Radiative decay properties of charmonia in nonrelativistic QCD

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2^{nd} Heavy Flavor Meet-2016

Plan of Talk

Motivation

- Discovered in 1974 at BNL and SLAC
- Then after other experimental facilities such as BES, CLEO, BaBar, LHC have also reported huge amount
- $\bar{P}ANDA$ at GSI will also provide the opportunities particularly in $c\bar{c}$ region.
- Powerful tool for understanding the strong interaction
- High mass makes possible to treat in non relativistic
- Decay properties provides the further insights

Mass Spectra

- We use the following Hamiltonia[1]

$$H = \sqrt{\mathbf{p}^2 + m_Q^2} + \sqrt{\mathbf{p}^2 + m_{\bar{q}}^2} + V(\mathbf{r}) \tag{1}$$

- The inter-quark potential is of the form[2,3,4]

$$V(r) = -\frac{\alpha_c}{r} + Ar^{\nu} \tag{2}$$

- Choose a hydrogenic trial wavefunction

$$R_{nl}(r) = \left(\frac{\mu^3 (n-l-1)!}{2n(n+l)!} \right)^{1/2} (\mu r)^l e^{-\mu r/2} L_{n-l-1}^{2l+1}(\mu r) \tag{3}$$

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[1] S.N. Gupta, J.M. Johnson, Phys. Rev. D 51(1), 168 (1995)

[2] A.K. Rai, R.H. Parmar, P.C. Vinodkumar, J. Phys. G: Nucl. Part. Phys. 28(8), 2275 (2002)

[3] A.K. Rai, J.N. Pandya, P.C. Vinodkumar, J. Phys. G: Nucl. Part. Phys. 31(12), 1453 (2005)

[4] A.K. Rai, B. Patel, P.C. Vinodkumar, Phys. Rev. C 78(5), 055202 (2008)

Mass Spectra

- Running coupling constant (α_s) is determined using the formula[5,6,7]

$$\alpha_s = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right)\ln\frac{M^2+m_B^2}{\Lambda^2}} \tag{4}$$

  where $M = 2m_Q m_{\bar{Q}}/\left(m_Q + m_{\bar{Q}}\right)$, $M_B = 0.95$ GeV and $\Lambda = 413$ MeV.
- By using suitable value of A, use virial theorem [8] to find the variational parameter μ.
- Solve the Schrodinger equation to obtain the ground state spin-averaged mass of the meson.
- Match the obtained spin-avergaed mass to experimental measurement using the equation

$$M_{CW,n} = \frac{\Sigma_J 2(2J+1)M_{nJ}}{\Sigma_J 2(2J+1)} \tag{5}$$

[5] A.M. Badalian, A.I. Veselov, B.L.G. Bakker, Phys. Rev. D 70, 016007 (2004)
[6] Y.A. Simonov, Physics of Atomic Nuclei 58, 107 (1995)
[7] D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Rev. D 79, 114029 (2009)
[8] D.S. Hwang, C. Kim, W. Namgung, Phys.Lett. B406, 117 (1997)

Mass Spectra

- Use the following equation to get excited state spectrum [9,10,11]

$$
\begin{aligned}
V_{SD}(\mathbf{r}) \quad = \quad & \left(\frac{\mathbf{L}\cdot\mathbf{S_Q}}{2m_Q^2}+\frac{\mathbf{L}\cdot\mathbf{S_{\bar{Q}}}}{2m_{\bar{Q}}^2}\right)\left(-\frac{dV(r)}{dr}+\frac{8}{3}\alpha_S\frac{1}{r^3}\right)+ \\
& \frac{4}{3}\alpha_S\frac{1}{m_Q m_{\bar{Q}}}\frac{\mathbf{L}\cdot\mathbf{S}}{r^3}+\frac{4}{3}\alpha_S\frac{2}{3m_Q m_{\bar{Q}}}\mathbf{S_Q}\cdot\mathbf{S_{\bar{Q}}}4\pi\delta(\mathbf{r})+ \\
& \frac{4}{3}\alpha_S\frac{1}{m_Q m_{\bar{Q}}}(3(\mathbf{S_Q}\cdot\mathbf{n})-(\mathbf{S_{\bar{Q}}}\cdot\mathbf{n})-\mathbf{S_Q}\cdot\mathbf{S_{\bar{Q}}})\frac{1}{r^3}, \\
& \text{where } \mathbf{n}=\frac{\mathbf{r}}{r}
\end{aligned}
\tag{6}
$$

[9] E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane, T.M. Yan, Phys. Rev. D 21, 203 (1980)

[10] D. Gromes, Z.Phys. C26, 401 (1984)

[11] S. Gershtein et al., Phys.Usp. 38, 1 (1995)

Table: S-Wave and P-Wave Masses (in GeV) of $c\bar{c}$ meson

ν	1^1S_0	1^3S_1	1^1P_1	1^3P_0	1^3P_1	1^3P_2	2^1S_0	2^3S_1	2^1P_1	2^3P_0	2^3P_1	2^3P_2	3^1S_0
0.5	3.000	3.092	3.313	3.292	3.302	3.323	3.352	3.375	3.519	3.494	3.507	3.531	3.541
0.7	2.980	3.100	3.373	3.341	3.357	3.389	3.427	3.464	3.666	3.623	3.644	3.687	3.697
0.9	2.960	3.109	3.429	3.383	3.406	3.451	3.495	3.547	3.808	3.742	3.775	3.842	3.846
1.0	2.950	3.112	3.450	3.398	3.424	3.477	3.522	3.583	3.872	3.792	3.832	3.911	3.912
1.1	2.942	3.116	3.473	3.414	3.444	3.503	3.549	3.619	3.936	3.843	3.889	3.983	3.979
1.3	2.926	3.123	3.513	3.441	3.477	3.550	3.597	3.683	4.055	3.933	3.994	4.116	4.102
1.5	2.912	3.129	3.547	3.461	3.504	3.590	3.636	3.739	4.162	4.009	4.085	4.239	4.212
1.7	2.899	3.134	3.576	3.477	3.526	3.625	3.668	3.788	4.257	4.073	4.165	4.394	4.309
1.9	2.887	3.141	3.603	3.491	3.547	3.658	3.696	3.832	4.345	4.129	4.237	4.453	4.396
2.0	2.882	3.144	3.615	3.497	3.556	3.673	3.708	3.852	4.385	4.153	4.269	4.501	4.436
[12]	2.984	3.097	3.511	3.415		3.556	3.639	3.686					
[13]	2.979	3.096	3.526	3.424	3.511	3.556	3.588	3.686	3.945	3.854	3.929	3.972	3.991
[14]	2.980	3.097	3.527	3.416	3.508	3.558	3.597	3.686	3.960	3.844	3.894	3.994	4.014

[12] K A Olive et al. (Particle Data Group)Chin. Phys. C 38, 090001(2014)

[13] D. Ebert R.N. Faustov and V. O. Galkin, Phys. Rev. **D 67** 014027 (2003)

[14] S. F. Redford and W. W. Repko, Phys. Rev. D **75**, 074031 (2007)

Decay Properties

- The color Compton Radius

$$r_c = \frac{N_c|e_Q|}{M_{P/V}} \tag{7}$$

where $N_c = 3$ and $e_Q \rightarrow$ charge of quark

Decay constant

Using the Van Royen Weisskopf relation [15]

$$f^2_{P/V} = \frac{3|R_{ns}(0)|^2}{\pi M_{P/V}} \bar{C}^2(\alpha_s) \tag{8}$$

where $\bar{C}^2(\alpha_s) \rightarrow$ first order QCD correction factor [16]

$$\bar{C}(\alpha_s) = 1 - \frac{\alpha_s}{\pi}\left(2 - \frac{m_Q - m_{\bar{Q}}}{m_Q + m_{\bar{Q}}} ln \frac{m_Q}{m_{\bar{Q}}}\right)$$

[15] R. Van Royen and V. F. Weisskopf, Nuovo Cinemento **50**, 617 (1967)

[16] E. Braaten and S. Fleming, Phys. Rev. **D 52**, 181 (1995)

Decay constant

Table: pseudoscalar decay constant in MeV

ν	1S		2S		3S	
	$f_p(r=0)$	$f_p(r=r_c)$	$f_p(r=0)$	$f_p(r=r_c)$	$f_p(r=0)$	$f_p(r=r_c)$
0.5	348	249	164	165	113	110
0.7	400	278	207	185	148	113
0.9	444	300	245	198	181	111
1.0	462	310	262	203	196	108
1.1	479	318	279	207	210	105
1.3	510	333	309	213	238	98
1.5	536	346	336	217	263	90
1.7	559	356	359	219	285	81
1.9	579	365	282	220	306	73
2.0	588	369	391	220	315	69
Expt.[a]	335±75					
NRQM[b c]	402		240		193	
BS[d]	292±25					

[a] Edwards K W et al. (CLEO collobration) 2001 Phys. Rev. Lett. 86 30

[b] Bali G S et al.2000 Phys. Rev. D 75 073016

[c] Bali G S et al.2000 Phys.Lett. B 343 1

[d] Cvetic G et al. 2004 Phys. Lett. B 596 84

Table: vector decay constant in MeV

ν	1S		2S		3S	
	$f_p(r=0)$	$f_p(r=r_c)$	$f_p(r=0)$	$f_p(r=r_c)$	$f_p(r=0)$	$f_p(r=r_c)$
0.5	353	251	168	165	113	111
0.7	409	282	208	186	148	114
0.9	456	307	247	200	182	112
1.0	476	217	265	205	197	110
1.1	495	327	282	210	212	108
1.3	529	241	313	217	240	102
1.5	559	358	341	222	265	95
1.7	585	370	365	226	288	88
1.9	609	381	388	229	310	82
2.0	620	386	399	230	319	79
Expt.[a b]	416±6		304±4		187±8	
NRQM[c]	393		293		258	
BS[d]	459±28		364±24		319±22	
RQM[e]	551		401			

[a] Yao W M et al. (Particle Data Group) 2006 J. Phys. G:Nucl. Part. Phys.33,1

[b] Amsler C et al. 2008 Phys. Lett. B 667 1

[c] Lakhiina O and Swanson E S 2006 Phys. Rev. D 74 014012

[d] Wang G L 2006 Phys. Lett. B 633 492

[e] Ebert D, Faustov R N and Galking V O 2003 Mod. Phys. Lett. A 18 1597

Digamma decay width

We compute the digamma decay width using the conventional Van Royen Weisskopf formula [17]

$$\Gamma_0(\gamma\gamma) = \frac{12\alpha_e^4 e_Q^4}{M_P^2} R^2(0) \tag{9}$$

According to Khan and Hoodbhoy [18] the important correction to the VRW formula are,

- Bound state effect $\Gamma_B = -2\frac{\epsilon_B}{M_P}\Gamma_0$
- quark/antiquark propagator $\Gamma_C = \frac{16}{3M_P^2}\frac{\nabla^2 R(0)}{R(0)}\Gamma_0$
- Radiative correction upto 1^{st} order $\Gamma_R = \frac{\alpha_S}{\pi}\left(\frac{\pi^2-20}{3}\right)\Gamma_0$
- Total decay width is given by

$$\Gamma_{\gamma\gamma} = \Gamma_0 + \Gamma_B + \Gamma_C + \Gamma_R$$

[17] R. Van Royen and V. F. Weisskopf, Nuovo Cinemento **50**, 617 (1967)

[18] Khan H and Hoodbhoy P., Phys. Rev **D 53**, 2534 (1996)

Dilepton Decay Width

We compute the dileptonic decay width using the conventional Van Royen Weisskopf formula [19]

$$\Gamma_{VW} = \frac{4\alpha_e^2 e_Q^2}{M_V^2} R^2(0) \tag{10}$$

Again the important correction to the VRW formula are,

- Radiative correction [20]

$$\Gamma_{rad} = \frac{-16}{3\pi} \alpha_S \Gamma_{VW} \tag{11}$$

- Correction due to quark propagator within the vector meson

$$\Gamma_{cor} = \frac{4}{3M_V^2} \frac{\nabla^2 R(0)}{R(0)} \Gamma_{VW} \tag{12}$$

[19] R. Van Royen and V. F. Weisskopf, Nuovo Cinemento **50**, 617 (1967)

[20] Celmaster W Phys. Rev. **D 19**, 1517 (1978); Khan H and Hoodbhoy P., Phys. Rev **D 53**, 2534 (1996)

Digamma and Dilepton decay width using NRQCD

The NRQCD expression for decay widths are given by [21]

$$\Gamma_{n^1S_0\to\gamma\gamma} = \frac{2Imf_{\gamma\gamma}(n^1S_0)}{\pi m_Q^2}|R_{n^1S_0}|^2 - \frac{N_c Img_{\gamma\gamma}(n^1S_0)}{\pi m_Q^4} Re(\bar{R_s^*}\bar{\nabla^2}R_s) + O(v^4)$$

$$\Gamma_{n^3S_1\to e^+e^-} = \frac{N_c Imf_{ee}(n^3S_1)}{\pi m_Q^2}|R_{n^3S_1}|^2 - \frac{N_c Img_{ee}(n^3S_1)}{\pi m_Q^4} Re(\bar{R_s^*}\bar{\nabla^2}R_s) + O(v^4)$$

$$\begin{aligned} Imf_{\gamma\gamma}(n^1s_0) &= \pi Q^4\alpha^2 \quad Img_{\gamma\gamma}(n^1s_0) = -\tfrac{4\pi Q^4}{3}\alpha^2 \\ Imf_{ee}(n^3s_1) &= \frac{\pi Q^2}{3}\alpha^2 \quad Img_{ee}(n^3s_1) = -\tfrac{4\pi Q^2}{9}\alpha^2 \end{aligned} \tag{13}$$

[21] G. T. Bodwin et al, Phys. Rev **D 51**, 1125 (1995); **55**, 5853 (1997)

Digamma decay width

Table: Digamma decay width in keV

ν	1S $\Gamma_{r=0}$	1S $\Gamma_{r=r_c}$	1S Γ_{NRQCD}	1S Γ_{NRQCD} $r=r_c$	2S $\Gamma_{r=0}$	2S $\Gamma_{r=r_c}$	2S Γ_{NRQCD}	2S Γ_{NRQCD} $r=r_c$	3S $\Gamma_{r=0}$	3S $\Gamma_{r=r_c}$	3S Γ_{NRQCD}	3S Γ_{NRQCD} $r=r_c$
0.5	5.53	2.48	2.40	1.12	4.31	1.26	1.15	1.16	1.90	1.11	0.70	0.67
0.7	7.38	3.38	3.52	1.70	6.57	1.89	1.72	1.38	4.08	1.66	1.19	0.70
0.9	9.13	4.27	4.74	2.18	8.84	2.54	2.29	1.50	7.13	2.12	1.71	0.64
1.0	9.93	4.69	5.36	2.44	8.95	2.82	2.56	1.54	8.90	2.31	1.99	0.61
1.1	10.71	5.12	5.95	2.63	11.05	3.18	2.83	1.56	10.84	2.47	2.26	0.56
1.3	12.18	5.93	7.15	3.06	13.13	3.79	3.30	1.57	15.12	2.70	2.78	0.47
1.5	13.53	6.69	8.30	3.45	15.14	4.35	3.72	1.56	19.70	2.81	3.34	0.38
1.7	14.76	7.42	9.40	3.82	17.04	4.86	4.10	1.52	24.56	2.82	3.65	0.30
1.9	15.91	8.25	10.47	4.17	18.80	5.35	4.45	1.49	29.57	2.76	4.0	0.23
2.0	16.46	8.43	10.97	4.33	19.84	5.53	4.60	1.46	32.07	2.70	4.15	0.20
ERHM[a b]	4.605											
BT[c]	4.664											
PL(Martin)[d]	7.210											
Log[e]	6.588											
Cornell[f]	12.931											

[a] J N Pandya and P C Vinodkumar, Pranama J.Phys. 57, 821

[b] J N Pandya, P C Vinodkumar, V M Bannur and S B Khadkikar, Eur. Phys. J A 4,83(1999)

[c] Buchmuller and Tye, Phys. Rev. D 24, 132(1981)

[d] A Martin, Phys. Lett. B93, 338 (1980)

[e] C Quigg and J L Rosner, Phys. Lett. B71, 153(1977)

[f] E Eichten, K Gottfried, T Kinoshita, K D Lane and T M Yan, Phys. Rev. D 17,3090(1978)

Dilepton decay width

Table: Dilepton decay width in keV

ν	1S $\Gamma_{r=0}$	1S $\Gamma_{r=r_c}$	1S Γ_{NRQCD}	1S Γ_{NRQCD} $r=r_c$	2S $\Gamma_{r=0}$	2S $\Gamma_{r=r_c}$	2S Γ_{NRQCD}	2S Γ_{NRQCD} $r=r_c$	3S $\Gamma_{r=0}$	3S $\Gamma_{r=r_c}$	3S Γ_{NRQCD}	3S Γ_{NRQCD} $r=r_c$
0.5	3.25	1.60	3.70	1.80	1.01	0.76	1.53	1.54	0.63	0.45	0.87	0.84
0.7	4.42	2.02	4.94	2.22	1.42	1.01	2.37	1.89	1.18	0.56	1.50	0.91
0.9	5.54	2.39	5.85	2.53	2.10	1.23	3.27	2.15	1.94	0.63	2.24	0.86
1.0	6.06	2.55	6.32	2.66	2.46	1.32	3.70	2.23	2.36	0.66	2.62	0.82
1.1	6.60	2.71	7.76	2.78	2.84	1.41	4.13	2.30	2.82	0.68	3.02	0.78
1.3	7.58	3.30	7.53	2.98	3.63	1.57	4.95	2.39	3.81	0.69	3.84	0.69
1.5	8.52	3.25	8.25	3.14	4.42	1.70	5.71	2.43	4.84	0.70	4.61	0.60
1.7	9.38	3.47	8.87	3.29	5.20	1.82	6.39	2.45	5.92	0.68	5.37	0.51
1.9	10.20	3.68	9.38	3.88	6.00	1.92	7.02	4.44	6.99	0.66	6.06	0.42
2.0	10.59	3.78	9.61	3.43	6.38	1.96	7.30	2.43	7.50	0.65	6.37	0.39
ERHM[a b]	2.214											
BT[c]	3.170											
PL(Martin)[d]	2.714											
Log[e]	3.057											
Cornell[f]	6.933											

[a] J N Pandya and P C Vinodkumar, Pranama J.Phys. 57, 821

[b] J N Pandya, P C Vinodkumar, V M Bannur and S B Khadkikar, Eur. Phys. J A 4,83(1999)

[c] Buchmuller and Tye, Phys. Rev. D 24, 132(1981)

[d] A Martin, Phys. Lett. B93, 338 (1980)

[e] C Quigg and J L Rosner, Phys. Lett. B71, 153(1977)

[f] E Eichten, K Gottfried, T Kinoshita, K D Lane and T M Yan, Phys. Rev. D 17,3090(1978)

Digluon Decay Width

The digluon decay width along with the NLO QCD radiative correction factor is [22]

$$\Gamma_{gg}(\eta_Q) = \frac{\alpha_s^2 M_{\eta_Q} |R_0(0)|^2}{3m_Q^3}[1 + 4.4(\alpha_s/\pi)] \tag{14}$$

[22] J.P. Lansberg and T.N. Pham, Phys. Rev. **D 79**, 094016 (2009); M. Mangano and A. Petrelli, Phys. Lett. **B 352**, 445 (1945)

Digluon Decay Width

Table: Digluon decay width in MeV

ν	1S		2S		3S	
	$\Gamma_{r=0}$	$\Gamma_{r=r_c}$	$\Gamma_{r=0}$	$\Gamma_{r=r_c}$	$\Gamma_{r=0}$	$\Gamma_{r=r_c}$
0.5	73.29	37.53	20.53	20.63	10.83	10.36
0.7	95.75	46.23	33.83	27.09	20.22	11.90
0.9	116.15	53.29	49.51	32.40	24.18	9.08
1.0	125.04	56.24	57.37	34.62	39.58	12.14
1.1	133.79	59.12	66.06	36.49	47.26	11.81
1.3	149.66	64.01	83.33	39.73	64.26	10.93
1.5	163.87	68.19	100.36	42.06	82.45	9.76
1.7	176.36	71.76	107.04	43.56	101.72	8.36
1.9	187.79	74.85	133.86	44.73	121.64	7.01
2.0	193.23	76.25	144.62	45.04	131.67	6.37
ERHM [a]	19.04		12.91		6.64	
[b]	26.7±3.0					
[c]	48.927					
pert [d]	15.70					
nonpert [e]	10.57					

[a] J.N Pandya et al.,Chin. Phys. C 39, 123101(2015)

[b] K.A Olive et al., (Particle Data Group) Chin. Phys. C 38, 090001(2014)

[c] A Parmar et al., Nucl. Phys. A 848, 299(2010)

Conclusion

- The masses of S-states and P-states are in agreement with the experimental results and other theoretical calculations.
- Introducing the relative correction improves the digamma dileptonic decay rates.
- The decay constant calculated here are close to experimental and other theoretical models.
- The properties calculated here are close to the experimental values between potential index $\nu = 1.0$ to 1.3.
- When the wave function calculations are done at color compton radius and the decay width is calculated the results have shown improvement.

Acknowledgments

- A. K. Rai acknowledges the financial sup- port extended by DST, India under SERB fast track scheme SR/FTP /PS-152/2012 and also to the research grant from Sardar Vallabhbhai National Institute of Technology-Surat, Gujarat (Dean R&C/1488/2013-14).
- JNP acknowledges the financial support the University Grants Commission of India under Major Research Project F.No.42-775/2013(SR).

Thank You