Heavy Quark Systems in Gluon Plasma

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Introduction

Heavy quarks: important probes of quark-gluon plasma

- Quarkonia peak structure above T_c; Dissolution temperature.
- Heavy quark in plasma: diffusion? Heavy-light mesons; flow.
- Energy loss of hard heavy quarks

Can flow of D, B, ... be understood as a diffusive process?
What do we know about quarkonia in equilibrium plasma?
Phenomenology in heavy ion collisions much harder

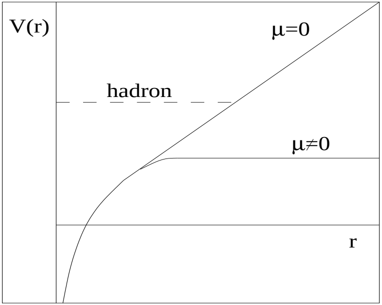
- Production: color octet passing through plasma?
- Coherent energy loss
- Time evolution of plasma. Lack of isotropy.
- Regeneration

(Strickland; Sharma; Singh; Bagchi; Bhatt; ...)

Quarkonia: finite temperature potential

Screening in plasma $\Longrightarrow$ reduced binding between $\bar{c}c$

T. Matsui & H. Satz, Phys. Lett. B 178 (1986) 416.



Free energy of a static $q\bar{q}$ pair at distance r

$$F_{T=0}(r) \sim -\frac{4\alpha}{3r} + \sigma r$$
$$F_{T>T_c}(r) \sim -\frac{4\alpha(T)}{3r} e^{-\mu(T)r}$$

A pattern of sequential suppression. $\eta_c, J/\psi$ dissolves by 1.2 T_c.

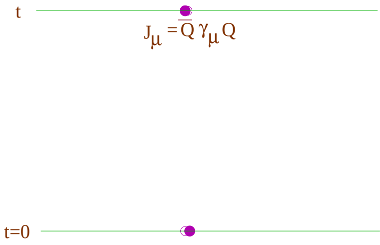
Karsch & Satz, 1991; Digal, Petreczky & Satz, 2001.

Theoretical framework for potential?

Definition of potential

Can we write down a potential which describes, e.g., the J/ψ peak in the dilepton channel?

Look at the correlation function



$$C(t) = \int d^3x \langle J_\mu(\vec{x}, t) J_\mu(\vec{0}, 0) \rangle_T$$

$$i\partial_t C(t; r, r') = \big(2M_Q + V(t; r, r')\big)\, C(t; r, r') + \mathcal{O}(1/M_Q) \qquad \text{Defn.}$$

$$V(r) = -\frac{4}{3}\alpha_s \left(\frac{e^{-m_D r}}{r} + m_D \right) - i\frac{8}{3}\alpha_s T\, \Phi(r)$$

M. Laine, O. Philipsen, P. Romatschke and M. tassler, JHEP 0703 (2007) 054.

The real part is same as the free energy before.

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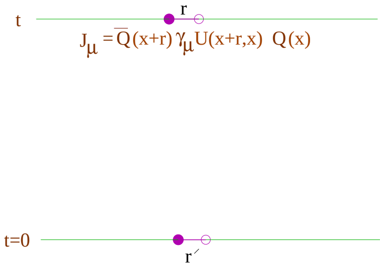

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More on potential

- Systematic study: *pNRQCD* framework.

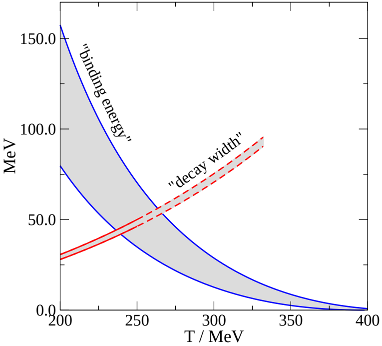
  Brambilla, Ghiglieri, Vairo, Petreczky, PRD 78 ('08) 014017.

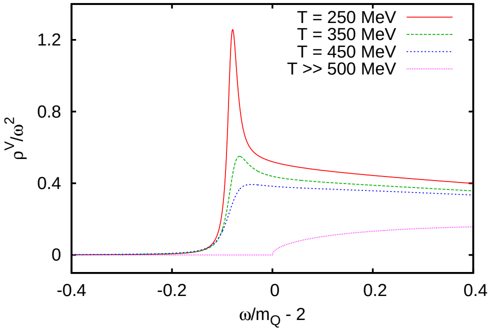
- Form of the imaginary part depends on the scale hierarchy. Related to collisional broadening.
- The potential has also been obtained from general framework of heavy quarks interacting with heatbath.

  Akamatsu, '12-14. Blaizot et al., 2015.

Results from potential

- Perturbative.
- $M_Q \gg T$
- Let us look for behavior of bottomonia.

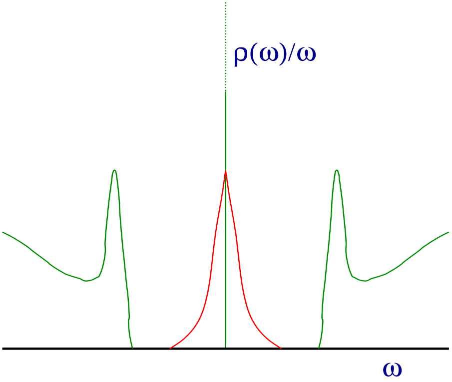




Burnier, Laine, Vepsalainen, JHEP ('11)

Direct lattice study?

We discussed getting the potential from the correlator, and using it to calculate the spectral function.
Cannot we get the spectral function directly from the correlator?
e.g., $\rho_{\bar{c}\gamma_i c}(\omega)$



Can we calculate ρ using lattice QCD?

Lattice analysis

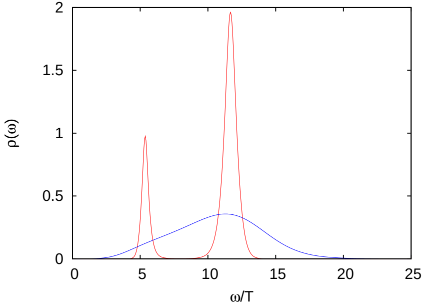
- Study the Euclidean field theory on discretized space-time
- Can calculate $G_{\bar{c}\gamma_i c}(\tau) = \langle \bar{c}\gamma_i c(\tau)\ \bar{c}\gamma_i c(0)\rangle_T$ nonperturbatively, using numerical techniques
- Invert $G_{\bar{c}\gamma_i c}(\tau) \ = \ \int d\omega\, \rho(\omega)\, \frac{\cosh \omega(\tau - \frac{1}{2T})}{\sinh \frac{\omega}{2T}}$.
- Highly unstable in the discretized theory and with $G(\tau)$ of finite accuracy
- Maximum entropy method: use prior information to keep in check uncontrolled directions in search space.
- Widely used for charmonia.
- First studies: 1S charmonia survive till quite deep in plasma, while the 1P states dissolve early.

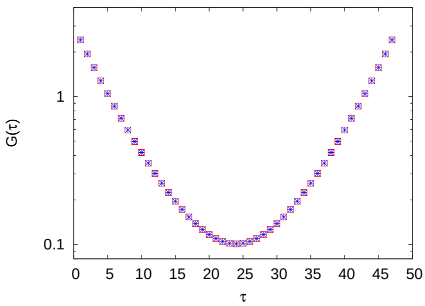
Datta, Karsch, Petreczky, Wetzorke, 2004; Asakawa & Hatsuda, 2004

- But correlators are completely consistent with other scenarios.

Umeda, 2007; Mocsy and Petreczky, 2007-2009.

Difficulty of analytic continuation





Bottomonia from lattice

- Study of bottomonia difficult as large discretization error ($m_b a \sim 1$)
- Study using NRQCD: $\Upsilon(1S)$ and $\eta_b(1S)$ survive till $> 2T_c$
  However, large width: ~ 400 MeV at 1.5 T_c

  G. Aarts, et al., JHEP 1111 (2011), 103; JHEP 1312 (2013) 064.

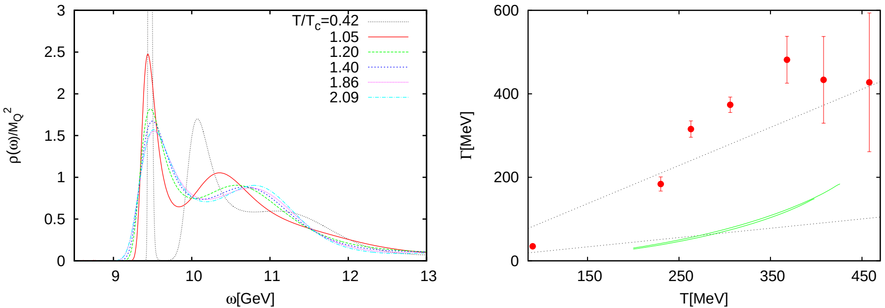
- χ_{b_0}, χ_{b_1} drastically modified in the plasma.

  Aarts et al.

- Much smaller width of $\Upsilon(1S)$ found in a recent study.

  S. Kim, P. Petreczky and A. Rothkopf, arXiv:1310.6461.

Bottomonia from NRQCD

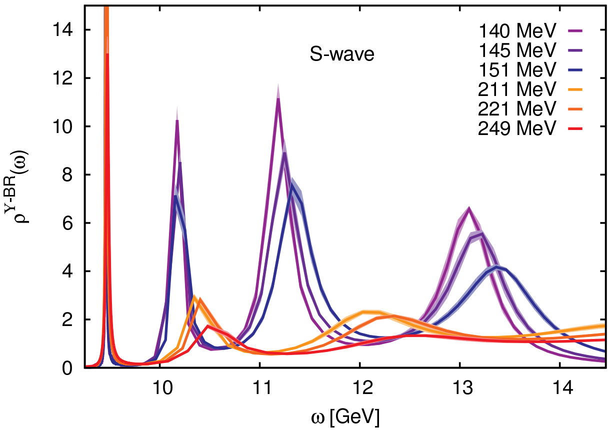


2-flavor QCD, $T_c \sim 219$ MeV

Aarts et al.

$\Gamma \sim 14\alpha_s^3 T$ in LO. Straight lines: α_s=0.25 and 0.4, resp.

Bottomonia with HISQ



Found very small width, < 25 MeV at $T = 249$ MeV.

Kim, Petreczky, Rothkopf, 1409.3630

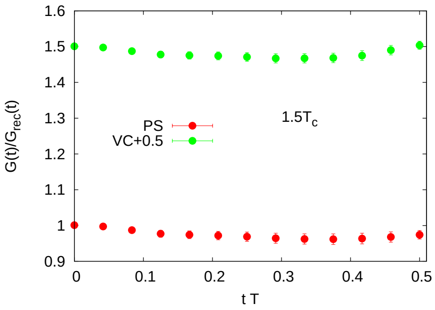
Relativistic bottom in gluon plasma

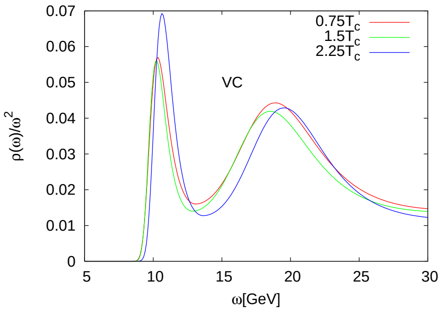
- We study relativistic bottom.
- Gluonic plasma: can use sufficiently fine lattices, $a^{-1} \sim 10$ GeV, and nonperturbatively $\mathcal{O}(a)$ improved bottom action.
- For thermal decay of bottomonia, thermal quarks not expected to be important.

D. Kharzeev & H. Satz, Phys. Lett. B334 (1994) 155

- Useful to compare the finite temperature correlator with correlator reconstructed from spectral function at low temperatures,
$G_{rec}(t;T) = \int d\omega \frac{\cosh \omega(t-1/2T)}{\sinh \omega/2T} \sigma(\omega, T' \approx 0)$

Υ at 1.5 T_c





No dramatic change on crossing T_c

ρ using maximum entropy method and free theory result as prior information.

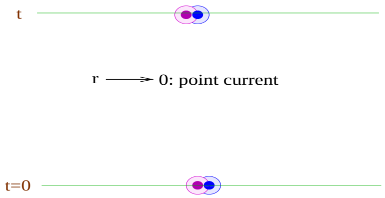
Indicative of very small change in $\Upsilon(1S)$ upto quite high temperatures.

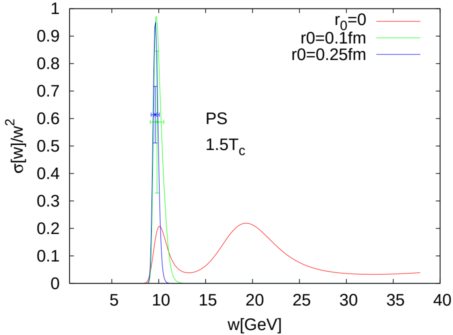
Point correlator not very sensitive to width, but indicative of small width < 200 MeV.

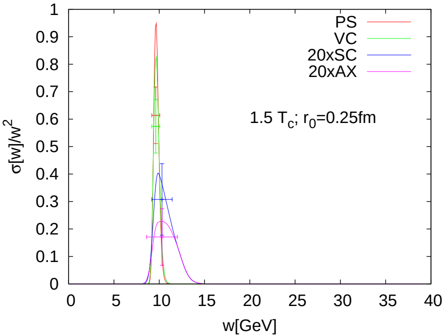
Smeared correlators

Use Gaussian smeared currents, to isolate the behavior of the lowest state

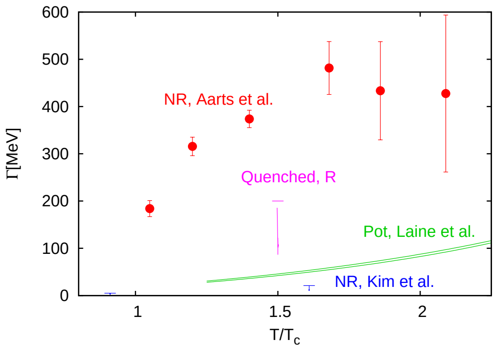
Note: this current does not directly connect to dilepton channel.





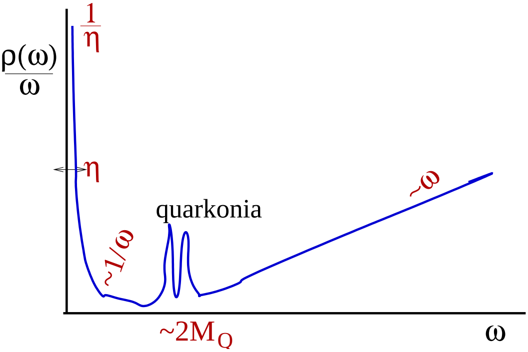


Width of $\Upsilon(1S)$



The diffusion part of the spectral function

$$\frac{\rho_{jj}(\omega)}{\omega}\Big|_{\omega\to 0} \sim \chi \frac{T}{\pi M} \frac{\eta}{\eta^2+\omega^2}$$



Difficult to extract from lattice correlator.

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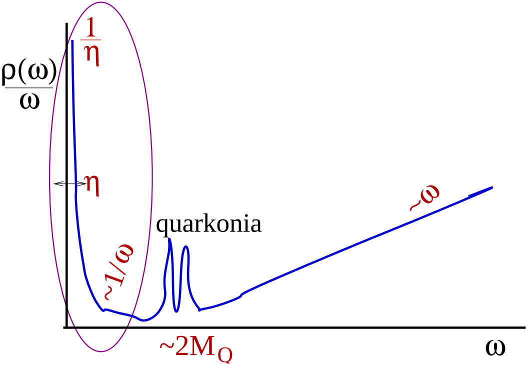
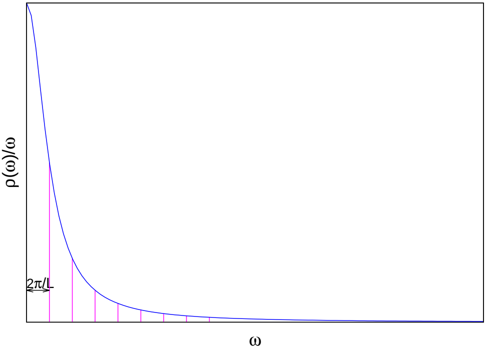


Difficult to extract from lattice correlator.

Diffusion part in lattice correlator

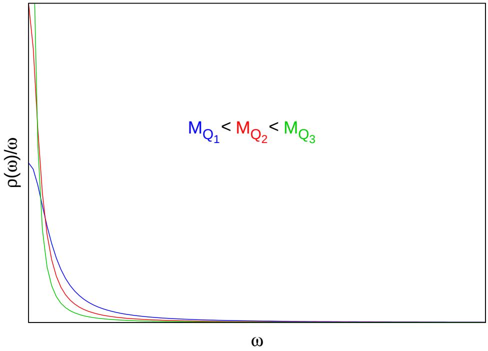
Finite lattice provides an infrared as well as ultraviolet cutoff.
Spectral function on lattice sum of discrete δ function peaks.



For extraction of diffusion part: $\frac{2\pi}{L} \ll \eta$
For $DT \sim 1/\pi$ this leads to $LT \gg 2M/T \sim 7$ for charm quark at 1.5 T_c

Extracting the diffusion part

$\rho(\omega)/\omega$

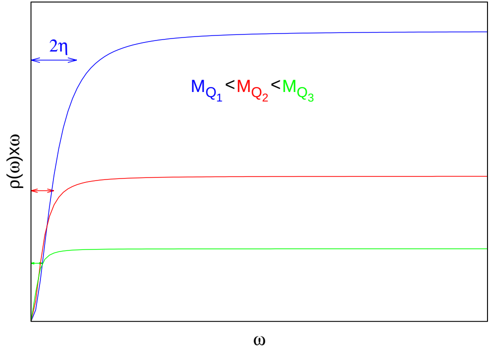


$\Rightarrow$ spectral function for $\frac{dJ_i}{dt}$

$$\omega^2 \rho_V(\omega) = \int dt \; e^{i\omega(t-t')} \int dx \; \frac{1}{2} \langle \dot{J}_i(\mathbf{x}, t) \, \dot{J}_i(\mathbf{0}, t') \rangle$$

Extracting the diffusion part

$\rho(\omega)/\omega \quad x\omega^2$



$\Rightarrow$ spectral function for $\frac{dJ_i}{dt}$

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Force-force correlator

$\dot{J}_i$ corresponds to the velocity operator for the heavy quark. We can therefore look at spectral function of $M_Q \dot{J}_i$ instead: corresponds to the force-force correlator.

$$\begin{aligned}\rho_F(\omega) &= M_Q^2 \int dt\; e^{i\omega(t-t')} \int dx\; \frac{1}{2}\langle \dot{J}_i(\mathbf{x},t)\, \dot{J}_i(\mathbf{0},t')\rangle \\ &= \int dt\; e^{i\omega(t-t')} \int dx\; \frac{1}{2}\langle F_i(\mathbf{x},t)\, F_i(\mathbf{0},t')\rangle\end{aligned}$$

Langevin description of heavy quark in plasma:

Svetitsky '88; Moore & Teaney '05; Rapp & van Hees '05; Mustafa '05

For thermal heavy quark, $M \gg T, \quad p \sim \sqrt{MT}$
Takes $\mathcal{O}(M/T)$ hard collisions to change momentum by $\mathcal{O}(1)$
Scattering with thermal quarks: uncorrelated momentum kicks

$$\frac{dp_i}{dt} = \xi_i(t) - \eta p_i, \qquad \langle \xi_i(t)\xi_j(t')\rangle = \kappa \delta_{ij}\delta(t-t')$$

Momentum diffusion coefficient

$$\langle p^2 \rangle = 3MT \;\rightarrow\; \eta \;=\; \frac{\kappa}{2MT} \qquad \langle x_i(t) x_j(t) \rangle \;=\; 2Dt\delta_{ij} \;\rightarrow\; D = \frac{2T^2}{\kappa}$$

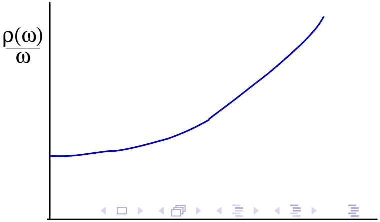
Static limit of ρ_F: color electric field correlator; Fluctuation dissipation theorem $\Rrightarrow$

$$\kappa \;=\; \lim_{\omega \to 0} \frac{2T}{\omega} \; \rho(\omega)$$

Caron-Huot, Laine & Moore, 0901.1195

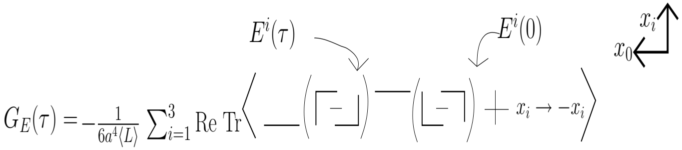
Calculated in HTL PT
Diffusion part $\propto \omega$ at small ω
However, very small κ/T^3, even negative at small temperatures.

Burnier, Laine, Langelage, Meher, 1006.0867



Nonperturbative determination of κ

Can calculate $\dot{J}_i(t)$ correlator on lattice.
In static limit: lattice discretized $E_c E_c$ correlator.



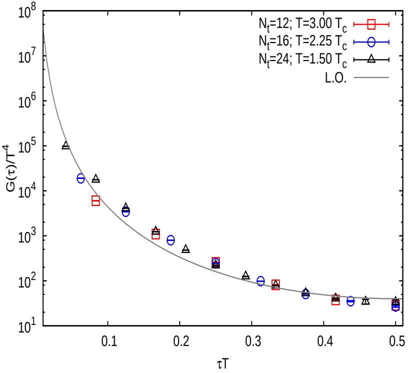
Can extract κ from such a correlator.

Francis, et al. ; Banerjee, Datta, Gavai, Majumdar (2012)

Temperatures $1 - 2T_c$ covered with lattices with $a^{-1} = 12 - 24T$
$LT = 2, 3, 4$ to look for finite volume effects
Finite volume effect negligible: underlines success of this strategy.

Banerjee et al (2012)

Nonperturbative determination of κ



Banerjee et al.

$$G_E^{\rm ren}(\tau) \;=\; Z_{E,{\rm Lat}}^2 \; G_E^{\rm Lat}(\tau)$$

$Z_E^{\rm Lat}$ calculated in one loop.

Christensen and Laine, arXiv:1601.01573

Model $\rho(\omega) = (\rho_{IR}(\omega), \rho_{UV}(\omega))$

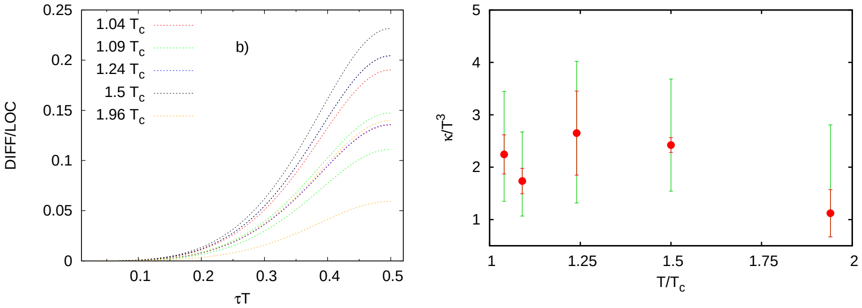
$\rho_{IR}(\omega) = \frac{\kappa}{2T}\;\omega$

Also try $\rho_{IR,1}(\omega) = \kappa \tanh \frac{\omega}{2T}$

$\rho_{UV}(\omega) = c.\frac{8}{9}\alpha\omega^3$

Also try lattice form of free spectral function.

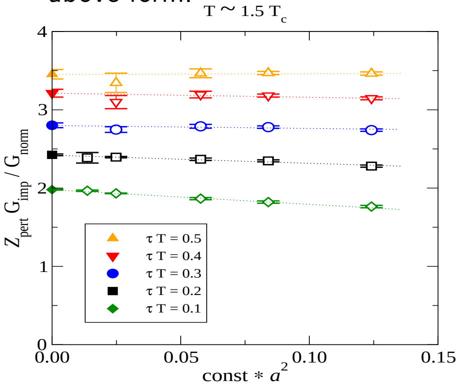
Result

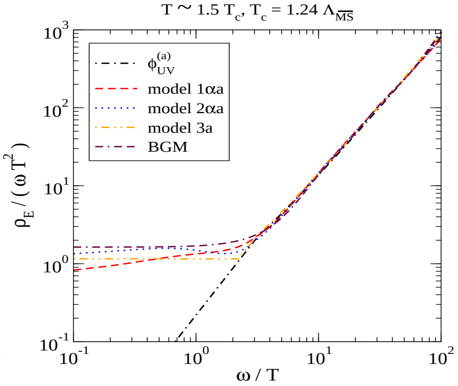


0.25
0.2
0.15
0.1
0.05
0
DIFF/LOC
1.04 T_c
1.09 T_c
1.24 T_c
1.5 T_c
1.96 T_c
b)
0.1
0.2
0.3
0.4
0.5
τT
5
4
3
2
1
κ/T^3
1
1.25
1.5
1.75
2
T/T_c

Another study

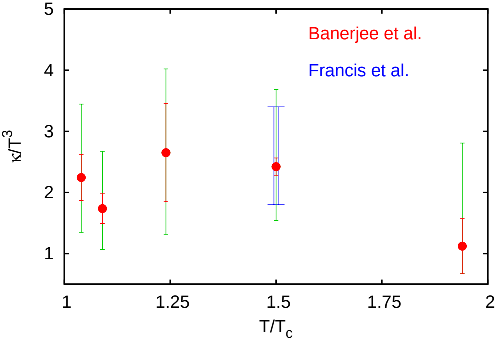
A detailed study and continuum extrapolation at 1.5 T_c.
Include a sinusoidal series to parametrize any difference from the above form.



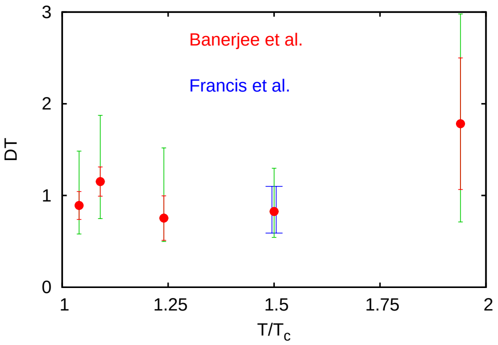


Francis, Kaczmarek, Laine, Neuhaus, Ohno, arxiv:1508.04543

Results for κ

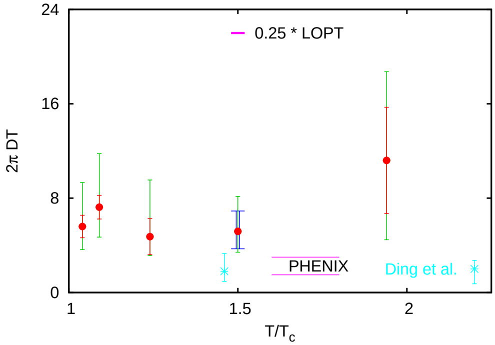


Results for D



DT near-flat in the temperature range 1.06-1.5 T_c

Diffusion coefficient and Phenix



D much smaller than leading order perturbation theory
NLO smaller by factor ~ 7 (Caron-Huot & Moore '08)
Rough agreement with Ding et al. (1204.4945)
In the right ballpark for Phenix.

Summary for Diffusion constant

- Estimate of κ stable, for gluon plasma.
- The result is very different from LOPT and is in the right ballpark to give confidence in diffusive explanation of heavy quark energy loss.
- Would be interesting to extend the study to higher temperatures, to investigate the approach to perturbative result.
- Unfortunately, a straightforward extension of the calculation to theory with dynamical quarks difficult: will require algorithmic breakthrough.