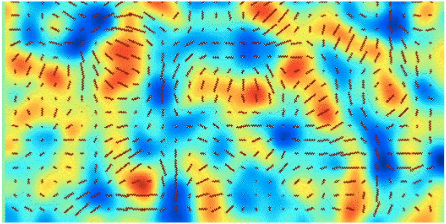
SIMULATIONS OF THE UNIVERSE IN GENERAL RELATIVITY

The N-Body Gauge



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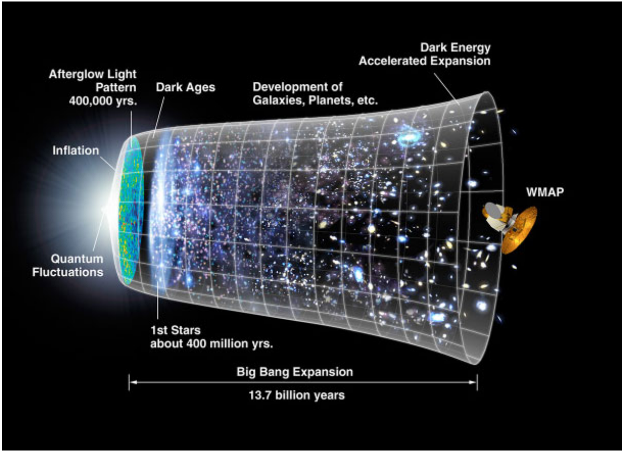
today

Motivation

A wealth of information about the origin of our Universe and the laws of nature is encoded in the cosmic large scale structure



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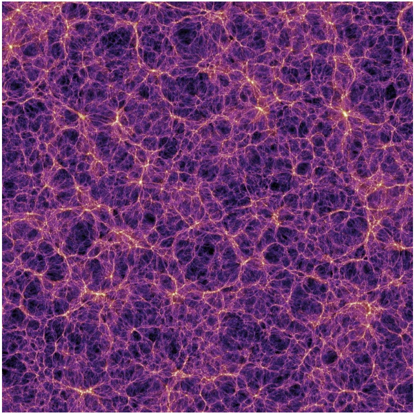
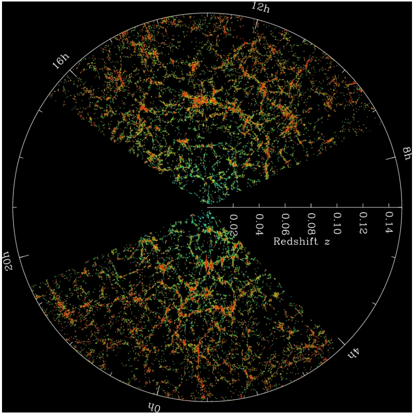
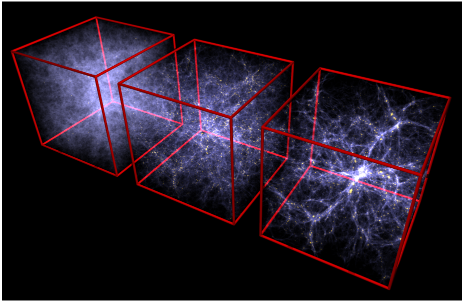




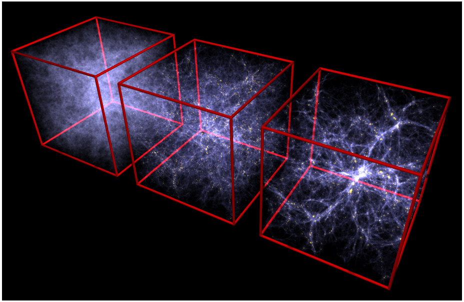
N-Body Simulations



Approximations

- Universe is filled by only dark matter
- dark matter distribution is represented by few macroscopic particles
- evolve motion of these particles in Newtonian gravity

N-Body Simulations



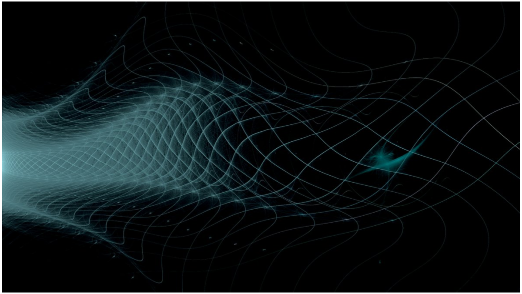
Steps

1. compute densities ρ based on particle positions
2. compute the gravitational potential $\nabla^2\Phi = -4\pi G\rho a^2$
3. move particles $\left(\frac{\partial}{\partial\eta} + \frac{\dot{a}}{a}\right) v = \nabla\Phi$

General Relativity

Complications

- need to simulate full curvature of Space-Time



Scalar perturbations:

Time potential	A
Space potentials	$H_{\mathrm{L}}, H_{\mathrm{T}}$
Shift vector	B

General Covariance

Gauge freedom

The equations of motion are covariant under general coordinate transformations

- choose a gauge in which the problem resembles the simple Newtonian picture
  - ➔ computation of the density in n-body: $H_{\mathrm{L}} = 0$
  - ➔ classical Poisson equation if $B = v$
- this completely fixes the n-body gauge, with H_{T} non-vanishing and locally curling up the three dimensional space

A lengthy computation in general relativity shows:

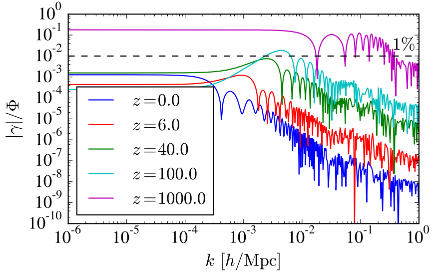
- relativistic corrections appear as an extra force acting on the particles $\left(\frac{\partial}{\partial\eta} + \frac{\dot{a}}{a}\right) v = \nabla\Phi + \nabla\gamma$, with $\Phi = \frac{\nabla^2}{3} H_{\mathrm{T}} - \frac{\dot{a}}{a}\left(B - \dot{H}_{\mathrm{T}}\right)$
- at leading order in the metric potentials in a pure dark matter Universe this extra force vanishes

Relativistic N-Body Simulation

A Newtonian n-body computes the relativistic evolution if

- the initial conditions is provided in the n-body gauge
- the output is interpreted in n-body gauge
- the coordinates of the simulation are understood as n-body gauge coordinates

- we keep the numerical efficiency of simple Newtonian codes
- we can quantify the size of the relativistic forces due to other components such as radiation



Summary

- the next generation of galaxy surveys is sensitive to effects beyond the Newtonian limit
  - ➔ requires simulations with relativity and more realistic cosmologies
- we specify a gauge uniquely suited for relativistic n-body simulations, keeping the numerical efficiency of Newtonian simulations
- we have modified the CLASS code to provide n-body gauge initial conditions
- we evaluate the impact of radiation in the early Universe

Outlook

- study relativistic forces beyond leading order
- include radiation correction in n-body codes
- gauge independent predictions for observables