Role of electrostatic charges in the calculation and measurement of AC losses in superconducting coils

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Outline

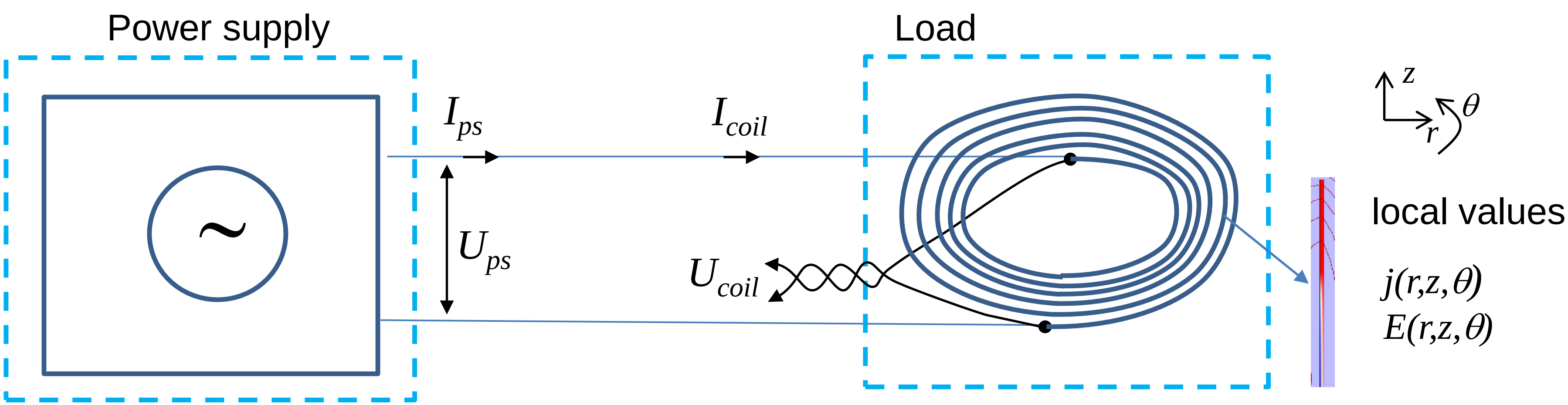
1) Two ways of AC loss determination

2) Computation of coil voltage – theory

3) Computation of coil voltage – virtual experiments and verification by real experiments

4) Conclusions

Two ways of AC loss computation in a SC coil



macroscopic point of view:

microscopic point of view

$$P_{ps}(t) = I_{ps}(t)U_{ps}(t) \quad \cong \quad \boxed{P_{coil}(t) = I_{coil}(t)U_{coil}(t)}$$

$$P_L(r,z,\theta) = j(r,z,\theta)E(r,z,\theta)$$

$$Q_{coil} = \int_0^T I_{coil}(t)U_{coil}(t)\mathrm{d}t \quad = \quad Q_L = \int_0^T \mathrm{d}t \int_V j(r,z,\theta)E(r,z,\theta)\mathrm{d}V$$

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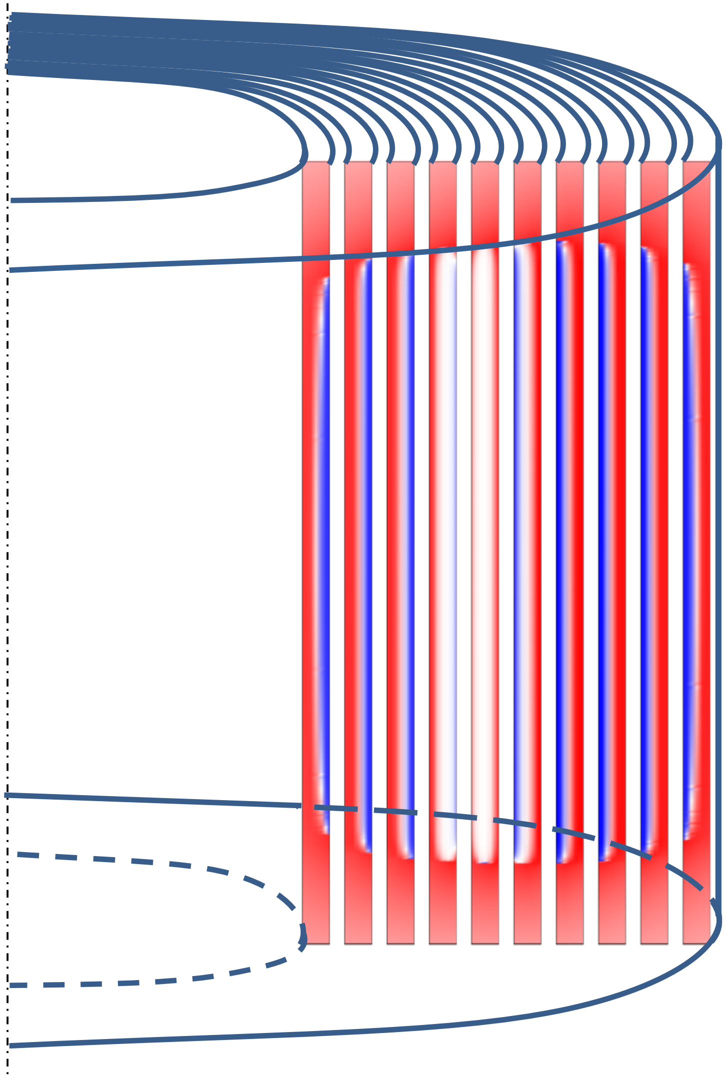
Expression for the local electric field

implementation of Maxwell equations (at low frequency limit)



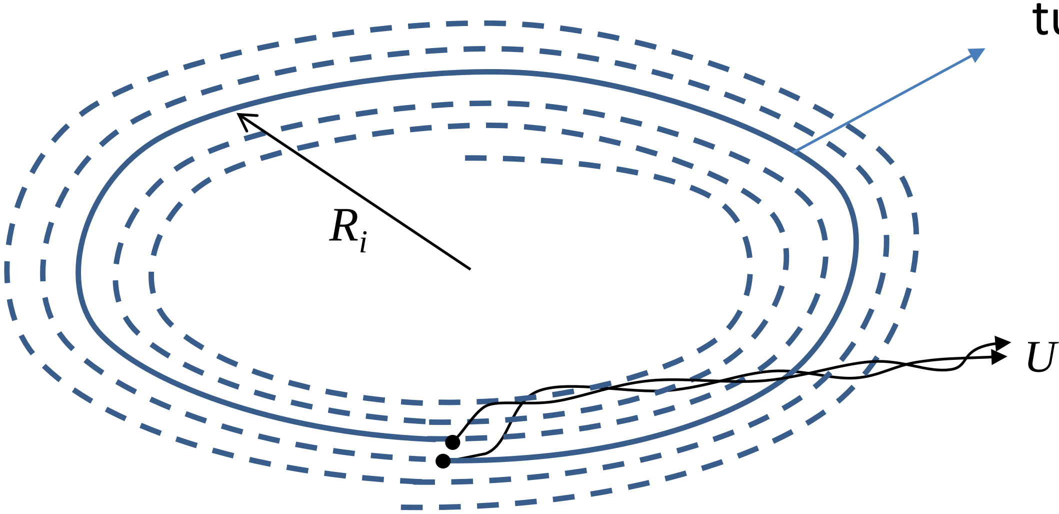
Electric field (force acting on electric charges $\vec{F} = q\vec{E}$)

$$\vec{E}(r) = -\frac{\partial \vec{A}(r)}{\partial t} - \nabla \varphi$$

Changing magnetic field („induced emf“)

Electrostatic potential created by electric charges

Evaluation of voltage measured on a coil turn



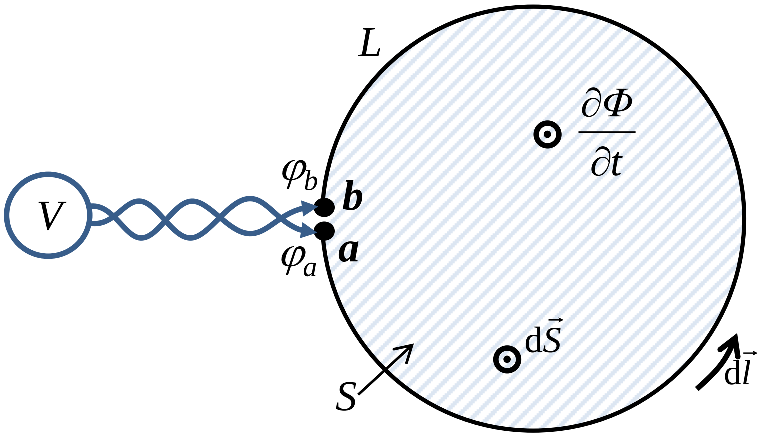
turn i : $L_i \cong 2\pi R_i$

$$U_E = \int_{L_i} \vec{E} \cdot \mathrm{d}\vec{l}$$

$$U_A = -\int_{L_i} \frac{\partial \vec{A}}{\partial t} \cdot \mathrm{d}\vec{l}$$

$$\boxed{U_\varphi = -\int_{L_i} \nabla \varphi \cdot \mathrm{d}\vec{l}}$$

Example 1: voltage induced in an open turn



$$\int_a^b \vec{E}\cdot \mathrm{d}\vec{l} = \int_a^b \left(-\frac{\partial \vec{A}}{\partial t} - \nabla\varphi\right)\cdot \mathrm{d}\vec{l}$$

$$\cong \int_S -\nabla\times\frac{\partial \vec{A}}{\partial t}\cdot \mathrm{d}\vec{S} + \int_{\mathrm{a}}^{\mathrm{b}} -\nabla\varphi\cdot \mathrm{d}\vec{l}$$

$$\int_a^b \vec{E}\cdot \mathrm{d}\vec{l} = -\frac{\partial \Phi}{\partial t} + \varphi(a) - \varphi(b)$$

voltage measured by the voltmeter $U = \varphi(a) - \varphi(b)$

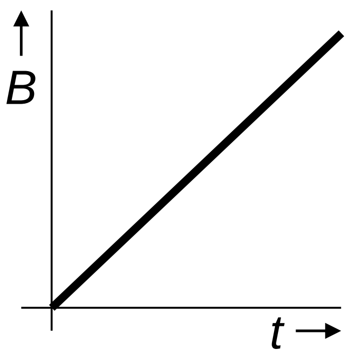
there is no current flowing in the turn, therefore $\vec{E} = 0$

as result $$\boxed{\varphi(a) - \varphi(b) = \frac{\partial \Phi}{\partial t}}$$

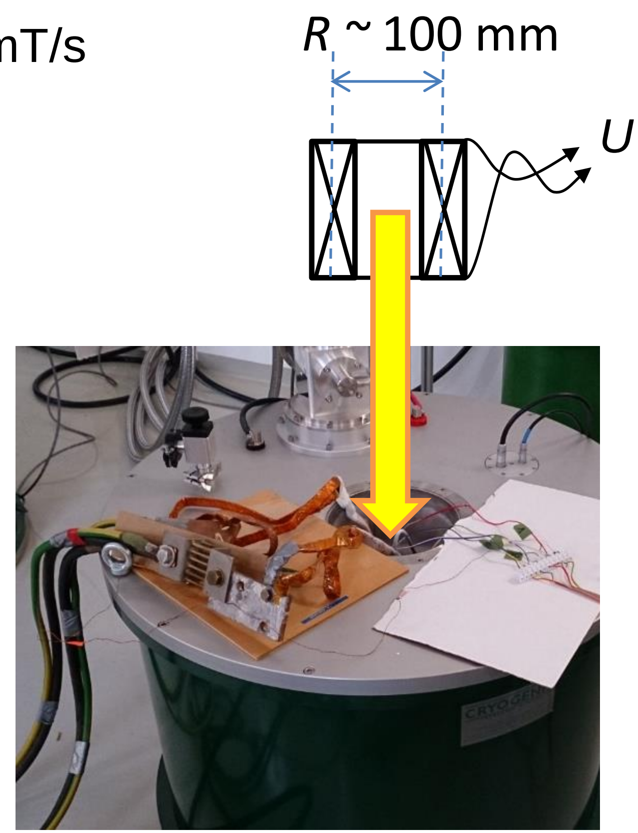
sign of the measured voltage is opposite to the induced emf

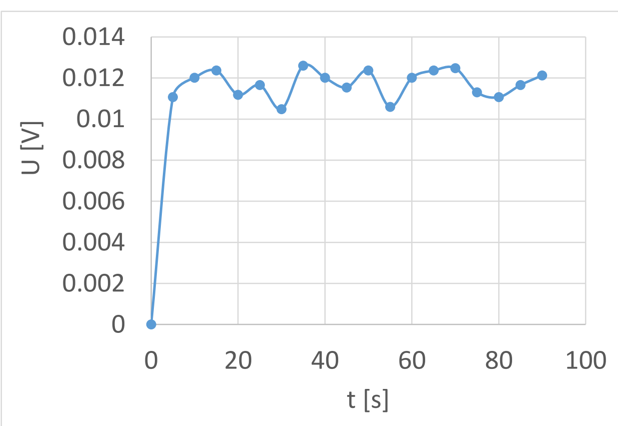
Example 1: voltage induced in an open turn

experimental verification:
SC magnet ϕ 200 mm bore at ambient temperature, with known field polarity and pick-up coil (Cu wire) with 450 turns (known sense of winding)

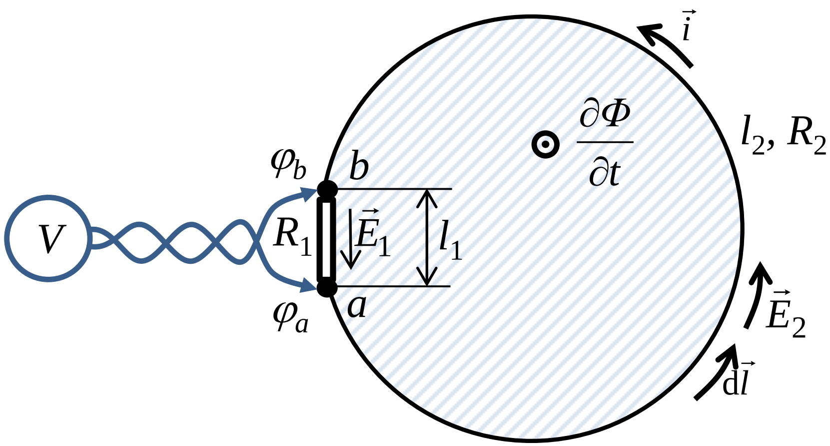


dB/dt ~ 3 mT/s





Example 2: metallic turn closed by resistance R_1



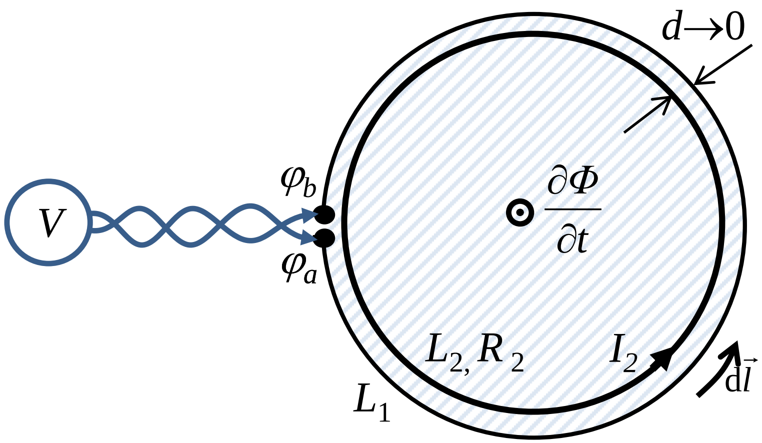
$$\vec{E}_1 = I\vec{i}\,\frac{R_1}{l_1}; \quad \vec{E}_2 = I\vec{i}\,\frac{R_2}{l_2}$$

$$\nabla\varphi_1 = -\frac{\partial \vec{A}}{\partial t} - I\vec{i}\,\frac{R_1}{l_1}; \quad \nabla\varphi_2 = -\frac{\partial \vec{A}}{\partial t} - I\vec{i}\,\frac{R_2}{l_2}$$

$$\left.\begin{aligned} \varphi_a - \varphi_b &= \int_b^a \nabla\varphi \cdot \mathrm{d}\vec{l} = \nabla\varphi_{1,i} l_1 = -\frac{\partial A_i}{\partial t} l_1 - I\,\frac{R_1}{l_1} \\ \varphi_a - \varphi_b &= \int_b^a \nabla\varphi \cdot \mathrm{d}\vec{l} = -\nabla\varphi_{2,i} l_2 = \frac{\partial A_i}{\partial t} l_2 + I\,\frac{R_2}{l_2} \end{aligned}\right\} \; -I(R_1 + R_2) = \frac{\partial \Phi}{\partial t}$$

$$\boxed{U = \varphi_a - \varphi_b = \frac{\partial \Phi}{\partial t}\left[\frac{l_2}{l_1 + l_2} - \frac{R_2}{R_1 + R_2}\right]}$$

Example 3: contactless measurement of electric field



closed (inner) metallic turn:

$$\int_{L_2} \vec{E}_2 \cdot \mathrm{d}\vec{l} = R_2 I_2$$

$$\int_{L_2} \vec{E}_2 \cdot \mathrm{d}\vec{l} = \int_{L_2} \left(-\frac{\partial \vec{A}}{\partial t} - \nabla \varphi \right) \cdot \mathrm{d}\vec{l} = -\frac{\partial \Phi}{\partial t}$$

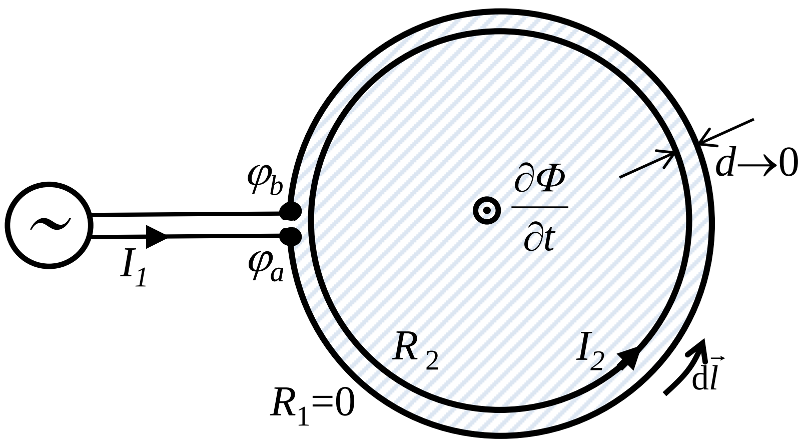
$$I_2 = -\frac{1}{R_2} \frac{\partial \Phi}{\partial t}$$

integral is 0 in closed loop

open (outer) turn with no current:

$$\boxed{U = \varphi_a - \varphi_b = \frac{\partial \Phi}{\partial t} = -R_2 I_2}$$

Example 4: Resistance-less ring energizing a conducting ring



energizing current in the outer ring: $I_1(t) = \alpha t$

steady state: $I_2 = \beta$

magnetic flux produced by both the currents:

$$\Phi = M_1 I_1 + M_2 I_2 \cong M(I_1 + I_2)$$

energizing ring: $\varphi_a - \varphi_b = M\alpha$ inner ring: $I_2 = -\frac{1}{R_2}\frac{\partial \Phi}{\partial t} = -\frac{\varphi_a - \varphi_b}{R_2} = -\alpha \frac{M}{R_2}$

furnished power: $P(t) = UI_1(t) = (\varphi_a - \varphi_b)\alpha t = M\alpha^2 t$

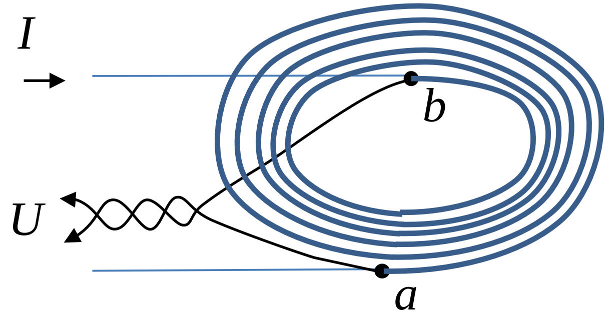
energy stored:

$$E_m(t) = \frac{M}{2}(I_1(t) + I_2)^2$$

$$\frac{\mathrm{d}E_m(t)}{\mathrm{d}t} = M(I_1(t) + I_2)\frac{\mathrm{d}I_1}{\mathrm{d}t} = M(\alpha t + \beta)\alpha = M\alpha^2 t + \overbrace{M\alpha\beta}^{=(\varphi_a - \varphi_b)I_2}$$

$$\boxed{P(t) = UI_1(t) = \frac{\mathrm{d}E_m(t)}{\mathrm{d}t} - (\varphi_a - \varphi_b)I_2 = \frac{\mathrm{d}E_m(t)}{\mathrm{d}t} + R_2 I_2^2}$$

Back to a superconducting coil



$$P(t) = U(t)I(t) = \frac{\mathrm{d}E_m(t)}{\mathrm{d}t} + P_{loss}(t)$$

$$U = \varphi_a - \varphi_b$$

numerical modeling: $\varphi_a - \varphi_b = \int_b^a \nabla\varphi . \mathrm{d}\vec{l}$

A-φ method: $\nabla\varphi$ is directly computed

H- formulation: $\nabla\varphi = -E(t,r) - \frac{\partial A(t,r)}{\partial t}$

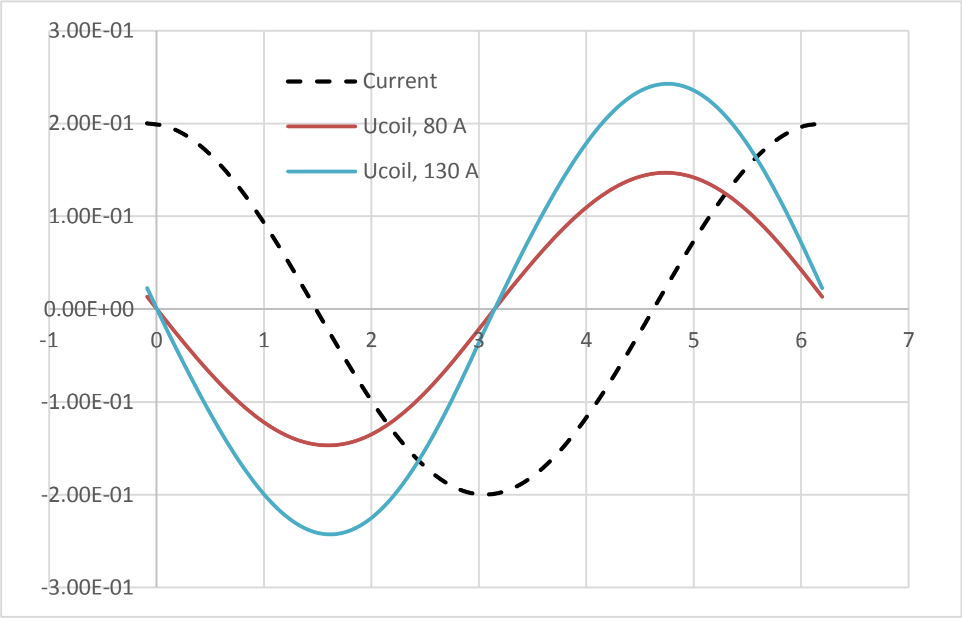
Experimental verification – 10 turn pancake coil from CC tape

f = 36 Hz

I_a [A]	Q [mJ] – A-FEM	Q [mJ] – H-FEM
80	1.87	1.82
130	9.06	9.10

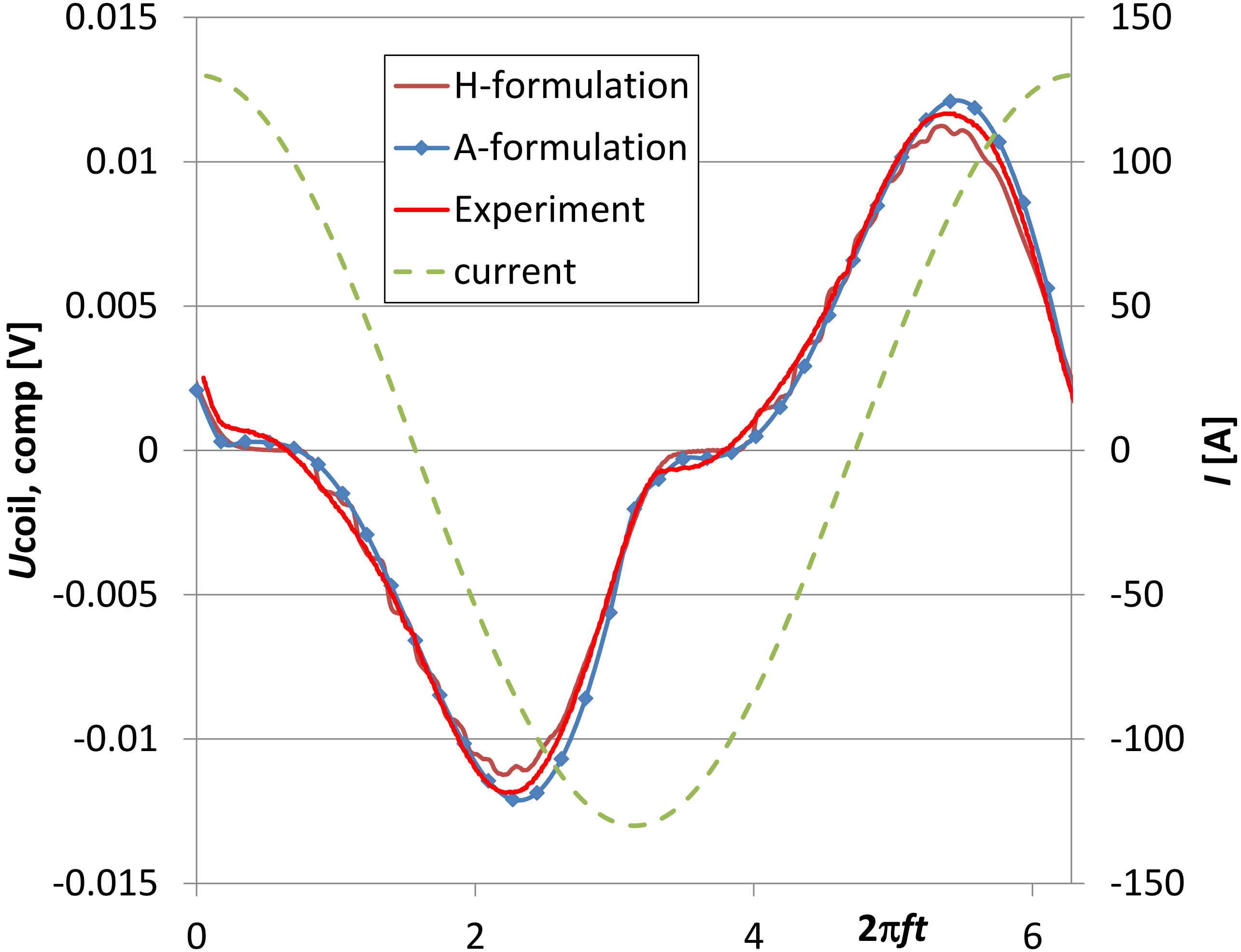
$j_c = const.$

comparison of calculated voltage waveforms: apparently only inductive signal



Experimental verification – 10 turn pancake coil from CC tape

after compensation of inductive signal: $$U_{coil,comp}(t) = U(t) + k_c \sin 2\pi ft$$



Conclusions

- The voltage necessary for AC loss evaluation in a superconducting coil is controlled by electrostatic charges on its terminations
- In experiment, this is directly what a standard voltmeter measures
- In numerical modeling, determination of electrostatic potential depends on the formulation:

  in *A*-φ formulation, $\nabla\varphi$ is calculated as an independent variable

  in *H*-formulation, macroscopic voltage should be derived

- Voltage on coil is a measurable quantity – allows detailed comparison of numerical model with experiments