Evaluation of absorbed fractions for beta-gamma radionuclides in ellipsoidal volumes of soft tissue through Geant4

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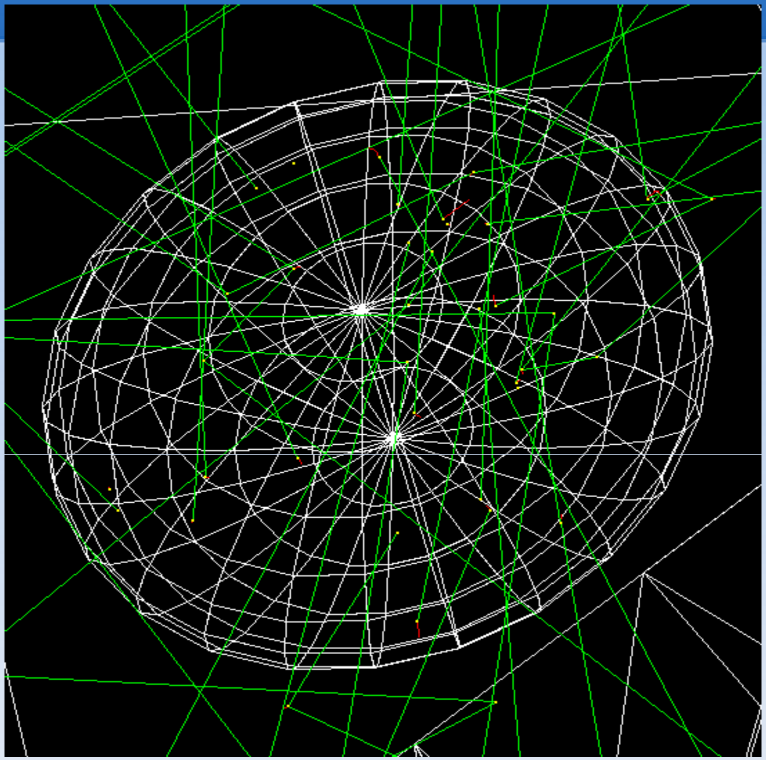
Aim

Beta-emitting radionuclides are employed for internal radiotherapy of tumors, hyperthyroidism, and other diseases.
Beta radioactivity is often accompained by gamma emission.
Following the MIRD approach, the average dose to a target tissue can be calculated as:

$$\overline{D} = D_\beta + D_\gamma = \frac{\tilde{A}\left(\Delta_\beta \varphi_\beta + \sum_i n_i E_i \varphi_{\gamma_i} \right)}{m}$$

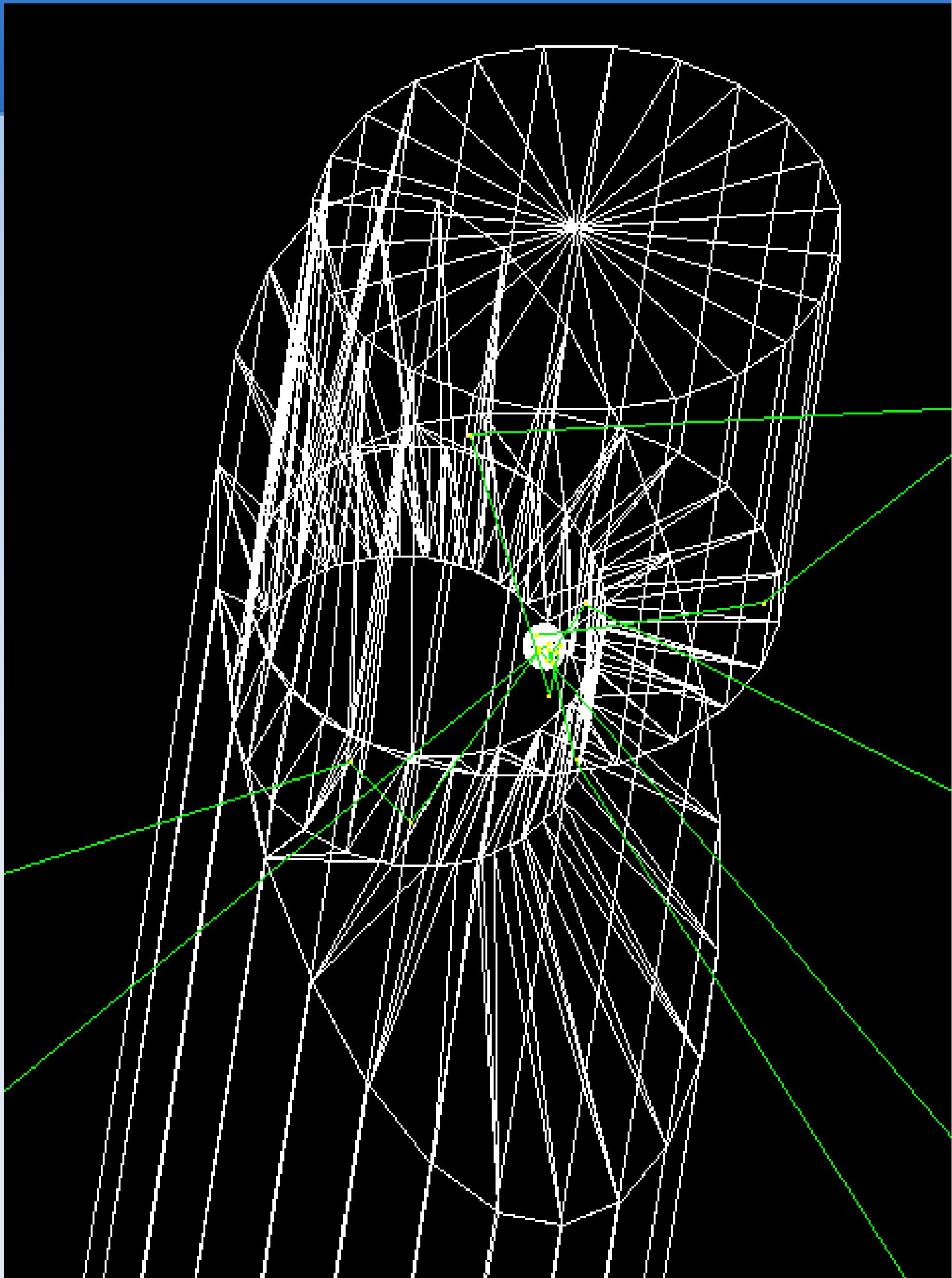
We studied the beta and gamma absorbed fractions ϕ for different beta spectra and photon energies in ellipsoidal volumes of soft tissue. Ellipsoids are the most frequently used geometries to model small uptaking target tissues, and also as approximate models of larger organs or tissues (kidneys, spleen...).

Geometry and materials



Oblate, prolate and scalene ellipsoids, and spheres, of soft tissue.
Density 1.04 g/cm^3
Composition: Soft tissue ICRP Pub. 89

Simplified antropomorphic phantom for scatter simulation



Ellipsoidal configurations

15 volumes from 0.001 cm^3 to 200 cm^3, each one with 10 geometries:

- 1 sphere of surface S_0
- 4 oblate ($a = b > c$) and 4 prolate ($a = b < c$) ellipsoids, with surfaces 1.25, 1.50, 1.75 times and twice S_0.
- 1 scalene ellipsoid with $c = 2b = 4a$ and $S = 1.26\ S_0$

c/a ratio for the oblate and prolate ellipsoids.

S/S_0	c/a oblates	c/a prolates
1.25	0.34	3.74
1.50	0.23	6.92
1.75	0.17	11.24
2.00	0.14	16.94

Definition of "Generalized Radius":

$$\rho = 3\frac{V}{S}$$

$$V = \frac{4}{3}\pi abc \qquad S = 4\pi\left(\frac{(ab)^p + (bc)^p + (ac)^p}{3}\right)^{1/p}$$

Radioactive sources

IMPLEMENTATION OF BETA SPECTRUM in PrimaryGeneratorAction.cc

Radionuclide	$T_{1/2}$ (days)	E_β (keV)	E_{mono} (keV)
^{199}Au	3.14	86.5	32.7
^{177}Lu	6.71	132.9	13.9
^{131}I	8.02	181.5	9.6
^{153}Sm	1.95	226.3	43.9
^{186}Re	3.78	326.1	14.7
^{90}Y	2.67	948.8	0.2

```
// Beta energy distribution

G4double epart;
G4int i;
G4double en[20], probList[20];
G4int nbins=20;

// Input of beta spectrum
std::ifstream cum("Iodio131.dat", std::ios::in);
for (i=0; i<=19; i++) cum >> en[i] >> probList[i];

G4RandGeneral GenDist(probList,nbins);
epart = 806.9*GenDist.shoot();  // shoots values using the engine

particleGun->SetParticleEnergy(epart*keV);
```

+ mono-energetic electrons (Auger or CE)
+ gamma rays

Decay data from Stabin and da Luz *Health Phys* **83** 471 (2002)

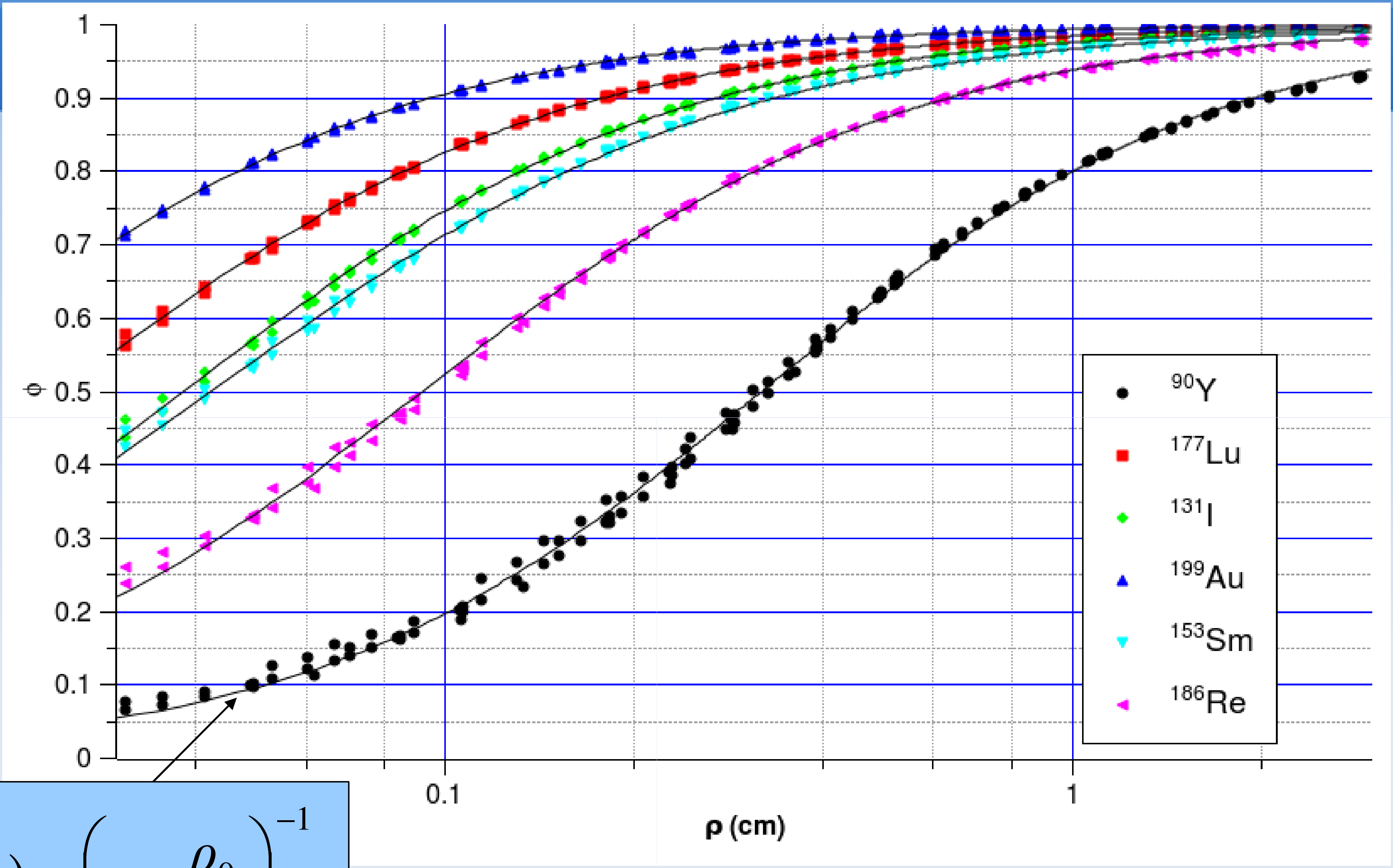
Validation for electrons

Table 4. Absorbed fractions for spheres: monoenergetic electrons, ^{90}Y and ^{131}I. Comparison with Siegel and Stabin (1994).

V (cm^3)	ϕ(0.2 MeV)	$\sigma_{0.2}$	ϕ(1 MeV)	σ_1	$\phi(^{90}$Y)	σ_Y	$\phi(^{131}$I)	σ_I
0.1	0.93	0.02	0.48	0.01	0.45	−0.01	0.90	−0.01
0.5	0.96	0.01	0.67	0.02	0.63	0	0.94	0
1	0.97	0.01	0.73	0.02	0.69	0.01	0.96	−0.01
2	0.98	0	0.79	0.01	0.75	0.01	0.97	−0.01
10	0.99	0	0.87	0.01	0.85	0.01	0.98	0
20	0.99	0	0.90	0	0.88	0.01	0.98	0
100	0.99	0	0.94	0	0.93	0	0.99	0

Siegel J A and Stabin M G (1994) Absorbed fractions for electrons and beta particles in spheres of various sizes *J. Nucl. Med.* ***35*** *152–6*

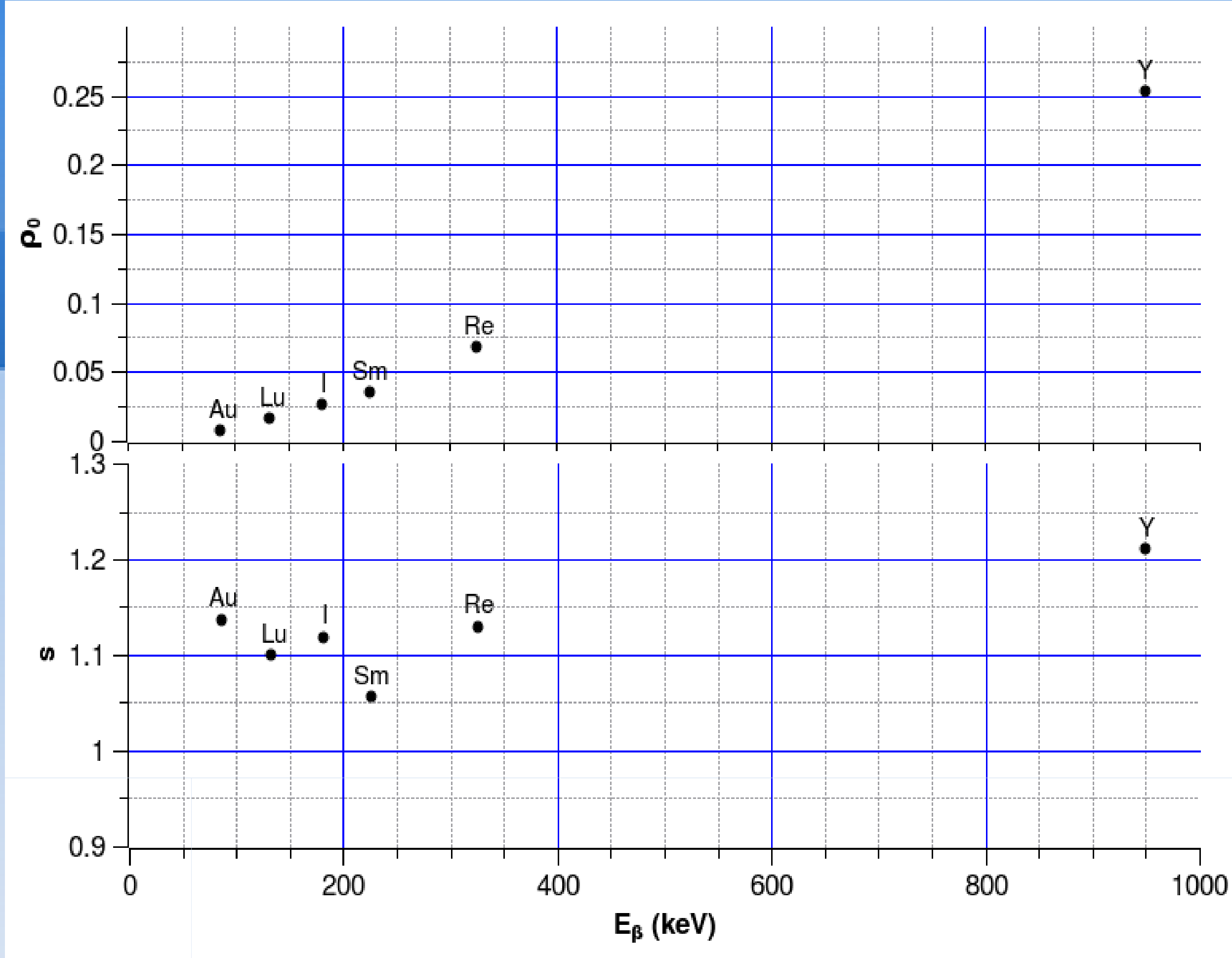
Beta absorbed fractions



$$\varphi(\rho) = \left(1 + \frac{\rho_0}{\rho^s}\right)^{-1}$$

2 fit parameters: ρ_0 and s

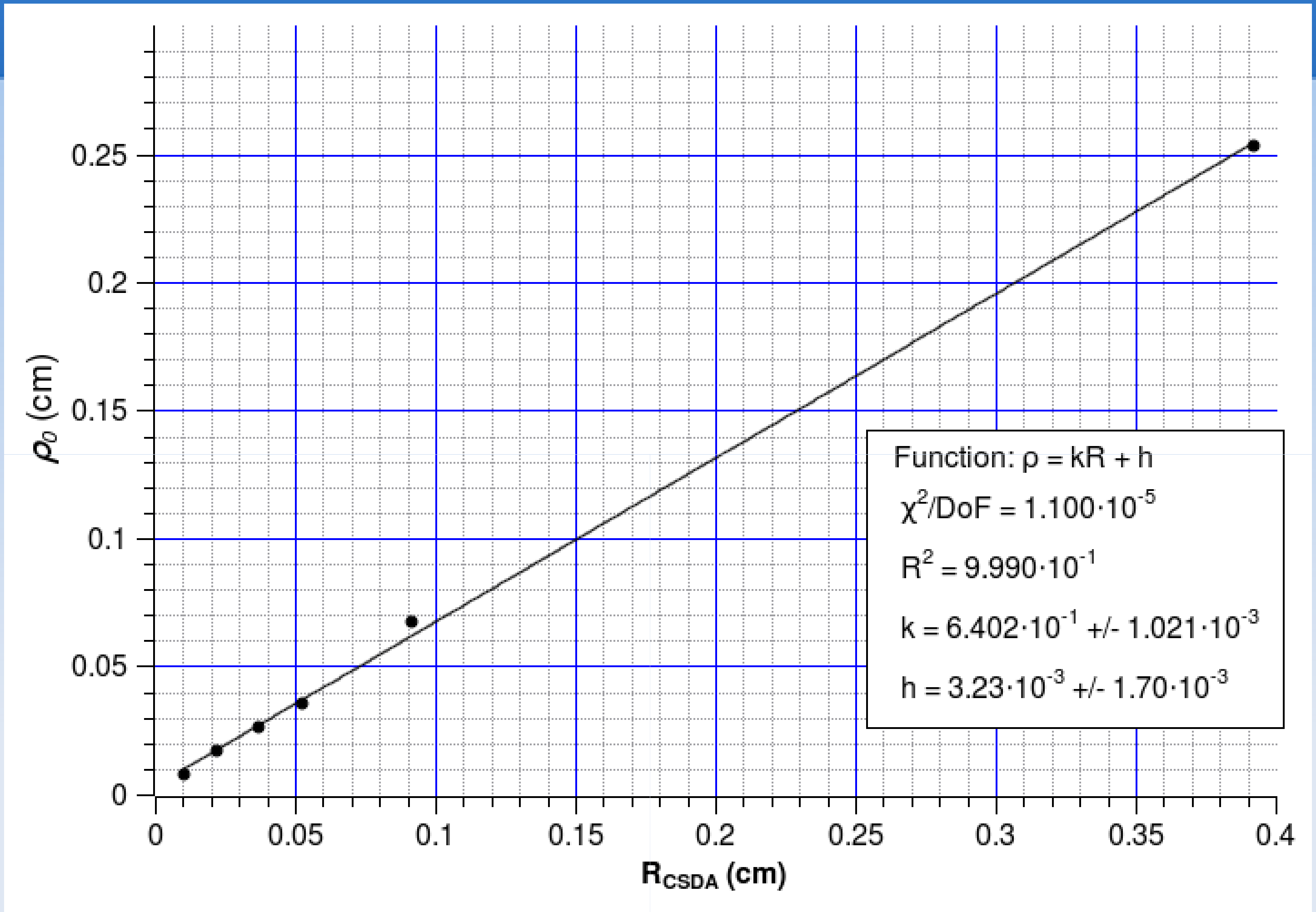
Fit parameters



$$\varphi(\rho) = \left(1 + \frac{\rho_0}{\rho^s}\right)^{-1}$$

Radionuclide	ρ_0	s	χ^2/DoF
^{199}Au	$7.72\ 10^{-3} \pm\ 9.0\ 10^{-5}$	$1.137 \pm\ 0.004$	$4.1\ 10^{-6}$
^{177}Lu	$1.70\ 10^{-2} \pm\ 1.1\ 10^{-4}$	$1.099 \pm\ 0.002$	$3.8\ 10^{-6}$
^{131}I	$2.60\ 10^{-2} \pm\ 1.9\ 10^{-4}$	$1.120 \pm\ 0.003$	$9.7\ 10^{-6}$
^{153}Sm	$3.54\ 10^{-2} \pm\ 3.2\ 10^{-4}$	$1.059 \pm\ 0.004$	$2.0\ 10^{-5}$
^{186}Re	$6.74\ 10^{-2} \pm\ 6.3\ 10^{-4}$	$1.133 \pm\ 0.004$	$4.7\ 10^{-5}$
^{90}Y	$2.53\ 10^{-1} \pm\ 1.7\ 10^{3}$	$1.211 \pm\ 0.005$	$8.8\ 10^{-5}$

Generalization for other nuclides



R_{CSDA} is the CSDA range in cm for electrons with $E_e = \langle E_\beta \rangle$

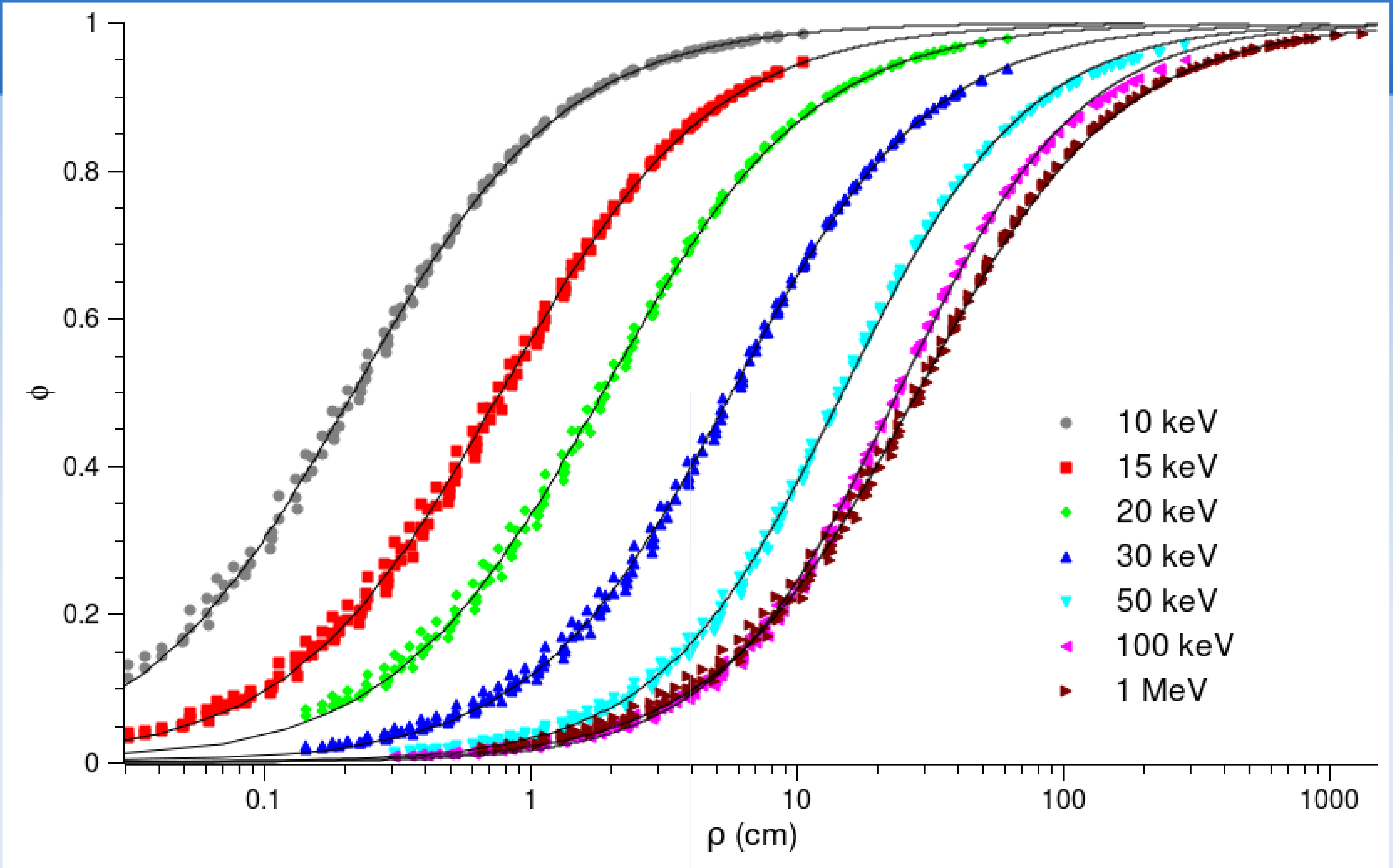
Validation for photons

Table 2. Absorbed fractions for photons uniformly distributed in spheres of unit density tissue: comparison with Stabin and Konijnenberg (2000).

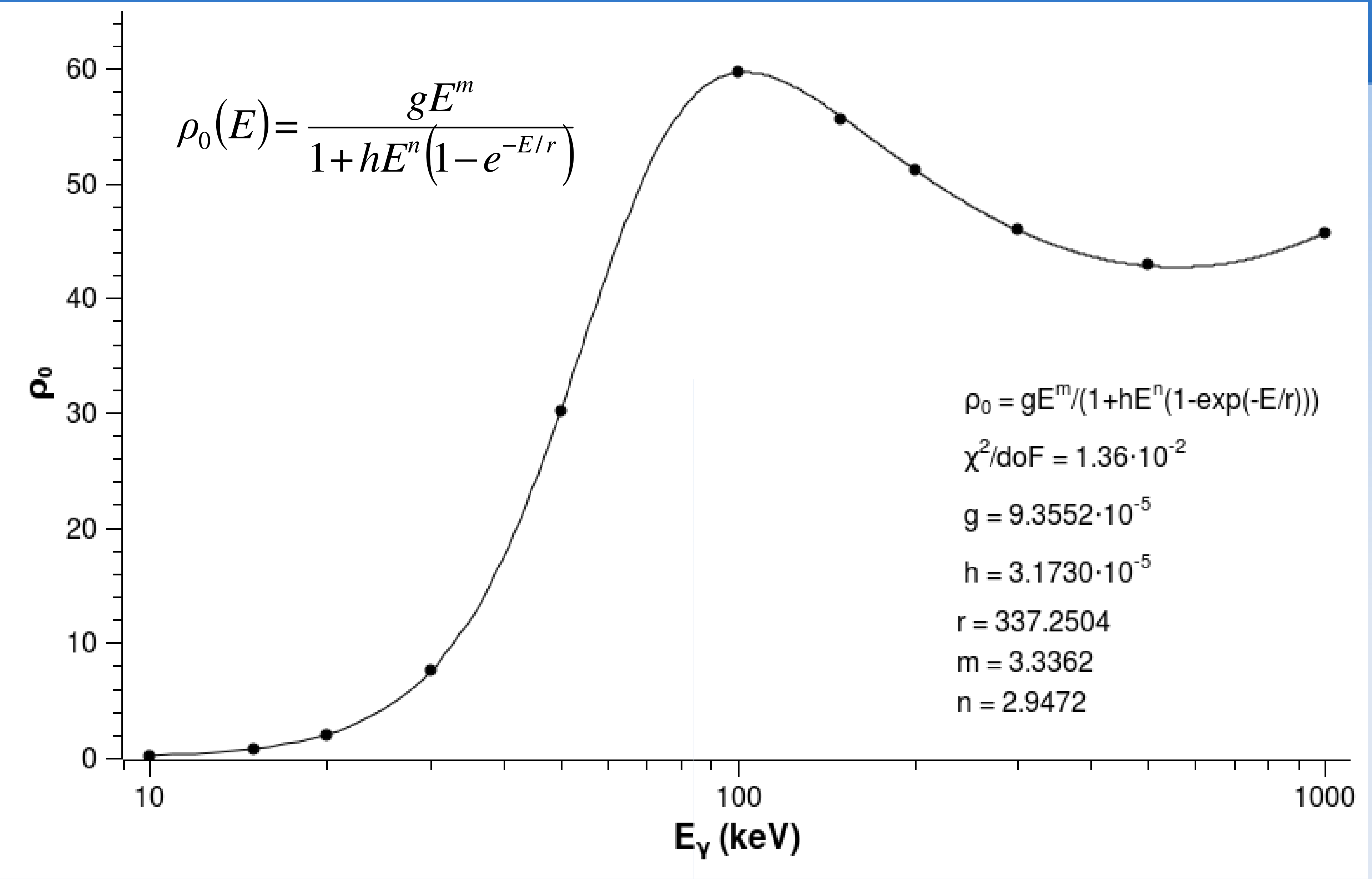
Vol. (cm^3)	R (cm)	ϕ_γ 80 (keV)	σ_{80}	ϕ_γ 140 (keV)	σ_{140}	ϕ_γ 364 (keV)	σ_{364}	ϕ_γ 662 (keV)	σ_{662}
1	0.62	0.012	0	0.013	0	0.015	0	0.014	0
2	0.78	0.016	0	0.016	0	0.019	0	0.018	0
6	1.13	0.023	0	0.024	0	0.027	0	0.026	0
10	1.34	0.028	0	0.028	0	0.032	0	0.031	0
20	1.68	0.037	0	0.036	0	0.041	−0.001	0.040	−0.001
60	2.43	0.056	−0.001	0.053	0	0.059	−0.001	0.057	−0.001

Stabin M G and Konijnenberg MW (2000) Re-evaluation of absorbed fractions for photons and electrons in spheres of various sizes *J. Nucl. Med.* ***41*** *149–60*

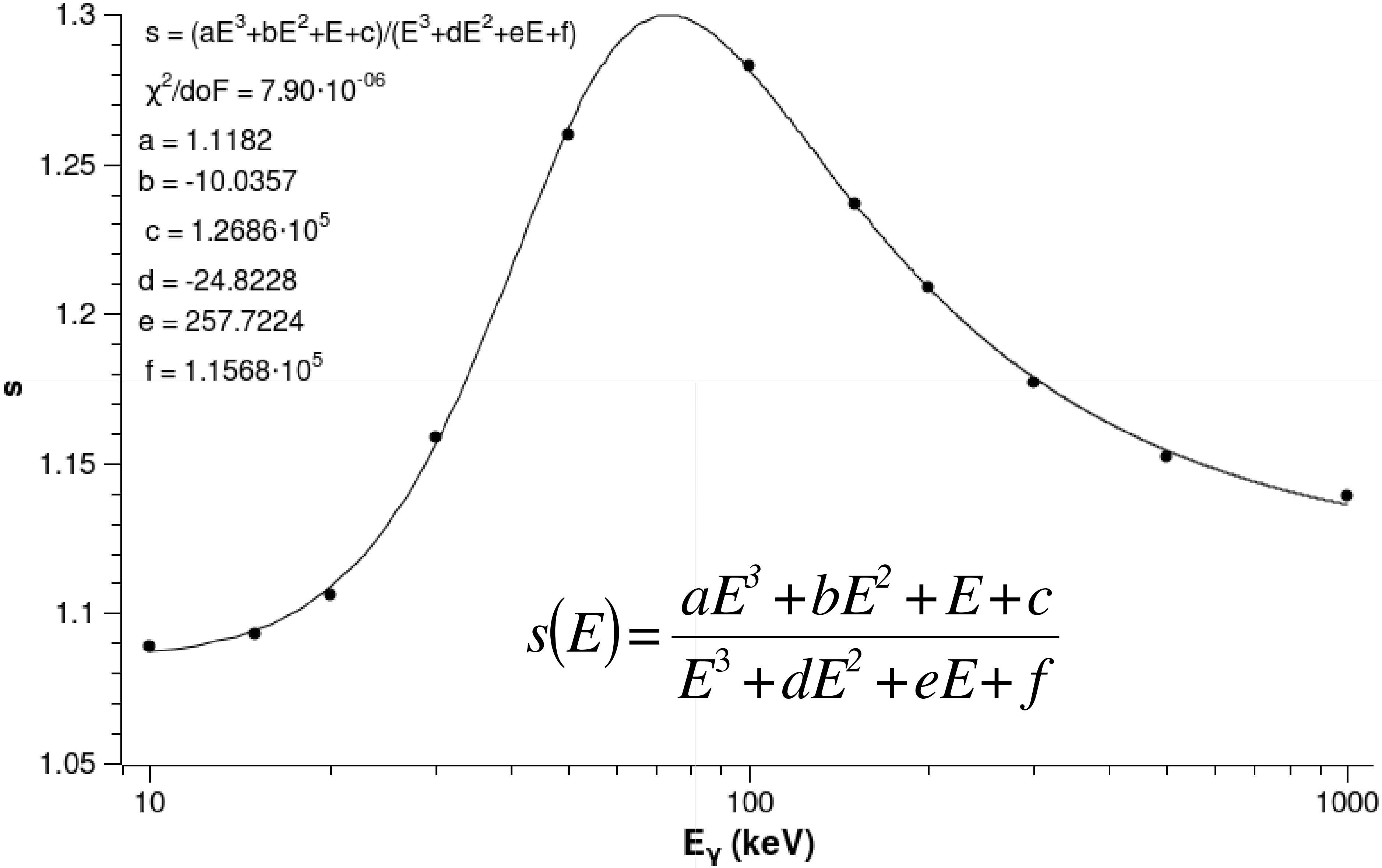
Absorbed fractions for photons



ρ_0 for 10-1000 keV photons



s for 10-1000 keV photons



$$s(E) = \frac{aE^3 + bE^2 + E + c}{E^3 + dE^2 + eE + f}$$

Dose calculation

$$\overline{D} = D_\beta + D_\gamma = \frac{\tilde{A}\left(\Delta_\beta \varphi_\beta + \sum_i n_i E_i \varphi_{\gamma_i}\right)}{m}$$

From our "model":

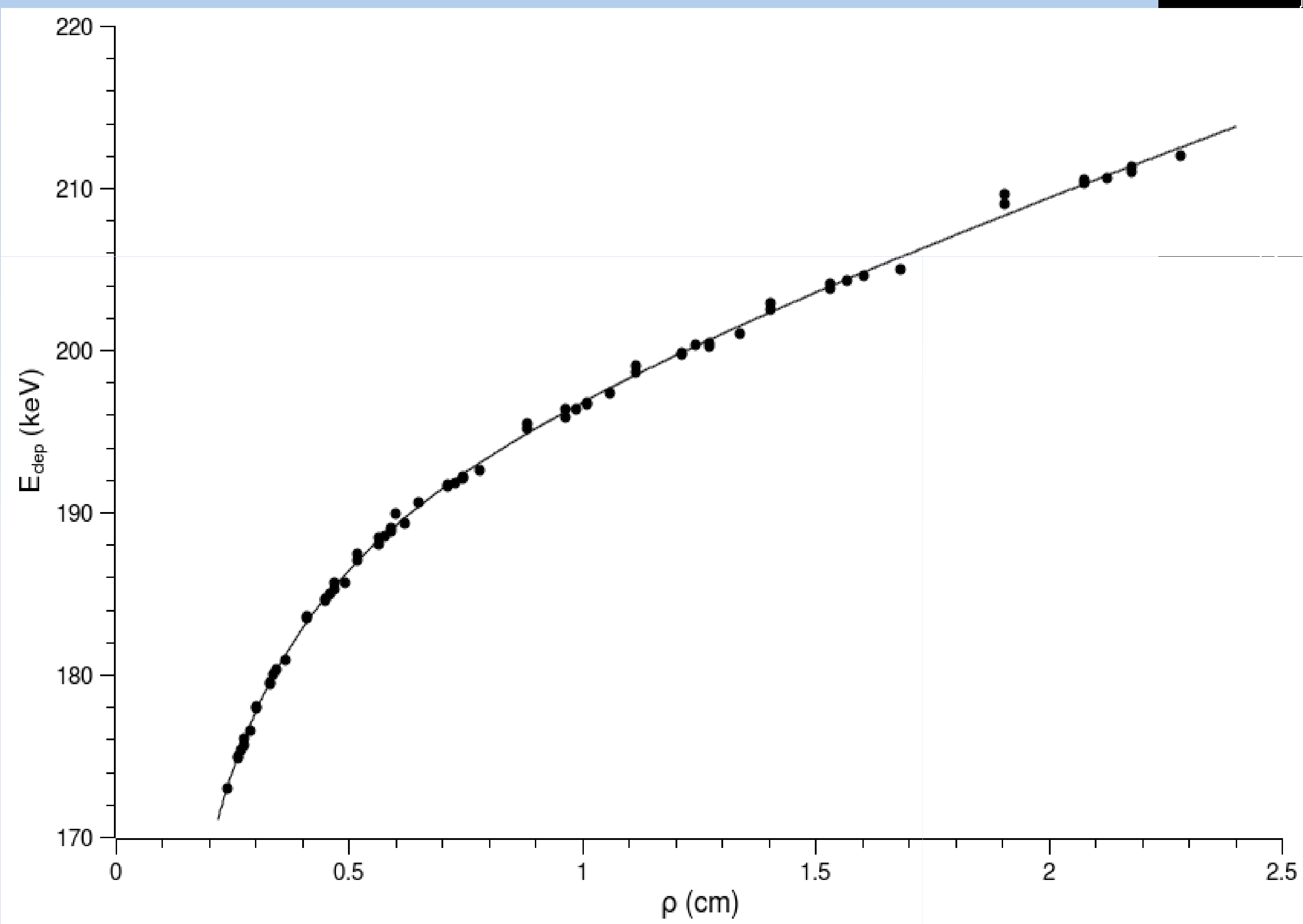
$\rho_{0,\beta}$ and s_β for betas

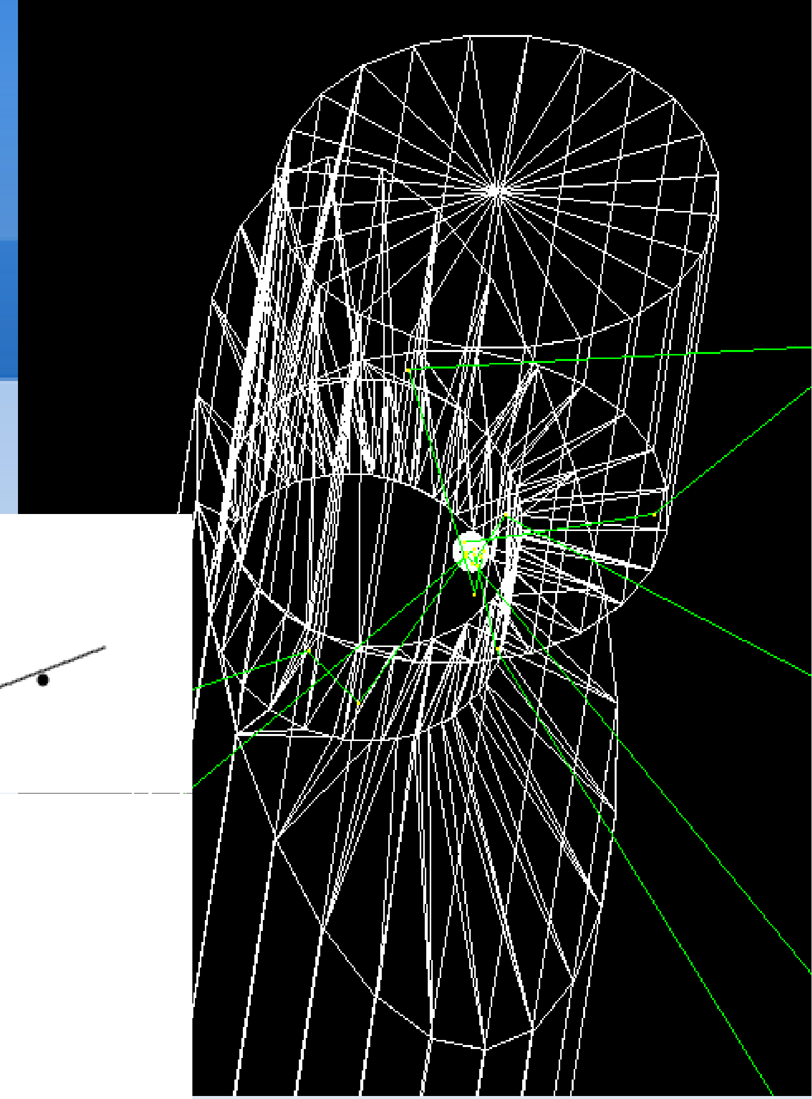
ρ_{0,γ_i} and s_{γ_i} for gammas

$$\overline{D} = \frac{\tilde{A}}{m} E_{dep} = \frac{\tilde{A}}{m}\left[\Delta_\beta \left(1 + \frac{\rho_{0,\beta}}{\rho^{s_\beta}}\right)^{-1} + \sum_i n_i E_i \left(1 + \frac{\rho_{0,\gamma_i}}{\rho^{s_{\gamma_i}}}\right)^{-1}\right]$$

a, b, c semiaxes → V and S → $3\frac{V}{S} = \rho$

Energy deposition for ^{131}I





Summary

- We demonstrated that the beta and gamma absorbed fractions in ellipsoidal volumes are functions of the "generalized radius".
- We introduced a bi-parametric function allowing to fit $\phi(\rho)$.
- Concerning betas, we obtained the parameters ρ_0 and *s* for six radionuclides, and we found a linear relationship between range and ρ_0, useful for other radionuclides.
- Concerning gammas, we chose two parametric forms for ρ_0(E) and *s*(E) valid in the broad 10-1000 keV energy interval.
- Once known the beta and gamma parameters, it is possible to calculate the average radiation absorbed dose to any ellipsoidal volume of practical interest.

Conclusions

Such an approach can be usefully applied for the evaluation of average absorbed dose to small intensely uptaking regions surrounded by scarcely uptaking tissue. In fact, in case of millimeter-scale ellipsoidal regions, an accurate volumetric evaluation can be carried out by means of MRI, CT, or ultrasonography, while the cumulated activity can be measured by scintigraphic imaging or emission tomography.

The method can be also useful for a simple, although approximate, evaluation of average dose to organs whose shape can be described by an ellipsoid; in such a case please remember that only the self-dose, i.e. the dose imparted by the radioactivity contained into the organ itself, with the assumption of homogeneous volume distribution, can be calculated.

The contribution due to the external radioactivity can be added following the MIRD approach for small as well as large volumes, while the only way to account for uptake inhomogeneities is to develop a direct Monte Carlo calculation, or to apply dose point-kernel methods, or to use pre-calculated voxel S-values

References

E. Amato, D. Lizio and S. Baldari
Absorbed fractions in ellipsoidal volumes for β^- radionuclides employed in internal radiotherapy
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Absorbed fractions for photons in ellipsoidal volumes
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