Mass spectra and decay modes of B_c^+ from relativistic confinement scheme

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2^{nd} Heavy Flavor Meet-2016

Outline

Motivation

- The only experimentally identified state having both heavy but different flavors.
- More stable as it can not decay into gluons.
- Discovered in 1998 by CDF collaboration through channel $B_c^{\pm} \rightarrow J/\psi l^{\pm} \nu$ [1]
- Also confirmed by $D0$ Collaboration [2] and LHCb Collaboration [3] in channel $B_c^{\pm} \rightarrow J/\psi \pi^{\pm}$
- World average mass 6.2756 ± 0.0011 GeV [4]

[1] F. Abe et al., Phys. Rev. Lett. **81**, 2432 (1998)
[2] V. Abazov et al., Phys. Rev. Lett. **101**, 012001 (2008)
[3] R. Aaij et al., Phys. Rev. Lett. **109**, 232001 (2012)
[4] K. A. Olive et al. (Particle Data Group) Chin. Phys. C, **38**, 090001 (2014)

Relativistic Harmonic Oscillator Model

- The Dirac equation

$$[\vec{\alpha}\cdot\vec{p}+\beta(m+S(r))+V(r)]\,\psi = i\frac{\partial\psi}{\partial t}$$

- For confinement, oscillator type of potential: Lorentz scalar and Lorentz vector → **RHM** for coloured quarks [5]

$$V(r)=\frac{1}{2}(1+\gamma_0)A^2r^2+B$$

- The Current Confinement Model was developed for confinement of gluons in the spirit of RHM [6,7]
- These both gives the satisfactory description of light hadron spectroscopy, baryon magnetic moments as well as glue-balls [8]

[5] S. B. Khadkikar and S. K. Gupta Phys. Lett. B **124**, 523 (1983)

[6] S. B. Khadkikar, Pramana Jnl of Phys. **24**, 63 (1985)

[7] S. B. Khadkikar, P. C. Vinodkumar Pramana, J. Phys. **29**, 39 (1987)

[8] Vinodkumar et al., DAE symposium on nuclear physics: contributed papers. Vol. **36B** (1993)

Dirac Equation with harmonic confinement

The Dirac equation with confinement potential

$$[\vec{\alpha}\cdot\vec{p}+\gamma_0 m_q+V(r)]\psi=E\psi$$

ψ can be expressed as

$$\psi=\begin{pmatrix}\chi\\ \phi\end{pmatrix}$$

The two component form of Dirac equation can be written as

$$\psi_q(r)=N_q\begin{pmatrix}\chi(r)\\ -\frac{i\vec{\sigma}\cdot\vec{\nabla}\chi}{E+m_q}\end{pmatrix}$$

and

$$\psi_{\bar{q}}(r)=N_q\begin{pmatrix}-\frac{\vec{\sigma}\cdot\vec{\nabla}\chi}{E+m_q}\\ i\chi(r)\end{pmatrix}$$

Non relativistic reduction of Dirac equation

Eliminating the lower component ϕ from bispinor totally through the transformation [9]

$$U\psi_q = \frac{1}{\left[1 + \frac{p^2}{(E+M_q)^2}\right]} \begin{pmatrix} 1 & \frac{\sigma\cdot\mathbf{p}}{|E+M_q|} \\ -\frac{\sigma\cdot\mathbf{p}}{|E+M_q|} & 1 \end{pmatrix} \begin{pmatrix} \chi_q \\ \phi_q \end{pmatrix} = \begin{pmatrix} \chi_q \\ 0 \end{pmatrix}$$

such that $\langle\psi_q|\psi_q\rangle = \langle\chi_q|\chi_q\rangle$ and are normalized

[9] S. B. Khadkikar et al, Phys. Lett. B **254**, 320 (1991)

The equation satisfied by the upper component of χ

$$\left[-\nabla^2 + \underline{(E+m_q)A^2}r^2\right]\chi = (E^2 - M_q^2)\chi$$

with size parameter $\Omega_N = (E_N + M_q)^{1/2} A$
The Energy eigen value $E_N^2 = M_q^2 + (2N+3)\Omega_N$, $N = 0, 1, 2...$
The radial solution is

$$R_{nl}(r) = \left[\frac{\Omega_N^{3/2}}{2\pi}\frac{n!}{\Gamma(n+l+\frac{3}{2})}\right]^{1/2} (\Omega_N^{1/2} r)^l exp\left(\frac{-\Omega_n r^2}{2}\right) L_n^{l+\frac{1}{2}}(\Omega_N r^2)$$

The single particle energy

$$E_N^3 - M_q E_N^2 - M_q^2 E_N + M_Q^3 - (2N+3)^2 A^2 = 0$$

Intrinsic Energy and Centre of Mass Correction

- The nonrelativistic reduction lead to the fact that the single particle energy obtained for both quarks and antiquarks are identical.
- Intrinsic energies of the participating quarks are obtained by subtracting their contribution to the centre of mass from their single particle energy

$$\epsilon_N(q)_{conf} = \sqrt{(2N+3)\Omega_N(q) + M_q^2 - \frac{3M_q}{M_1+M_2}\Omega_0(q)}$$

- Last term is centre of mass correction
- Applicable to hadronic systems

Mass Spectra

The mass of meson in the N energy eigenstate and J spin state [10]

$$M_N^J = \sum_{i=1}^{2} \epsilon_N(q_i)_{conf} + \sum_{i<j=1}^{2} \epsilon(q_i, q_j)_{coul} + \sum_{i<j=1}^{2} \epsilon_N^J(q_i, q_j)_{S.D.}$$

[10] Vinodkumar et al, Euro. J. Phys. A **4**, 83 (1999)

Residual Colour Coulomb Interaction

- Coulomb interaction

$$\epsilon(q_1, q_2)_{coul} = \left\langle NS \middle| V_{coul}(r) = \frac{k\alpha_s^{eff}}{r} \middle| NS \right\rangle$$

α_s^{eff} $\rightarrow$ effective strong running coupling coefficient related to perturbative $\alpha_s(\mu)$: $\alpha_s^{eff} = \frac{\alpha_s(\mu)}{\omega_n}$

where $\omega_n \rightarrow$ color dielectric coefficient[11], expressed as

$$\omega_n = \sqrt{\frac{\alpha_s^3(\mu)}{\alpha_m(\mu)}} \left\{ 2^{\frac{n-1}{2}} [2^{n+1} - (-1)^n] \right\}^{-1}$$

$\alpha_m(\mu) \rightarrow$ flavor dependent dimensionless model parameter and is a measure of confinement strength, expressed as

$$\alpha_m(\mu_q) = \left[\frac{A^{2/3}}{\mu_q} \right]$$

$\alpha_s(\mu) \rightarrow$ strong running coupling constant.

[11] Gottfried K. and Weisskopf V. F., "Concept of particle physics -Vol II" Oxford University Press, New York (1986) p. 307-08

The wave functions for the two quark systems by retaining the nature of single particle wave function but with a two particle size parameter

$$\Omega_N(q_i q_j) = C_N(q_i)\Omega_N(q_i) + C_N(q_j)\Omega_N(q_j)$$

$C_N(q_i)$ can be related to the Moshinsky transformation coefficients[12] and are parameterised in terms of the ratio of two dimensionless parameters α_m and α_s

The additivity of the size parameter is weighted with the residual interaction strength.

$$\Omega_N(q_q, q_j) = \frac{C_0}{2^n} \sum_{i=1}^{2} \frac{\alpha_s(\mu_i)}{\alpha_m(\mu_i)} \Omega_N(q_i)$$

where

$n \rightarrow$ radial quantum number

$C_0 \rightarrow$ constant weight factor

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[12] M. Moshinsky, The Harmonic Oscillator in Modern Physics : From Atoms to Quarks, Gordon and Breach Science Publishers, New York 1969

Confined one gluon exchange potential

- In most of the confinement models studied, gluon exchange effects are incorporated into the theory through OGEP and the gluon propagators used are similar to the free photon propagators of QED.
- COGEP has spin-spin + spin-orbit + tensor terms
- Spin-spin interaction is computed using the spin hyperfine interaction of the residual confined one gluon exchange potential

$$V_{\sigma_i \cdot \sigma_j} = \frac{\alpha_s(\mu) N_i^2 N_j^2}{4} \frac{\lambda_i \lambda_j}{[E_i + m_i][E_j + m_j]} \left[4\pi\delta^3(r) - C^4 r^2 D_1(r)\right] \left(-\frac{2}{3}\sigma_i \sigma_j\right)$$

- And the spin-orbit interaction of confined one gluon exchange potential

$$V_{q_1 q_2}^{LS} = \frac{\alpha_s}{4} \frac{N_1^2 N_2^2}{(E_1 + M_1)(E_2 + M_2)} \frac{\lambda_i \cdot \lambda_j}{2r_{12}} \left[4\vec{L} \cdot \vec{S}(D_0'(r_{12}) + 2D_1'(r_{12}))\right]$$

$N_{1/2}$ are the RHM normalisation constants

$$N_{1/2} = \sqrt{\frac{2(E_{1/2} + M_{1/2})}{3E_{1/2} + M_{1/2}}}$$

D_0 and D_1 are the confined one gluon propagator

$$D_0(r) = \frac{\Gamma(1/4)}{4\pi^{3/2}} c(cr)^{-3/2} W_{1/2,-1/4}(c^2 r^2)$$
$$D_1(r) = \frac{\Gamma(1/4)}{4\pi^{3/2}} c(cr)^{-3/2} W_{0,-1/4}(c^2 r^2)$$

where,
$c \rightarrow$ confinement strength of gluon
$W \rightarrow$ *Whittaker* functions

Parameters used for computation

Table: Parameters fitted to compute masses

Confinement mean field parameter	A	$=$	2166 $MeV^{3/2}$
Coulomb parameter	k	$=$	0.19
Weight factor (used in size parameter)	C_0	$=$	1.47
Mass of b quark	m_b	$=$	4637 MeV
Mass of c quark	m_c	$=$	1426 MeV
Running coupling constant	α_s	$=$	0.255

Table: Mass of Bc meson in MeV

State	present	Ref[a]	Ref[b]	Ref[c]	Ref[d]	Ref[e]	Ref[f]	Ref[g]
1^1S_0	6256	6278	6271	6272	6253	6264	6286	6280±30±190
1^3S_1	6314	6331	6338	6333	6317	6337	6341	6321±20
1^3P_0	6709	6748	6706	6699	6683	6700	6701	6727±30
1^3P_1	6755	6767	6741	6743	6717	6730	6737	6743±30
1^3P_0	6801	6769	6750	6750	6729	6736	6760	6765±30
1^1P_1	6847	6775	6768	6761	6743	6747	6772	6783±30
2^1S_0	6929	6863	6855	6842	6867	6856	6882	6960±80
2^3S_1	6968	6873	6887	6882	6902	6899	6914	6990±80
2^3P_0	7156	7139	7122	7094	7088	7108		
2^3P_1	7169	7155	7145	7134	7113	7135		
2^3P_0	7182	7156	7150	7147	7124	7142		
2^1P_1	7195	7162	7164	7157	7134	7153		
3^1S_0	7308	7244	7250	7226				
3^3S_1	7326	7249	7272	7258				

[a] N. Devlani et al.,Eur. Phys. J. A **50**, 154 (2014)
[b] S. Godfrey, Phys. Rev. D **70**, 054017 (2004)
[c] D. Ebert et al, Eur. Phys. J. C **71**, 1825 (2011)
[d] S. Gershtein, V. Kiselev et al,Phys. Usp. **38**, 1 (1995)
[e] E.J. Eichten et al, Phys. Rev. D **49**, 5845 (1994)
[f] L. P. Fulcher, Phys. Rev. D **60**, 074006 (1999)
[g] C.T. H. Davies et al, Phys. Lett. B **382**, 131 (1996)

M1 transition

The M1 transition width is given by [13]

$$\Gamma_{i\to f+\gamma} = \frac{\alpha}{3}\mu^2\omega^3(2J_f+1)\left|\left\langle f\left|j_0\left(\frac{\omega r}{2}\right)\right|i\right\rangle\right|^2$$

$\omega \to$ photon energy

$$\omega = \frac{M_i^2 - M_f^2}{2M_i}$$

$\mu \to$ magnetic moment

$$\mu = \frac{m_b e_c - m_c e_c}{m_b + m_c}$$

[13] N. Brambilla et al., Eur. Phys. J. C **71**, 1534 (2011)

Table: M1 transition partial width (in eV)

Transition	present	Ref[14]	Ref[15]	Ref[16]	Ref[17]	Ref[18]
$1^3S_1 \rightarrow 1^1S_0 + \gamma$	135	80	33	59	60	134.5
$2^3S_1 \rightarrow 2^1S_0 + \gamma$	41	10	17	12	10	28.9
$3^3S_1 \rightarrow 3^1S_0 + \gamma$	4	3				

[14] S. Godfrey, Phys. Rev. D **70**, 054017 (2004)

[15] D. Ebert et al, Phys. Rev. D **67**, 014027 (2003)

[16] L. P. Fulcher, Phys. Rev. D **60**, 074006 (1999)

[17] S. S. Gershtein et al., Phys. Rev. D **51**, 3613 (1995)

[18] E. J. Eichten and C. Quigg, Phys. Rev. D **49**, 5845 (1994)

Conclusion

- Mass spectrum of B_C^+ has been obtained and the results match with the other theoretical models.
- The model parameters and wave function can be used to compute various decay modes.
- $M1$ transition for S wave have been computed.
- Computation of other decay properties such as electric transitions, semi-leptonic and hadronic decays are underway.

Acknowledgment

This work is supported by the University Grants Commission of India under Major Research Project F.No.42-775/2013(SR)

Thank You