Evolutionary Algorithm for Particle Trajectory Reconstruction

Oskar Wyszyński

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Jagiellonian University

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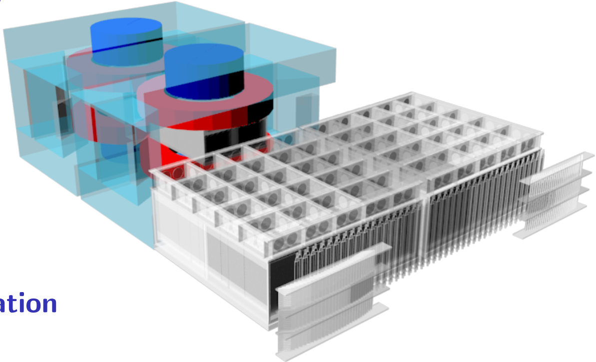


Algorithm Overview

Model - what is it?

Natural fern ?

or

Barnsley fern ?

$$f_1(x,y) = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.16 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix},$$

$$f_2(x,y) = \begin{bmatrix} 0.85 & 0.04 \\ -0.04 & 0.85 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0.00 \\ 1.60 \end{bmatrix},$$

$$f_3(x,y) = \begin{bmatrix} 0.20 & -0.26 \\ 0.23 & 0.22 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0.00 \\ 1.60 \end{bmatrix},$$

$$f_4(x,y) = \begin{bmatrix} -0.15 & 0.28 \\ 0.26 & 0.24 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0.00 \\ 0.44 \end{bmatrix},$$

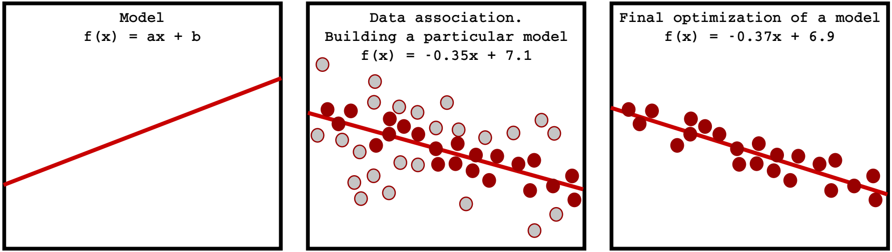
$$P(f_1) = 1\%, P(f_2) = 85\%, P(f_3) = 7\%, P(f_4) = 7\%$$

Pattern Recognition

Pattern Recognition

$=$

Model + Data Association + Optimization



The concept

The general idea of the evolutionary trajectory reconstruction:

- Generative model of trajectory
- Discriminative model of background
- Combinatorial search (Part of data association)
- Continuous optimization using evolutionary strategy

Models

Generative model (Trajectory)

$$\overbrace{P(\hat{\mathbf{\Theta}}|\mathcal{S}(\mathbf{c}))}^{\text{Posterior probability}} \propto \underbrace{P(\hat{\mathbf{\Theta}})}_{\text{Prior probability}} \overbrace{P(\mathcal{S}(\mathbf{c})|\hat{\mathbf{\Theta}})}^{\text{Likelihood}} \quad (1)$$

where:
$\mathbf{c} = (x, y, z, s_1, \ldots, s_n)$ stands for a cluster with center of gravity located at (x, y, z) along with charge deposition ADC signals $(s_1, \ldots, s_n)$ in the cluster. The symbol $\hat{\mathbf{\Theta}}$ denotes an estimation for a track parameter vector $\mathbf{\Theta} = p \oplus o$, where $p = (p_x, p_y, p_z)$ denotes the particle momentum vector at a starting point $o = (x, y, z)$ and $\oplus$ denotes concatenation operator $(dim(\mathbf{\Theta}) = dim(p) + dim(o))$. The operator $\mathcal{S} : T \cup C \to \mathbb{R}^3$ produces a three dimensional Euclidean space vector

$$\mathcal{S}(\xi) = (\xi_x, \xi_y, \xi_z) \quad (2)$$

where C and T are a cluster and a track parameter vector space respectively.

Likelihood (Trajectory)

The likelihood that a point $\mathbf{x}$ belongs to the track with parameters Θ is defined as follows:

$$p(\mathcal{S}(\mathbf{c})|\Theta) = p(\mathcal{S}(\mathbf{c})|\bar{\Theta}, \Sigma) \sim \mathcal{N}(\bar{\Theta}, \Sigma)$$

where:

- $" \sim "$ denotes equality in distribution
- $\bar{\Theta}$ represents Θ track parameters extrapolated to $\mathbf{c}_z$ position
- Σ is a covariance matrix (typically diagonal)

Discriminative Model (Background)

$$p(Bg|\mathbf{x}) = H_{Background}(\max F_{ADC}(\mathbf{x}))$$

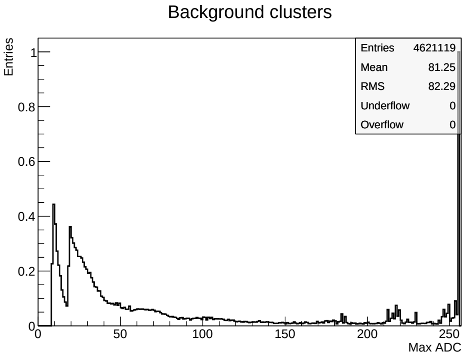
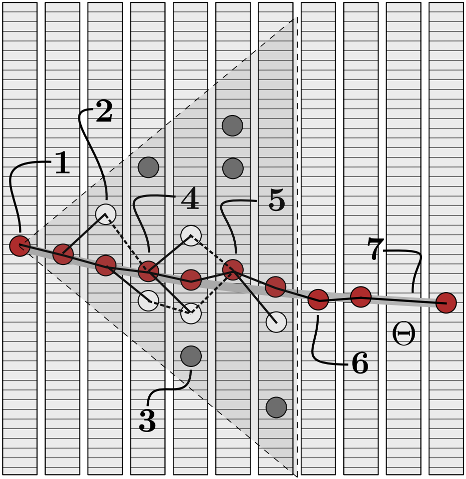


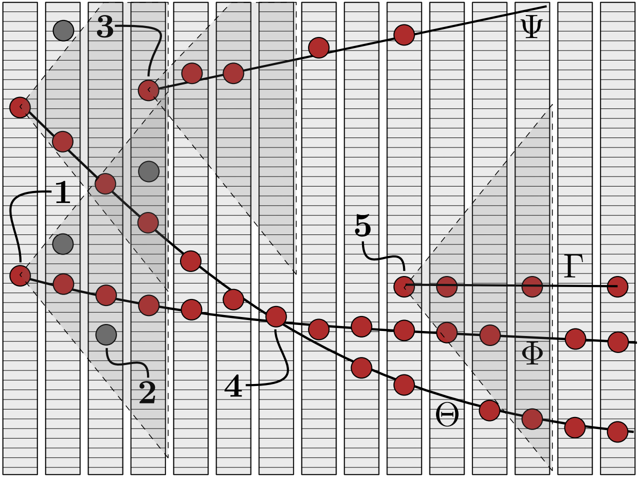
Figure: Histogram $H_{Background} = H_{All} - H_{Tracks}$

Data Association

Probability is calculated for the path from leaf to the tree root.

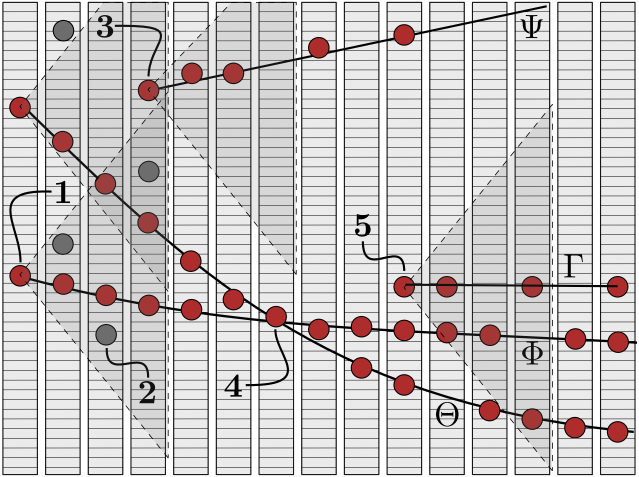


1. Tree root
2. $P(Bg|\mathbf{c}) < P(\Theta|\mathbf{c})$
   but isn't the most probable one.
   It will become a new seed.
3. $P(Bg|\mathbf{c}) > P(\Theta|\mathbf{c})$
   classified as the background.
   It will become a new seed.
4. Good track cluster. It has three potential children.
5. Looks for the most probable parent.
6. The track is formed. Cluster is attached to the track. White parent clusters are ignored.
7. The track is growing.



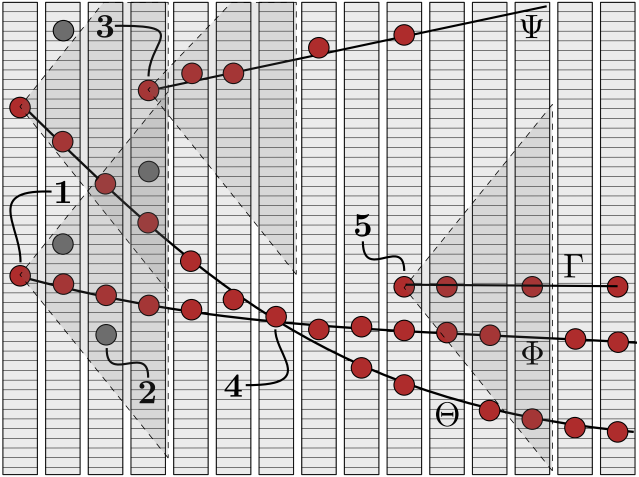
Situation 1:

No candidates, create a new seed. The seed becomes the track Φ, because it meets suitable clusters.



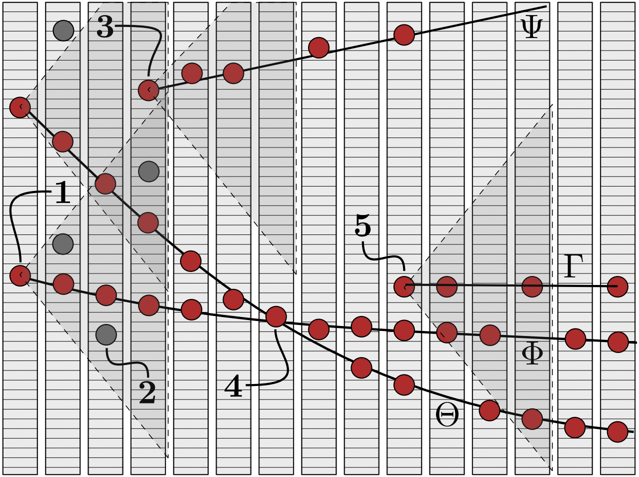
Situation 2:

$P(Bg|\mathbf{c}) > P(\Phi|\mathbf{c})$
so the new seed is created. However it isn't converted to the track candidate.
The seed becomes a noise.



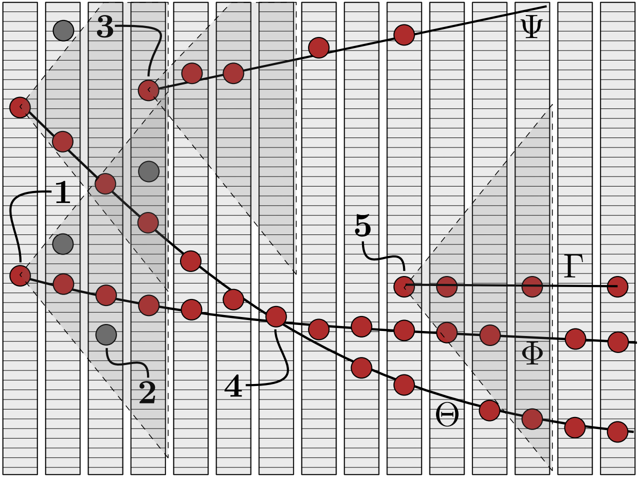
Situation 3:

$P(Bg|\mathbf{c}) > P(\Theta|\mathbf{c})$
No suitable candidate, create a new seed. The seed becomes the track Ψ.



Situation 4:

$P(\Phi|\mathbf{c}) > P(Bg|\mathbf{c})$
and
$P(\Theta|\mathbf{c}) > P(\Phi|\mathbf{c})$
so the cluster is attached to the track Θ. The parameter Θ is optimized using the new cluster.



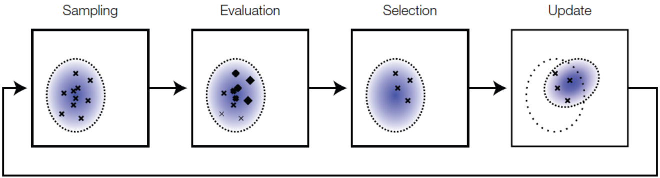
Situation 5:

$P(Bg|\mathbf{c}) > P(\Phi|\mathbf{c})$ no suitable candidate, create a new seed. The seed becomes the track Γ.

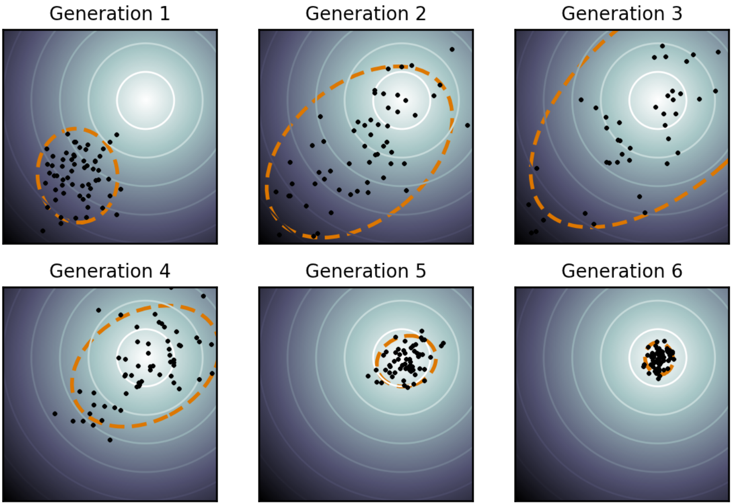
Parameter Optimization

Covariance Matrix Adaptation Evolution Strategy

The CMA-ES (Covariance Matrix Adaptation Evolution Strategy) is an evolutionary algorithm for difficult non-linear non-convex black-box optimisation problems in continuous domain.



Optimization of two parameters, e.g. linear function.



Error/Fitness/Cost function is shown as a gradient.

Further reading

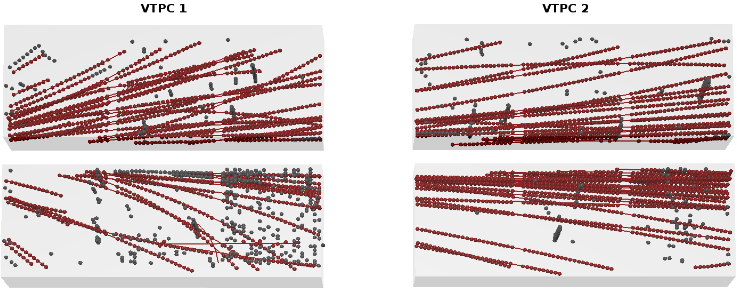
N. Hansen and A. Ostermeier.
Adapting arbitrary normal mutation distributions in evolution strategies: The covariance matrix adaptation.
In Proceedings of the 1996 IEEE International Conference on Evolutionary Computation, pages 312317. IEEE, 1996.

N. Hansen and A. Ostermeier.
Completely derandomized self-adaptation in evolution strategies.
Evolutionary Computation, 9(2):159195, 2001.

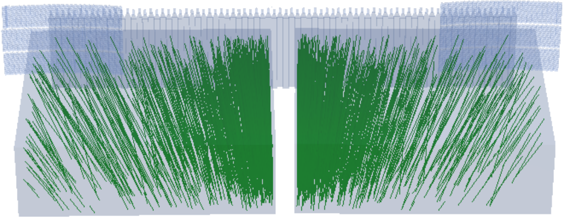
Results

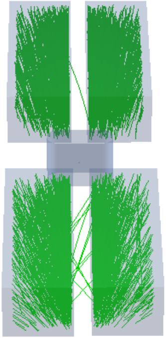
Thanks to Courtesy of the NA61/SHINE collaboration

The efficiency and fake track rate were calculated using simulated events with superimpose real detector noise.



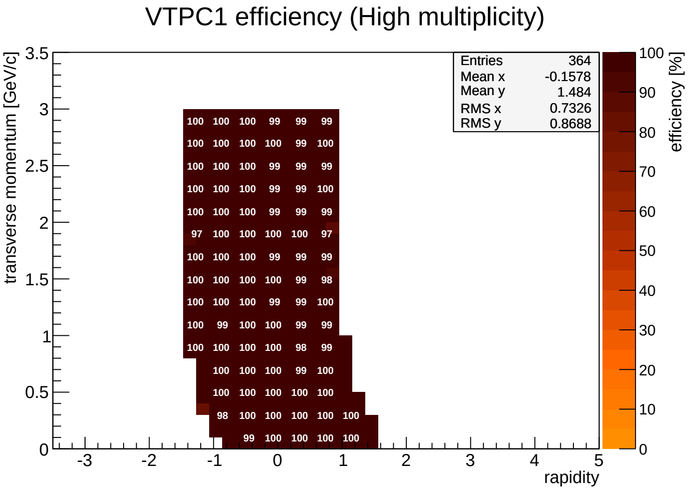
Medium multiplicity: $\frac{dn}{dy} = 17.2$





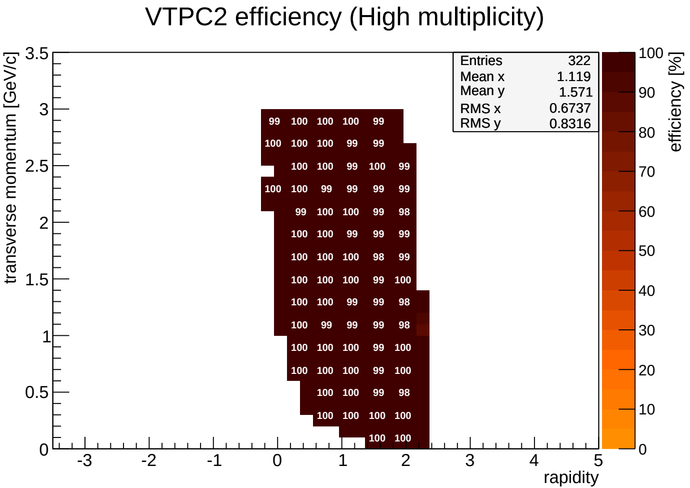
Medium multiplicity: $\frac{dn}{dy} = 423.1$

Results



High multiplicity: $\frac{dn}{dy} = 423.1$

Results



High multiplicity: $\frac{dn}{dy} = 423.1$

More details

More details about the method can be found in:

O. Wyszynski.
Evolutionary Algorithm for Particle Trajectory Reconstruction within Inhomogeneous Magnetic Field in the NA61/SHINE Experiment at CERN SPS.
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DOI: 10.4467/20838476SI.15.015.3997