SEMI-ANALYTIC TECHNIQUES FOR BARYOGENESIS
Plenty of BSM models out there which have various 0T motivations

Would like to know the cosmological consequences of a BSM model

Ideally you would like to do scans

If you would like to know where EWBG can be accommodated in your model you will encounter two non-trivial calculations:

- The dynamics of the phase transition
- The behaviour of number densities during such a phase transition
- Ok EDMs are tricky too but let's leave those for another time!
SUPER QUICK REVIEW OF HOW TO DERIVE TRANSPORT EQUATIONS

- CTP formalism
- Schwinger dyson equations
- Sources in terms of self energies
- Vev Insertion approximation
- General structure of equations
SUPER QUICK REVIEW OF HOW TO DERIVE TRANSPORT EQUATIONS

- CTP formalism

\[ \tilde{G}(x, y) = \begin{pmatrix} G^+(x, y) & -G^<(x, y) \\ G^<(x, y) & -G^+(x, y) \end{pmatrix} \]

FIG. 2: Time contour in the real time formalisms.
SUPER QUICK REVIEW OF HOW TO DERIVE TRANSPORT EQUATIONS

- Schwinger dyson equations

\[
\tilde{G}(x, y) = \tilde{G}^0(x, y) + \int d^4w \int d^4z \ \tilde{G}^0(x, w) \tilde{S}(w, z) \tilde{G}(z, y)
\]

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\]
SUPER QUICK REVIEW OF HOW TO DERIVE TRANSPORT EQUATIONS

- Sources in terms of self energies

\[ \frac{\partial n}{\partial X_0} + \nabla \cdot j(X) = \int d^3z \int_{\infty}^{X_0} dz_0 \left[ \Sigma^>(X, z)G^<(z, X) - G^>(X, z)\Sigma^<(z, X) 
+ G^<(X, z)\Sigma^>(z, X) - \Sigma^<(X, z)G^>(z, X) \right] \]

- “VEV insertion approximation”

\[ \partial_{\mu} J_{\mu}^I = S_{t}^{CP} + S_{t}^{CP} + S_{t}^{sph} \]
SEMI-ANALYTIC SOLUTION TO TRANSPORT EQUATIONS

- Diffusion approximation $\vec{j} = D\vec{\nabla}n$
SEMI-ANALYTIC SOLUTION TO TRANSPORT EQUATIONS

- Diffusion approximation \( \vec{j} = D \vec{\nabla} n \)
- Relation between chemical potentials and number densities

\[
n_i = \frac{k_i (m_i / T) T^2}{6} \mu_i
\]
SEMI-ANALYTIC SOLUTION TO TRANSPORT EQUATIONS

- Diffusion approximation: $\vec{j} = \vec{v} n$
- Relation between chemical potentials and number densities

$$n_i = \frac{k_i (m_i / T)}{6} \frac{T^2}{\mu_i}$$

- Solve in rest frame of bubble wall: $z = |v_w t - x|$
- Solve in two regions $z<0$ and $z>0$
- Convenient rewriting of coefficients $\rightarrow a^i_{jk} \partial^i n_j$
MSSM example

\[ a^i_{Q1} \partial^i Q + a^i_{T1} \partial^i T = 0 \]
\[ a^i_{Q2} \partial^i Q + a^i_{T2} \partial^i T + a^i_{H2} \partial^i H = 0 \]
\[ a^i_{Q3} \partial^i Q + a^i_{T3} \partial^i T + a^i_{H3} \partial^i H = \Delta(z) \]

\[ T = \frac{1}{a^2_{Q1}} \sum_{\pm} \frac{1}{\kappa_+ - \kappa_+} e^{\kappa_+ y} \left[ \int^y e^{-\kappa_+ y} \left( a^i_{Q1} \frac{\partial^i Q}{\partial y^i} \right) dy - \beta_i \right] \]

\[ T = -a^i_{Q1} \partial^i k \]
\[ Q = a^i_{T1} \partial^i k \]

\[ T = \sum_{i=1}^6 A_T(\alpha_i) x_i e^{\alpha_i z} \left( \int^z e^{-\alpha_i y} \Delta(y) dy - \beta_i \right) \]
Assume vev dependent transport equations “switched off” in the symmetric phase

Let's relax that assumption to take into account the space time varying vacuum

Numerically one finds about a 25% correction in the BAU for the first perturbation
SOLVING BUBBLE WALL PROFILES

- Turn our attention toward tunnelling
- Involves solving the classical equations of motion

\[
\frac{\partial^2 \phi_i}{\partial \rho^2} + \frac{(d-1)}{\rho} \frac{\partial \phi_i}{\partial \rho} - \frac{\partial V}{\partial \phi_i} = 0
\]

- For \( \phi(0) \sim \phi_{\text{true}} \) and \( \phi(\infty) = \phi_{\text{false}} \)
- Our approach: Start with an Ansatz and create a perturbative series around it: \( \phi_i(\rho) \sim A_i(\rho) + \varepsilon_i(\rho) \)
SOLVING BUBBLE WALL PROFILES

Now a uniquely defined set of inhomogenous Eqns!

\[
\begin{align*}
\frac{\partial^2 \phi_i}{\partial \rho^2} + \frac{2}{\rho} \frac{\partial \phi_i}{\partial \rho} - \frac{\partial V \phi_j}{\partial \phi_i} &= 0 \\
\frac{\partial^2 A_i}{\partial \rho^2} + \frac{2}{\rho} \frac{\partial A_i}{\partial \rho} - \frac{2}{\rho} \frac{\partial \epsilon_i}{\partial \rho} &= \frac{\partial^2 V \phi_k}{\partial \phi_i \partial \phi_j} \bigg|_{A_j, \epsilon_j} + \frac{\partial^2 V \phi_k}{\partial \phi_i \partial \phi_j} |_{A_j, \epsilon_j} \\
\frac{\partial^2 \epsilon_i}{\partial \rho^2} + \frac{2}{\rho} \frac{\partial \epsilon_i}{\partial \rho} - \frac{\partial^2 V \phi_k}{\partial \phi_i \partial \phi_j} |_{A_j, \epsilon_j} &= \frac{\partial V \phi_i}{\partial \phi_i} |_{A_j} - \frac{\partial^2 A_i}{\partial \rho^2} - \frac{2}{\rho} \frac{\partial A_i}{\partial \rho} \\
\frac{\partial^2 \epsilon_i}{\partial \rho^2} + \frac{2}{\rho} \frac{\partial \epsilon_i}{\partial \rho} - \frac{\partial^2 V \phi_k}{\partial \phi_i \partial \phi_j} |_{A_j, \epsilon_j} &= B_i(\rho)
\end{align*}
\]
1-DIMENSIONAL ANSATZ

- Initial Ansatz solution to a single differential equation along a one dimensional path between the true and false vacuum

\[ V(\phi) = M^2 \phi^2 + b\phi^3 + \lambda \phi^4 \]
\[ \varphi = \phi_m \phi \]
\[ V(\varphi) = \frac{(3 - 4\alpha)}{2} \varphi^2 + \varphi^3 - \alpha E \varphi^4 \]
\[ S_E = 4\pi \frac{\phi_m^3}{\sqrt{E}} f(\alpha) \]

- \( \alpha \in [0.5, 0.75] \)
- Use the Tanh ansatz

\[ \varphi = \frac{1}{2} \left( 1 - \text{tanh} \left( \frac{\rho - \delta(\alpha)}{L_\omega(\alpha)} \right) \right) \]
2-DIMENSIONAL PERTURBATIONS

- Can derive perturbations to the Ansatz numerically or analytically
  \[ \tilde{M}_{ij}(\rho) \approx M_{ij}(0) - M_{ij}(\infty)\Theta(z - b) \]

- Can use same techniques as solving the transport equations

\[ \varepsilon_{k}^{>,<} = \sum_{i} A_{k}(\alpha_{i})h_{ij}\frac{e^{\mu_{i}}}{\rho} \left( \int_{0}^{\rho} te^{-\alpha_{i}t}B_{j}^{>,<}(t)dt - \beta_{i}^{>,<} \right) \]

- Same as with transport equations define correction to the mass matrix as \( \varepsilon_{i} \rightarrow \varepsilon_{i} + \delta_{i} \) and \( M_{ij}(\rho) = \tilde{M}_{ij}(\rho) + \eta_{ij}(\rho) \)

\[ \delta\varepsilon_{k}^{>,<} = \sum_{i} A_{k}(\alpha_{i})h_{ij}\frac{e^{\mu_{i}}}{\rho} \left( \int_{0}^{\rho} te^{-\alpha_{i}t}(\varepsilon_{i}\eta_{ij})^{>,<}t\rho_{i}dt - \delta\beta_{i}^{>,<} \right) \]
EXAMPLE POTENTIAL

- Use the example potential from CosmoTransitions
  \[ V(x, y) = (x^2 + y^2) \left[ 1.8(x - 1)^2 + 0.2(y - 1)^2 - \delta \right] \]

- 1-D potential rescaled
  \[ \frac{V(u, 0)}{|E|} = 0.36u^2 - u^3 + 0.57u^4 \]

- 1-D Ansatz
  \[
  \begin{align*}
  x(\rho) &= 1.046 \left( 1 - \tanh\left[ \frac{x - 0.437}{1} \right] \right) \\
  y(\rho) &= 1.663 \left( 1 - \tanh\left[ \frac{x - 0.437}{1} \right] \right)
  \end{align*}
  \]
EXAMPLE POTENTIALS
EXAMPLE POTENTIALS
CONVERGENCE QUESTIONS

- Newtons Method known to have two convergence flaws:
  - Oscillating solutions  \( \epsilon_i = -\epsilon_{i+1} \)
  - No Convergence if the derivative is zero

- Can show that oscillating solutions do not occur

- Analogue to a null derivative is when mass matrix is zero.

\[
\begin{align*}
\frac{\partial^2[A_i + \epsilon_i + \epsilon_{i+1}]}{\partial \rho^2} + \frac{2 \partial[A_i + \epsilon_i + \epsilon_{i+1}]}{\rho} \frac{\partial}{\partial \rho} + \frac{\partial V}{\partial \phi} \bigg|_{A_i + \epsilon_i + \epsilon_{i+1}} &= 0 \\
\frac{\partial^2[A_{i+1} + \epsilon_{i+1}]}{\partial \rho^2} + \frac{2 \partial[A_{i+1} + \epsilon_{i+1}]}{\rho} \frac{\partial}{\partial \rho} + \frac{\partial V}{\partial \phi} \bigg|_{A_{i+1} + \epsilon_{i+1}} &= 0 \\
\frac{\partial^2 A_i}{\partial \rho^2} + \frac{2 \partial A_i}{\rho} \frac{\partial}{\partial \rho} + \frac{\partial V}{\partial \phi} \bigg|_{A_i} &= 0
\end{align*}
\]

\[
\epsilon_i = \int_0^\rho \frac{dy}{y^2} \int y^2 B(x) dx + \beta_0 + \frac{\beta^{-1}_i}{\rho}
\]
CONCLUSION

- Found semi Analytic methods for calculating two of the most numerically intensive problems in baryogenesis calculations
- Aim to eventually make scanning for arbitrary models available
- For now you can get more details about these techniques here
- 1510.03901v3. and Arxiv 16xx.xxxxx