Errors and optics study of a permanent magnet quadrupole system

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And

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MEDical application @ ELI-Beamlines

25th International Conference on Magnet Technology,
Amsterdam 27 August – 01 September 2017
Laser-driven ion beams

- Large proton number: $10^{10} \div 10^{13}$
- Short bunch duration: few psec
- High Beam Current: kA
- \textit{Low Emittance!}: $5 \times 10^{-3} \pi \text{ mm mrad}$
  (microscale spot size but...)
- Wide Angular Aperture: $10 \sim 20^\circ$
  (if we are lucky!)
- High Energy Spread: $\Delta E/E \gg 10\%$
- Low shot-to-shot reproducibility
Laser-driven ion beams

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- High Energy Spread: $\Delta E/E >> 10\%$
- Low shot-to-shot reproducibilt
- High dose-rate per bunch: $\sim 10^9$ Gy/sec

*PIC simulations by J. Psikal
Expected @ ELI Beamlines*
Laser-driven hadrontherapy

Conventional hadrontherapy facilities:
- High complexity for the beam, acceleration, transport and delivery
- High cost

Laser-based hadrontherapy facilities:
- Compactness (hospital-room size)
- Cost-reduction (optical gantry)
- Innovative treatment modalities:
  - Variable energies in the accelerator (no degraders needed)
  - Hybrid treatment (protons, ions, electrons, gamma-rays, neutrons)
  - In-situ diagnostics (PET, X-rays)
  - Low emittance: normal-tissue sparing?
  - High fluence rate (ultrashort pulses): higher RBE???

Cell irradiation experiments with laser-driven protons
- Doria et al., AIP Advances (2012)

http://newscenter.lbl.gov/2010/10/18/ion-beam-therapy/
Laser-driven hadrontherapy
ELIMAIA & ELIMED

ELI Multidisciplinary Applications of laser-Ion Acceleration

Graphics by J. Grosz
Beam line elements:
1) Collection system
2) Selection system
3) Standard transport elements (quadrupoles and steerers)
4) in air dosimetry and irradiation

Beam line features:
1) Tunability (deliver ion beams from 5 up to 60 MeV/u) with a controllable energy spread (5% up to 20%) and $10^6$-$10^{11}$ ions/pulse
2) Large acceptance
3) Flexibility to meet different User requirements
Magnets for laser-driven particles

- 20 mm long dipole
- 50 mm gap
- C-shape
- NdFeBo magnets + iron yoke
Magnets for laser-driven particles

- 20 mm long dipole
- 50 mm gap
- C-shape
- NdFeBo magnets + iron yoke

- Electron spectrometer!

The general idea of laser-people is:

“I need X Tesla, just put a random magnet there and it will work”
Magnets for laser-driven particles

- 20 mm long dipole
- 50 mm gap
- C-shape
- NdFeBo magnets + iron yoke

Sample: 2

- 10 - 10 MeV
- 40 mrad uniform divergence

Sample: 1

- 10 - 40 MeV
- 2 mrad uniform divergence
- 40 mrad pointing down

shot #81

N 6.4 bar

shot #225

N 6.4 bar

Laser axis

shot #317

> 40 MeV electrons

N 3.2 bar

shot #317

> 40 MeV electrons

N 3.2 bar
Magnets for laser-driven particles

Sample: 2-10 Mev
40 mrad uniform divergence

Sample: 10-40 Mev
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N 6.4 bar

Shot #225
N 3.2 bar

Project supported by:
EUROPEAN UNION
European Structural and Invested Funds
Operational Programme Research,
Development and Education
Magnets for laser-driven particles

- 20 mm long dipole
- 50 mm gap
- C-shape
- NdFeBo magnets + iron yoke

Electron spectrometer!

Radia Field uniformity ~30%!!!
# Magnets for laser-driven particles

F. Schillaci et al., JINST 10 T05001 (2015)
F. Schillaci et al., JINST 11 T07005 (2016)

- 20 mm long dipole
- 50 mm gap
- C-shape
- NdFeBo magnets + iron yoke

<table>
<thead>
<tr>
<th></th>
<th>An (Skew)</th>
<th>Bn (Normal)</th>
<th>an</th>
<th>bn</th>
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<td>%</td>
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<td>37%</td>
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</table>
Permanent Magnet prototype test results @ LOA (Fr)

F. Schillaci et al., JINST 10 T05001 (2015)
F. Schillaci et al., JINST 11 T07005 (2016)

Low Energy cutoff | Gal'Chromic | Tracewin simulation | Simion simulation
--- | --- | --- | ---
4 MeV | ![Image](image1.png) | ![Image](image2.png) | ![Image](image3.png)
5.18 MeV | ![Image](image4.png) | ![Image](image5.png) | ![Image](image6.png)
6.17 MeV | ![Image](image7.png) | ![Image](image8.png) | ![Image](image9.png)
7.05 MeV | ![Image](image10.png) | ![Image](image11.png) | ![Image](image12.png)
7.83 MeV | ![Image](image13.png) | ![Image](image14.png) | ![Image](image15.png)
8.56 MeV | ![Image](image16.png) | ![Image](image17.png) | ![Image](image18.png)
• Quadrupole features
• Error source in magnets and modelling
• Fixing the tolerances
• Beam transport (simulations and experiment)
• Quadrupole features
• Error source in magnets and modelling
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• Beam transport (simulations and experiment)
Quadrupole layout

4 PMQs features (simulations)

- 2 elements 40 mm long
- 2 elements 80 mm long
- 22 mm bore – 20 mm clearance
- 100T/m field gradient
- NdFeBo N50 permanent magnets
- Gradient homogeneity: -6% @ R = 8mm
- Integrated gradient homogeneity: -1% @ R = 8mm
- Harmonic content $B_n/B_2 < 2$

- Cost-effective prototype
Quadrupole layout

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Quadrupole layout

Magnetic design and manufacturing

Mechanics designed and manufactured at INFN
2D Harmonic analysis

2D simulations:
• r0 = 8 mm radius reference circle for B-field post-processing and harmonic analysis
2D Harmonic analysis

2D simulations:
- $r_0 = 8$ mm radius reference circle for B-field post-processing and harmonic analysis
- Modulus of induction $|B|$ should be constant
2D Harmonic analysis

2D simulations:

- $r_0 = 8 \text{ mm}$ radius reference circle for B-field post-processing and harmonic analysis
- Modulus of induction $|B|$ should be constant
- Radial component $B_{\text{rad}} = B_x (x/r_0) + B_y (y/r_0)$ should be purely sinusoidal
2D Harmonic analysis

2D simulations:

- \( r_0 = 8 \text{ mm} \) radius reference circle for B-field post-processing and harmonic analysis
- Modulus of induction \(|B|\) should be constant
- Radial component \( B_{rad} = B_x (x/r_0) + B_y (y/r_0) \) should be purely sinusoidal
- Fourier expansion of \( B_{rad} \) gives the magnitude of the harmonic components \( C_n \):

\[
C_n = \frac{1}{N} \sum_{k=1}^{N-1} B_{rad k} \frac{1}{r_0} \exp \left( i k \left( \frac{2 \pi n N}{N} \right) \right)
\]

- Deviations from ideal behaviour affect the field quality and the beam transport can show filamentation, emittance growth, steering
• Quadrupole features
• Error source in magnets and modelling
• Fixing the tolerances
• Beam transport (simulations and experiment)
• Magnetization of permanent magnets (remanence, magnetization angle, ...)

• Manufacturing errors (assembly, pole shimming, ...)

• Alignment (skew components)

• Eddy currents (see my talk *Status and realization of an high efficiency transport beamline for laser-driven ion beamline* [Wed-Mo-Or19])

• ...

If one or more error sources are introduced symmetry is broken!

In order to minimize the errors the tolerances have to be stated for each possible error source.

**The tighter are the tolerances the higher will be the cost!**
Error source in a magnet

- Magnetization of permanent magnets (remanence, magnetization angle, ...)
- Manufacturing errors (assembly, pole shimming, ...)
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In order to minimize the errors the tolerances have to be stated for each possible error source.

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The goal here is to have no more than 3% of total harmonic component
How to introduce errors in simulations:

**Remanence:** The remanence $M_r$ of each rare-earth piece is multiplied by a random number, $\text{rand}_1$, with a fixed seed depending on the block identification number and on the ordinal number of the magnetic configuration produced (401 in total).

$\text{rand}_1$ is uniformly distributed around the mean value 1 with a range of $\pm 0.03$ and $\pm 0.06$, making the remanent magnetization increasing or decreasing up to 3% and 6%.

**Assembly:**
How to introduce errors in simulations:

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**Assembly:** The mechanical assembly errors is simulated introducing a different displacement for each block controlled by a random number $rand_2$ with fixed seed. The direction has been forced to avoid overlapping of the magnets (iron parts are considered fixed). The T-like pieces between two poles are treated as three independent blocks, even if they will be realized as a single one; this allows to take into account not only errors due to the assembly but also errors due to the machining of these parts. $rand_2$ is defined as uniformly distributed around the mean value 0 with a range of ±0.1 and ±0.2. In this way each block is shifted from the ideal position up to 100 mm in the first case and up to 200 mm in the second case.
How to introduce errors in simulations:

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$rand2$ is been defined as uniformly distributed around the mean value 0 with a range of ±0.1 and ±0.2. In this way each block is shifted from the ideal position up to 100μm in the first case and up to 200μm in the second case.
The radial displacement of the pole at 45° produces a small decrease in the peak of $B_{\text{rad}}$ at the same angle. The loss of symmetry produces a dipole contribution in the opposite direction of the pole shift. The real and imaginary parts of the coefficient $C_1$ are equal to each other even if the field is analysed at different reference radii, which means that the phase of the dipole component is $\theta = \arctan(B_1/A_1) = 45^\circ$, namely in the direction of the displaced pole.
Model validation II
Ideal $B_1 = 0.092$ units

If the pole is in its ideal position but its remanence is increased by a factor of two there is a strong increase in the peak of $B_{rad}$ as the loss of symmetry produces a dipole contribution in the same direction of the pole magnetization direction.
OUTLINE

- Quadrupole features
- Error source in magnets and modelling
- Fixing the tolerances
- Beam transport (simulations and experiment)
Random Errors

400 different simulations per range of variation of $M_r$ and magnet position
Random Errors

400 different simulations per range of variation of $M_r$ and magnet position

Ideal case results

| Harmonic n | $|C_n|$ Value (units of $10^4$) | $B_n$ Value (units of $10^4$) |
|-----------|-------------------------------|-------------------------------|
| Sum       | 1.22%                         | 1.114%                        |
Random Errors

400 different simulations per range of variation of $M_r$ and magnet position

The normal content ($B_n$) does not increase significantly with the increasing of the errors.

The complex harmonics ($C_n$) are strongly affected by the errors and their contribution is about 3% of the main harmonic if the errors range in the wider interval.
Random Errors

Combining errors on $M_r$ and magnet position

each magnetic configuration is reproduced on all the different geometric configurations (400 x 400 simulations)

![Graph showing harmonic content vs. configurations]
Random Errors

Effects on the field quality

Magnetic center nominal position

Magnetic center shift
Random Errors

Effects on the field quality

Magnetic measurement
Random Errors

Effects on the field quality

Magnetic measurement
• Quadrupole features
• Error source in magnets and modelling
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• Beam transport (simulations and experiment)
Beam Transport Simulations
Beam Transport Simulations

Ideal case

Perturbed case

1.4% Emittance growth

<table>
<thead>
<tr>
<th></th>
<th>Ideal Quadrupole System</th>
<th>Perturbed Quadrupole System</th>
<th>$\Delta e/e_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space</td>
<td>$e_i$ (rms)</td>
<td>$e_p$ (rms)</td>
<td></td>
</tr>
<tr>
<td>$(x,x')$</td>
<td>$0.6572,\pi,\text{mm.mrad}$</td>
<td>$0.6661,\pi,\text{mm.mrad}$</td>
<td>0.0135</td>
</tr>
<tr>
<td>$(y,y')$</td>
<td>$0.9322,\pi,\text{mm.mrad}$</td>
<td>$0.9360,\pi,\text{mm.mrad}$</td>
<td>0.0041</td>
</tr>
<tr>
<td>$(x',y')$</td>
<td>$5.1267,\text{mrad}^2$</td>
<td>$5.9272,\text{mrad}^2$</td>
<td>0.1561</td>
</tr>
<tr>
<td>$(x,y)$</td>
<td>$0.2583,\text{mm}^2$</td>
<td>$0.3163,\text{mm}^2$</td>
<td>0.2245</td>
</tr>
<tr>
<td>Centroid position</td>
<td>$dx = -0.0016,\text{mm}, dy = 0.0016,\text{mm}$</td>
<td>$dx = 0.2304,\text{mm}, dy = 0.0003,\text{mm}$</td>
<td></td>
</tr>
</tbody>
</table>
Beam Transport Simulations

**Ideal case**

- 0.6572 π mm mrad
- 0.9322 π mm mrad
- 5.1267 mrad²
- 0.2583 mm²
- Centroid position: \(dx = -0.0016 \text{ mm}, dy = 0.0016 \text{ mm}\)

**Perturbed case**

- 0.6661 π mm mrad
- 0.9360 π mm mrad
- 5.9272 mrad²
- 0.3163 mm²
- Centroid position: \(dx = 0.2304 \text{ mm}, dy = 0.0003 \text{ mm}\)

<table>
<thead>
<tr>
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<th>(\Delta \varepsilon / \varepsilon_i)</th>
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<td>(x', y')</td>
<td>5.127 mrad²</td>
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<td>(x, y)</td>
<td>0.2583 mm²</td>
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</table>

- 1.4% Emittance growth
- 0.5° bigger angular aperture and more filamentations
Beam Transport Simulations

Ideal case

- 1.4% Emittance growth
- Not negligible steering effect on the radial plane (as expected)

Perturbed case

- 0.5° bigger angular aperture and more filamentations

<table>
<thead>
<tr>
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Centroid position

For Ideal Quadrupole System:

$$dx = -0.0016 \text{ mm}, \ dy = 0.0016 \text{ mm}$$

For Perturbed Quadrupole System:

$$dx = 0.2304 \text{ mm}, \ dy = 0.0003 \text{ mm}$$
Beam Transport Test

Wide big spot size for cell irradiation
Beam Transport Test @ LOA (Fr)

3.5 MeV

<table>
<thead>
<tr>
<th>Gaf Position</th>
<th>Film dimension</th>
<th>GafChromatic</th>
<th>TraceWin simulation</th>
<th>Simion simulation</th>
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</thead>
<tbody>
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<td>225 mm</td>
<td>29.6x29.6 mm</td>
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<tr>
<td>255 mm</td>
<td>29.6x29.6 mm</td>
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<td>305 mm</td>
<td>29.6x29.6 mm</td>
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<tr>
<td>395 mm</td>
<td>29.6x51.6 mm</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>445 mm</td>
<td>29.6x29.6 mm</td>
<td></td>
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<td></td>
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<tr>
<td>930 mm</td>
<td>57.8x57.8 mm</td>
<td></td>
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</table>

6.5 MeV

<table>
<thead>
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<th>Gaf Position</th>
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</table>
Beam Transport Test
@ LOA (Fr)

<table>
<thead>
<tr>
<th>3.5 MeV</th>
<th>6.5 MeV</th>
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<tbody>
<tr>
<td><strong>Gaf Position</strong></td>
<td><strong>Film Dimension</strong></td>
</tr>
<tr>
<td>225 mm</td>
<td>29.6x29.6 mm</td>
</tr>
<tr>
<td>255 mm</td>
<td>29.6x29.6 mm</td>
</tr>
<tr>
<td>305 mm</td>
<td>29.6x29.6 mm</td>
</tr>
<tr>
<td>395 mm</td>
<td>29.6x51.6 mm</td>
</tr>
<tr>
<td>445 mm</td>
<td>29.6x29.6 mm</td>
</tr>
<tr>
<td>930 mm</td>
<td>57.8x57.8 mm</td>
</tr>
</tbody>
</table>

The centroid is shifted and the beam is rotated

B1 + skew + missalignment
Conclusion

- A model to study random errors in PMQs is proposed
- Validated in simple cases
- Effects of the harmonic contents on beam dynamics results in agreement with the dipole component produced by the loss of symmetry due to the introduction of imperfection on magnets
- The method results to be robust and reliable
- This model is useful to state tolerances on magnet assembly
- The model is completely general and can include any kind of error source... if you have enough time to run and analyse thousands of simulations
Thank you for your attention

http://www.eli-beams.eu/

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Thanks to W. Beeckman (SigmaPhi) for advice and discussion
**Quadrupole layout**

**4 PMQs features (simulations)**

- 2 elements 40 mm long
- 2 elements 80 mm long
- 22 mm bore – 20 mm clearance
- 100T/m field gradient
- NdFeBo N50 permanent magnets
- Gradient homogeneity: -6% @ R = 8mm
- Integrated gradient homogeneity: -1% @ R = 8mm
- Harmonic content $B_n/B_2 < 2$
- **Cost-effective prototype**
Random Errors

400 different simulations per range of variation of $M_r$ and magnet position

The normal content ($B_n$) does not increase significantly with the increasing of the errors. The complex harmonics ($C_n$) are strongly affected by the errors and their contribution is about 3% of the main harmonic if the errors range in the wider interval.