Searching for perfect fluid in ultracold fermionic gas

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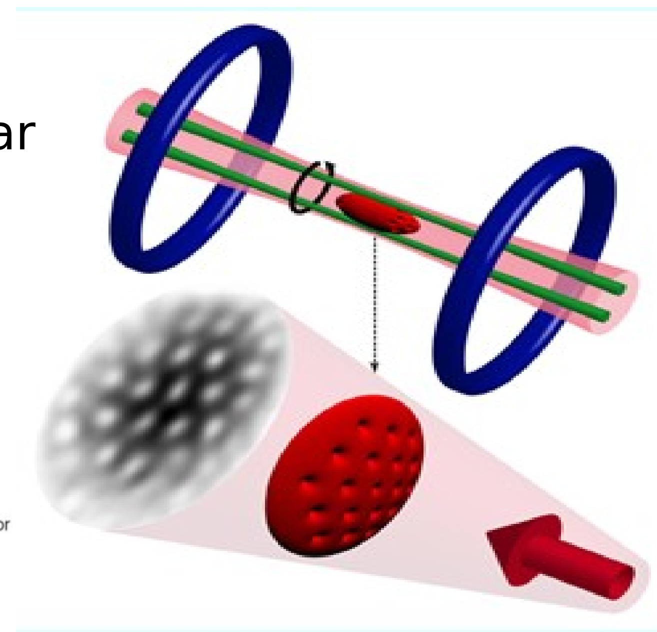
Warsaw, Poland, 06-11-2015

ultracold fermionic gas =

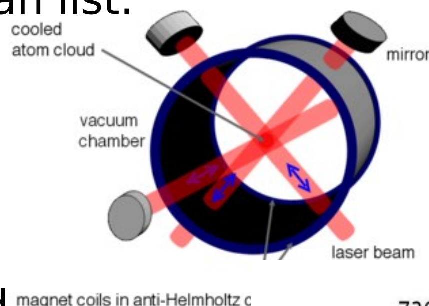
very dilute gas of atoms (half-integer total angular momentum) confined in external potential and cooled down to very low temperatures!

[System is metastable – life time ~ min.]



Among the control parameters we can list:

- temperature,
- density,
- type of atoms (bosons, fermions),
- mixture,
- polarization,
- shape of confining potential (cloud, optical lattices)
- dimensionality
- **interaction between atoms can be tuned** over a wide range (via Feshbach resonances) - the most unique ability!



QUANTUM SIMULATORS / EMULATORS

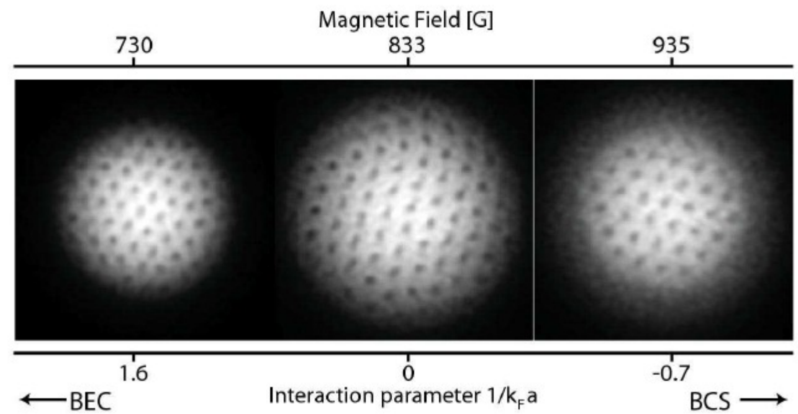
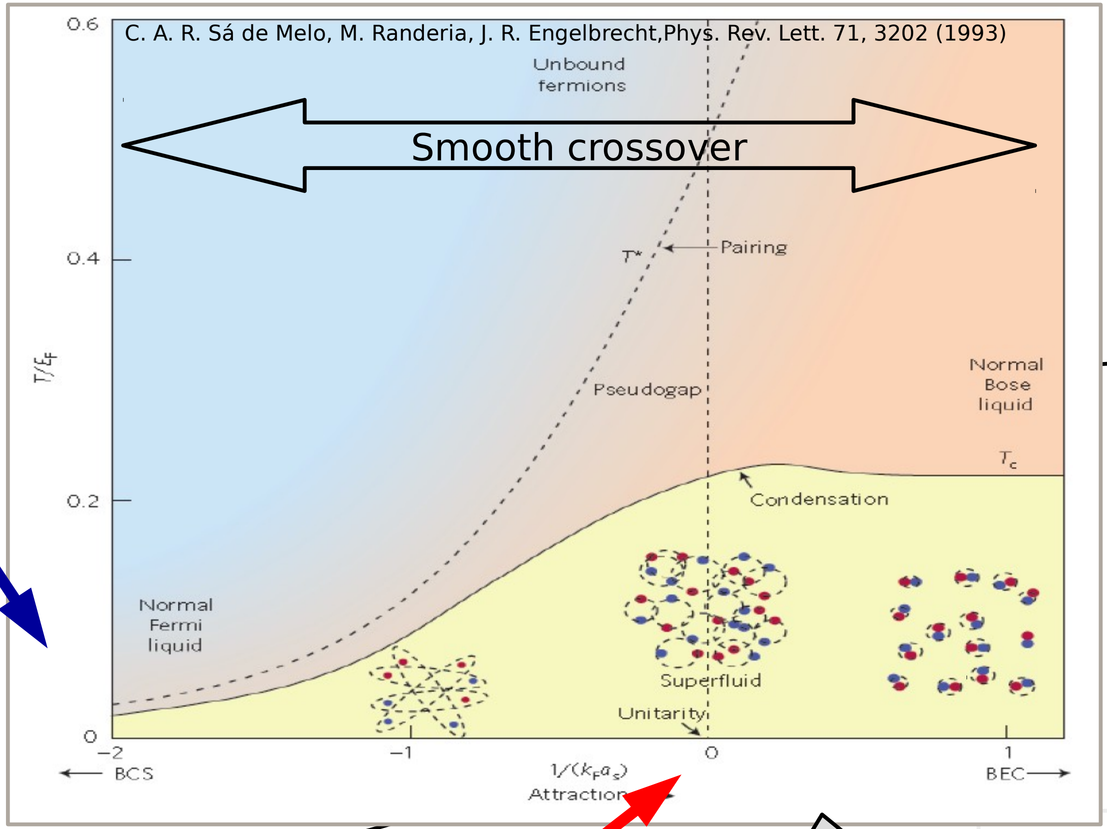


FIG. 36 Vortex lattice in a rotating gas of ^{6}Li precisely at the Feshbach resonance and on the BEC and BCS side. Reprinted with permission from Zwierlein *et al.* (2005).



Free Fermi gas
[only normal state]

Noninteracting bosons
[condensate + normal state]

Universal physics

Unitary Fermi gas
[superfuid + pseudogap + normal state]

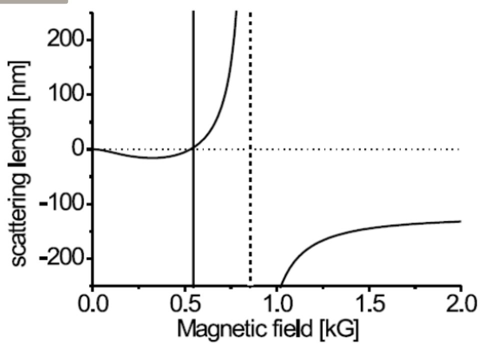


FIG. 4: Magnetic field dependence of the scattering length in ^{6}Li, showing a broad Feshbach resonance at $B_0 \simeq 834$ G and a narrow Feshbach resonance at $B_0 \simeq 543$ G (can not be resolved on this scale). From Bourdel *et al.* (2003).

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Universality of UFG

Low energy scattering:

$$f(k) \simeq \frac{1}{-\frac{1}{a} + \frac{1}{2} r_{\text{eff}} k^2 - ik} \to \frac{i}{k}$$

Universal form

$\sigma_0(k) \leq 4\pi/k^2$

Total cross section is saturated

Effective parameters defining interaction *(coupling constants)*

$$\begin{cases} k_F r_{\text{eff}} \to 0 \\ k_F a \to \pm\infty \end{cases}$$

System is **dilute** but...

strongly interacting!

- Unitary limit:
  no interaction length scale...
- Universal physics...
  - Cold atomic gases
  - Neutron matter
  - High-Tc superconductors
  - Heavy ion collisions
- Simple, but hard to calculate!
  (Bertsch Many Body X-challenge)

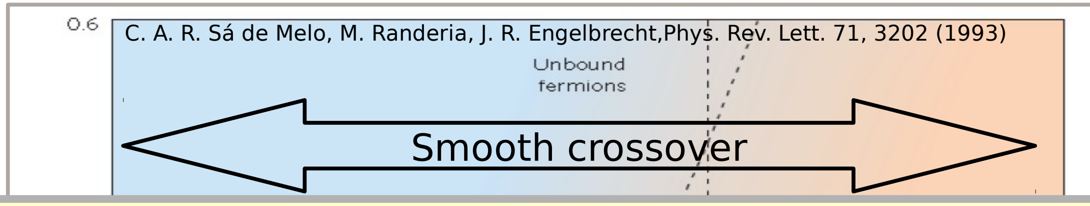
The only relevant scale is mean interparticle distance $n^{1/3}$ – as for free Fermi gas!

All thermodynamic quantities should be universal function of the Fermi energy e_F and of the ratio $k_B T/e_F$

Universal dimensionless function

$$E(T) = \xi(k_B T/\varepsilon_F) E_{\text{fg}}$$

UT



Free Fermi gas

Universal physics

The ultracold atoms provide an **ideal laboratory** for very precise experimental and theoretical studies of an enormous range of **quantum mechanical phenomena**.

- ...
- verification of various hypothetical bounds for transport coefficients (like viscosities) having their roots directly in quantum mechanics
- ...

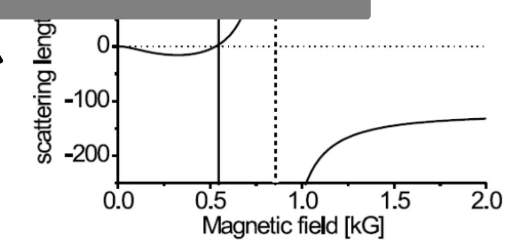


FIG. 4: Magnetic field dependence of the scattering length in ^{6}Li, showing a broad Feshbach resonance at $B_0 \simeq 834$ G and a narrow Feshbach resonance at $B_0 \simeq 543$ G (can not be resolved on this scale). From Bourdel *et al.* (2003).

Unitary Fermi gas
[superfuid + pseudogap + normal state]

KSS conjecture

[Kovtun, Son, Starinets, PRL (2005)]

shear viscosity

entropy density

$$\frac{\eta}{s} \geq \frac{1}{4\pi}\frac{\hbar}{k_B}$$

Minimum defines a "perfect" fluid

Bound has been proposed on the basis of string theory.

Valid for large class of (string) theories.

Saturated for the case of strongly coupled theory.

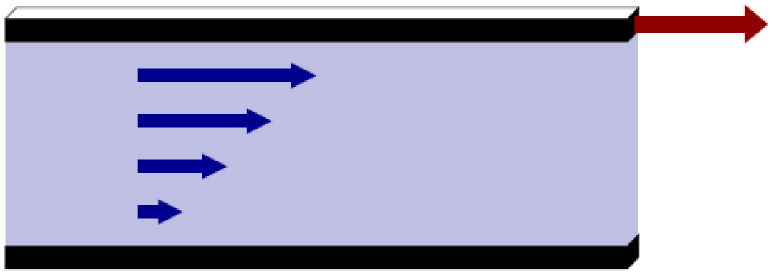
Idea: generate system with lowest possible viscosity

Other propositions:

- Hydrodynamic bound for η/n (originating from quantum fluctuations)
  [Chafin and Schafer PRA 2013; Romatschke and Young PRA 2013]
- Conductivity bound σ/x
  [Kovtun and Ritz PRD 2008]
- Existence of minimal value for spin diffusion coefficient
  [Bruun, New J. Phys. 2011]

Shear viscosity

The shear viscosity: determines “friction” force F per unit area A created by a shear flow



$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

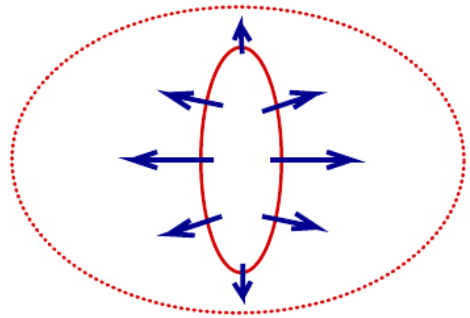
For incompressible fluid or if ξ=0: kinetic energy dissipated per unit time

$$\dot{E}_{\text{kin}} = -\tfrac{1}{2}\eta \int \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \right)^2 \mathrm{d}V$$

<u>Kinetic theory (Boltzmann equation) prediction</u>: $\eta = n\bar{p}l_{\text{mfp}}$

For ideal hydrodynamic ($\eta \to 0$ and $\xi \to 0$)

→ **elliptic flow**



Perfect fluid Ingredients:

- **Strongly interacting system**
- …

Note: in genral $\eta \to 0$ and $\xi \to 0 \neq$ ideal hydrodynamic

Viscosity coefficients

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \cdot \vec{j}^{\epsilon} = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0$$

Stress tensor

The **hydrodynamics** of a non-relativistic one-component (non-superfluid) fluid is governed by the conservation laws.

6 hydrodynamic variables: **v**, ρ, ϵ, P

In order to close equations:

$$P = P(\epsilon, \rho)$$

ideal

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \delta\Pi_{ij}$$

☜ dissipation (1st order)

Shear viscosity

Bulk viscosity

$$\delta\Pi_{ij} = -\eta\left(\nabla_i v_j + \nabla_j v_i - \frac{2}{3}\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{v}\right) - \zeta\delta_{ij}(\boldsymbol{\nabla}\cdot\boldsymbol{v})$$

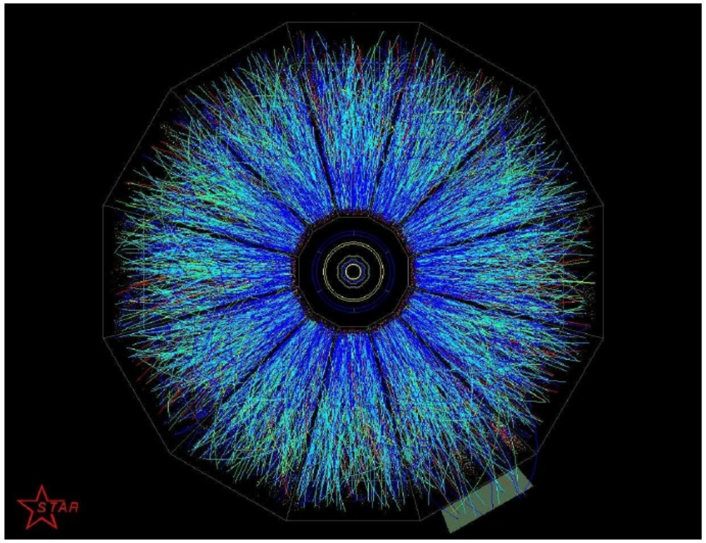
$$j_i^{\epsilon} = v_i(\epsilon + P) + v_j\delta\Pi_{ij} - \kappa\nabla_i T$$

Thermal conductivity

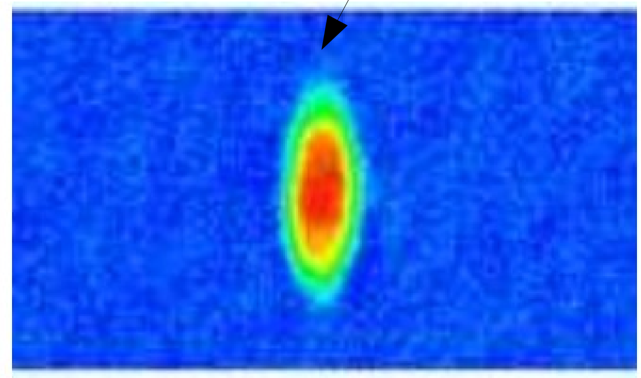
ideal

Systems with low viscosity...

UFG – limit of infinite coupling constant ak_F



Hottest and coldest systems...



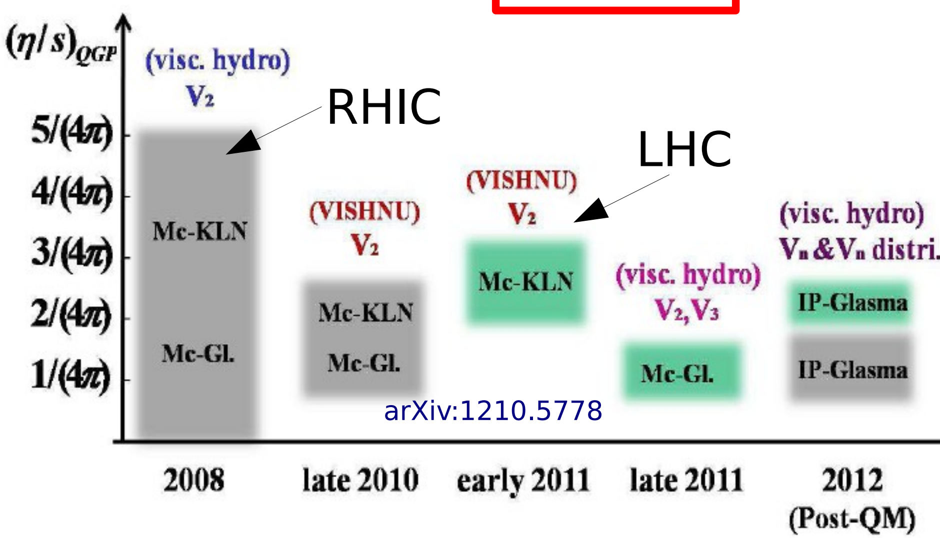
Trapped Atoms

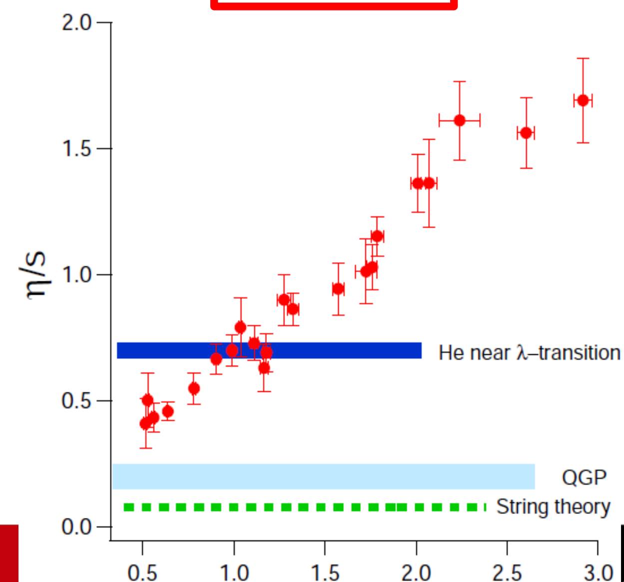
$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$

(T=0.1 neV) $\eta/s \leq 0.5$

QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

(T=180 MeV) $\eta/s \leq 0.4$





J. Low Temp. Phys. 150, 567 (2008)

Natural units for shear viscosity

$$Pa \cdot s = \frac{N}{m^2} \cdot s = J \cdot s \cdot \frac{1}{m^3}$$

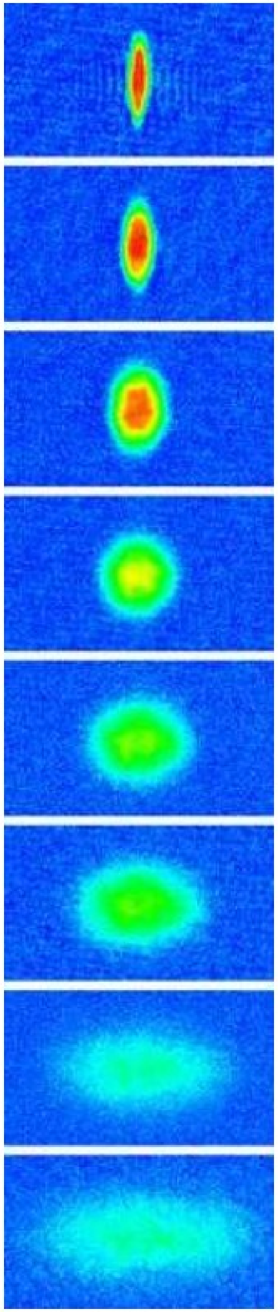
Natural dimensionless quantity is: $\frac{\eta}{n\hbar}$ ← Typically this quantity we use for non-relativistic systems

In the case of a relativistic fluid
the number of particles is not conserved.
As a consequence the quantity n is not well defined...

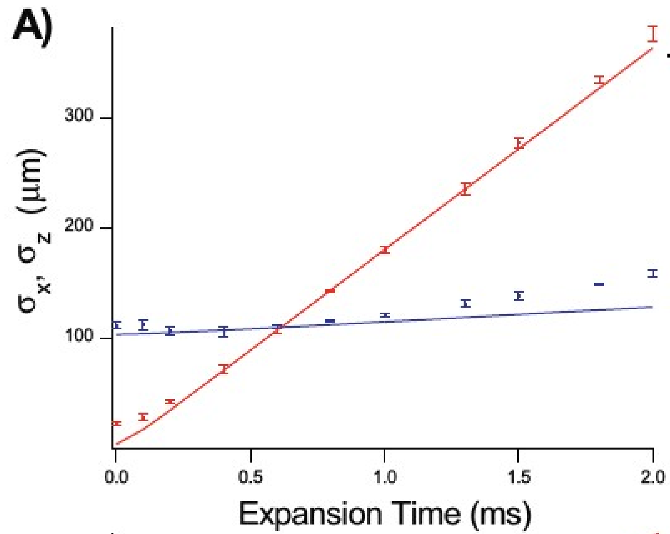
But $\frac{\eta k_B}{s\hbar}$ is also dimensionless...

This ratio is well defined in both the relativistic and non-relativistic case, and typically $s \sim n k_B$

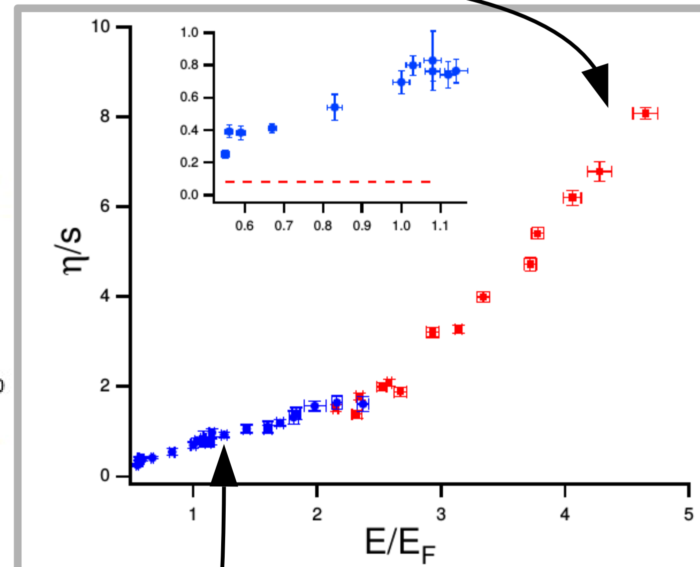
Measurements



O'Hara et al. (2002)

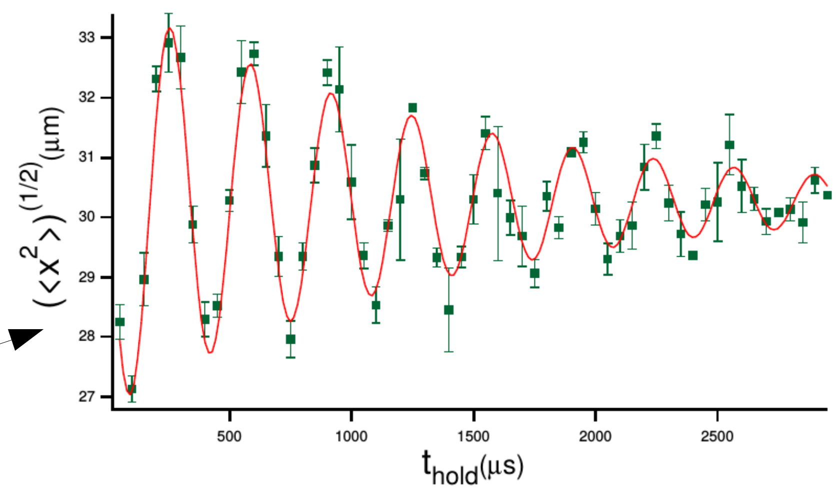


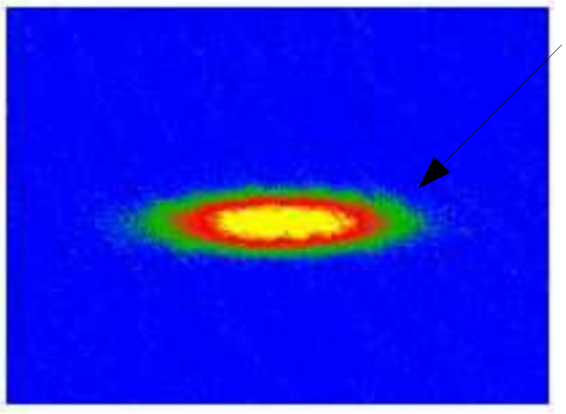
New Journal of Physics 13 (2011) 075007



Observations of elliptic flow
[less accurate for small viscosities]

Observation of the radial breathing mode excited by releasing the cloud for a short time and then recapturing it
[more accurate for small viscosities]





Non-uniform system!

$$\varepsilon_F(n) = \frac{\hbar^2(3\pi^2 n)^{2/3}}{2m}$$

$$\theta = \frac{k_B T}{\varepsilon_F}$$ Reduced temperature

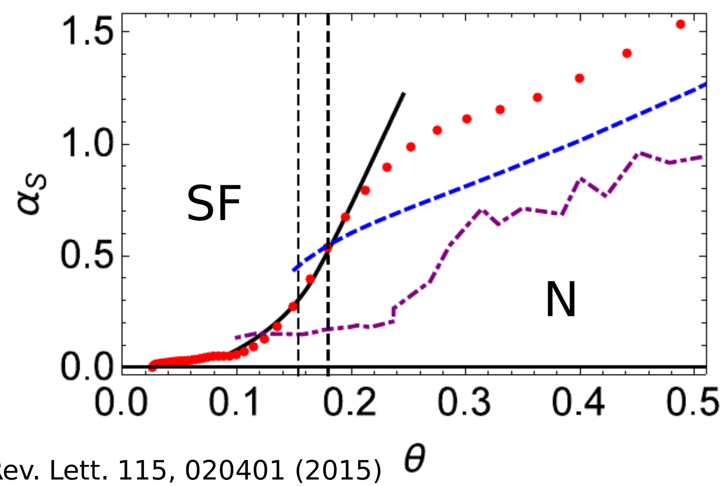
Position dependent

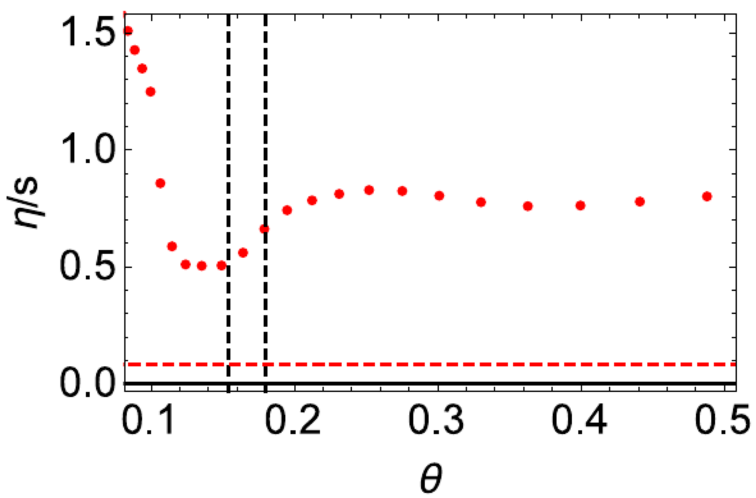
$$\eta = \eta(\theta)$$

Experiments provides averaged value

$$\langle \eta/n \rangle = \frac{1}{N\hbar}\int \eta(\boldsymbol{r})\, d^3\boldsymbol{r}$$

Extraction of the shear viscosity for uniform system from non-uniform measurements *(model dependent)*





Phys. Rev. Lett. 115, 020401 (2015)

Shear viscosity from *ab-initio* calculations (QMC)

Input:

- Interaction – only requirement $a \to \infty$ and $r_{\rm eff} \to 0$ (many possible forms – take the most convenient for you)

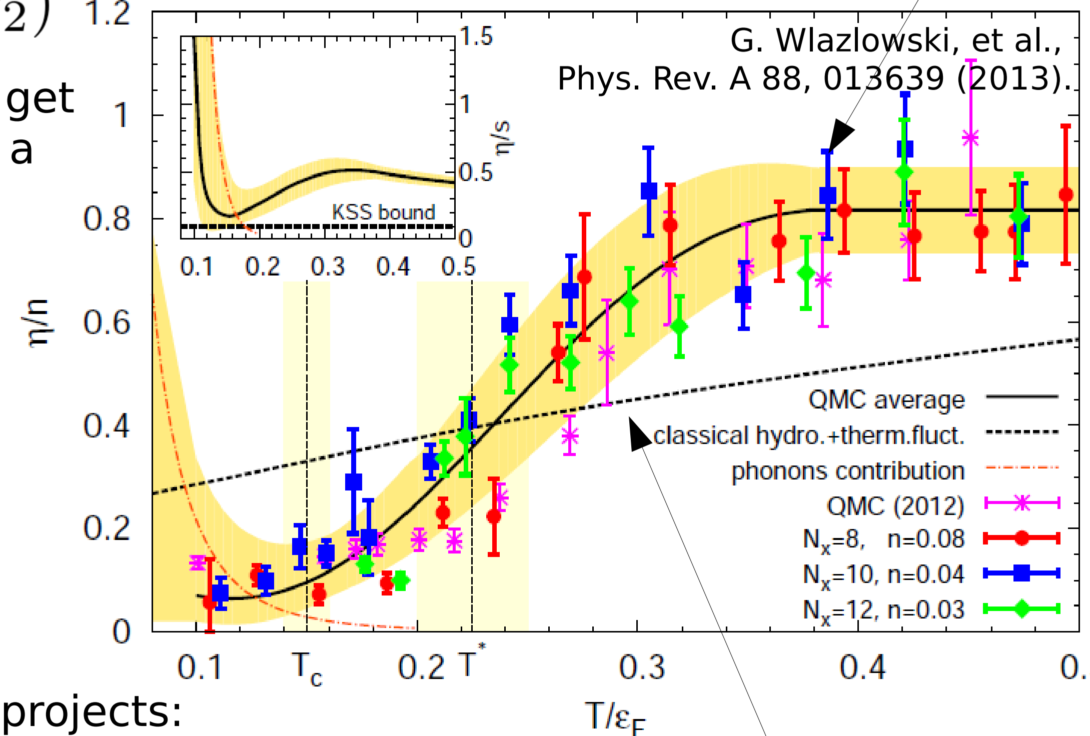
Theoretical uncertainties ~10%

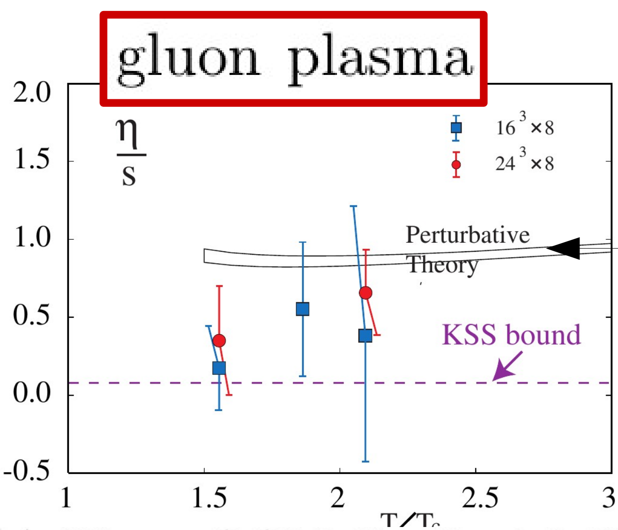
$$V(\boldsymbol{r}_1 - \boldsymbol{r}_2) = -g\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

For these interaction QMC **free from sign problem**!

tune to get desired a

- Analytical results, like tail asymptotics, sum rules...





A. Nakamura, S. Sakai, Phys. Rev. Lett. **94**, 072305 (2005).

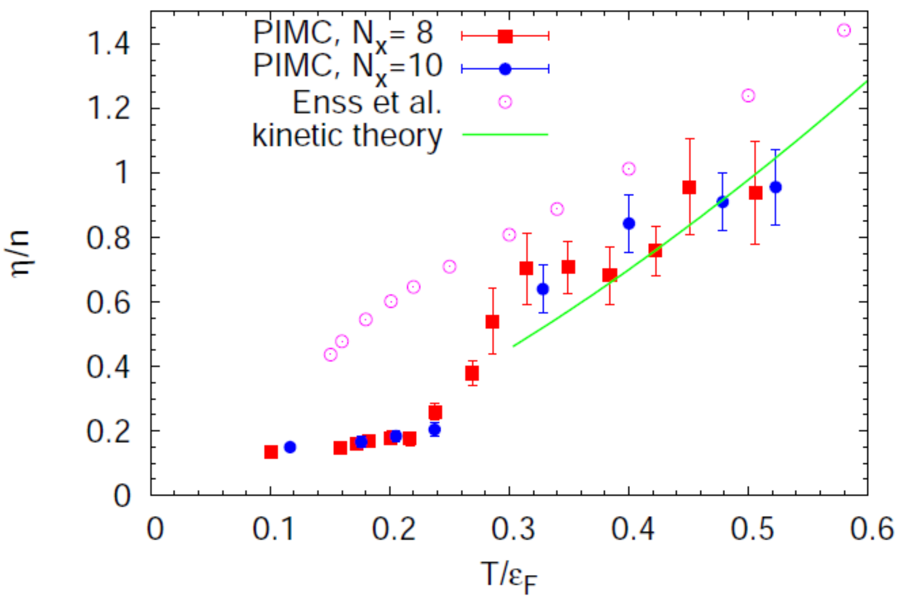
Similar projects: lattice computation of the shear viscosity for QGP

(note size of uncertainties)

(Calculations for <u>quark</u>-gluon plasma failed...)

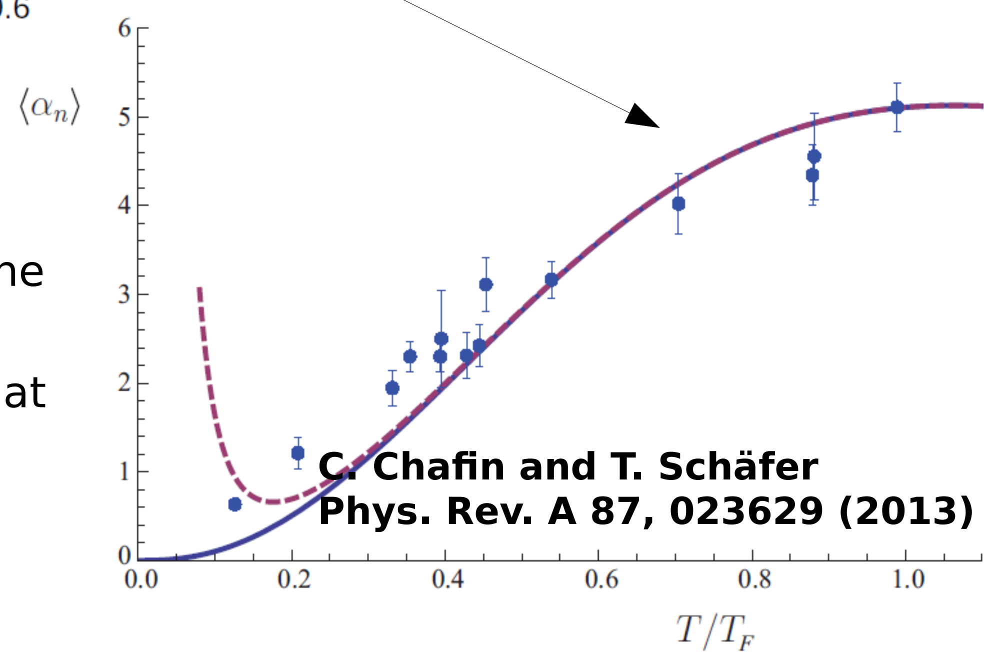
"hydrodynamic bound" violated!

Shear viscosity from QMC



Kinetic theory seems to be valid for temperatures above $0.4e_F$ $(2.5T_c)$

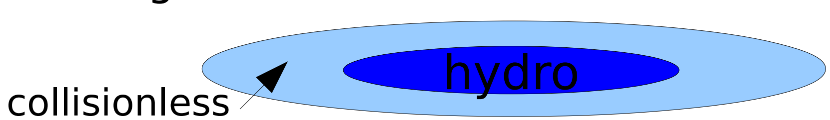
Trap averaged of the kinetic theory results



C. Chafin and T. Schäfer Phys. Rev. A 87, 023629 (2013)

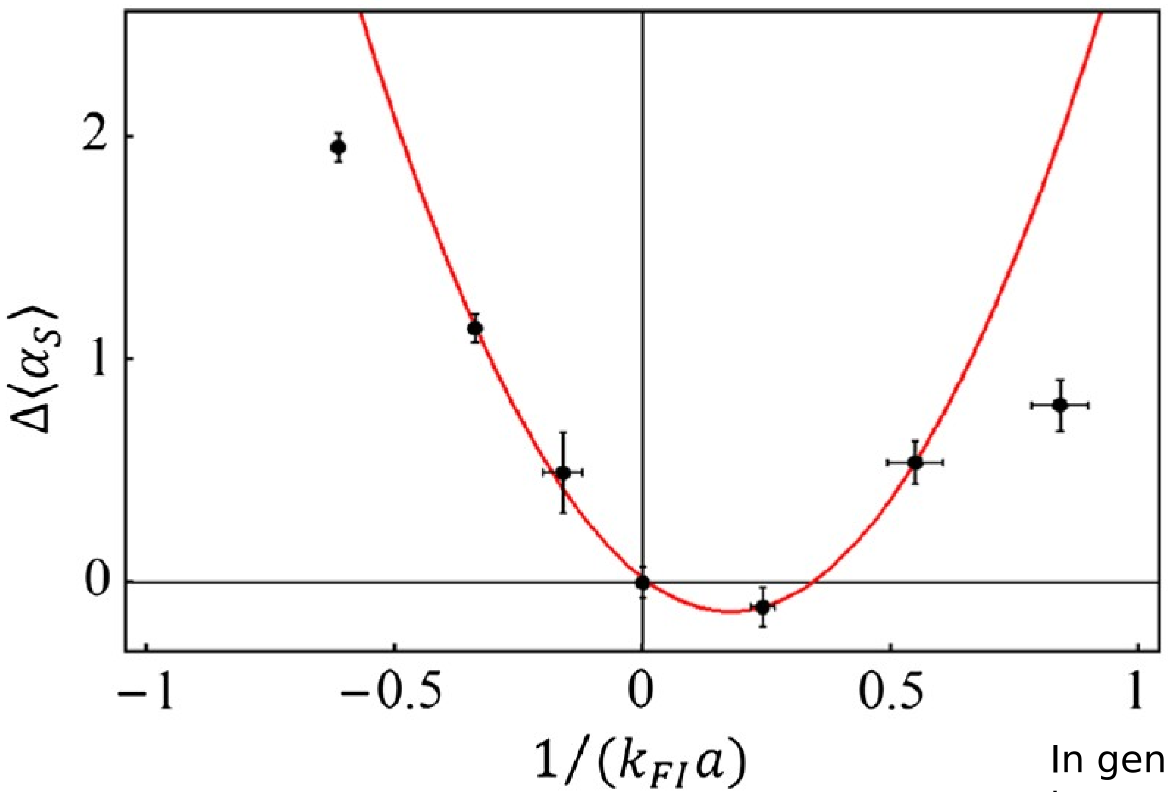
Problem with averaging procedure of the uniform results

Hydrodynamic description breaks down at the edges.

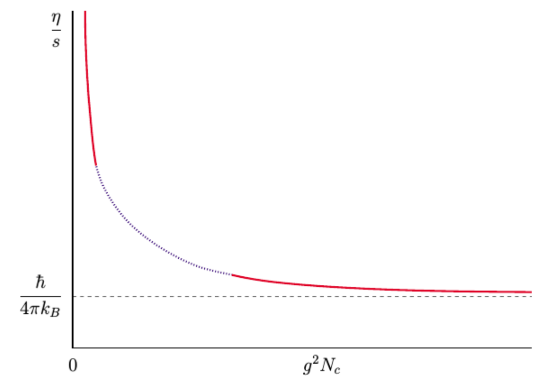


Anomalous Minimum in the Shear Viscosity of a Fermi Gas

E. Elliott,[1,2] J. A. Joseph,[1] and J. E. Thomas[1]
[1]*Department of Physics, North Carolina State University, Raleigh, North Carolina 27695, USA*
[2]*Department of Physics, Duke University, Durham, North Carolina 27708, USA*
(Received 6 November 2013; revised manuscript received 21 April 2014; published 10 July 2014)



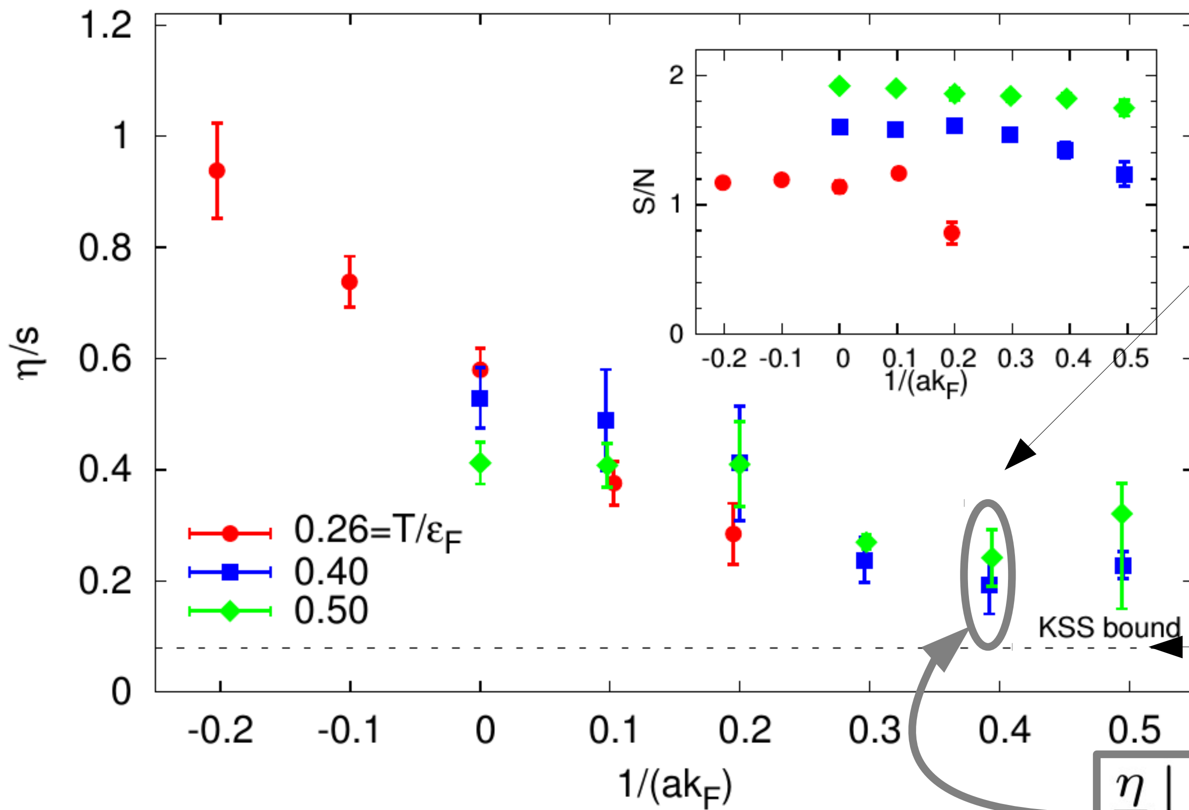
Minimum of
the shear viscosity
not for UFG
(as suggested by KSS)



[Kovtun, Son, Starinets, PRL (2005)]

In general position of the minimum is temperature dependent

Shear viscosity from QMC near unitary



Experimental finding also obtained in QMC simulations

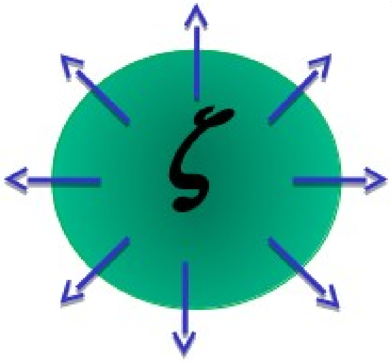
$$\frac{1}{4\pi}\frac{\hbar}{k_B} \approx 0.08\frac{\hbar}{k_B}$$

$$\left.\frac{\eta}{s}\right|_{min} \approx 0.2\frac{\hbar}{k_B}$$

Conclusion:
We have not found systems that violates KSS bound...

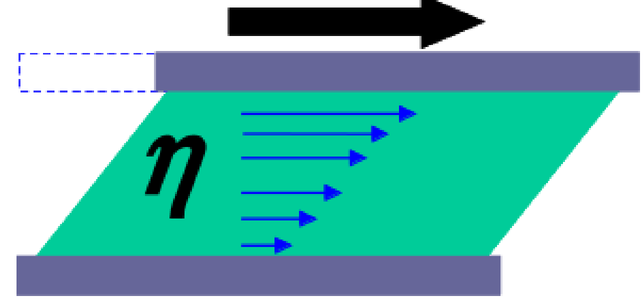
Recent theoretical prediction:
KSS bound can be voilated in "bad metals"
[Pakhira,McKenzie, Phys. Rev. B 92, 125103 (2015)]

Bulk viscosity



Works against radial expansion

But for UFG:



Works against building gradients (anisotropy)...

No intrinsic length scale ⇒ Uniform expansion keeps the unitary gas in equilibrium

Consequence:

uniform expansion does not produce entropy = bulk viscosity is zero!

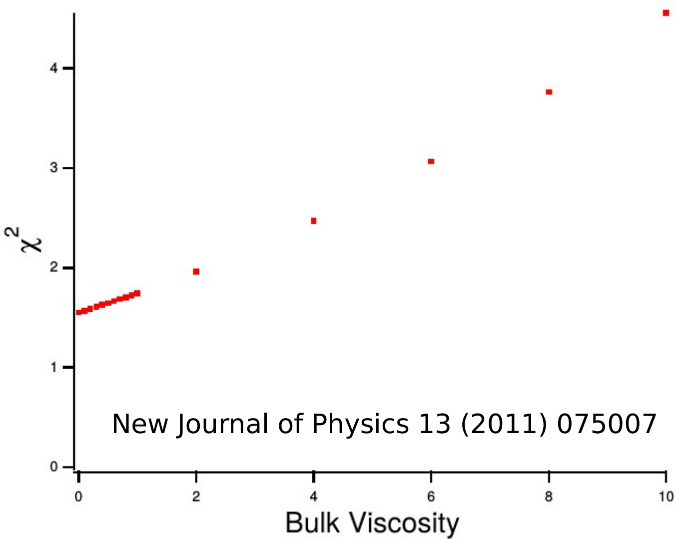
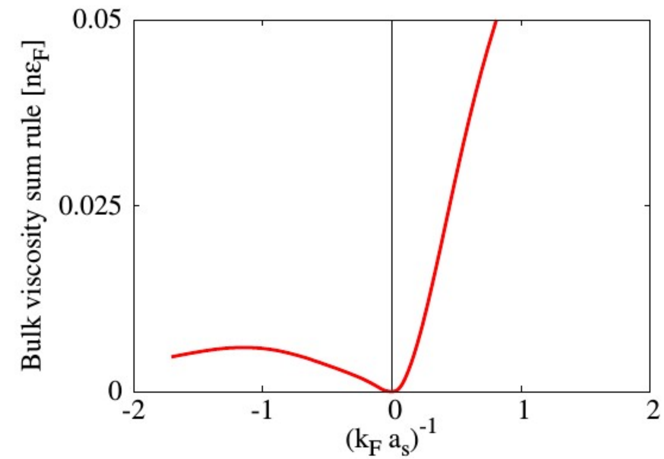


Figure 5. χ^2 per degree of freedom versus bulk viscosity with shear viscosity as the only free parameter.



Phys.Rev.A81:053610,2010

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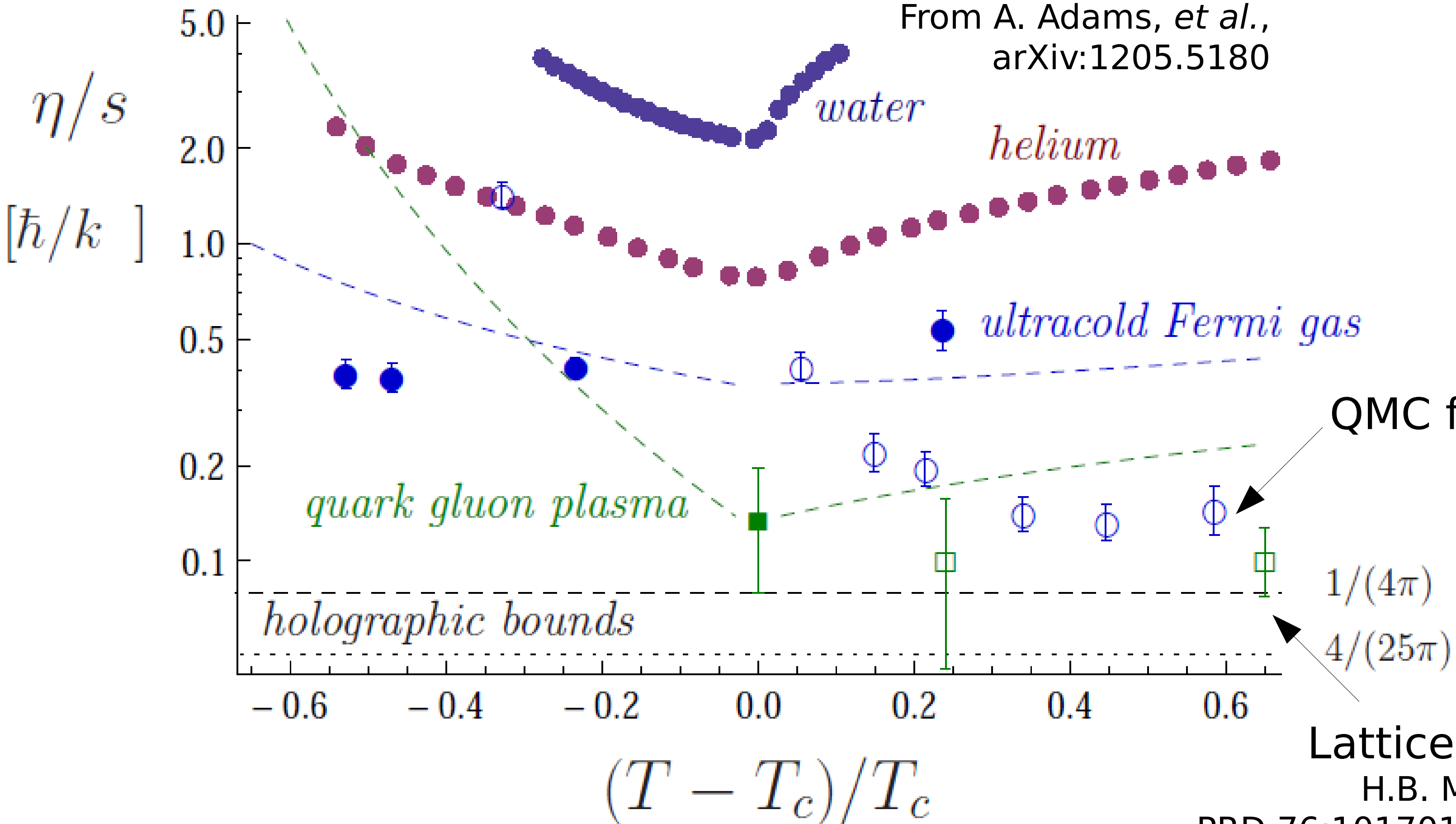
Conclusions

- Ultracold atoms can be tuned to get a system with lowest viscosity
- Computation of transport coefficients from first principles for fermionic system is feasible
- No violation of KSS bound
- Lowest value for η/s is about two times above KSS bound (QMC result)
- System with lowest viscosity realized for systems shifted towards BEC side of resonance

Searching for perfect fluid...



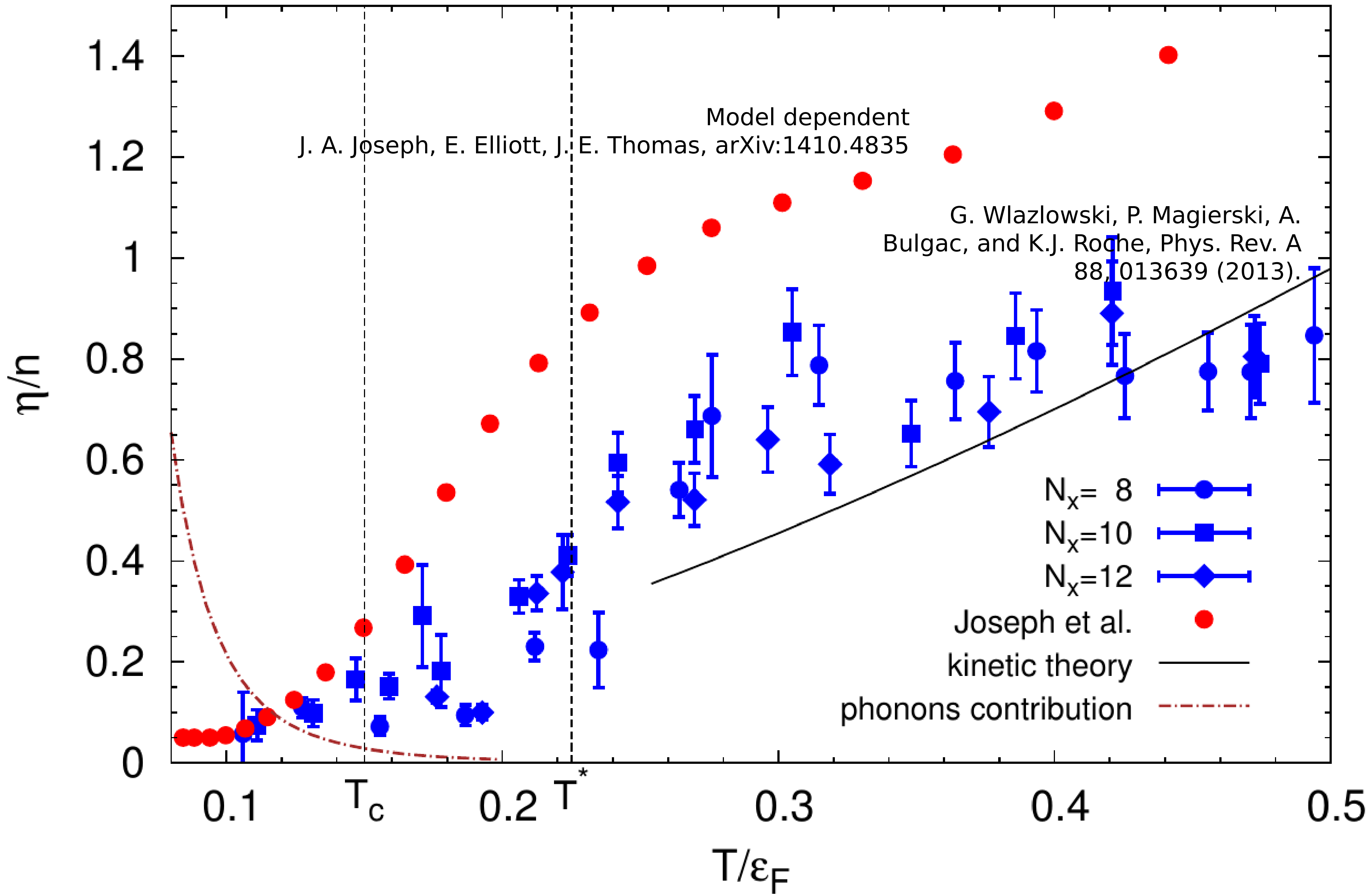
From A. Adams, *et al.*, arXiv:1205.5180

QMC for UFG

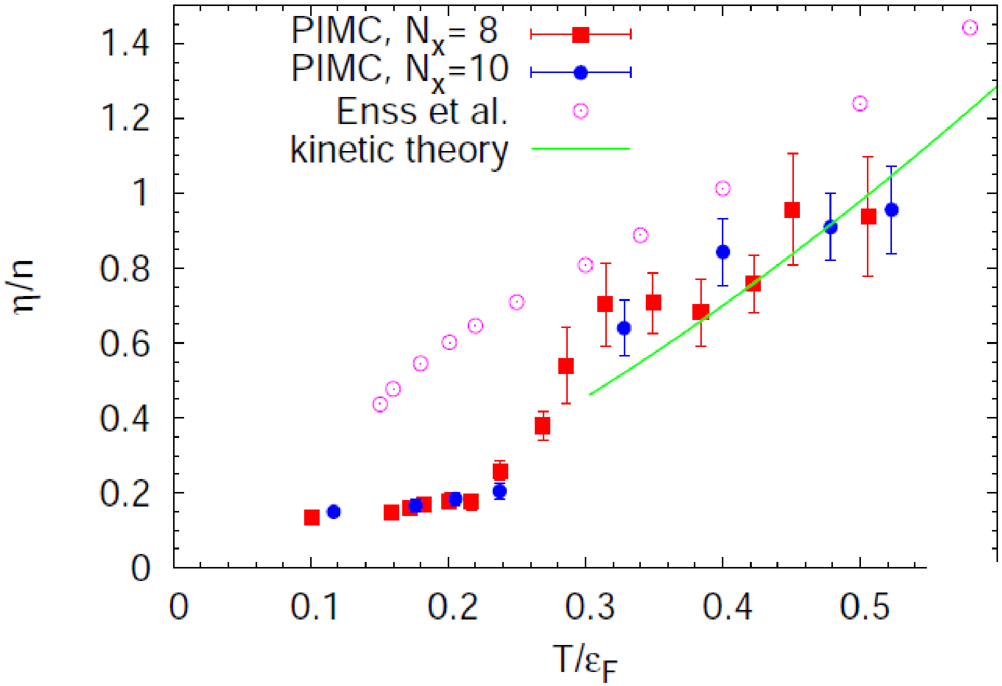
Lattice QCD
H.B. Meyer, PRD 76:101701, 2007

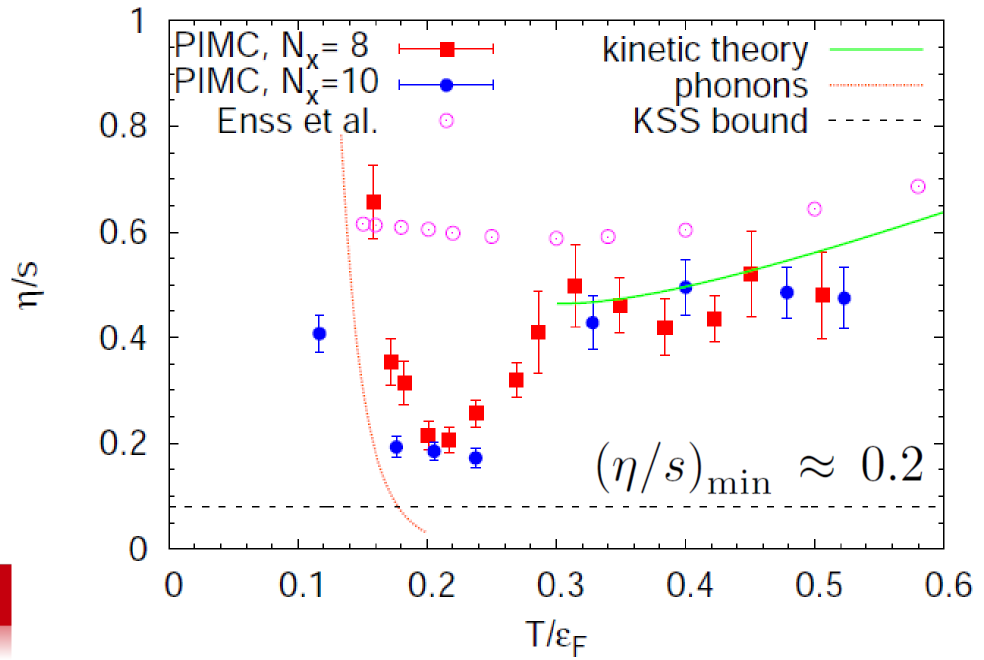
THANK YOU ...

physics wut


Model dependent
J. A. Joseph, E. Elliott, J. E. Thomas, arXiv:1410.4835
G. Wlazlowski, P. Magierski, A. Bulgac, and K.J. Roche, Phys. Rev. A 88, 013639 (2013).
η/n
T/ε_F
T_c
T*
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0.1
0.2
0.3
0.4
0.5
N_x= 8
N_x=10
N_x=12
Joseph et al.
kinetic theory
phonons contribution

Shear viscosity from QMC





G. Wlazłowski, P. Magierski, J.E. Drut, Phys. Rev. Lett. 109:020406, 2012

fluid	P [Pa]	T [K]	η [Pa·s]	η/n [$\hbar$]	η/s [$\hbar/k_B$]
H_2O	$0.1 \cdot 10^6$	370	$2.9 \cdot 10^{-4}$	85	8.2
^{4}He	$0.1 \cdot 10^6$	2.0	$1.2 \cdot 10^{-6}$	0.5	1.9
H_2O	$22.6 \cdot 10^6$	650	$6.0 \cdot 10^{-5}$	32	2.0
^{4}He	$0.22 \cdot 10^6$	5.1	$1.7 \cdot 10^{-6}$	1.7	0.7
^{6}Li ($a = \infty$)	$12 \cdot 10^{-9}$	$23 \cdot 10^{-6}$	$\leq 1.7 \cdot 10^{-15}$	≤ 1	≤ 0.5
QGP	$88 \cdot 10^{33}$	$2 \cdot 10^{12}$	$\leq 5 \cdot 10^{11}$		≤ 0.4

Table 1. Viscosity η, viscosity over density, and viscosity over entropy density ratio for several fluids. Data for water and helium taken from [6, 7] and [8], data for Li and the quark gluon plasma (QGP) will be explained in Sect. 5. For water and helium we show data at atmospheric pressure and temperatures just below the boiling point and the λ transition, respectively. These data points roughly correspond to the minimum of η/n at atmospheric pressure. We also show and data near the tri-critical point which roughly corresponds to the global minimum of η/s. Note that the quark gluon plasma does not have a well defined density.

From Rept.Prog.Phys.72:126001,2009

Physical system:
unitary Fermi gas (unpolarized)

$$\hat{H}_0 \equiv \sum_{\boldsymbol{p},\lambda=\uparrow,\downarrow} \frac{p^2}{2m}\, \hat{a}^\dagger_\lambda(\boldsymbol{p})\, \hat{a}_\lambda(\boldsymbol{p}) - g \sum_i \hat{n}_\uparrow(\boldsymbol{r}_i)\, \hat{n}_\downarrow(\boldsymbol{r}_i)$$

$$\frac{1}{g} = -\frac{m}{4\pi\hbar^2 a} + \frac{k_c m}{2\pi^2\hbar^2}$$

UFG: System is dilute
but strongly interacting!

$$0 \leftarrow k_F r_0 \ll 1 \ll k_F a \rightarrow \infty$$

NONPERTURBATIVE REGIME!

Method: Path Integral Monte Carlo

$$\langle O \rangle_0 = \frac{1}{Z} \mathrm{Tr}\left\{ \hat{O} \exp[-\beta(\hat{H}_0 - \mu \hat{N})] \right\}$$

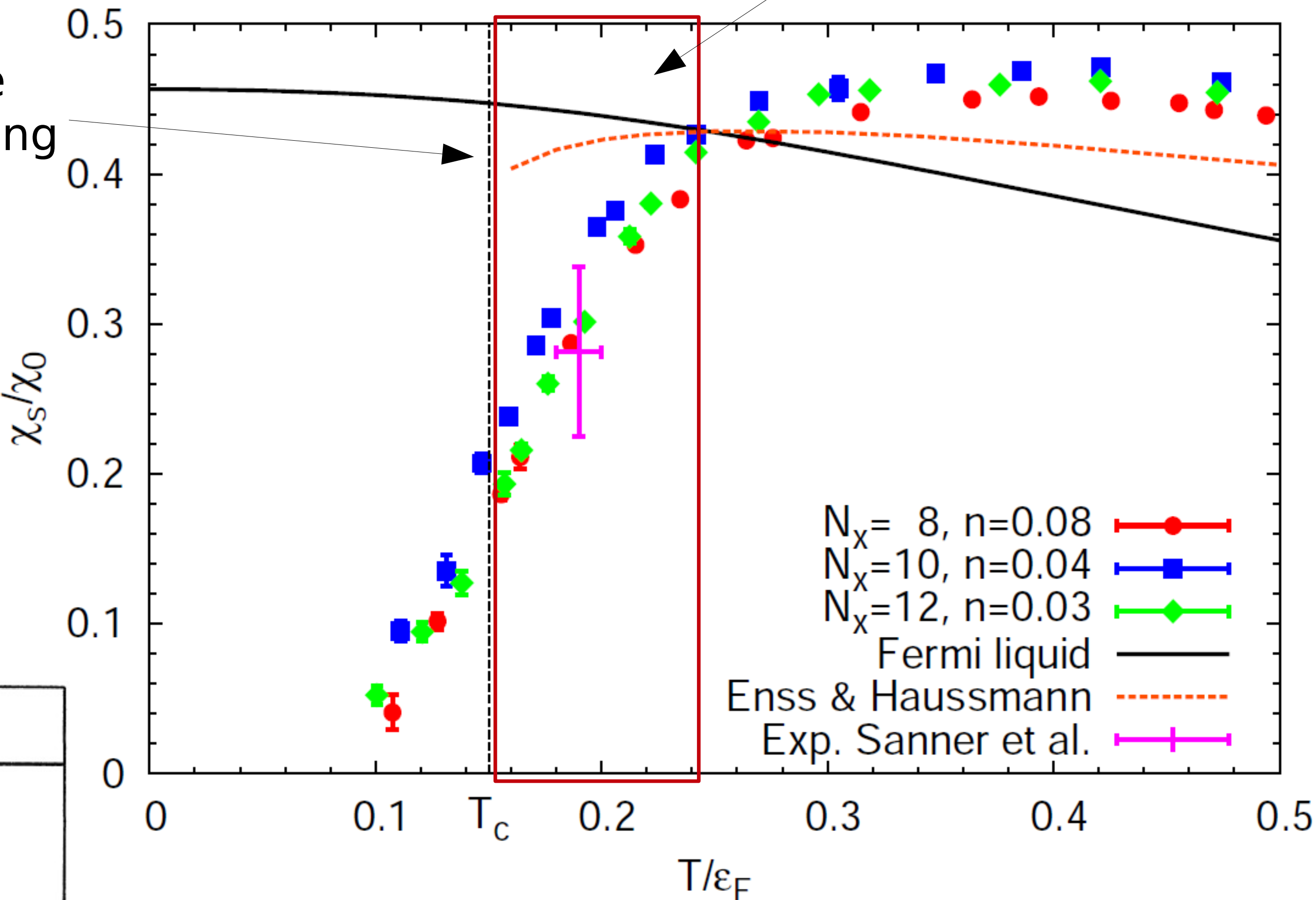
$$Z = \mathrm{Tr}\left\{ \exp[-\beta(\hat{H}_0 - \mu \hat{N})] \right\}$$

1. The system is placed on a cubic spatial lattice
2. Trotter-Suzuki decomposition to expand imaginary time evolution operator $\exp[-\beta(\hat{H}_0 - \mu \hat{N})]$
3. The interaction is represented by means of a Hubbard-Stratonovich transformation
4. Evaluation of the emerging path-integral via Metropolis importance sampling – **NO SIGN PROBLEM**

Static spin susceptibility

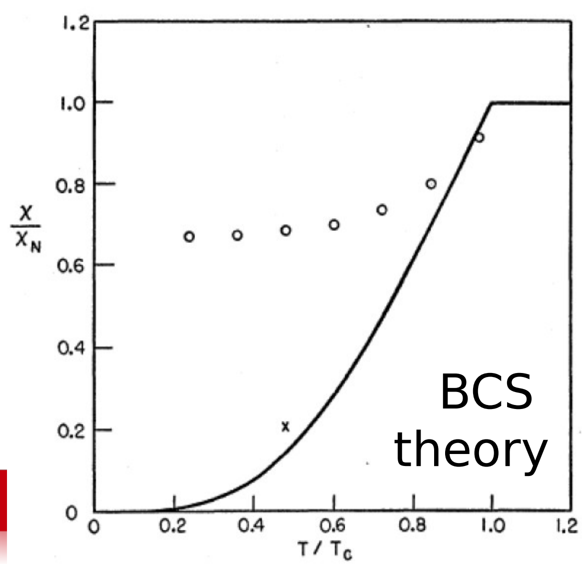
PSEUDOGAP REGIME

Critical temperature from finite size scaling analysis



$$\chi_s = \partial(n_\uparrow - n_\downarrow)/\partial(\mu_\uparrow - \mu_\downarrow)$$

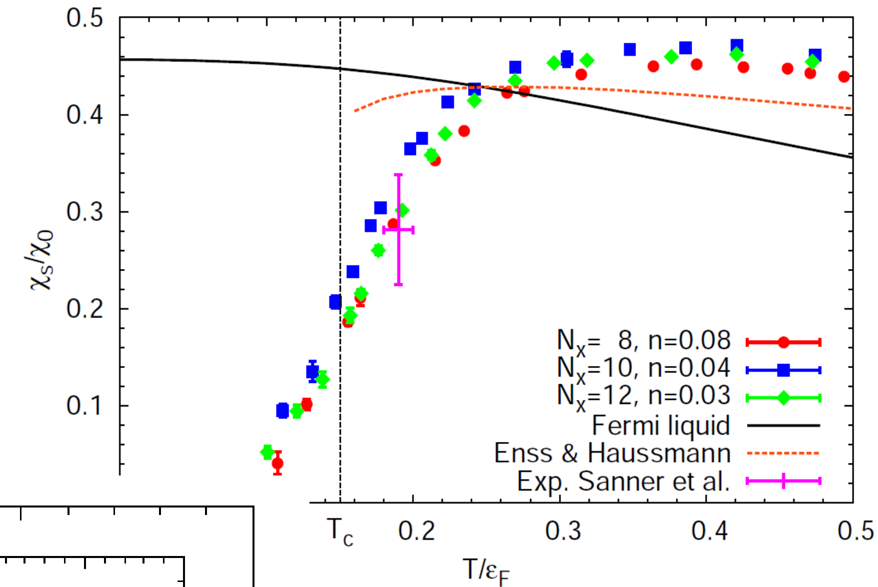
G. Wlazłowski, P. Magierski, J.E. Drut, A. Bulgac, K.J. Roche, Phys. Rev. Lett. 110, 090401 (2013)

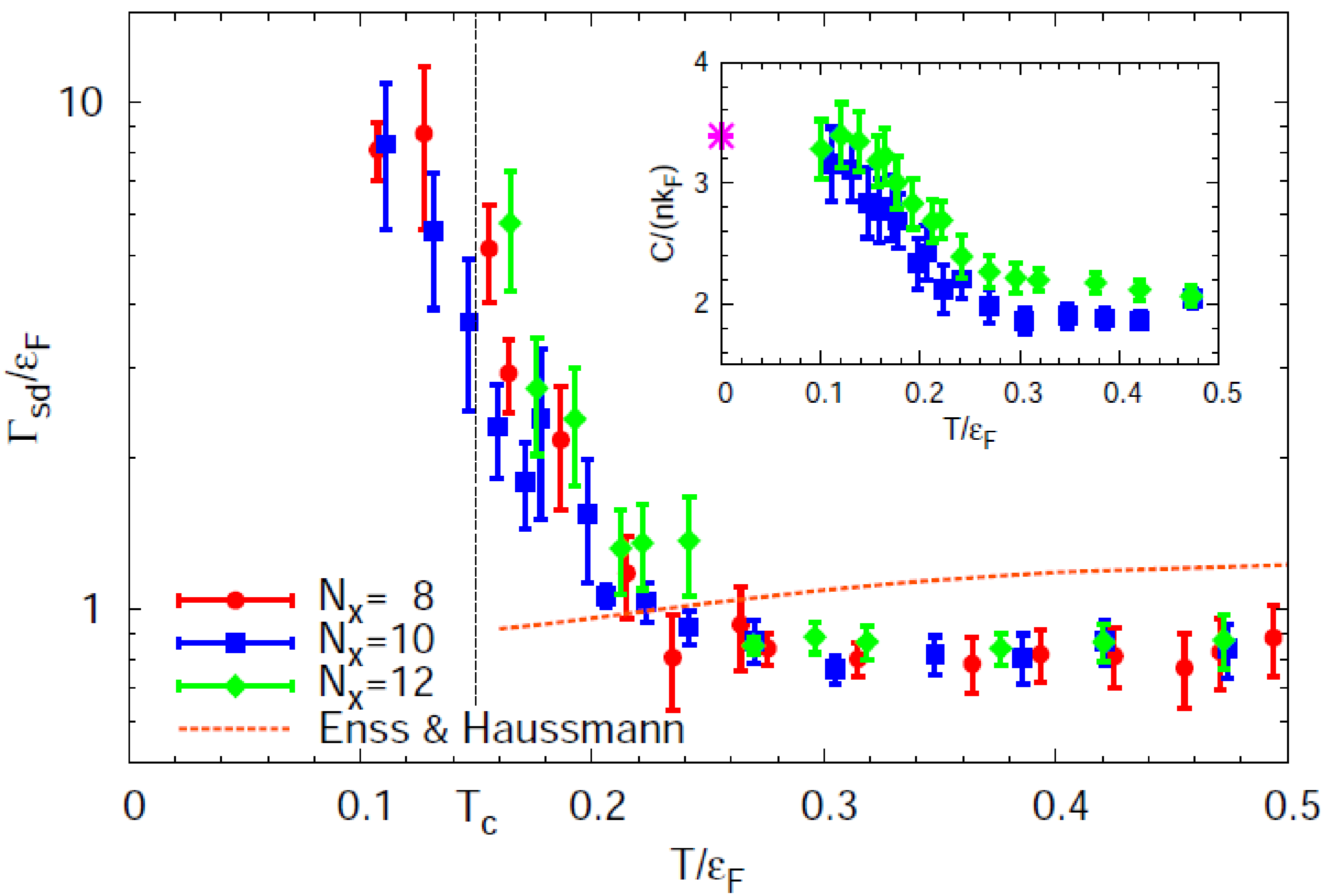


Spin drag rate

$$\Gamma_{sd} = n/\sigma_s$$

Enhancement appears consistently with the spin susceptibility suppression





G. Wlazłowski, P. Magierski, J.E. Drut, A. Bulgac, K.J. Roche, PRL 110, 090401 (2013)

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Spin diffusion coefficient

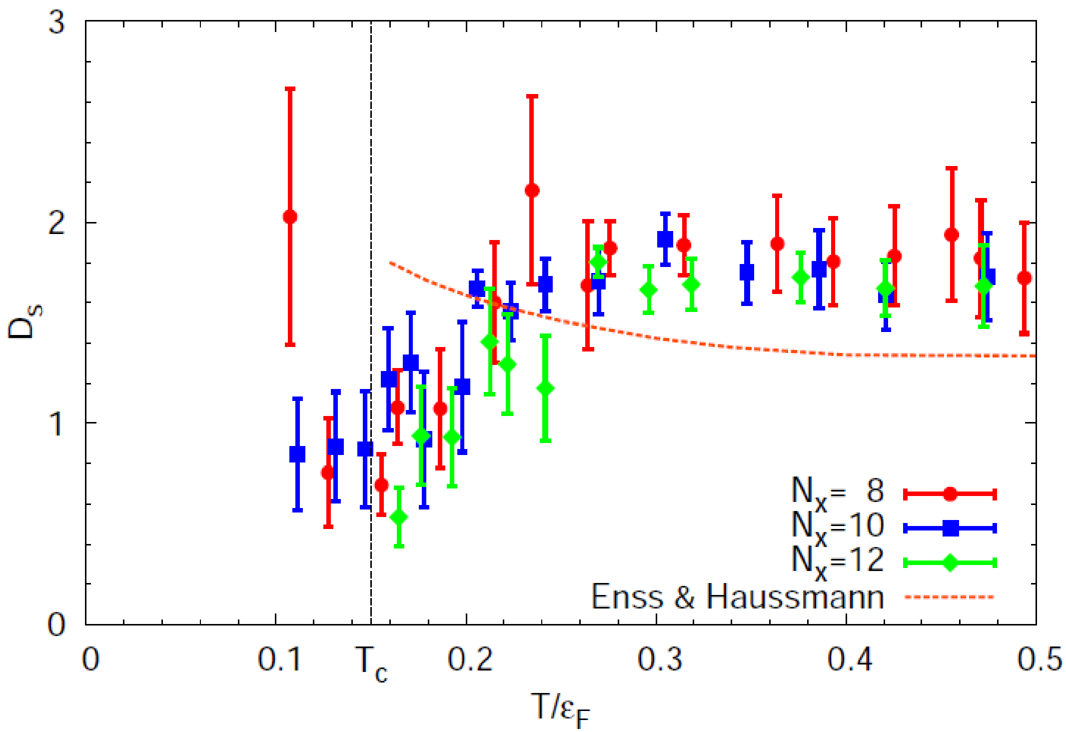


FIG. 4: (Color online) The spin diffusion coefficient obtained by the Einstein relation $D_s = \sigma_s/\chi_s$ as function of temperature. The notation is identical to Fig. 3.

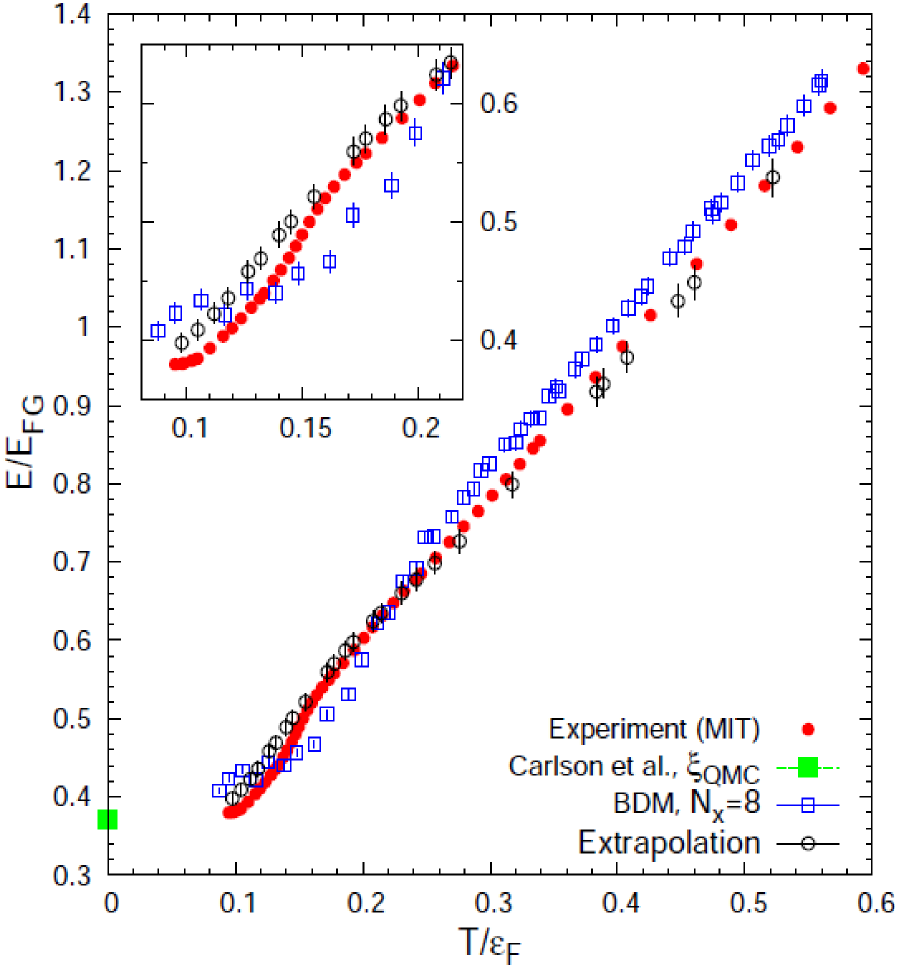
$$D_s \sim vl$$

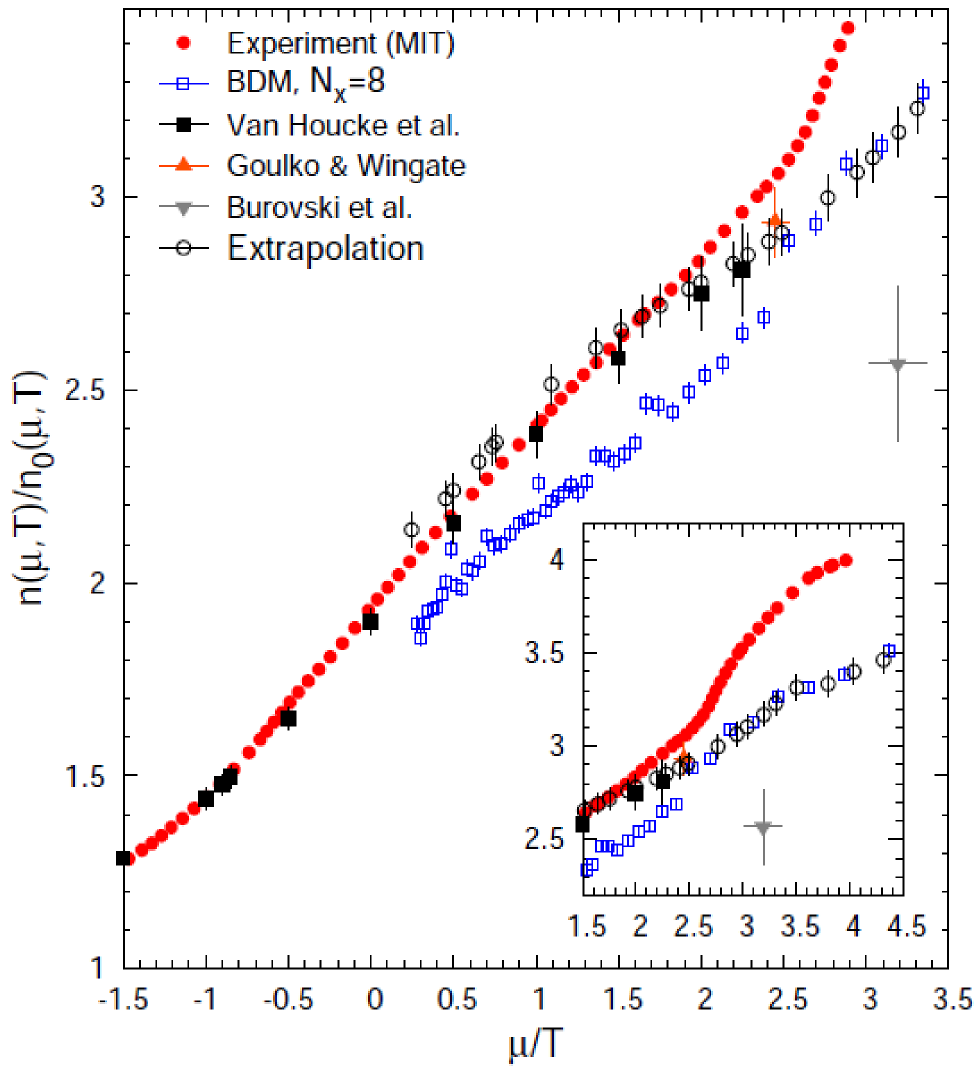
$$v \sim p_F \sim n^{1/3}$$

$$l \sim n^{-1/3}$$

$$D_s \sim 1$$

EoS for UFG

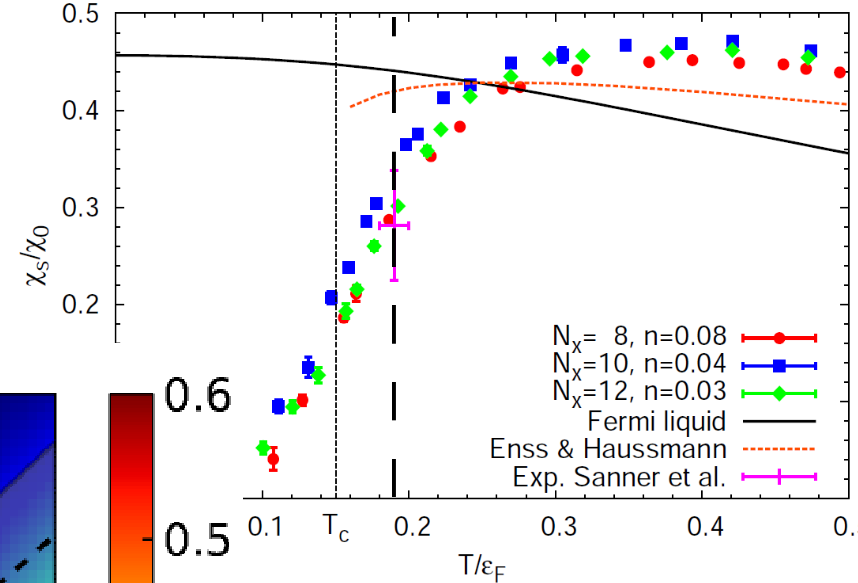


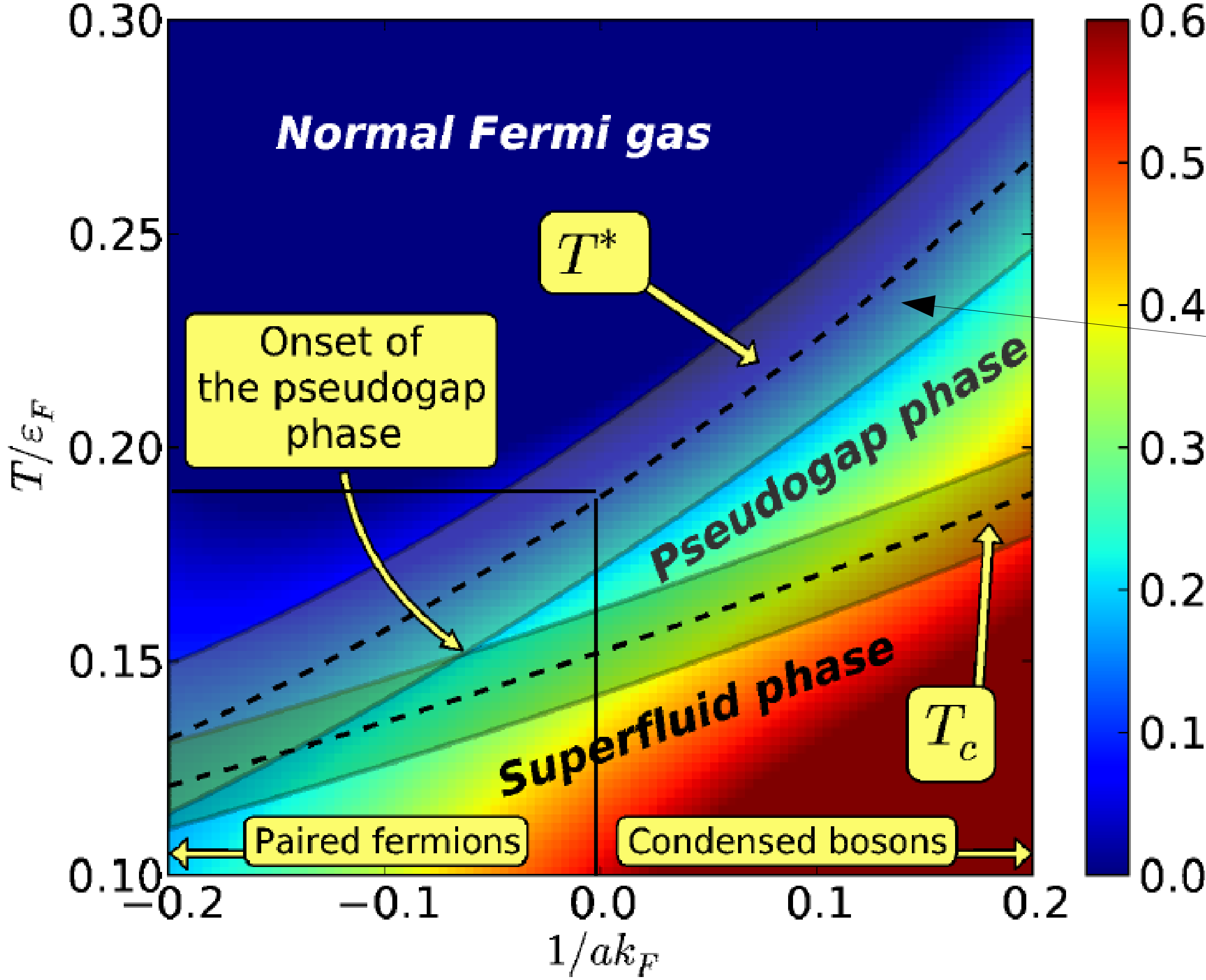


J.E. Drut, T.A. Lähde, G. Wlazłowski, P. Magierski, Phys. Rev. A 85, 051601(R) (2012)

T_c & T^*

P. Magierski, G. Wlazłowski, A. Bulgac
Phys. Rev. Lett. 107, 145304 (2011)





Lower limit for T^*:
due to finite resolution of the analytic continuation procedure

Holographic Duality

- duality relating quantum field theory (QFT) and gravity
- it maps the quantum physics of strongly correlated many-body systems to the classical dynamics of black hole horizons in one higher dimension
  - thus translates problems in quantum many-body physics into equivalent problems in classical gravity – sometimes easier to solve
  - the physics of black hole horizons in general relativity (GR) is largely independent of the details of the black hole

Strongly coupled thermal field theory on R^4	$\Leftrightarrow$	Weakly coupled string theory on AdS_5 black hole
CFT temperature	$\Leftrightarrow$	Hawking temperature of black hole
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon