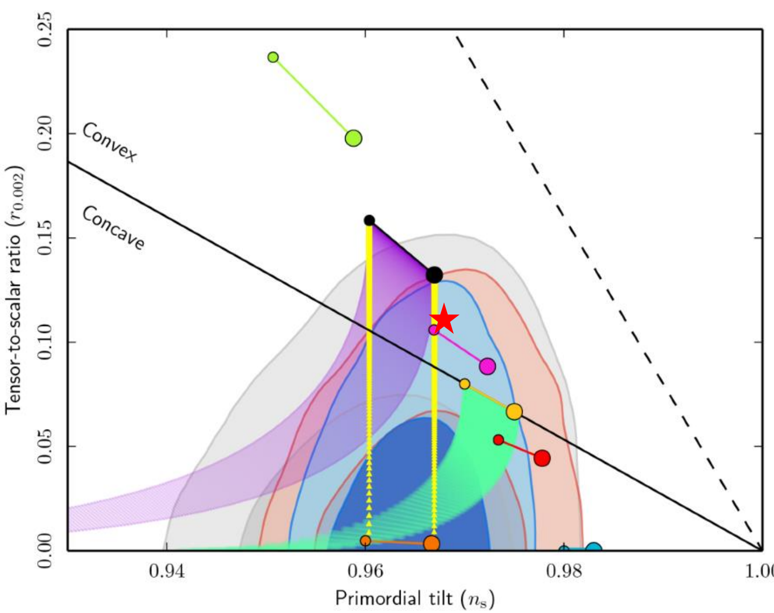
Scalar spectral index and tensor-to-scalar ratio

$$n_s - 1 = -\frac{3(p+1)}{(p+3)N+p}, \quad r = \frac{64\sqrt{2}}{3\sqrt{3}}\frac{p}{(p+3)N+p},$$

For $p=2$ we find $n_s = 0.970$, $r = 0.11$ saturating the tensor-to-scalar ratio.

Inflation with sub-strong-coupling-scale field excursion is possible even when the tensor-to-scalar ratio is large, saturating the observed bound.

Kunimitsu, Suyama, Watanabe, & JY (2015)



Other models of Potential driven Generalized G-inflation

Gravitationally enhanced friction model (New Higgs inflation type) $h_4 = const$

$$\mathcal{L} = \frac{M_P^2 R}{2} + X - V + \frac{1}{2M^2} G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

Strong coupling scale is $\Lambda^{h_4=\mathrm{const.}}_{\mathrm{strong\ coupling}} \sim (M_P H^2)^{1/3}$

Field excursion is

$$\Delta\phi \lesssim \phi_{N=60} \simeq 0.68 \left(\frac{M}{10^9 \mathrm{GeV}}\right) \left(\frac{\mathcal{P}_\zeta}{2.2 \times 10^{-9}}\right)^{-\frac{5}{6}} (M_P H^2)^{\frac{1}{3}}$$

$$M < 10^9 \,\mathrm{GeV}$$

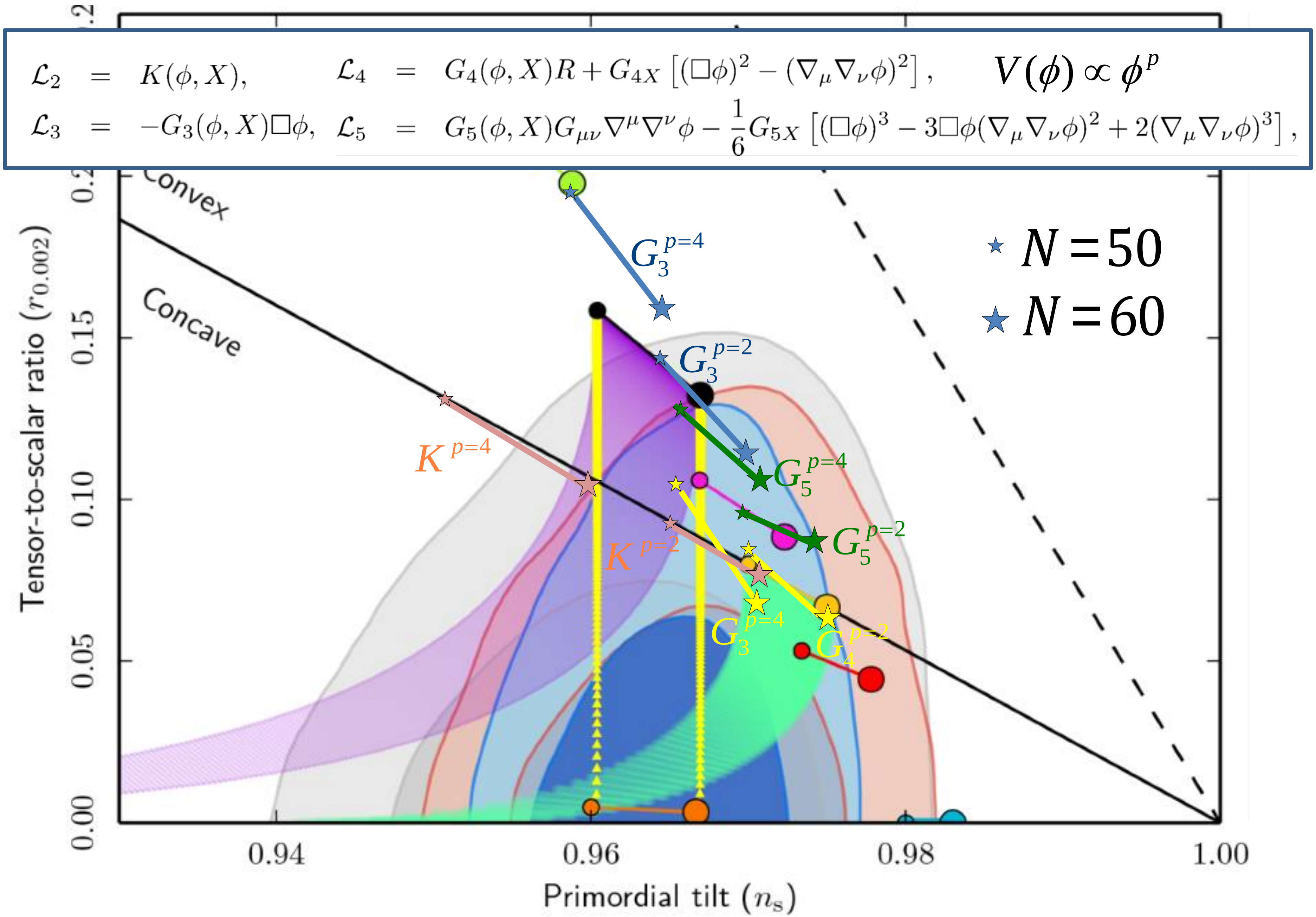
$$n_s - 1 = -\frac{4(p+1)}{2(p+2)N + p}, \qquad r = \frac{16p}{2(p+2)N + p}$$

$$n_s = 0.972\,, \qquad r = 0.066$$

Observables of Potential-driven Generalized G-inflation model

$$\mathcal{L}_2 = K(\phi, X), \qquad \mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \qquad V(\phi) \propto \phi^p$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \quad \mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right],$$



CONCLUDING REMARKS

For a long time, the issue of inflation model construction was how we may realize small parameters ($/ \square\, 10^{-13}$) to reconcile with the smallness of observed CMB anisotropy.

Supersymmetry is helpful to preserve the smallness of parameters.

“Currently favored” models contain large parameters, and particle theorists are invited to explain their origin from fundamental theory.

We can preserve flatness of the inflaton potential and achieve sub-strong-coupling scale inflation with the help of higher-order derivative interactions leading to Generalized G-inflation.

Nevertheless, the underlying physics is clearer in SUSY inflation models.

上州人

上州無智亦無才
剛毅木訥易被欺
唯以正直接萬人
至誠依神期勝利

鑑三

Backup slides

Quantum Consistency

Screening effect due to nontrivial background dynamics
A simpler example :

$$\mathcal{L} \sim Z(\partial\phi)^2 + m^2\phi^2 + \lambda\phi^3$$
$$\sim (\partial\tilde{\phi})^2 + \frac{m^2}{Z}\tilde{\phi}^2 + \frac{\lambda}{Z^{3/2}}\tilde{\phi}^3$$

$$Z(\bar{\phi}, \partial\bar{\phi}, \dots) \sim \mathrm{const.}$$
$$Z \gg 1$$
$$\tilde{\phi} \sim \sqrt{Z}\phi$$

→coupling constants and quantum corrections are effectively reduced, and an internally consistent theory is obtained without resort to any symmetry

Higher order interaction

- あくまで3次の計算からの外挿(数学的証明ではない)
  - 元のHorndeski actionと同様、n次の相互作用項で2つ微分がかかるζは
    たかだか(n-2)個→ Hの数はたかだか(2n-4)個
  - 正準規格化で ζ 1つにつき $\sqrt{\epsilon}M_P$が分母に
  - 全体には ϵM_P^2がかかる(次元、de Sitter limitで消える)

→係数は $\sim \dfrac{1}{\epsilon^{n/2-1}M_P^{n-2}H^{2n-4}} = \dfrac{1}{\left(\sqrt{\epsilon}M_P H^2\right)^{n-2}}$

よってstrong coupling scaleは低くとも

$$\Lambda_{\text{strong coupling}} \sim \left(\sqrt{\epsilon}M_P H^2\right)^{1/3}$$

と予想される。

Power law divergenceは信頼できない

- まず、regularization schemeに依存する、存在しない場合もある

- 次に、出てくる係数も低エネルギーの理論からは原理的に計算出来ないはずである。低エネルギー理論のスケール m 、新しい物理のスケール Λ 、中間スケール Λ' について $m \ll \Lambda' \ll \Lambda$ となるヒエラルキーを考える。真の理論において、たとえばmass termは

$$\mu^2 = am^2 + b\Lambda^2$$

- たとえば低エネルギー理論からpower law divergentな寄与を計算できるが、Λ' までを低エネルギーから計算したとしても

$$\mu^2 = \mu^2_{E<\Lambda'} + \mu^2_{E\geq\Lambda'} = (am^2 + c\Lambda'^2) + (c'\Lambda'^2 + b\Lambda^2)$$

となり、真の理論は Λ' に依存しないことから $c + c' = 0$

- 低エネルギーから計算されるのはcであって、真の理論のbがcと関係する理由はない。一方、Log発散の場合は係数が同じ。

(Burgess, London 1992)

Heat Kernelからの計算

Heat Kernel のトレースを R と V で展開すると、

$$\mathrm{Tr}K(\tau) = \int \frac{d^4x\sqrt{g_{\mathrm{eff}}}}{(4\pi\tau^2)^2}\left[1 + \tau\left(\frac{1}{6}R_{\mathrm{eff}} - \widetilde{V}''\right) + \tau^2\left\{\left(\frac{1}{6}R_{\mathrm{eff}} - \widetilde{V}''\right) f_1(-\tau\Box_{\mathrm{eff}})\widetilde{V}'' \right.\right.$$
$$\left.\left. + \widetilde{V}'' f_2(-\tau\Box_{\mathrm{eff}})R_{\mathrm{eff}} + R_{\mathrm{eff}} f_3(-\tau\Box_{\mathrm{eff}})R_{\mathrm{eff}} + R^{\mu\nu}_{\mathrm{eff}} f_4(-\tau\Box_{\mathrm{eff}})R^{\mathrm{eff}}_{\mu\nu}\right\} + \mathcal{O}(R^3_{\mathrm{eff}}, \widetilde{V}''^3)\right]$$

$$f_1(\xi) \equiv \int_0^1 e^{-\xi u(1-u)} du, \qquad f_2(\xi) \equiv -\frac{f_1(\xi)}{6} - \frac{f_1(\xi) - 1}{2\xi}$$
$$f_3(\xi) \equiv \frac{f_1(\xi)}{32} + \frac{f_1(\xi) - 1}{8\xi} - \frac{f_4(\xi)}{8}, \qquad f_4(\xi) \equiv \frac{f_1(\xi) - 1 + \frac{1}{6}\xi}{\xi^2}.$$

発散部分を求めるためには、τ で展開する

$$\mathrm{Tr}K(\tau) = \int \frac{d^4x\sqrt{g_{\mathrm{eff}}}}{(4\pi\tau^2)^2}\left[1 + \tau\left\{\frac{1}{6}R_{\mathrm{eff}} - \widetilde{V}''\right\}\right.$$
$$\left. + \tau^2\left\{\frac{1}{2}\widetilde{V}''^2 - \frac{1}{6}\widetilde{V}'' R_{\mathrm{eff}} + \frac{1}{120}R^2_{\mathrm{eff}} + \frac{1}{60}R^{\mathrm{eff}}_{\mu\nu}R^{\mu\nu}_{\mathrm{eff}}\right\} + \mathcal{O}(\tau^3, R^3_{\mathrm{eff}}, \widetilde{V}''^3)\right]$$

Generalized G-inflation (aka Horndeski theory)

- The most general single scalar action with second order field equations

Horndeski (1974)
Deffayet, et al. (2011)
Kobayashi, et al. (2011)

$$S=\sum_{i=2}^{5}\int d^4x\,\sqrt{-g}\mathcal{L}_i$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi,X),\\
\mathcal{L}_3 &= -G_3(\phi,X)\Box\phi,\\
\mathcal{L}_4 &= G_4(\phi,X)R+G_{4X}\left[(\Box\phi)^2-(\nabla_\mu\nabla_\nu\phi)^2\right],\\
\mathcal{L}_5 &= G_5(\phi,X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi-\frac{1}{6}G_{5X}\left[(\Box\phi)^3-3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2+2(\nabla_\mu\nabla_\nu\phi)^3\right],
\end{aligned}$$

Expand in terms of $X=-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

$$K(\phi,X)=-V(\phi)+\mathcal{K}(\phi)X+\frac{1}{2}h_2(\phi)X^2,\qquad G_i(\phi,X)=g_i(\phi)+h_i(\phi)X$$

Generalized G-inflation

expand the free functions in terms of X

$$K(\phi, X) = -V(\phi) + \mathcal{K}(\phi)X + \frac{1}{2}h_2(\phi)X^2, \qquad G_i(\phi, X) = g_i(\phi) + h_i(\phi)X$$

For slow-roll inflation

$$H^2 \simeq \frac{V}{6g_4}, \qquad 3HJ \simeq -V_\phi + 12H^2 g_{4\phi}$$

$$J \simeq (\mathcal{K} + h_2 X)\dot{\phi} + 6H(h_3 X + H h_4 \dot{\phi} + H^2 h_5 X)$$

Generalized G-inflation

$$S_2 = \int d^4x a^3 \left[\mathcal{G}_S \dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\partial\zeta)^2\right]$$

$$\mathcal{F}_S \simeq \frac{X}{H^2}(\mathcal{K} + h_2 X + 6H^2 h_4) + \frac{4\dot{\phi}X}{H}(h_3 + H^2 h_5), \quad \mathcal{F}_T \simeq 2g_4,$$
$$\mathcal{G}_S \simeq \frac{X}{H^2}(\mathcal{K} + 3h_2 X + 6H^2 h_4) + \frac{6\dot{\phi}X}{H}(h_3 + H^2 h_5), \quad \mathcal{G}_T \simeq 2g_4.$$

$$\mathcal{P} \simeq \frac{1}{2}\frac{\mathcal{G}_S^{\frac{1}{2}}}{\mathcal{F}_S^{\frac{3}{2}}}\frac{H^2}{4\pi^2} \qquad r = 16\left(\frac{\mathcal{F}_S}{\mathcal{F}_T}\right)^{3/2}\left(\frac{\mathcal{G}_S}{\mathcal{G}_T}\right)^{-1/2}$$

$$\sum_{i=2}^{5} \mathcal{E}_i = 0,$$

where

$$\begin{aligned}
\mathcal{E}_2 &= 2XK_X - K, \\
\mathcal{E}_3 &= 6X\dot{\phi}HG_{3X} - 2XG_{3\phi}, \\
\mathcal{E}_4 &= -6H^2G_4 + 24H^2X(G_{4X} + XG_{4XX}) - 12HX\dot{\phi}G_{4\phi X} - 6H\dot{\phi}G_{4\phi}, \\
\mathcal{E}_5 &= 2H^3X\dot{\phi}\,(5G_{5X} + 2XG_{5XX}) - 6H^2X\,(3G_{5\phi} + 2XG_{5\phi X})\,.
\end{aligned}$$

$$\sum_{i=2}^{5} \mathcal{P}_i = 0, \tag{3·6}$$

where

$$\mathcal{P}_2 = K, \tag{3·7}$$

$$\mathcal{P}_3 = -2X\left(G_{3\phi} + \ddot{\phi}G_{3X}\right), \tag{3·8}$$

$$\begin{aligned}
\mathcal{P}_4 = {}& 2\left(3H^2 + 2\dot{H}\right)G_4 - 12H^2XG_{4X} - 4H\dot{X}G_{4X} - 8\dot{H}XG_{4X} - 8HX\dot{X}G_{4XX} \\
&+2\left(\ddot{\phi} + 2H\dot{\phi}\right)G_{4\phi} + 4XG_{4\phi\phi} + 4X\left(\ddot{\phi} - 2H\dot{\phi}\right)G_{4\phi X},
\end{aligned} \tag{3·9}$$

$$\begin{aligned}
\mathcal{P}_5 = {}& -2X\left(2H^3\dot{\phi} + 2H\dot{H}\dot{\phi} + 3H^2\ddot{\phi}\right)G_{5X} - 4H^2X^2\ddot{\phi}G_{5XX} \\
&+4HX\left(\dot{X} - HX\right)G_{5\phi X} + 2\left[2\,(HX)^{\cdot} + 3H^2X\right]G_{5\phi} + 4HX\dot{\phi}G_{5\phi\phi}.
\end{aligned} \tag{3·10}$$

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3 J\right) = P_\phi, \tag{3·}$$

where

$$J = \dot{\phi}K_X + 6HXG_{3X} - 2\dot{\phi}G_{3\phi} + 6H^2\dot{\phi}\left(G_{4X} + 2XG_{4XX}\right) - 12HXG_{4\phi X}$$
$$+2H^3X\left(3G_{5X} + 2XG_{5XX}\right) - 6H^2\dot{\phi}\left(G_{5\phi} + XG_{5\phi X}\right), \tag{3·}$$

$$P_\phi = K_\phi - 2X\left(G_{3\phi\phi} + \ddot{\phi}G_{3\phi X}\right) + 6\left(2H^2 + \dot{H}\right)G_{4\phi} + 6H\left(\dot{X} + 2HX\right)G_{4\phi X}$$
$$-6H^2XG_{5\phi\phi} + 2H^3X\dot{\phi}G_{5\phi X}. \tag{3·13}$$

Gravity strong coupling

$$S_{\gamma^3} = \int d^4x a^3 \left[\frac{\dot{\phi} X h_5}{12} \dot{\gamma}_{ij} \dot{\gamma}_{jk} \dot{\gamma}_{ki} + \frac{\mathcal{F}_T}{4a^2} \left(\gamma_{ik}\gamma_{jl} - \frac{1}{2}\gamma_{ij}\gamma_{kl} \right) \partial_k \partial_l \gamma_{ij} \right],$$

$$\rightarrow \qquad \Lambda_{\text{strong}} \sim \sqrt{\frac{M_{PH}}{\epsilon}}$$

$$\mathcal{L} = \frac{1}{2}M_P^2 R + X + \frac{1}{M^3} X\Box\phi - V(\phi)$$

$$g_{\mu\nu}^{\rm eff}(\phi_0) = {\rm diag}(A, B, B, B)$$

$$A = -\left(1 - \frac{2\ddot{\phi}}{M^3} - \frac{4H\dot{\phi}}{M^3}\right)^{\frac{3}{2}} \left(1 - \frac{6H\dot{\phi}}{M^3}\right)^{-\frac{1}{2}}$$

$$B = a^2 \left(1 - \frac{2\ddot{\phi}}{M^3} - \frac{4H\dot{\phi}}{M^3}\right)^{\frac{1}{2}} \left(1 - \frac{6H\dot{\phi}}{M^3}\right)^{\frac{1}{2}}$$

$$R_{00}^{\rm eff} = -\frac{3\ddot{B}}{2B} + \frac{3\dot{B}^2}{4B^2} + \frac{3\dot{A}\dot{B}}{4AB}$$

$$R_{i0}^{\rm eff} = 0$$

$$R_{ij}^{\rm eff} = \left(\frac{\dot{A}\dot{B}}{4A^2} - \frac{\ddot{B}}{2A} - \frac{\dot{B}^2}{4AB}\right)\delta_{ij}$$

$$R_{\rm eff} = -\frac{3\ddot{B}}{AB} + \frac{3\dot{A}\dot{B}}{2A^2B}$$

$$\widetilde{V}'' = \frac{\sqrt{-g}}{\sqrt{-g_{\rm eff}}} V''$$

Concluding Remarks

1 It is often claimed that a positive tensor spectral index would falsify inflation.

This is true only for the standard inflation with canonical kinetic energy and for K inflation.

2 Energy conditions (strong, weak, null) have been believed to be satisfied.

Normal matter and radiation do, but dark energy may not.

3 If the Null Energy Condition is violated the energy density and the Hubble parameter may increase in time even in expanding phase.

The Universe may start in a low energy state, rather than the highest energy quantum state.

4 There have been several attempts to realize such a scenario so far with or without inflation.

But all of them suffered from instabilities at some point.

5 We have constructed a specific example of a model that realizes the following scenario without any instabilities:

a) The Universe starts with asymptotically Minkowski space.

b) It starts expansion spontaneously and smoothly connected to inflationary regime.

c) Inflation is terminated at some point and the Universe is reheated by gravitational particle production.

Stability is assured thanks to the higher spatial derivative terms which appear in the Beyond Horndeski theory.

6 The form of the action and the way of analysis is so strange that it is very difficult to convince you (and myself) that the model is relevant.

7 Nevertheless, if future observations of tensor perturbations found a positive tensor spectral index, we might have to consider such class of models seriously.

Generalized G-inflation

The most general single-field inflation φ with second order field equations

$$S=\sum_{i=2}^{5}\int \mathcal{L}_i\sqrt{-g}\,d^4x$$

Generalized Galileon = Horndeski Theory

(Kobayashi, Yamaguchi, JY 2011)

$$\mathcal{L}_2=\boxed{K(\phi,X)}$$

$$\mathcal{L}_3=-\boxed{G_3(\phi,X)}\Box\phi$$

$$\mathcal{L}_4=\boxed{G_4(\phi,X)}R+G_{4X}\left[(\Box\phi)^2-\left(\nabla_\mu\nabla_\nu\phi\right)^2\right]$$

$$\mathcal{L}_5=\boxed{G_5(\phi,X)}G_{\mu\nu}\nabla^\mu\nabla^\nu\phi-\frac{1}{6}G_{5X}\left[(\Box\phi)^3-3(\Box\phi)\left(\nabla_\mu\nabla_\nu\phi\right)^2+2\left(\nabla_\mu\nabla_\nu\phi\right)^3\right]$$

4 arbitrary functions of ϕ and $X\equiv-\frac{1}{2}(\partial\phi)^2$

Generalized G-inflation

The most general single-field inflation with second order field equations

$$S=\sum_{i=2}^{5}\int \mathcal{L}_i \sqrt{-g}\,d^4x$$

Generalized Galileon = Horndeski Theory

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$G_4 \supset M_{Pl}^2/2$ gives the Einstein action

$$\mathcal{L}_4 = \boxed{G_4(\phi, X)}R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

Generalized G-inflation is a framework to study the most general single-field inflation model with second-order field equations.

G-inflation model

$$K(\phi, X) - G(\phi, X)\Box\phi$$

Example: Generalized Higgs Inflation

$$\mathcal{L} = \frac{M_{Pl}^2}{2} R - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4 + \Delta\mathcal{L}$$

The standard Higgs potential is too steep as it is.
Several remedies have been proposed.

$$\Delta\mathcal{L} = -\frac{\xi}{2} R\phi^2$$ Original (original idea: Spokoiny 1984 Cervantes-Cota and Dehnen 1995)

$$\Delta\mathcal{L} = -\frac{1}{2M_{NH}^2} G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$ New Higgs (Germani & Kehagias 2010)

$$\Delta\mathcal{L} = -\frac{\phi^{2n}}{M_{RK}^{2n}}(\partial\phi)^2$$ Running Kinetic (Nakayama & Takahashi 2010)

$$\Delta\mathcal{L} = -\frac{\phi}{M_{HG}} X\Box\phi$$ Higgs G (Kamada, Kobayashi, Yamaguchi & JY 2011)

Example: Generalized Higgs Inflation

$$\mathcal{L} = \frac{M_{Pl}^2}{2} R - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4 + \Delta\mathcal{L}$$

All of them are variants of Generalized G-inflation

$$K(\phi, X) = -V(\phi) + \mathcal{K}(\phi) X + \ldots \qquad G_I(\phi, X) = -g_I(\phi) + h_I(\phi) X + \ldots$$

$$\Delta\mathcal{L} = -\frac{\xi}{2} R\phi^2$$

Original $r = 3\times10^{-3}$

$$g_4(\phi) = \frac{M_{Pl}^2}{2} - \frac{\xi}{2}\phi^2$$

$$\Delta\mathcal{L} = -\frac{1}{2M_{NH}^2} G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

New Higgs $r = 0.1$

$$h_4(\phi) = \frac{1}{M_{NH}^2}$$

$$\Delta\mathcal{L} = -\frac{\phi^{2n}}{M_{RK}^{2n}}(\partial\phi)^2$$

Running Kinetic $r = 0.05 - 0.16$

$$\mathcal{K}(\phi) = 1 + \frac{\phi^{2n}}{M_{RK}^{2n}}$$

$$\Delta\mathcal{L} = -\frac{\phi}{M_{HG}} X \Box\phi$$

Higgs G $r = 0.05 - 0.13$

$$g_3(\phi) = \frac{\phi}{M_{HG}}$$

Running Einstein

$$h_5(\phi) = \frac{\phi}{M_{RE}}$$

Physics behind G inflation : The Galileon

(Nicolis, Rattazzi, & Trincherini 2009)

Higher derivative theory with a 2nd order field equation was formulated using the Galilean shift symmetry (constant shift of the velocity) in flat spacetime

$$\partial_\mu\phi \longrightarrow \partial_\mu\phi + b_\mu \quad \text{namely,} \quad \phi \longrightarrow \phi + c + b_\mu x^\mu$$

and named Galileon.

$$(\partial_\mu\partial_\nu\phi)^2 = \partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi,$$
$$(\partial_\mu\partial_\nu\phi)^3 = \partial_\mu\partial_\nu\phi\partial^\nu\partial^\lambda\phi\partial_\lambda\partial^\mu\phi$$

$$\begin{aligned}
\mathcal{L}_1 &= \phi \\
\mathcal{L}_2 &= (\partial\phi)^2 \\
\mathcal{L}_3 &= (\partial\phi)^2\,\Box\phi \\
\mathcal{L}_4 &= (\partial\phi)^2\left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right] \\
\mathcal{L}_5 &= (\partial\phi)^2\left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\partial_\mu\partial_\nu\phi)^2 + 2\,(\partial_\mu\partial_\nu\phi)^3\right]
\end{aligned}$$

It is constructed by contracting 2nd derivatives with totally antisymmetric tensors, so it has only up to 8th order derivative terms in 4 spacetime dimension.

In fact the most general scalar+gravity theory that yields second-order field eqs. was discovered by Horndeski already in 1974. (revisited by Charmousis et al. 1106.2000)

$$\mathcal{L}_H = \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}\left[\kappa_1 \nabla^\mu\nabla_\alpha\phi R_{\beta\gamma}{}^{\nu\sigma} + \left(\tfrac{2}{3}\kappa_{1X}\nabla^\mu\nabla_\alpha\phi + 2\kappa_{3X}\nabla_\alpha\phi\nabla^\mu\phi\right)\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi + \kappa_3\nabla_\alpha\phi\nabla^\mu\phi R_{\beta\gamma}{}^{\nu\sigma}\right]$$

$$+\delta^{\alpha\beta}_{\mu\nu}\left[F R_{\alpha\beta}{}^{\mu\nu} + 2F_X\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi + 2\kappa_8\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\right] - 6\left(F_\phi - X\kappa_8\right)\Box\phi + \kappa_9$$

with $\delta^{\alpha\beta\gamma}_{\mu\nu\sigma} = 3!\delta^{[\alpha}_{\mu}\delta^{\beta}_{\nu}\delta^{\gamma]}_{\sigma}$ $\quad F_X = 2\left(\kappa_3 + 2X\kappa_{3X} - \kappa_{1\phi}\right)$ $\quad \kappa_1, \kappa_3, \kappa_8, \kappa_9(\phi, X)$

We have found that the Generalized Galileon is equivalent to Horndeski theory by the following identification.

$$\int^X$$

Generalized Galileon=Horndeski Theory

$$G_3 = 6F_\phi - 2X\kappa_8 - 8X\kappa_{3\phi} + 2\int^X dX'(\kappa_8 - 2\kappa_{3\phi}),$$

$$G_4 = 2F - 4X\kappa_3,$$

$$G_5 = -4\kappa_1,$$

(Kobayashi, Yamaguchi, JY 2011)

Tensor Perturbations

- The quadratic action $\alpha = \beta = \mathcal{R} = 0$

$$S_T^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\vec{\nabla} h_{ij})^2\right]$$

$$\mathcal{F}_T := 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right], \quad > 0$$

$$\mathcal{G}_T := 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right] \quad > 0$$

for stability

The "sound" velocity $c_T^2 \equiv \mathcal{F}_T / \mathcal{G}_T$ deviates from unity if $G_{4X} \neq 0$, $G_{5X} \neq 0$ or $G_{5\phi} \neq 0$.

- Tensor spectral index and amplitude

$$n_T = 3 - 2\nu_T = -\frac{4\varepsilon + 3f_T - g_T}{2(1 - \varepsilon - s_T)}$$

Blue spectrum if $4\varepsilon + 3f_T - g_T < 0.$

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}, \quad f_T \equiv \frac{\dot{\mathcal{F}}_T}{H\mathcal{F}_T}, \quad g_T \equiv \frac{\dot{\mathcal{G}}_T}{H\mathcal{G}_T}, \quad s_T \equiv \frac{\dot{c}_T}{Hc_T}.$$

$$\mathcal{P}_T(k) \cong \frac{1}{4\pi^2}\frac{H^2}{\mathcal{F}_T c_T}\bigg|_{SHC}$$

Kinetically driven G-inflation: A simple example

- $K(\phi, X) \equiv K(X),\ \ G_3(\phi, X) \equiv gX \equiv X/M^3,\ G_4 = M_{Pl}^2/2,$ and $G_5 = 0.$

  seek for a solution with $H = \text{const.}$ and $\dot\phi = \text{const.}$

$$3M_{Pl}^2 H^2 = \rho, \quad M_{Pl}^2\left(3H^2 + \cancel{2\dot H}\right) = -p$$

with $\rho = 2K_X X - K + 3G_{3X} H\dot\phi^3 - \cancel{2G_{3\phi}} X$

$$p = K - 2\left(\cancel{G_{3\phi}} + \cancel{G_{3X}\ddot\phi}\right)X$$

For $\rho = -p = -K = \text{const.} > 0$ we set $\boxed{D \equiv K_X + 3gH\dot\phi = 0}$

- The simplest solution

$$K(X) \equiv -X + \frac{X^2}{2M^3\mu} \quad \Longrightarrow \quad X \simeq M^3\mu,\ \ H^2 \simeq \frac{M^3\mu}{6M_{Pl}^2}.$$

μ=const. during de Sitter inflation

These theories may violate the NEC without any instabilities.
This issue has been studied by us in terms of perturbations generated in the Generalized G-inflation using the same Lagrangian.

$$\mathrm{d}s^2 = -N^2 \mathrm{d}t^2 + \gamma_{ij} \left(\mathrm{d}x^i + N^i \mathrm{d}t\right) \left(\mathrm{d}x^j + N^j \mathrm{d}t\right)$$

$$N = 1 + \alpha, \quad N_i = \partial_i \beta,$$

$$\gamma_{ij} = a^2(t) e^{2\mathcal{R}} \left(\delta_{ij} + h_{ij} + \frac{1}{2} h_{ik} h_{kj} \right)$$

$$h_{ii} = 0 = h_{ij,j}$$

work in the unitary gauge where the scalar field is homogeneous $\phi = \phi(t)$.

Tensor Perturbations

- The quadratic action $\alpha = \beta = \mathcal{R} = 0$

$$S_T^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\vec{\nabla} h_{ij})^2\right]$$

$$\mathcal{F}_T := 2\left[G_4 - X\left(\ddot{\phi} G_{5X} + G_{5\phi}\right)\right], \quad > 0$$

$$\mathcal{G}_T := 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right] \quad > 0$$

for stability

The "sound" velocity $c_T^2 \equiv \mathcal{F}_T / \mathcal{G}_T$ deviates from unity if $G_{4X} \neq 0$, $G_{5X} \neq 0$ or $G_{5\phi} \neq 0$.

- Tensor spectral index and amplitude

$$n_T = 3 - 2\nu_T = -\frac{4\varepsilon + 3f_T - g_T}{2(1-\varepsilon - s_T)}$$

Blue spectrum if $4\varepsilon + 3f_T - g_T < 0$.

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}, \quad f_T \equiv \frac{\dot{\mathcal{F}}_T}{H\mathcal{F}_T}, \quad g_T \equiv \frac{\dot{\mathcal{G}}_T}{H\mathcal{G}_T}, \quad s_T \equiv \frac{\dot{c}_T}{Hc_T}.$$

$$\mathcal{P}_T(k) \simeq \frac{1}{4\pi^2}\frac{H^2}{\mathcal{F}_T c_T}\bigg|_{SHC}$$

Curvature Perturbations

We adopt the unitary gauge in which ϕ is homogeneous, $\delta\phi = 0$.

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\partial_i\beta dt dx^i + a^2(1+2\mathcal{R})d\mathbf{x}^2$$

Curvature Perturbation

As usual,

① Expand the action to the second order.

② Eliminate α and β using constraint equations.

③ Obtain a quadratic action for $\mathcal{R}$.

$$S_S^{(2)} = \frac{1}{2}\int dt d^3x a^3\left[\mathcal{G}_S\dot{\mathcal{R}}^2 - \frac{\mathcal{F}_S}{a^2}(\nabla\mathcal{R})^2\right],$$

$$\mathcal{G}_T\dot{\mathcal{R}} = \Theta\alpha,$$

$$\frac{\nabla^2}{a^2}\left(\mathcal{G}_T\mathcal{R} + a^2\Theta\beta\right) = \Sigma\alpha + 3\Theta\dot{\mathcal{R}}$$

where

$$\mathcal{F}_S := \frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a}{\Theta}\mathcal{G}_T^2\right) - \mathcal{F}_T,$$

$$\mathcal{G}_S := \frac{\Sigma}{\Theta^2}\mathcal{G}_T^2 + 3\mathcal{G}_T.$$

with

$$\begin{aligned}
\Sigma := \; & XK_X + 2X^2K_{XX} + 12H\dot{\phi}XG_{3X} \\
& +6H\dot{\phi}X^2G_{3XX} - 2XG_{3\phi} - 2X^2G_{3\phi X} - 6H^2G_4 \\
& +6\Big[H^2\left(7XG_{4X} + 16X^2G_{4XX} + 4X^3G_{4XXX}\right) \\
& -H\dot{\phi}\left(G_{4\phi} + 5XG_{4\phi X} + 2X^2G_{4\phi XX}\right)\Big] \\
& +30H^3\dot{\phi}XG_{5X} + 26H^3\dot{\phi}X^2G_{5XX} \\
& +4H^3\dot{\phi}X^3G_{5XXX} - 6H^2X\big(6G_{5\phi} \\
& +9XG_{5\phi X} + 2X^2G_{5\phi XX}\big), \\
\Theta := \; & -\dot{\phi}XG_{3X} + 2HG_4 - 8HXG_{4X} \\
& -8HX^2G_{4XX} + \dot{\phi}G_{4\phi} + 2X\dot{\phi}G_{4\phi X} \\
& -H^2\dot{\phi}\left(5XG_{5X} + 2X^2G_{5XX}\right) \\
& +2HX\left(3G_{5\phi} + 2XG_{5\phi X}\right).
\end{aligned}$$

$$S_S^{(2)} = \frac{1}{2}\int dt d^3x a^3 \left[\mathcal{G}_S \dot{\mathcal{R}}^2 - \frac{\mathcal{F}_S}{a^2} (\nabla \mathcal{R})^2 \right],$$

No ghosts, No gradient instability if $\mathcal{G}_S > 0, \quad c_S^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S} > 0.$

In k inflation where $G_3 = G_5 = 0,\ G_4 = M_{Pl}^2 / 2$ hold,
we find $\mathcal{F}_S = M_{Pl}^2 \varepsilon = -M_{Pl}^2 \dot{H} / H^2$, which means that
$\dot{H} > 0$ is prohibited by the stability condition.
But in G-inflation $\dot{H} > 0$ is possible.

☆ Scalar spectral index and amplitude

$$n_S - 1 = 3 - 2\nu_S = -\frac{4\varepsilon + 3f_S - g_S}{2(1 - \varepsilon - s_S)}$$

$$\mathcal{P}_S(k) \cong \frac{1}{4\pi^2} \frac{H^2}{\mathcal{F}_S c_S}\bigg|_{SHC}$$

★ The tensor-to-scalar ratio

$$r = \frac{\mathcal{P}_T(k)}{\mathcal{P}_S(k)} = 16\frac{\mathcal{F}_S}{\mathcal{F}_T}\frac{c_S}{c_T}$$

Standard inflation $\quad \mathcal{L} = X - V[\phi] \quad$ $r = 16\varepsilon$

k-inflation $\quad \mathcal{L} = K\left(\phi, X\right) \quad$ $r = 16\varepsilon c_S$

G-inflation $\quad \mathcal{L} = K(\phi, X) - G(\phi, X)\Box\phi \quad$ $r = 16\mathcal{F}_S c_S$

Generalized G-inflation $\quad r = 16\frac{\mathcal{F}_S}{\mathcal{F}_T}\frac{c_S}{c_T}$

NOT slow-roll suppressed

Small sound speed and large tensor-to-scalar ratio are compatible.

Conventional way of thinking

As we trace evolution of the Universe backwards in time, the size of the Universe becomes smaller and smaller, and the energy density gets larger and larger, eventually reaching the Planck density.

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - g_{\mu\nu} p \qquad ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) < 0 \quad \text{if} \quad \rho + 3p > 0$$

$$\dot{\rho} = -3H(\rho + p). \qquad \dot{H} = -4\pi G(\rho + p)$$

$$\dot{\rho} \leq 0. \quad \text{if} \quad \rho + p \geq 0$$