Supersymmetry: to be or not to be?

SUSY 2016

University of Melbourne, 3 - 8 July 2016

Hermann Nicolai

MPI für Gravitationsphysik, Potsdam

(Albert Einstein Institut)

- **Why Supersymmetry?**

- **Why *low energy* ($N = 1$) Supersymmetry?**

- **What are the theoretical perspectives for SUSY if it does *not* show up at LHC?**

- **Quantum gravity: emergent space and time $\Rightarrow$ fate of space-time supersymmetry?**

- **Is there symmetry 'beyond' (maximal) SUSY?**

- **Once again: $N = 8$ Supergravity?**

Why Supersymmetry?

- Overcome Coleman Mandula No Go Theorem (1968) $\Rightarrow$ merging space-time with internal symmetries.
- Needed for UV completion of Standard Model?
- Needed for UV finite theory of Quantum Gravity?
- Strings (membranes) need supersymmetry!
- Identify geometrical origin of fermions (superspace).

NB: ***Local*** **supersymmetry** $\equiv$ **supergravity** because

$$\{Q^i_\alpha, \bar{Q}_{\dot{\beta}j}\} = 2\delta^i_j \sigma^\mu_{\alpha\dot{\beta}} P_\mu$$

$\Rightarrow$ if *all* fundamental symmetries are local (gauge) symmetries $\Rightarrow$ local supersymmetry $\Rightarrow$ local translations $\equiv$ diffeomorphisms $\Rightarrow$ general covariance and gravity!

Supersymmetric QFT

Neglecting central charges $Z_{ij}, Z^{ij} \equiv (Z_{ij})^*$, the most general supersymmetry algebra is [Haag,Lopuszanski,Sohnius (1975)]

$$[P_\mu, P_\nu] = 0 \ , \ [M_{\mu\nu}, P_\rho] = \eta_{\nu\rho} P_\mu - \eta_{\mu\rho} P_\nu \ , \dots$$
$$[P_\mu, Q^i_\alpha] = [P_\mu, \bar{Q}_{\dot{\alpha} i}] = 0 \ , \ [M^{\mu\nu}, Q^i_\alpha] = \sigma^{\mu\nu}_{\alpha\beta} Q^{\beta i} \ , \dots$$
$$\{Q^i_\alpha, Q^j_\beta\} = \{\bar{Q}_{\dot{\alpha} i}, \bar{Q}_{\dot{\beta} j}\} = 0 \ , \ \{Q^i_\alpha, \bar{Q}_{\dot{\beta} j}\} = 2\delta^i_j \sigma^\mu_{\alpha\dot{\beta}} P_\mu$$

N-extended supersymmetry (for $i, j = 1, \dots, N$) merges *spacetime and internal symmetries* when $N \geq 2$.

Other possibilities:

- **Conformal supersymmetry $\{Q^i_\alpha\} \to \{Q^i_\alpha, S^i_\alpha\}$**
- **AdS supersymmetry $(\Lambda < 0)$: AdS$_{d+1}$ $\equiv$ SConf$_d$**

NB: $\Lambda > 0$ and supersymmetry don't go together!

Representations (Supermultiplets)

Global (= rigid) supersymmetry: $s \leq 1 \quad \leftrightarrow N \leq 4$

$N = 4$ **multiplet:** $\quad 1\times[1] \oplus 4\times\left[\frac{1}{2}\right] \oplus 6\times[0]$

Local supersymmetry (supergravity) $s \leq 2 \quad \leftrightarrow N \leq 8$

$N = 8$ **multiplet:** $\quad 1\times[2] \oplus 8\times\left[\frac{3}{2}\right] \oplus 28\times[1] \oplus 56\times\left[\frac{1}{2}\right] \oplus 70\times[0]$

Maximal multiplets are CPT self-conjugate $\rightarrow$
reduces outer automorphism group from $U(N)$ **to** $SU(N)$
In particular, scalar fields are complex self-dual:

$$\phi^{ij} \equiv (\phi_{ij})^* = \frac{1}{2}\epsilon^{ijkl}\phi_{kl} \quad \textbf{for } N = 4$$

$$\phi^{ijkl} \equiv (\phi_{ijkl})^* = \frac{1}{24}\epsilon^{ijklmnpq}\phi_{mnpq} \quad \textbf{for } N = 8$$

Why Low Energy ($N = 1$) Supersymmetry?

- $N > 1$ supersymmetry does not admit chiral fermions, at least not with fundamental gauge bosons.
- Hierarchy problem: Fundamental scalar fields $\Rightarrow$ quadratic divergences. SUSY QFT has only logarithmic divergences $\Rightarrow$ stabilize (but do not explain) hierarchy between electroweak and Planck scale?
- Strongly suggested by string compactification, e.g. may emerge from heterotic string upon compactification on some Calabi-Yau manifold.
- This appears to be the only option if we want to see supersymmetry at $\mathcal{O}$(TeV) colliders! (Looking under the lamp post...)

How to break Supersymmetry?

→ still no compelling mechanism!

- Spontaneous breaking not sufficient (unlike for SM).
- Break 'softly' by introducing *explicit* mass terms.

NB: *time-dependent* (*e.g.* cosmological) backgrounds always break supersymmetry!

In the larger perspective, need to embed symmetry breaking mechanism into superstring theory:

- Below M_{Planck} superstrings give way to $N = 1$ QFT.
- From there on discard 'stringy' excitations and proceed with a standard SUSY QFT and supergravity
- Problem of breaking SUSY is even more acute in superstring theory (tachyons, runaway dilaton, SUSY breaking *vs.* modular invariance, UV finiteness?)

What if $N = 1$ Supersymmetry is not there?

- **Move up SUSY breaking scale to > 10 TeV range?**
  But: with higher and higher exclusion limits the case for $N=1$ SUSY to solve hierarchy problem weakens considerably!
- **Asymptotic safety: no SUSY needed?**
- **Conformal symmetry to solve hierarchy problem?**
- **Axions or light heavy neutrinos as DM particles?**

In this talk, more radical proposal:

- **Fate of space-time supersymmetry in quantum gravity scenarios with *emergent* space and time?**
- **Symmetry 'beyond' supersymmetry: E_{10} and $K(E_{10})$?**
- **Linking up maximal SUSY with 'real physics' may require novel symmetries, such as E_{10} and $K(E_{10})$.**

Exceptionality and Maximal Supergravity

Main message: duality symmetries are more important than space-time symmetries (and SUSY!).

- **Maximal theories:** $E_{n(n)}$ **for** $D = 11 - n$ [Cremmer,Julia(1979)]

$N = 8$ Supergravity

[Cremmer,Julia(1979); B. deWit, HN (1981)]

Unique theory (modulo 'gauging'), *most symmetric* known field theoretic extension of Einstein's theory!

$$1\times[2] \oplus 8\times\left[\tfrac{3}{2}\right] \oplus 28\times[1] \oplus 56\times\left[\tfrac{1}{2}\right] \oplus 70\times[0]$$

- ***Diffeomorphisms and local Lorentz symmetry***
- $N = 8$ ***local supersymmetry***
- $SU(8)$ ***R symmetry (local or rigid)***
- ***Linearly or non-linearly realised duality symmetry*** $E_{7(7)}$

28 electric + 28 (dual) magnetic vectors in 56 of $E_{7(7)}$**.**

70 scalar fields described by 56-bein $\mathcal{V}(x) \in E_{7(7)}/SU(8)$

$$\mathcal{V}(x) \to \mathcal{V}'(x) = g\mathcal{V}(x)h(x) \; , \quad g \in E_{7(7)} \, , \; h(x) \in SU(8)$$

Exceptionality and Maximal Supergravity

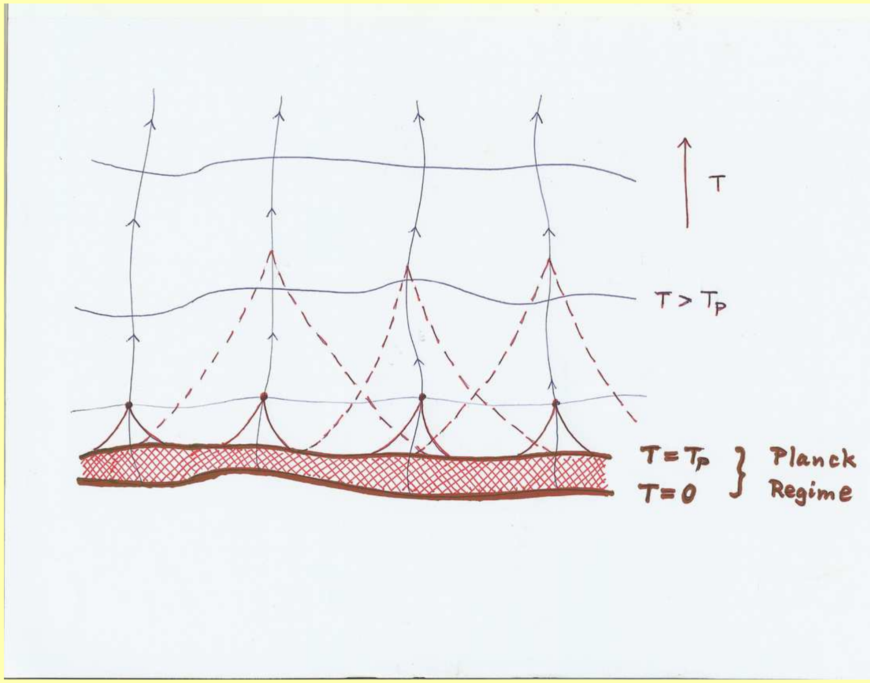
Main message: duality symmetries are more important than space-time symmetries (and SUSY!).

- **Maximal theories:** $E_{n(n)}$ **for** $D = 11 - n$ [Cremmer,Julia(1979)]

Below $D = 3$ **symmetries become *infinite-dimensional*:**

- $E_{9(9)} \equiv E_8^{(1)}$**: a solution generating symmetry acting on moduli space** $\mathcal{M} = E_{9(9)}/K(E_9)$.
- **... suggests** $E_{10(10)}$ **for** $D = 1$**: no space, only time?**
- $\Rightarrow$ **trade space-time for duality symmetries.**

E_{10}: The Basic Picture



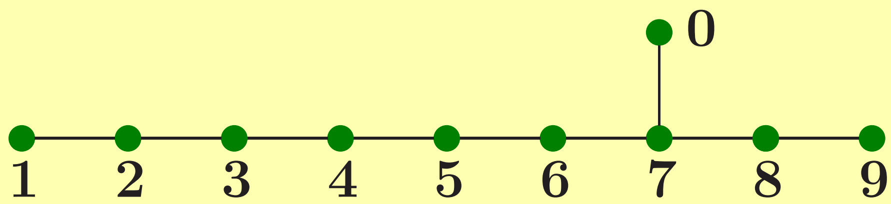
Conjecture: **for $0 < T < T_P$ space-time 'de-emerges', and space-time based (quantum) field theory is replaced by quantised 'spinning' $E_{10}/K(E_{10})$ σ-model.**

[Damour,Henneaux,Kleinschmidt, HN: since 2002]

What is E_{10}?

The nice thing about it is that no one knows .... [Murat Günaydin, unpublished]

E_{10} is the 'group' associated with the Kac-Moody Lie algebra $\mathfrak{g} \equiv \mathfrak{e}_{10}$ defined via the Dynkin diagram [e.g. Kac]



Defined by generators $\{e_i, f_i, h_i\}$ and relations via Cartan matrix A_{ij} ('Chevalley-Serre presentation')

$$
\begin{aligned}
[h_i, h_j] &= 0, & [e_i, f_j] &= \delta_{ij} h_i, \\
[h_i, e_j] &= A_{ij} e_j, & [h_i, f_j] &= -A_{ij} f_j, \\
(\mathrm{ad}\, e_i)^{1-A_{ij}} e_j &= 0 & (\mathrm{ad}\, f_i)^{1-A_{ij}} f_j &= 0.
\end{aligned}
$$

$\mathfrak{e}_{10}$ is the free Lie algebra generated by $\{e_i, f_i, h_i\}$ modulo these relations $\rightarrow$ infinite dimensional as A_{ij} is *indefinite* $\rightarrow$ Lie algebra of *exponential growth* !

Exceptionality and Maximal Supergravity

Main message: duality symmetries are more important than space-time symmetries (and SUSY!).

- **Maximal theories:** $E_{n(n)}$ **for** $D = 11 - n$ [Cremmer,Julia(1979)]

Below $D = 3$ **symmetries become** ***infinite-dimensional*****:**

- $E_{9(9)} \equiv E_8^{(1)}$**: a solution generating symmetry acting on moduli space** $\mathcal{M} = E_{9(9)}/K(E_9)$.
- **... suggests** $E_{10(10)}$ **for** $D = 1$**: no space, only time?**
- **⇒ trade space-time for duality symmetries.**
- E_{10} **'knows all' about maximal supersymmetry:**
  - contains dualities of maximal supergravities
  - supermultiplets: M theory, mIIA and IIB
  - allows to reconstruct full dynamics

$SL(10)$ level decomposition of $\mathbf{E}_{10}$

- **Decomposition w.r.t. $SL(10)$ subgroup in terms of $SL(10)$ tensors $\rightarrow$ *level expansion***

$$\alpha = \ell\alpha_0 + \sum_{j=1}^{9} m^j \alpha_j \quad \Rightarrow \quad E_{10} = \bigoplus_{\ell\in\mathbb{Z}} E_{10}^{(\ell)}$$

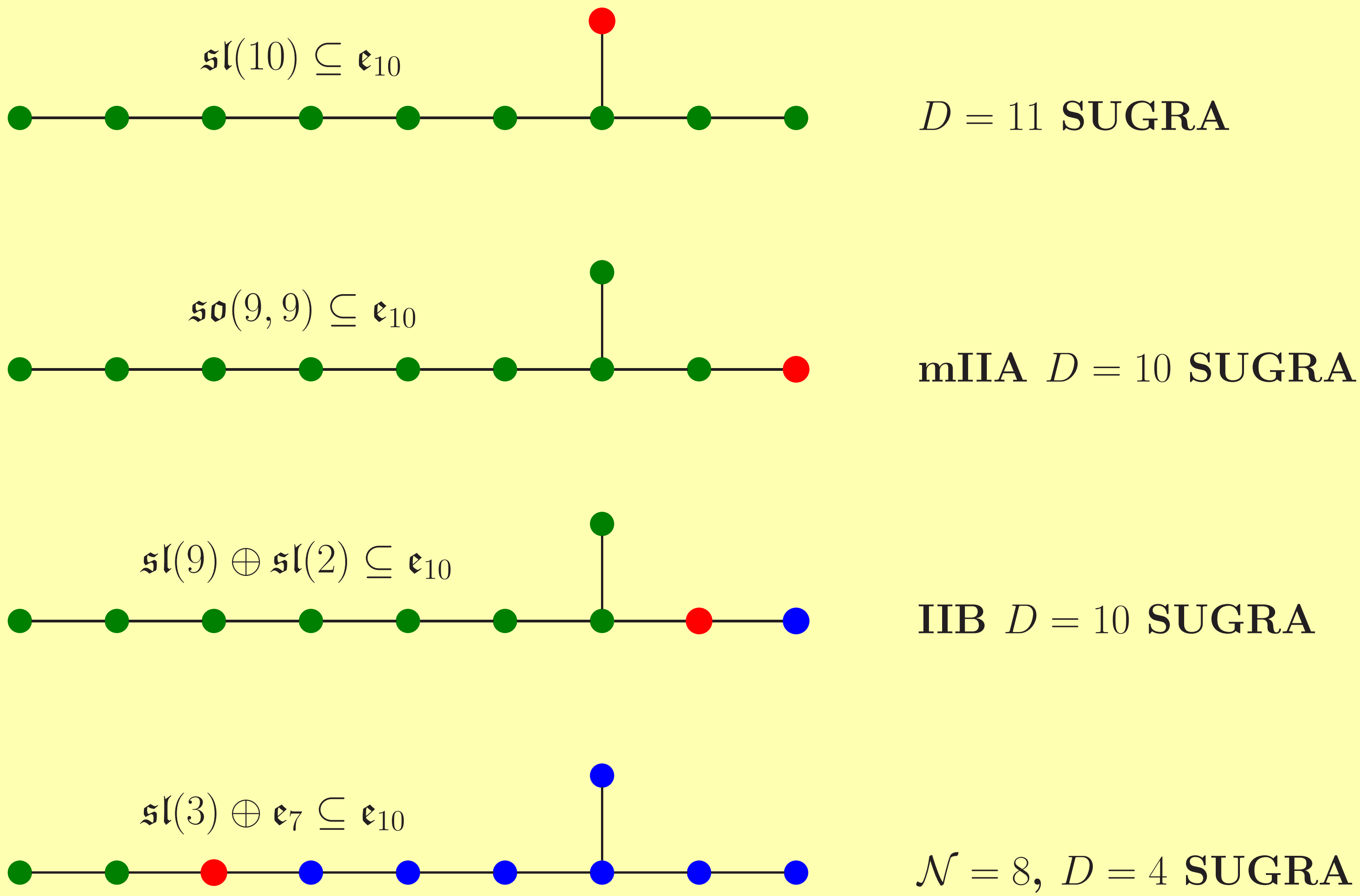
- **Up to $\ell \leq 3$ basic fields of $D = 11$ SUGRA together with their magnetic duals (spatial components)**

$\ell = 0$	G_{mn}	**Graviton**
$\ell = 1$	A_{mnp}	**3-form**
$\ell = 2$	$A_{m_1\dots m_6}$	**dual 6-form**
$\ell = 3$	$h_{m_1\dots m_8\vert n}$	**dual graviton**

- **Analysis up to level $\ell \leq 28$ yields 4 400 752 653 representations (Young tableaux) of $SL(10)$** [Fischbacher,HN:0301017]

- **Lie algebra structure (structure constants, etc.) understood only up to $\ell \leq 4$. Also: no matter where you stop it will get even more complicated beyond!**

$\mathbf{E}_{10}$ Versatility



Fermions and $\mathbf{K(E_{10})}$

Important point: maximal supersymmetric theories *not* based on (hypothetical) superextensions of E_n:

- **There is no proper superextension of E_n for any n.**
- **For $D \geq 3$ supergravity fermions transform in *maximal compact subgroup* $K(E_n) \subset E_{n(n)}$, e.g.**

$$K(E_7) \equiv SU(8) \qquad \text{fermions} \in \mathbf{8} \text{ and } \mathbf{56}$$

$$K(E_8) \equiv Spin(16)/Z_2 \qquad \text{fermions} \in \mathbf{16}_v \text{ and } \mathbf{128}_c$$

- **The associated (double-valued) fermion representations are not 'liftable' to E_n representations**
- **Fermionic sector of M theory governed by $K(E_{10})$?**
- $K(E_{10})$ **unifies R symmetries, *e.g.* IIA and IIB fermions.**

Back to $N=8$: recent developments

Very recent work has shown that $N=8$ supergravity

- **is much more finite than expected (behaves like $N=4$ super-Yang-Mills up to four loops)**

  [Bern,Carrasco,Dixon,Johansson, Roiban, PRL103(2009)081301]

- **... and could thus be finite to all orders!**
- **However: efforts towards five loops seem to be stuck.**

In string theory as well there appear difficulties starting *at five loops*: super-moduli space is no longer 'split' [Grushevsky,Witten,...]

Even if $N=8$ Supergravity is finite there remain many open questions (*e.g.* concerning *non-perturbative* quantum gravity). But ... there is a strange coincidence:

$56-8=3\times 16 \Rightarrow$ if no new fundamental spin-$\frac{1}{2}$ degrees of freedom are found at LHC, the following proposal could become relevant:

$N = 8$ Supergravity: a strange coincidence?

$SO(8) \to SU(3)\times U(1)$ **breaking and 'family-color locking'**

$$
\begin{array}{rlll}
(u\,,\,c\,,\,t)_L & : & \mathbf{3}_c \times \bar{\mathbf{3}}_f \to \mathbf{8} \oplus \mathbf{1}\,, & +\frac{1}{2} = \frac{2}{3} - q \\
(\bar{u}\,,\,\bar{c}\,,\,\bar{t})_L & : & \bar{\mathbf{3}}_c \times \mathbf{3}_f \to \mathbf{8} \oplus \mathbf{1}\,, & -\frac{1}{2} = -\frac{2}{3} + q \\
(d\,,\,s\,,\,b)_L & : & \mathbf{3}_c \times \mathbf{3}_f \to \mathbf{6} \oplus \bar{\mathbf{3}}\,, & -\frac{1}{6} = -\frac{1}{3} + q \\
(\bar{d}\,,\,\bar{s}\,,\,\bar{b})_L & : & \bar{\mathbf{3}}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{6}} \oplus \mathbf{3}\,, & +\frac{1}{6} = \frac{1}{3} - q \\
(e^-,\mu^-,\tau^-)_L & : & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}\,, & -\frac{5}{6} = -1 + q \\
(e^+,\mu^+,\tau^+)_L & : & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}\,, & +\frac{5}{6} = 1 - q \\
(\nu_e\,,\,\nu_\mu\,,\,\nu_\tau)_L & : & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}\,, & -\frac{1}{6} = -q \\
(\bar{\nu}_e\,,\,\bar{\nu}_\mu\,,\,\bar{\nu}_\tau)_L & : & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}\,, & +\frac{1}{6} = q
\end{array}
$$

Supergravity and Standard Model assignments agree if spurion charge is chosen as $q = \frac{1}{6}$ [Gell-Mann (1983)]

Realized at $SU(3)\times U(1)$ **stationary point!** [Warner,HN, NPB259(1985)412]

Fixing the spurion charge with $\mathbf{K(E_{10})}$

[Meissner,HN: Phys.Rev.D91(2015)065029; Kleinschmidt,HN: 1504.01586]

Spurion charge shift can be realised via $\mathbf{U(1)}_q$

$$\mathcal{I} = \frac{1}{2}\big(T \wedge \mathbf{1} \wedge \mathbf{1} + \mathbf{1} \wedge T \wedge \mathbf{1} + \mathbf{1} \wedge \mathbf{1} \wedge T + T \wedge T \wedge T\big)$$

acting on 56 fermions χ^{ijk} in $\mathbf{8} \wedge \mathbf{8} \wedge \mathbf{8}$ of SU(8), with $T = \varepsilon \otimes \mathbf{1}_4$ (imaginary unit in SU(3) $\times$ U(1) breaking).

$\mathcal{I}$ is *not* in SU(8) $\equiv K(\mathrm{E}_7)$ ... but it is in $\mathbf{K(E_{10})}$!

The proof requires over-extended root of $\mathbf{E}_{10} \Rightarrow$ no way to realise q-shift with finite-dimensional R symmetries!

Also: $\mathbf{K(E_{10})} \supset W(E_{10}) \supset W(E_7) \supset PSL_2(7)$
$\rightarrow$ a new family symmetry? [cf.: Chen,Perez,Ramond,1412.6107]

A new way to connect up the Planck scale?

- **Obvious need to go *beyond* $N = 8$ supergravity – but not exactly in the 'stringy way'.**
- **Family SU(3)$_f$ does *not* commute with SU(2)$_w$?**
- ***No detour* via low energy ($N = 1$) SUSY needed?**
- **K(E$_{10}$) contains transformations that act chirally on $D = 4$ fermions → extension to full SM symmetries?**
- **NB: SU(2) is the maximal anomaly free subgroup of R symmetry group SU(8)** [Derendinger, PLB151(1985)203]
  However, U(1)$_Y$ assignments don't fit → need another (anomaly-free) deformation within K(E$_{10}$)?

It would be rather striking if K(E$_{10}$) were needed to relate $N = 8$ supergravity to Standard Model fermions...

Outlook

- **All results obtained so far indicate that E_{10} requires a setting beyond known concepts of space and time.**
- **In this case space-time, and with it, general covariance and *space-time supersymmetry* would have to be emergent.** $\Rightarrow$
- **Conventional ($\equiv$ space-time) SUSY not sufficient?**
- **Can E_{10} supersede SUSY as a unifying principle?**

Outlook

- All results obtained so far indicate that E_{10} requires a setting beyond known concepts of space and time.
- In this case space-time, and with it, general covariance and *space-time supersymmetry* would have to be emergent. $\Rightarrow$
- Conventional ($\equiv$ space-time) SUSY not sufficient?
- Can E_{10} supersede SUSY as a unifying principle?
- Despite the existence of (at least) 10^{272000} string vacua

  [most recent figures from: Taylor,Wang:1511.03209; Schellekens:1601.02462]

  $N = 8$ Supergravity remains the only theory that (after complete breaking of supersymmetry) gives 48 spin-$\frac{1}{2}$ fermions, and nothing more.

Supersymmetry will have a role to play in the unification program ... but maybe not quite in the way that we have thought!

THANK YOU