TELE script

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TELE script

- Code to automatically produce 4 lens telescopes with given magnification
- used in FFADA program
- rewritten from FORTRAN to Python
- based on calculations for 3 thin lenses $\rightarrow$ requires:
  - first drift length (e.g. L^*)
  - last drift length (little to no impact on result when kept short)
  - first quadrupole (roughly defined by L^* and maximum gradient)
- $\Rightarrow$ ideal for FFS design

4 Lens System

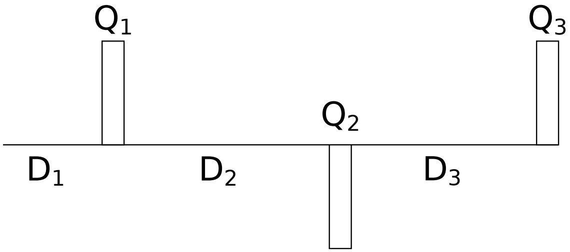
$$M_{tot} = D_4 \cdot Q_4 \cdot D_3 \cdot Q_3 \cdot D_2 \cdot Q_2 \cdot D_1 \cdot Q_1 \cdot D^*$$

L^* is given, first guess for Q_1 is defined by L^* and max. gradient, D_4 chosen freely (result depends only weakly on D_4), M_{tot} defined by magnification

$$\Rightarrow D_4^{-1} M_{tot} D^{*-1} Q_1^{-1} = Q_4 \cdot D_3 \cdot Q_3 \cdot D_2 \cdot Q_2 \cdot D_1 = M_{3\text{ lens}}$$

$\rightarrow$ reduced 4 lens system to 3 lens system

3 thin lenses



$$Q_3 \cdot D_3 \cdot Q_2 \cdot D_2 \cdot Q_1 \cdot D_1 = M_{3\text{ lens}} \quad \text{in x and y} \tag{1}$$

→ 8 nonlinear equations to solve (numerically expensive) with many unphysical solutions (e.g. negative drifts)

→ using iterative methode by Y-C Chao (1992) instead

Iterative methode

Step 1:

$$Q_2 \cdot D_2 = M^1 \tag{2}$$

$$\Rightarrow \begin{bmatrix} 1 & L_2 \\ K_2 & 1 + K_2 L_2 \end{bmatrix} = M^1$$

$$\Rightarrow L_2 = M^1_{1,2} \quad K_2 = M^1_{2,1}$$

$$\Rightarrow \begin{bmatrix} 1 & M^1_{1,2} \\ M^1_{2,1} & 1 + M^1_{1,2} M^1_{2,1} \end{bmatrix} = M^1 \tag{3}$$

Step 2:

$$M^1 = Q_2 \cdot D_2 = D_3^{-1} M^2 Q_1^{-1} \tag{4}$$

Using Eq. 3 we get expressions for L_3,K_1,L_2,K_2 as functions of matrix M^2

Step 3:

$$M^2 = D_3 \cdot Q_2 \cdot D_2 \cdot Q_1 = Q_3^{-1} M^3 D_1^{-1} \tag{5}$$

similar to Step 2 just with larger equations

> *"The algebra involved in this step is too massive to be reproduced here."*
>
> Y-C Chao

Iterative methode

REFERENCE

1. O. Napoly, Thin lens telescopes for final focus systems, CERN/LEP–TH/89–69, CLIC Note 102.

APPENDIX A EXPLICIT FORMS OF X, Y, AND Z IN EQ. (3.18)

By careful manipulation of Eq. (1.2), taking advantage of the simplified forms for the sum and difference of its x-part and y-part, one arrives at two independent and relatively simple equations for L_1 and K_3 as follow:

$$\begin{aligned} [\underline{a}^2 - 2(\overline{a}-1)\overline{a}]\tilde{K}_3\tilde{L}_1 + (\overline{a}-1)\hat{B}\tilde{K}_3 + (\overline{a}-1)\hat{C}\tilde{L}_1 + \underline{a}(\overline{a}-1)^2 &= 0, \\ [(\overline{a}-1)\,\overline{a}^2 - \underline{a}^2]\ \tilde{K}_3\tilde{L}_1 - (\overline{a}-1)\overline{a}\hat{B}\tilde{K}_3 & \\ -(\overline{a}-1)\overline{a}\hat{C}\tilde{L}_1 + [(\overline{a}-1)\hat{C}\hat{B} - \underline{a}^3(\overline{a}-1)] &= 0 \end{aligned} \qquad (A.1)$$

where

$$\widetilde{K_3} = \underline{a}K_3 - \overline{c}\ , \qquad \widetilde{L_1} = \underline{a}L_1 - \underline{b}$$

$$\hat{B} = (\underline{a}\overline{b} - \overline{a}\underline{b})\ , \qquad \hat{C} = (\underline{a}\underline{c} - \overline{a}\overline{c})\ ,$$

$$\overline{a} = \frac{M_{x11}+M_{y11}}{2}\ , \qquad \underline{a} = \frac{M_{x11}-M_{y11}}{2}\ ,$$

$$\overline{b} = \frac{M_{x12}+M_{y12}}{2}\ , \qquad \underline{b} = \frac{M_{x12}-M_{y12}}{2}\ ,$$

$$\overline{c} = \frac{M_{x21}+M_{y21}}{2}\ , \qquad \underline{c} = \frac{M_{x21}-M_{y21}}{2}\ .$$

Elimination of say, $\tilde{K}_3$ in Eq. (A.1) thus leads to the following equation:

$$A_l\ \tilde{L}_1^{\ 2} + B_l\ \tilde{L}_1 + C_l = 0\ , \qquad (A.2)$$

where

$$A_l = (\overline{a}-1)\ (\overline{a}-\underline{a})\ (\overline{a}+\underline{a})\,\hat{C}\ ,$$

$$B_l = -(2\overline{a}^2 - 2\overline{a} - \underline{a}^2)\,\hat{B}\,\hat{C} - \underline{a}(\overline{a}^4 - 2\overline{a}^3 - 2\underline{a}^2\overline{a}^2 + \overline{a}^2 + \underline{a}^2\overline{a} + \underline{a}^4 + \underline{a}^2)\ ,$$

$$C_l = (\overline{a}-1)\,\hat{B}(\hat{B}\hat{C} + \underline{a}\overline{a}^2 - \underline{a}\overline{a} - \underline{a}^3)\ .$$

The above expression is really more symmetric in the matrix elements if one uses the explicit forms of $\underline{a}$ and $\overline{a}$:

$$A_l = \frac{\hat{C}M_{x11}M_{y11}(M_{y11}+M_{x11}-2)}{2}\ ,$$

$$\begin{aligned} B_l = {} & \frac{-\hat{B}\hat{C}(M_{y11}^2 + 6M_{x11}M_{y11} - 4M_{y11} + M_{x11}^2 - 4M_{x11})}{4} \\ & -\frac{(M_{y11}-M_{x11})}{16}\ (M_{y11}^3 - 8M_{x11}^2M_{y11}^2 + 7M_{x11}M_{y11}^2 \\ & - 4M_{y11}^2 + 7M_{x11}^2M_{y11} + M_{x11}^3 - 4M_{x11}^2)\ , \end{aligned}$$

$$C_l = \frac{-\hat{B}(M_{y11}+M_{x11}-2)\ (2M_{x11}M_{y11}^2 - M_{y11}^2 - 2M_{x11}^2M_{y11} + M_{x11}^2 - 4\hat{B}\hat{C})}{8}\ .$$

Finally X, Y, and Z in equation Eq. (3.18) are given by

$$X = 2A_l\ , \qquad Y = -B_l\ , \qquad \text{and} \qquad Z = B_l^2 - 4A_lC_l\ . \qquad (A.3)$$

The complexity of Eqs. (A.2) and (3.19) does not pose any problem in terms of coding into a FORTRAN program and maintaining high degrees of accuracy and efficiency.

Iterative methode

Eq. (3.18) into equations obtained at earlier stages of the iteration with fewer elements:

$$K_3 = -\frac{-2 + M_{x22} + M_{y22} - M_{x21}L_1 - M_{y21}L_1}{-M_{x12} + M_{y12} + M_{x11}L_1 - M_{y11}L_1} ,$$

$$K_1 = \frac{K_{1D}}{D} , \qquad K_2 = \frac{K_{2D}}{D} ,$$

$$L_2 = \frac{L_{2D}}{D} , \qquad L_3 = \frac{L_{3D}}{D} , \tag{3.19}$$

where

$$K_{1D} = -M_{x21} - M_{y21} + M_{x11}K_3 - M_{y11}K_3 ,$$

$$L_{2D} = -(M_{y12} - M_{y11}L_1)(M_{x22} - M_{x12}K_3 - (M_{x21} - M_{x11}K_3)L_1) + (M_{x12} - M_{x11}L_1)(M_{y22} + M_{y12}K_3 - (M_{y21} + M_{y11}K_3)L_1) ,$$

$$K_{2D} = (M_{y21} + M_{y11}K_3)(M_{x22} - M_{x12}K_3 - (M_{x21} - M_{x11}K_3)L_1) + (M_{x21} - M_{x11}K_3)(M_{y22} + M_{y12}K_3 - (M_{y21} + M_{y11}K_3)L_1) ,$$

$$L_{3D} = -M_{x12} + M_{y12} + M_{x11}L_1 - M_{y11}L_1 ,$$

$$D = -M_{x22} + M_{y22} + M_{x12}K_3 + M_{y12}K_3 + M_{x21}L_1 - M_{y21}L_1 - M_{x11}K_3L_1 - M_{y11}K_3L_1 .$$

Iterative methode

- no simple criterion to ensure physical solutions
- no proof the results are always real numbers
- explicit solution $\rightarrow$ fast numerical calculation

Finding physical solutions

- physical solution (i.e. positive drift lengths) not guaranteed → introduction of *Figure Of Merit*

$$FOM = \frac{1}{\Sigma_i L_i \cdot \Sigma_j K_j}$$

- maximizing FOM = best balance between overall length and integrated Quadrupole strength (chromaticity)
- TELE increases/decreases length of first quadrupole (without changing gradient) till a local maximum of the FOM is found
- physical solution still not guaranteed but more likely
- optimum solution also not guaranteed due to search for local maximum

Literature

- O. Napoly, B. Dunham, 'FFADA, Computer Design of Final Focus Systems for Linear Colliders', EPAC94, London, England, June 1994, p. 698
- Y-C Chao, J. Irwin, 'Solution of a Three-Thin-Lens System', SLAC-PUB-5834 (1992)
- B. Zotter, 'Design Considerations for a Chromatically Corrected Final Focus System for TeV Colliders', CLIC-Note-64 (1988)
- O. Napoly, 'Thin Lens Telescopes for Final Focus Systems', CLIC Note 102 (1989)