Higher derivative couplings in supergravity

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IUAP meeting

Antwerp, 18 December 2015

Introduction: AIM

Classify supergravity-matter couplings in $\mathrm{D} \leq 6$ space-time dimensions to construct general higher-derivatives supergravity couplings

Introduction: Ingredients

- Supersymmetry (SUSY)
- Supergravity (SUGRA)
- higher-derivatives couplings

focus on motivations and some recent 5D results in this talk

Why SUSY/SUGRA?

- Supersymmetry is the only symmetry consistent with special relativity combining fermions and bosons, unifying particles of matter with the mediators of forces!
- If you lift SUSY to a local, gauge, spacetime symmetry $\Longrightarrow$ Supergravity: naturally unify matter with gravity under a simple symmetry principle.

Why SUSY/SUGRA?

Phenomenologically, SUSY $\Longrightarrow$ some of the best proposals for solving many open problems of fundamental physics:

- At the quantum level, renormalisable supersymmetric models posses improved ultraviolet behaviour (cancellation of fermionic and bosonic loops)
- SUSY extensions of the standard model: no quadratic corrections to the Higgs mass; unification of running coupling constants
- low energy SUSY help solving the SM hierarchy problem. (why Electro-Weak scale so low compared to Plank scale)
- SUSY helps solving the instability of the SM vacuum
- Naturally contain dark matter candidates

Why SUSY/SUGRA? String Theory

We ultimately want to find a consistent theory of QUANTUM GRAVITY!

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STRING THEORY

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SUGRA

- Best framework to quantise gravity: STRING THEORY (ST)
  dynamics of fundamental strings instead of point particles
- Consistency of String Theory imply Supersymmetry
- Supergravity is low-energy effective theory of string theory.

SUSY, solvability and mathematical physics

SUSY theories are often easier to solve than non supersymmetric ones. $\Longrightarrow$ large mathematical physics framework for analytic results and guidance in understanding more realistic theories. Examples:

- insight into the strong coupling dynamics and the mechanism of confinement in gauge theories (4D $\mathcal{N}=2$ SUSY and Seiberg-Witten theory): algebraic geometry related to confinement and instantons.

Also Direct connection of SUSY sigma-models and complex geometry:

- 4 supercharges $\Longrightarrow$ Kähler geometry;
- 8 supercharges $\Longrightarrow$ hyper-Kähler geometry;
- 2D sigma-models $\Longrightarrow$ Generalized-Complex-Geometry.

important for pure mathematics and string compactifications

SUSY, solvability: Localization

Last few years: SUSY theories on curved backgrounds
Reason: localize the infinite dimensional path integral of various observables in QFT (Wilson loops, Partition Functions...) to finite dimensional *treatable* integrals once it is possible to:

1) map computation to a curved background (typical for CFT);
2) There is preserved off-shell SUSY on such backgrounds.

off-shell: symmetry that closes without using E.O.M.

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computational techniques to find plethora of
new non-perturbative results in SUSY QFT!

- renewed interest in constructing off-shell SUSY theories on curved backgrounds in $2 \leq \mathrm{D} \leq 6$.
- Classification problem for SUSY backgrounds of off-shell sugra.

SUSY/SUGRA and AdS/CFT

Of great importance in last two decades
maximally supersymmetric 4D $\mathcal{N} = 4$ Yang-Mills theory:

- first example of a finite 4D conformal field theory (CFT)
- predominant role in string theory once the AdS/CFT conjecture formulated in 1997
- AdS/CFT relate dynamics of gravity, or string theory, on a maximally symmetric anti de Sitter background in D+1 dimensions to the dynamics of a D-dimensional CFT on the boundary.
- Original conjecture:4D $\mathcal{N} = 4$ SYM $\leftrightarrow$ Type IIB SUGRA/strings in $AdS_5 \times S^5$ background

AdS/CFT became laboratory used to shed new light on:

- the quantum behavior of black holes;
- the confinement of gauge theories;
- the dynamics of the quark-gluon plasma;
- strongly-coupled condensed matter.

SUGRA, Strings and higher-derivatives

Supergravity is an extension of Einstein GR through Rarita-Schwinger (for spin-3/2 gravitini), described by the Lagrangian

$$L_{SG} = L_{EH} + L_{gravitini} + \dots \,, \quad L_{EH} \propto \mathcal{R} \,, \quad L_{gravitini} \propto \bar{\Psi}_\mu \gamma^{\mu\nu\rho} \mathcal{D}_\nu \Psi_\rho$$

String theory effective action: Einstein GR modified by an infinite series of higher derivative terms in the Riemann tensor (which might strongly affect low energy physics, see cosmology as example)

$$L^{low}_{string} = L_{SG} + \sum [\mathcal{D}^p \mathcal{R}^q_{\dots}]$$

- SUSY extension of higher derivatives terms is poorly understood but
- crucial for low energy string theory
- needed for precision tests of AdS/CFT: crucial for computing higher-order corrections to black-hole entropy within AdS/CFT
- even the simple $\mathcal{R}^2_{\dots}$ case is not fully understood in general

higher-derivatives?

Even before String Theory became predominant, higher derivatives gravity attracted attention for over 50 years:

- renormalization of QFT in curved spacetime requires counterterms containing $\mathcal{R}^2$ [Utiyama & DeWitt (62)]
- Renormalizable (not unitary) $\alpha(\mathcal{W}_{abcd})^2 + \beta(\mathcal{R}_{ab})^2 + \gamma\mathcal{R}^2$, [Stelle (77)]
- $\mathcal{R} + \mathcal{R}^2$ Starobinsky model of inflation [Starobinsky (80)]
- Counterterms for UV divergencies in SUGRA (see open debate on finiteness in $\mathcal{N} = 8$ sugra)

Interestingly, SUGRA models, like no-scale SUGRA and SUSY extensions of the Starobinsky model, are solid inflationary candidates for CMB data.

$\mathcal{R} + \mathcal{R}^2$ SUGRA related to remarkable "Pure de Sitter supergravity", constructed for the first time this year: unifies susy breaking and positive cosmological constant [Bergshoeff, Freedman, Kallosh, Van Proeyen (2015)]

higher-derivatives? 3D Massive (super)gravity

In 3D space-time dimensions it is even possible to construct a unitary higher-derivative theory of (massive) gravity, New-Massive-Gravity (NMG), [Bergshoeff-Hohm-Townsend ('09)] based on

$$\simeq \Lambda + \mathcal{R} + \varepsilon^{abc}\omega_a \mathcal{R}_{ab} + \mathcal{R}^2 + (\mathcal{R}_{ab})^2$$

Λ cosmological constant and $\varepsilon^{abc}\omega_a \mathcal{R}_{ab}$ action for conformal gravity

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toy-model for quantum gravity with finite higher-derivatives series.

Relevant in studying AdS/CFT, black holes microstates...

how higher-derivatives SUGRA?

Once convinced about the interest towards higher derivative supergravity the question is: how to classify higher-derivatives SUGRA?

Best approach would be to possess a formalism that guarantees manifest supersymmetry in a model independent way similar to how tensor calculus in special relativity and electromagnetism or Einstein General Relativity, is fully covariant for global and local Poincaré.

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An off-shell approach to SUGRA, when available, can be used to generate general couplings of supergravity and matter.

Two possibility on the market:

- superconformal tensor calculus [de Wit, Van Holten, Van Proeyen, ...]
  See "Supergravity" book by [Freedman & Van Proeyen]
- superspace approaches
  See [Gates, Grisaru, Roček, Siegel (83)], [Buchbinder, Kuzenko (98)]

how off-shell SUGRA?

Let us here focus on superspace approaches which powerfully work to:

- Provides general and manifestly supersymmetric action principles and even cohomological "superform" techniques to construct and classify SUSY invariants.
- Describe general off-shell supermultiplets with finitely or infinitely many component fields.
- A formalism to generate hyper-Kähler and quaternionic Kähler metrics (see relation with complex geometry)
- Systematic approach to higher derivative actions.

As examples let's review some results for 5D SUGRA
(has various features relevant for various dimensions).
GTM collaboration: D. Butter, S. Kuzenko, U. Lindström, J. Novak, ...

interlude on:

Supersymmetry and Superspace

Supersymmetry (SUSY)

- is a continuos symmetry between particles generated by fermionic spinorial charges Q that anticommute on the momentum P

$$\{Q, Q\} \simeq P_a$$

- As supersymmetry extend the Poincaré group to a supergroup with the fermionic charges Q
- Superspace extend the Minkowski space-time to a supermanifold with extra fermionic Grassmannian coordinates θ

$$\{\theta, \theta\} = [x, \theta] = 0$$

- – Minkowski space-time: $\{x\}$ = Poincaré/SO(1,D-1)
  – Flat superspace: $z = \{x,\ \theta\}$ = Super-Poincaré/SO(1,D-1)
- supersymmetry transformations are generated linearly in superspace as translations of θ coordinates: (γ=gamma-matrices in D-dimensions)

$$\theta' = \theta + \epsilon \ , \quad x' = x - \mathrm{i}(\epsilon\,\gamma\,\theta)$$

Superfields

In formulating supersymmetric field theory and supergravity you then lift fields to Superfields, functions on superspace:

$$\Psi(x,\theta) = C(x) + \theta\xi(x) + \cdots + (\theta)^{\mathcal{N}\times n}B(x) \ ,$$

$\mathcal{N} =$ "number" of supersymmetries, $n =$ dimension spinor representation

- most compact way to organize in simple objects all the component fields of supermultiplets
- keep together physical and auxiliary fields of supermultiplets. auxiliaries (nondynamical) typically needed to have off-shell SUSY
- useful to classify multiplets as differential constraints on superfields.

How $\mathcal{N} = 1$ SUSY in 5D?

- In 5D we have 8 real supercharges

In the case of SUSY with 8 real supercherges a powerful off-shell formalism in superspace used to engineer general matter multiplets:

- Projective superspace (PS) [Karlhede, Lindström, Roček ('84)], useful in studying sigma models and explicit construction of hyper-Kähler and quaternionic-Kähler metrics.
  PS extends superspace with an extra internal bosonic variable in $\mathbb{C}P^1$

use of extended superspace become almost mandatory if one wants to deal with most general off-shell multiplets with ∞ auxiliary fields

How $\mathcal{N} = 1$ SUGRA in 5D?

SUGRA in projective superspace was actually unknown till 2007 when

- Study simplest curved case in 5D: $\mathrm{AdS}^{5|8}$ [Kuzenko, GTM, ('07)]

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- Then 5D $\mathcal{N} = 1$ SUGRA [Kuzenko, GTM, ('07), ('08)]
- Generalize to 4D $\mathcal{N} = 2$ SUGRA [Kuzenko, Lindström, Roček, GTM, ('08), ('09)]
- 2D $\mathcal{N} = (4, 4)$ SUGRA [GTM, ('09)]
- 3D $\mathcal{N} = 3, 4$ SUGRA [Kuzenko, Lindström, GTM, ('11)]
- 6D $\mathcal{N} = 1$ SUGRA [Linch, GTM, ('12)]

How covariant off-shell sugra-matter systems?

- Step One:
  First find a covariant geometrical description of the off-shell conformal SUGRA "Weyl-multiplet" embedded in superspace.
- Step Two:
  Similarly to superconformal tensor calculus, couple conformal supergravity to compensator multiplets and generate Poincaré supergravity + general sugra-matter couplings .
  5D Projective multiplets become crucial for full off-shell

Step two was pushed forward in pioneering works by [Butter ('09),('11)]
for 4D $\mathcal{N} = 1, 2$ conformal supergravity
gauging the entire superconformal algebra in superspace
and 3D $\mathcal{N}$-extended [Butter, Kuzenko, Novak & GTM ('13)]
and 5D $\mathcal{N} = 1$ [Butter, Kuzenko, Novak & GTM ('14)]

Conformal gravity-matter approach to gravity

SUGRA as conformal gravity+matter: reminescent of standard gravity

- Conformal gravity invariance: general coordinate + Weyl-rescaling.
  Equivalently: theory of local conformal group $\{P_a, M_{ab}; \mathbb{D}, K_a\}$.
- Einstein GR $\Longleftrightarrow$ conformal gravity + compensating field φ
- Start with the conformal gravity-matter action

$$S = +\frac{1}{2}\int \mathrm{d}^D x\sqrt{-g}\left(\partial_m\varphi\partial^m\varphi - \frac{D-2}{4(D-1)}\varphi^2\mathcal{R}\right)$$

invariand under

$$\delta g_{mn} = -2\sigma g_{mn}\ , \quad \delta\varphi = \frac{D-2}{2}\sigma\varphi$$

- Upon gauge-fixing $\varphi = \frac{2}{\kappa}\sqrt{\frac{D-1}{D-2}} = const$

one gets the Einstein-Hilbert action

$$S = -\frac{1}{2\kappa^2}\int \mathrm{d}^D x\sqrt{-g}\mathcal{R}$$

5D conformal supergravity in conformal superspace

Step 1:

define a new 5D $\mathcal{N} = 1$ conformal superspace

- Supergeometry based on a manifest gauging of the entire 5D superconformal group $\mathrm{F}^2(4)$
- Superspace realization of the so called 5D superconformal tensor calculus
  (gauging of superconformal group on standard space-time and component fields)

Step 2:

Projective superspace and supergravity-matter coupligs

Take a weight-2 projective superfield $\mathcal{L}^{(2)}(z, cv) = v^2 \mathcal{L}^{(2)}(z, v)$
Associated with $\mathcal{L}^{(2)}(v)$ is the action

$$S[\mathcal{L}^{(2)}] = \frac{1}{2\pi} \oint_\gamma (v, \mathrm{d}v) \int \mathrm{d}^5x \mathrm{d}^8\theta \, E \, C^{(-4)} \mathcal{L}^{(2)}$$

- $S[\mathcal{L}^{(2)}]$ inv. under local superconformal transformations
- Although in general non-trivial, can be reduced to a standard integral over standard fields
  but loose manifest SUSY invariance and become expressions that might fill pages depending on the model...

Models

Now play and construct actions of covariant projective multiplets conformally coupled to sugra. Compensators for dilatation and SU(2)
Poincaré gauged supergravity:

$$\mathcal{L}^{(2)}_{\rm SG} = \frac{1}{4} V\, H^{(2)}_{\rm VM} + G^{(2)} \ln G^{(2)} + \kappa V G^{(2)} \equiv \mathcal{L}^{(2)}_{\rm V} + \mathcal{L}^{(2)}_{\rm L} + \mathcal{L}^{(2)}_{\rm VL}$$

with composite for "Vector Multiplet" $(V \leftrightarrow A_m,\ W \leftrightarrow F_{mn})$

$$H^{ij}_{\rm VM} = {\rm i}(\nabla^{\hat\alpha(i} W)\, \nabla^{j)}_{\hat\alpha} W + \frac{\rm i}{2} W \nabla^{\hat\alpha(i} \nabla^{j)}_{\hat\alpha} W \;, \qquad \nabla^{(i}_{\hat\alpha} H^{jk)}_{\rm VM} = 0$$

once fixed dilatation with $W = 1$, leads to the component lagrangian

$$\mathcal{L}_{\rm SG} = -\frac{1}{2}\mathcal{R} + \frac{8}{3}\kappa^2 + \cdots$$

Einstein-Hilbert plus cosmological constant $\Lambda = -\frac{8}{3}\kappa^2 < 0$
(plus U(1) graviphotons, gravitini etc...)

SUSY Curvature Squared Lagrangians?

curvature squared Lagrangian:

$$\mathcal{L}_{R^2} \propto a\,\mathcal{C}^{\hat{a}\hat{b}\hat{c}\hat{d}}\mathcal{C}_{\hat{a}\hat{b}\hat{c}\hat{d}} + b\,\mathcal{R}^{\hat{a}\hat{b}}\mathcal{R}_{\hat{a}\hat{b}} + c\,\mathcal{R}^2$$

- Weyl tensor: $C_{\hat{a}\hat{b}}{}^{\hat{c}\hat{d}} = \mathcal{R}_{\hat{a}\hat{b}}{}^{\hat{c}\hat{d}} - \frac{4}{3}\delta_{[\hat{a}}{}^{[\hat{c}}\mathcal{R}_{\hat{b}]}{}^{\hat{d}]} + \frac{1}{6}\delta_{[\hat{a}}{}^{[\hat{c}}\delta_{\hat{b}]}{}^{\hat{d}]}\mathcal{R}$ with $\mathcal{R}_{\hat{a}\hat{b}}{}^{\hat{c}\hat{d}}$ component Riemann tensor
- Ricci tensor: $\mathcal{R}_{\hat{a}}{}^{\hat{b}} := \mathcal{R}_{\hat{a}\hat{d}}{}^{\hat{b}\hat{d}}$
- Ricci scalar: $\mathcal{R} := \mathcal{R}_{\hat{a}}{}^{\hat{a}}$

Weyl^2

Analogue of a “vector” compensator V, $W \neq 0$

$$\mathcal{L}^{(2)}_{\rm Weyl} = V H^{(2)}_{\rm Weyl} = V H^{ij}_{\rm Weyl} v_i v_j$$

with composite built only out of the gravitational Weyl multiplet

$$H^{ij}_{\rm Weyl} := -\frac{\rm i}{2} W^{\hat\alpha\hat\beta\hat\gamma\, i} W_{\hat\alpha\hat\beta\hat\gamma}{}^{j} + \frac{3{\rm i}}{2} W^{\hat\alpha\hat\beta} X_{\hat\alpha\hat\beta}{}^{ij} - \frac{3{\rm i}}{4} X^{\hat\alpha i} X^{j}_{\hat\alpha} \,, \qquad \nabla^{(i}_{\hat\alpha} H^{jk)}_{\rm Weyl} = 0$$

where

$$W_{\hat\alpha\hat\beta\hat\gamma}{}^{k} := \nabla^{k}_{(\hat\alpha} W_{\hat\beta\hat\gamma)} \,, \qquad X^{i}_{\hat\alpha} := \frac{2}{5} \nabla^{\hat\beta i} W_{\hat\beta\hat\alpha} \,, \qquad X_{\hat\alpha\hat\beta}{}^{ij} := -\frac{1}{4} \nabla^{\hat\gamma(i} \nabla^{j)}_{\hat\gamma} W_{\hat\alpha\hat\beta}$$

Reduce to components and $W = 1$ gauge leads to

$$\mathcal{L}_{\rm Weyl} \propto \mathcal{C}^{\hat a\hat b\hat c\hat d} \mathcal{C}_{\hat a\hat b\hat c\hat d} + \cdots$$

- In components first construction in [Hanaki, Ohashi, Tachikawa ('06)]

scalar^2

$$\mathcal{L}^{(2)}_{\rm scal} = V H^{(2)}_{\rm VM}[\mathbb{W}]$$

where composite linear multiplet

$$H^{ij}_{\rm VM}[\mathbb{W}] = {\rm i}(\nabla^{\hat\alpha(i}\mathbb{W})\,\nabla^{j)}_{\hat\alpha}\mathbb{W} + \frac{\rm i}{2}\mathbb{W}\nabla^{\hat\alpha(i}\nabla^{j)}_{\hat\alpha}\mathbb{W}$$

with composite vector multiplet field strength

$$\mathbb{W} = \frac{\rm i}{16} G\nabla^{\hat\alpha i}\nabla^{j}_{\hat\alpha}\left(\frac{G_{ij}}{G^2}\right)\,, \qquad \nabla^{(i}_{\hat\alpha}\nabla^{j)}_{\hat\beta}\mathbb{W} = \frac{1}{4}\varepsilon_{\hat\alpha\hat\beta}\nabla^{\hat\gamma(i}\nabla^{j)}_{\hat\gamma}\mathbb{W}$$

Reduce to components and $W = 1$ gauge leads to

$$\mathcal{L}_{\rm scal} \propto \mathcal{R}^2 + \cdots$$

- In components [Ozkan, Pang ('13)]

Ricci^2

$$\mathcal{L}^{(2)}_{\rm Ric} = -VG^{(2)}[\log W]$$

where the linear multiplet is function of the vector multiplet field strength

$$G^{(2)}[\log W] = -\frac{\rm i}{8}\Delta^{(4)}\nabla^{(-2)}\log W$$

Reduce to components and $W = 1$ gauge leads to

$$\mathcal{L}_{\rm Ricci} \propto \mathcal{R}^{\hat a\hat b}\mathcal{R}_{\hat a\hat b} + \cdots$$

- First time constructed for the standard Weyl Multiplet in [Butter, Kuzenko, Novak & GTM ('14)]

Was believed not to exist, and constructed only in "dilaton-Weyl multiplet" [Bergshoeff, Rosseel, Sezgin ('11)], [Ozkan, Pang ('13)]

Conclusion

In [Butter, Kuzenko, Novak & GTM (14)]

- developed a new off-shell formulation for five-dimensional (5D) conformal supergravity obtained by gauging the 5D superconformal algebra in superspace.
- Using the conformal superspace approach, we showed how to reproduce practically all off-shell constructions derived so far in literature, including the supersymmetric extensions of R^2 terms, thus demonstrating the power of our formulation.
- Furthermore, we construct for the first time a supersymmetric completion of the Ricci tensor squared term using the standard Weyl multiplet coupled to an off-shell vector multiplet.

Higher derivatives in 3D and other

- The R^2 case proves that we have a powerful "top-down" approach to construct higher derivative invariants.
- the new approach open classification and construction of new higher derivative invariants: R^n $(n > 2)$; $\mathcal{D}^m R^p$...
- In [Kuzenko, Novak & GTM (15)] we classified all the R^2 terms for $\mathcal{N} = 1, 2, 3$ supergravity in 3D, $\mathcal{N} = 3$ was new.
  Continuation of previous results [GTM, ... (11)–(15)] where for the first time the 3D $\mathcal{N} = 3, 4, 5, 6$ conformal supergravity actions were constructed. Hence construction of general $\mathcal{N} = 1, 2, 3$ massive 3D supergravity
- How about R^3 in 6D? This is of importance in studying renormalization group flows and dilaton effective action of 6D QFT, CFT and AdS/CFT, String Theory, M-Theory, ...