SEMI-ANALYTIC TECHNIQUES FOR BARYOGENESIS

OVERVIEW

- Plenty of BSM models out there which have various 0T motivations
- Would like to know the cosmological consequences of a BSM model
- Ideally you would like to do scans
- If you would like to know where EWBG can be accommodated in your model you will encounter two non-trivial calculations
  - The dynamics of the phase transition
  - The behaviour of number densities during such a phase transition
  - Ok EDMs are tricky too but lets leave those for another time!

SUPER QUICK REVIEW OF HOW TO DERIVE TRANSPORT EQUATIONS

- CTP formalism
- Schwinger dyson equations
- Sources in terms of self energies
- Vev Insertion approximation
- General structure of equations

SUPER QUICK REVIEW OF HOW TO DERIVE TRANSPORT EQUATIONS

- CTP formalism

$$\widetilde{G}(x,y) = \begin{pmatrix} G^{t}(x,y) & -G^{<}(x,y) \\ G^{>}(x,y) & -G^{\bar{t}}(x,y) \end{pmatrix}$$

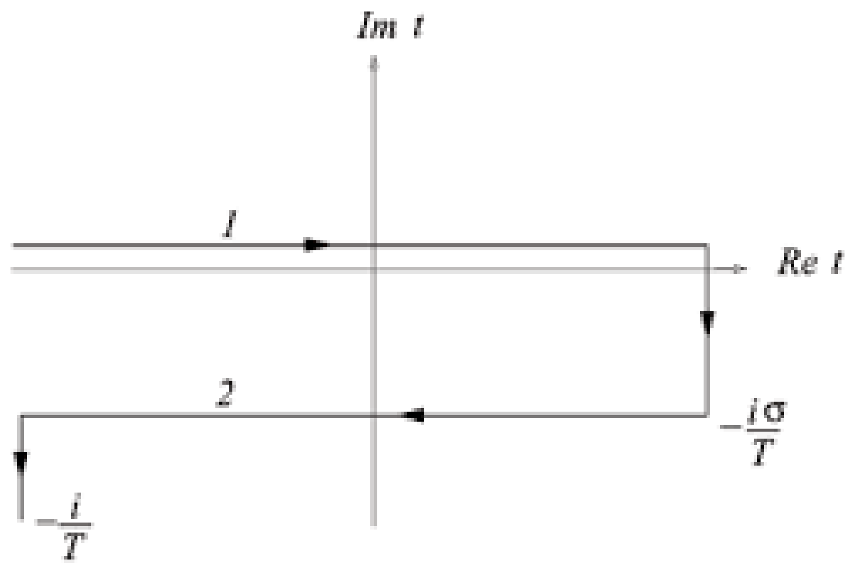

FIG. 2: Time contour in the real time formalisms.

SUPER QUICK REVIEW OF HOW TO DERIVE TRANSPORT EQUATIONS

- Schwinger dyson equations

$$\widetilde{G}(x,y) = \widetilde{G}^0(x,y) + \int d^4w \int d^4z \; \widetilde{G}^0(x,w)\widetilde{\Sigma}(w,z)\widetilde{G}(z,y)$$
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SUPER QUICK REVIEW OF HOW TO DERIVE TRANSPORT EQUATIONS

- Sources in terms of self energies

$$\frac{\partial n}{\partial X_0} + \nabla \cdot \mathbf{j}(X) = \int d^3z \int_{-\infty}^{X_0} dz_0 \Big[\Sigma^>(X,z)G^<(z,X) - G^>(X,z)\Sigma^<(z,X) + G^<(X,z)\Sigma^>(z,X) - \Sigma^<(X,z)G^>(z,X)\Big]$$

- "VEV insertion approximation"

$$\partial_\mu J_i^\mu = S_i^{CP} + S_i^{\not{CP}} + S_i^{\rm sph}$$

SEMI-ANALYTIC SOLUTION TO TRANSPORT EQUATIONS

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SEMI-ANALYTIC SOLUTION TO TRANSPORT EQUATIONS

- Diffusion approximation: $\vec{J} = \vec{\nabla} n$
- Relation between chemical potentials and number densities

$$n_i = \frac{k_i(m_i/T)\, T^2}{6} \mu_i$$

- Solve in rest frame of bubble wall: $z = |v_w t - x|$
- Solve in two regions z<0 and z>0
- Convenient rewriting of coefficients $\rightarrow a^i_{jk} \partial^i n_j$

MSSM EXAMPLE

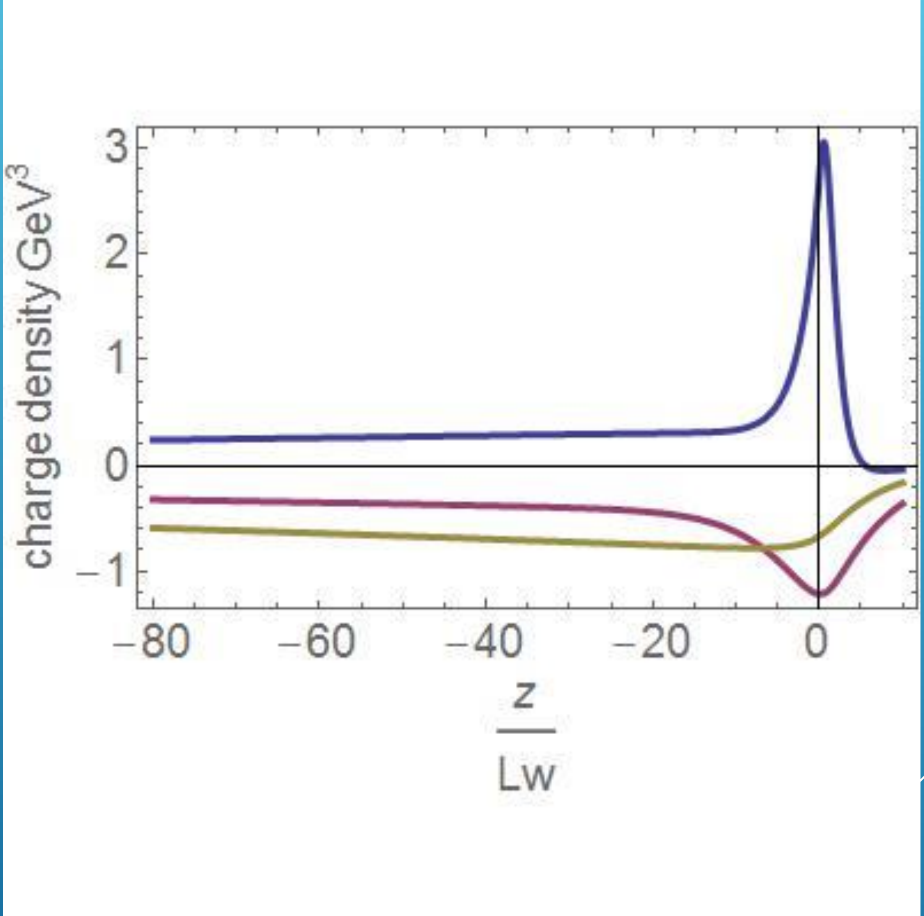
- MSSM example

$$\begin{aligned} a^i_{Q1}\partial^i Q + a^i_{T1}\partial^i T &= 0 \\ a^i_{Q2}\partial^i Q + a^i_{T2}\partial^i T + a^i_{H2}\partial^i H &= 0 \\ a^i_{Q3}\partial^i Q + a^i_{T3}\partial^i T + a^i_{H3}\partial^i H &= \Delta(z) \end{aligned}$$

$$T = \frac{1}{a^2_{T1}} \sum_{\pm} \frac{1}{\kappa_\mp - \kappa_\pm} e^{\kappa_\pm z} \left[\int^z e^{-\kappa_\pm y} \left(a^i_{Q1} \frac{\partial^i Q}{\partial y^i} \right) dy - \beta_i \right]$$

$$\begin{aligned} T &= -a^i_{Q1}\partial^i k \\ Q &= a^i_{T1}\partial^i k \,. \end{aligned}$$

$$T = \sum_{i=1}^{6} A_T(\alpha_i) x_i e^{\alpha_i z} \left(\int^z e^{-\alpha_i y} \Delta(y) dy - \beta_i \right)$$



PERTURBATIONS TO SOLUTIONS

- Assume vev dependent transport equations "switched off" in the symmetric phase
- Lets relax that assumption to take into account the space time varying vacuum

$$
\begin{aligned}
H &= H_0 + \delta_1 H + \delta_2 H + \cdots \\
T &= T_0 + \delta_1 T + \delta_2 T + \cdots \\
Q &= Q_0 + \delta_1 Q + \delta_2 Q + \cdots
\end{aligned}
$$

$$
\begin{aligned}
a^i_{T1}\partial^i\delta_1 T + a^i_{Q1}\partial^i\delta_1 Q &= 0 \\
\delta a^0_{H2}(z)H_0 + \delta a^0_{Q2}(z)Q_0 + \delta a^0_{T2}(z)T_0 + & \\
a^i_{H2}\partial^i\delta_1 H + a^i_{Q2}\partial^i\delta_1 Q + a^i_{T2}\partial^i\delta_1 T &= 0 \\
\delta a^0_{H3}(z)H_0 + \delta a^0_{Q3}(z)Q_0 + \delta a^0_{T3}(z)T_0 + & \\
a^i_{H3}\partial^i\delta_1 H + a^i_{Q3}\partial^i\delta_1 Q + a^i_{T3}\partial^i\delta_1 T &= \epsilon(z)
\end{aligned}
$$

$$
a^i_{T1}\partial^i\delta_1 T + a^i_{Q1}\partial^i\delta_1 Q = 0
$$

- Exact same format as we just solved!
- Numerically one finds about a 25% correction in the BAU for the first perturbation

SOLVING BUBBLE WALL PROFILES

- Turn our attention toward tunnelling
- Involves solving the classical equations of motion

$$\frac{\partial^2 \phi_i}{\partial \rho} + \frac{(d-1)}{\rho}\frac{\partial \phi_i}{\partial \rho} - \frac{\partial V}{\partial \phi_i} = 0$$

- For $\phi(0) \sim \phi_{true}$ and $\phi(\infty) = \phi_{false}$
- Our approach: Start with an Ansatz and create a perturbative series around it: $\phi_i(\rho) \sim A_i(\rho) + \varepsilon_i(\rho)$

SOLVING BUBBLE WALL PROFILES

$$\frac{\partial^2 \phi_i}{\partial \rho^2} + \frac{2}{\rho}\frac{\partial \phi}{\partial \rho} - \frac{\partial V \phi_j}{\partial \phi_i} = 0$$

$$\frac{\partial^2 A_i}{\partial \rho^2} + \frac{\partial^2 \epsilon_i}{\partial \rho^2} + \frac{2}{\rho}\frac{\partial A_i}{\partial \rho} + \frac{2}{\rho}\frac{\partial \epsilon_i}{\partial \rho} = \frac{\partial V \phi_j}{\partial \phi_i}\bigg|_{A_j} + \frac{\partial^2 V \phi_k}{\partial \phi_i \partial \phi_j}|_{A_j}\epsilon_j$$

$$\frac{\partial^2 \epsilon_i}{\partial \rho^2} + \frac{2}{\rho}\frac{\partial \epsilon_i}{\partial \rho} - \frac{\partial^2 V \phi_k}{\partial \phi_i \partial \phi_j}|_{A_j}\epsilon_j = \frac{\partial V \phi_j}{\partial \phi_i}\bigg|_{A_j} - \frac{\partial^2 A_i}{\partial \rho^2} - \frac{2}{\rho}\frac{\partial A_i}{\partial \rho}$$

$$\frac{\partial^2 \epsilon_i}{\partial \rho^2} + \frac{2}{\rho}\frac{\partial \epsilon_i}{\partial \rho} - \frac{\partial^2 V \phi_k}{\partial \phi_i \partial \phi_j}|_{A_j}\epsilon_j = B_i(\rho)$$

- Now a uniquely defined set of inhomogenous Eqns!

1-DIMENSIONAL ANSATZ

- Initial Ansatz solution to a single differential equation along a one dimensional path between the true and false vacuum

$$V(\phi) = M^2\phi^2 + b\phi^3 + \lambda\phi^4$$

$$\varphi = \phi_m \phi$$

$$V(\varphi) = \frac{(3-4\alpha)}{2}E\varphi^2 + E\varphi^3 - \alpha E\varphi\varphi^4$$

$$S_E = 4\pi\frac{\phi_m^3}{\sqrt{|E}}f(\alpha)$$

- $\alpha \in [0.5,0.75]$
- Use the Tanh ansatz

$$\varphi = \frac{1}{2}\left(1 - \tanh[\frac{\rho - \delta(\alpha)}{L_w(\alpha)}]\right)$$

2-DIMENSIONAL PERTURBATIONS

- Can derive perturbations to the Ansatz numerically or analytically

$$\bar{M}_{ij}(\rho) \approx M_{ij}(0) - M_{ij}(\infty)\Theta(z-b)$$

- Can use same techniques as solving the transport equations

$$\epsilon_k{}^{>,<} = \sum_i A_k(\alpha_i) h_{ij} \frac{e^{\rho\alpha_i}}{\rho} \left(\int_0^\rho t e^{-\alpha_i t} B_j{}^{>,<}(t) dt - \beta_i{}^{>,<} \right)$$

- Same as with transport equations define correction to the mass matrix as $\epsilon_i \rightarrow \varepsilon_i + \delta_i$ and $M_{ij}(\rho) = \bar{M}_{ij}(\rho) + \eta_{ij}(\rho)$

$$\delta\epsilon_k{}^{>,<} = \sum_i A_k(\alpha_i) h_{ij} \frac{e^{\rho\alpha_i}}{\rho} \left(\int_0^\rho t e^{-\alpha_i t} \left(\epsilon_i \eta_{ij}\right)^{>,<} t dt - \delta\beta_i{}^{>,<} \right)$$

EXAMPLE POTENTIAL

- Use the example potential from CosmoTransitions

$$V(x, y) = (x^2 + y^2)\left[1.8(x-1)^2 + 0.2(y-1)^2 - \delta\right]$$

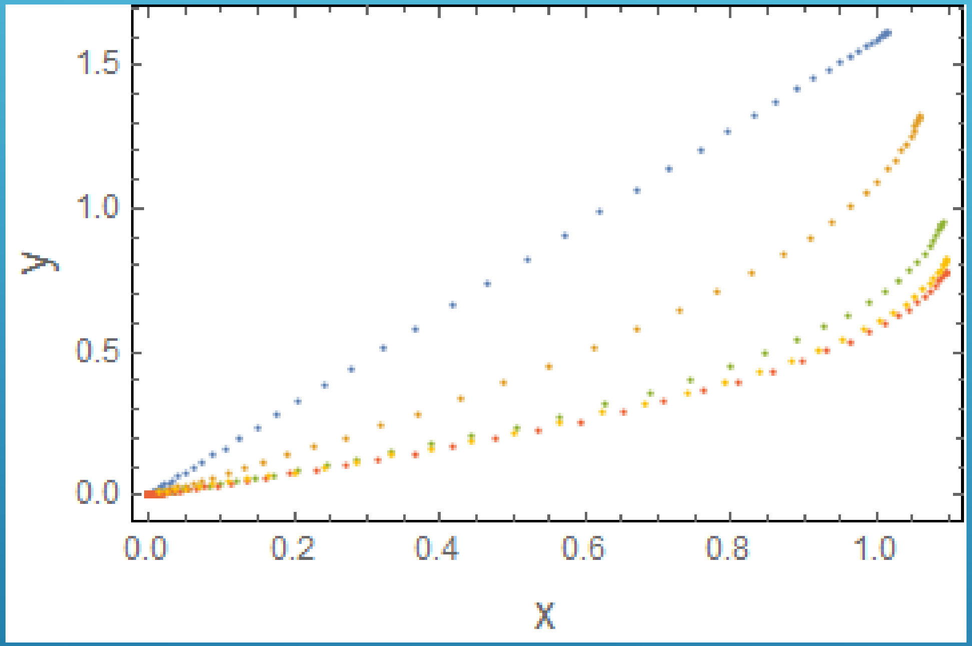
- 1-D potential rescaled

$$\frac{V(u,0)}{|E|} = 0.36u^2 - u^3 + 0.57u^4$$

- 1 D Ansatz

$$x(\rho) = 1.046\left(1 - \tanh[\frac{x - 0.437}{1}]\right)$$
$$y(\rho) = 1.663\left(1 - \tanh[\frac{x - 0.437}{1}]\right)$$

EXAMPLE POTENTIALS



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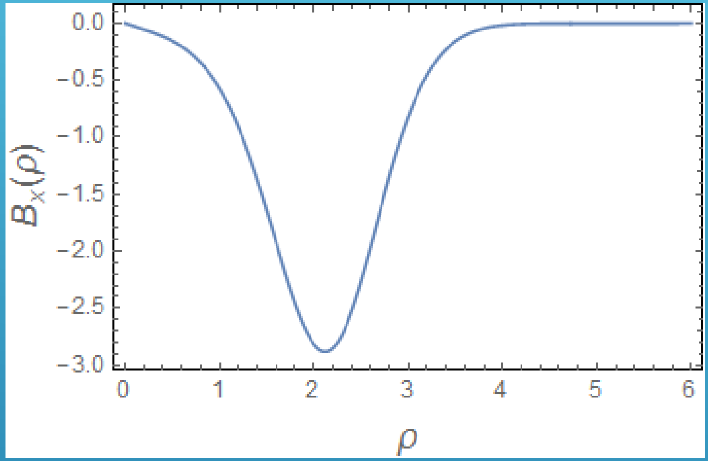

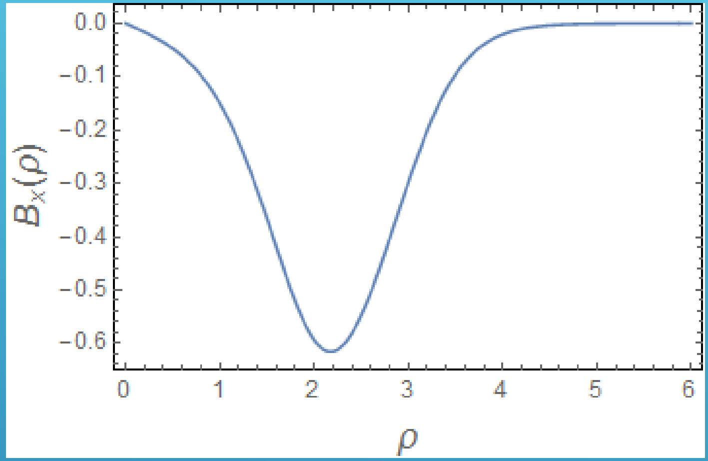

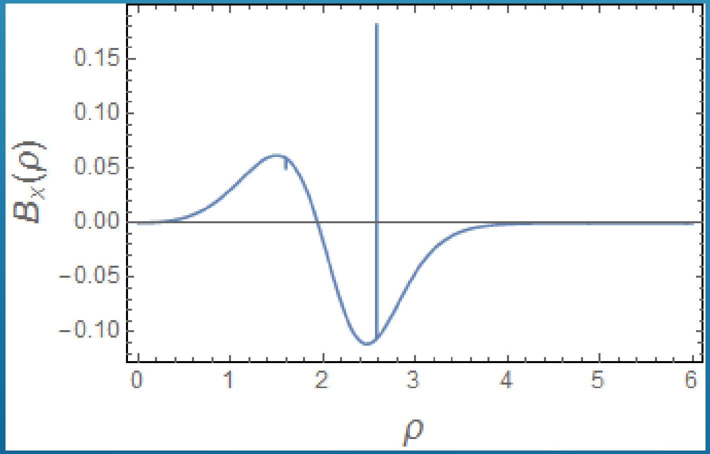

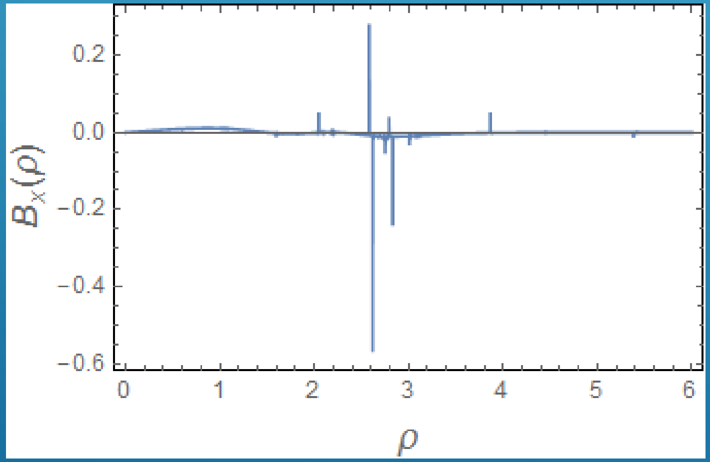

CONVERGENCE QUESTIONS

- Newtons Method known to have two convergence flaws:
  - Oscillating solutions $\epsilon_i = -\epsilon_{i+1}$
  - No Convergence if the derivative is zero
- Can show that oscillating solutions do not occur

$$\frac{\partial^2[A_{i+1}+\epsilon_{i+1}]}{\partial\rho^2}+\frac{2}{\rho}\frac{\partial[A_{i+1}+\epsilon_{i+1}]}{\partial\rho}+\left.\frac{\partial V}{\partial\phi}\right|_{A_{i+1}+\epsilon_{i+1}}=0$$

$$\frac{\partial^2[A_i+\epsilon_i+\epsilon_{i+1}]}{\partial\rho^2}+\frac{2}{\rho}\frac{\partial[A_i+\epsilon_i+\epsilon_{i+1}]}{\partial\rho}+\left.\frac{\partial V}{\partial\phi}\right|_{A_i+\epsilon_i+\epsilon_{i+1}}=0$$

$$\frac{\partial^2 A_i}{\partial\rho^2}+\frac{2}{\rho}\frac{\partial A_i}{\partial\rho}+\left.\frac{\partial V}{\partial\phi_i}\right|_{A_i}=0$$

- Analogue to a null derivative is when mass matrix is zero.

$$\epsilon_i=\int^{\rho}\frac{dy}{y^2}\int^{y}x^2B(x)dx+\beta_0+\frac{\beta_{-1}}{\rho}$$

CONCLUSION

- Found semi Analytic methods for calculating two of the most numerically intensive problems in baryogenesis calculations
- Aim to eventually make scanning for arbitrary models available
- For now you can get more details about these techniques here
- 1510.03901v3. and Arxiv 16xx.xxxxx