

Since 2010 to present, several sub-size magnet assemblies, designed as test beds for the validation of impregnated Nb₃Sn-based coil technology, have been tested at the Superconducting Magnet Test Facility (SM18) at CERN. These Short Model Coils (SMC) and Racetrack Model Coils (RMC) have been used to study two types of Rutherford cables foreseen for the coils of the Nb₃Sn magnets in the framework of the HL-LHC and High Field Magnets program of CERN. During several test campaigns, the Rod Restack Process (RRP) and the Powder-In-Tube (PIT) conductors have been characterized in terms of performance and Quench Propagation Velocity (QPV). Moreover, Hot Spot Temperature (HST) increase during quenches has been estimated from the analysis of the registered voltage and current signals.

In this work, the multi-physics problem of quench propagation in Nb₃Sn cables is addressed under adiabatic conditions by means of a set of analytical formulae and several Finite Element Models (FEM) with different level of complexity in ANSYS Mechanical APDL, COMSOL Multiphysics and MATLAB. These models are aimed at describing the conductor behaviour in terms of HST and QPV observed during the tests of racetrack coils at SM18.

Adiabatic Quench Models

Quench Propagation Velocity (QPV) and Hot Spot Temperature (HST)

3-D Model (ANSYS APDL)

$$C_i \frac{\partial T_i}{\partial t} = \nabla \cdot (\bar{K}_i \cdot \nabla T_i) + f_i + e_i, \quad i = \{Cu, Nb_3Sn, G10\}$$

$$\text{Charge Conservation: } \nabla \cdot \vec{J}_i = 0$$

$$\text{Constitutive Relation: } -\nabla V_i = \vec{E}_i = \rho_i \vec{J}_i$$

$$\rho_{Nb_3Sn} = \begin{cases} \gg \rho_{Cu}, & \text{if } J_{Nb_3Sn} > J_c(T_{Nb_3Sn}, B, \epsilon) \\ \ll \rho_{Cu}, & \text{if } J_{Nb_3Sn} \leq J_c(T_{Nb_3Sn}, B, \epsilon) \end{cases}$$

$$\text{Heat Generation: } f_i = \vec{E}_i \cdot \vec{J}_i$$

1-D Lumped Model (MATLAB)

1-D Heat Transport:

$$A_i C_i \frac{\partial T_i}{\partial t} = \frac{\partial}{\partial x} \left(A_i k_i \frac{\partial T_i}{\partial x} \right) + \sum_{j=1}^{N_c} g_{ij} (T_j - T_i) + A_i (f_i + e_i)$$

Thermal Conductances:

$$g_{ij} = \frac{p_{ij} k_i(T_i) k_j(T_j)}{k_i(T_i) d_{ji} + k_j(T_j) d_{ij}}$$

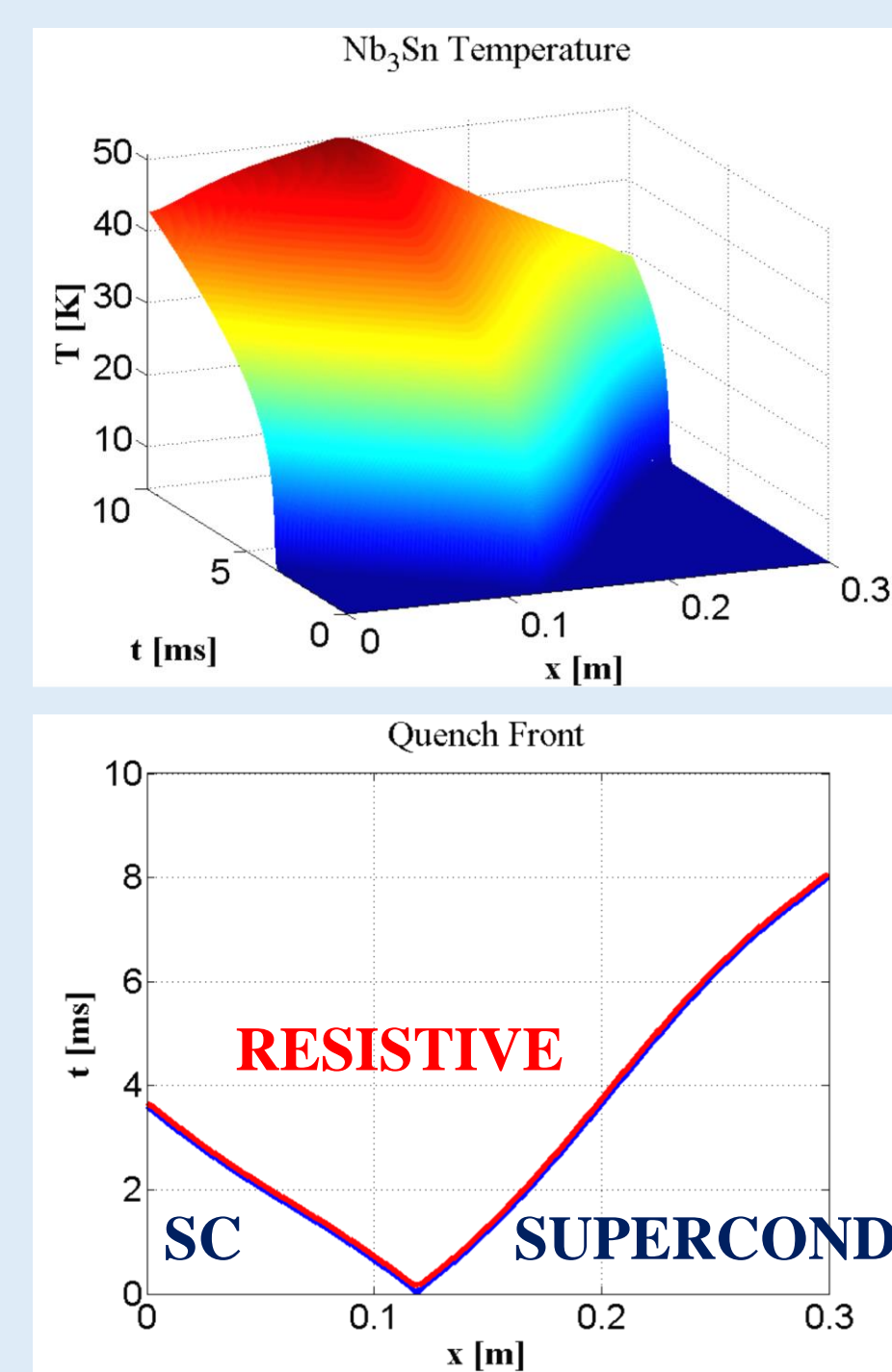
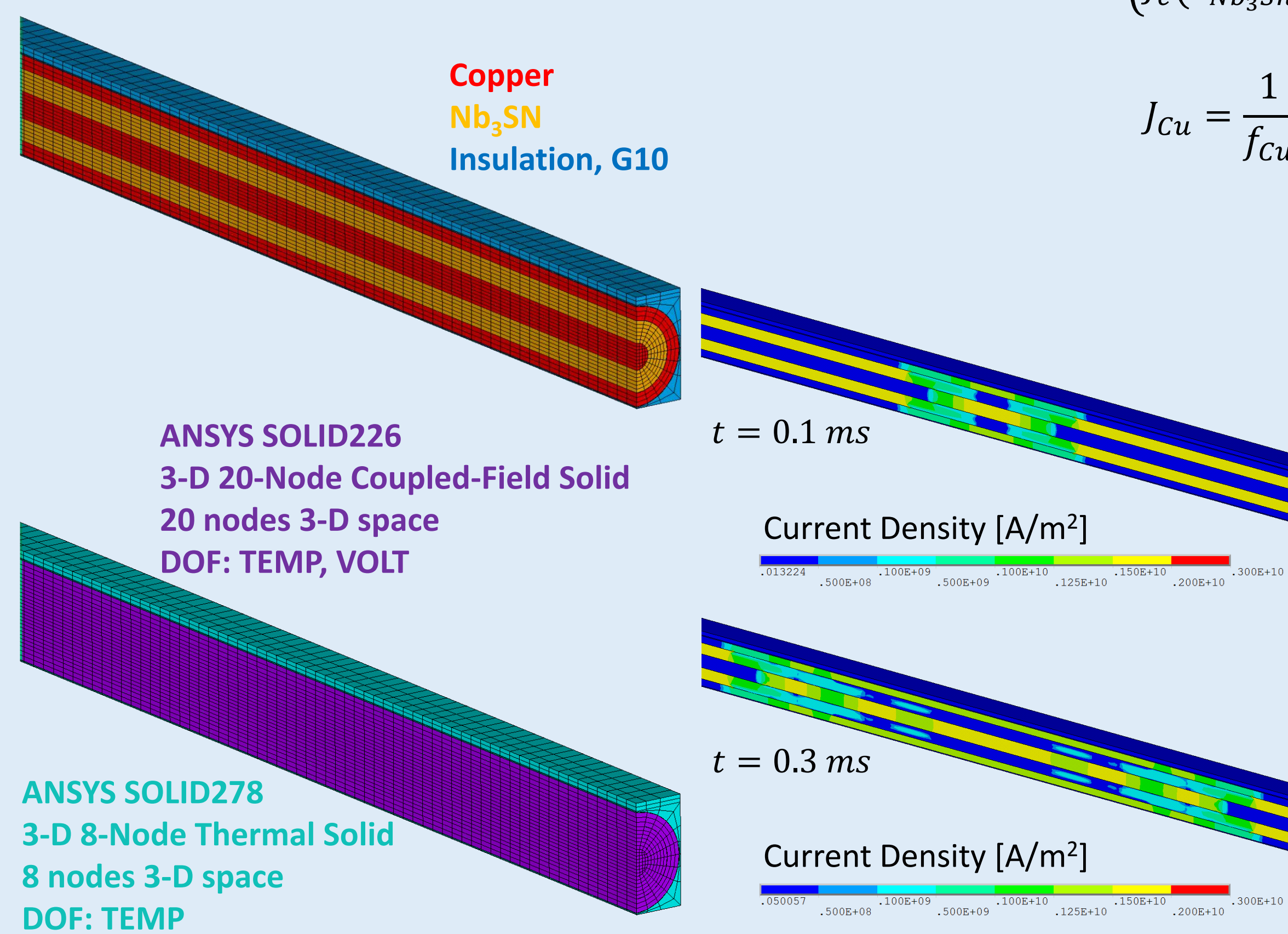
Heat Generation:

$$f_i = E_i J_i = \rho_{Cu} J_{Cu} i$$

Current Distribution:

$$J_{Nb_3Sn} = \begin{cases} \frac{I}{S_{Nb_3Sn}}, & \text{if } I > J_c(T_{Nb_3Sn}, B, \epsilon) S_{Nb_3Sn} \\ J_c(T_{Nb_3Sn}, B, \epsilon), & \text{if } I \leq J_c(T_{Nb_3Sn}, B, \epsilon) S_{Nb_3Sn} \end{cases}$$

$$J_{Cu} = \frac{1}{f_{Cu}} \left(\frac{I}{A_{Nb_3Sn} + A_{Cu}} - f_{sc} J_{Nb_3Sn} \right)$$



Hot Spot Temperature (without axial propagation)

The axial thermal conduction can be neglected provided that the dimensionless parameter

$$\lambda = \frac{2k_{av}(T_{HS})[T_c - T_{HS}(t)]}{f_{Cu} \rho_{Cu}(T_{HS})} \left[\frac{A_{Cu}}{l(t)I(t)} \right]^2 \ll 1$$

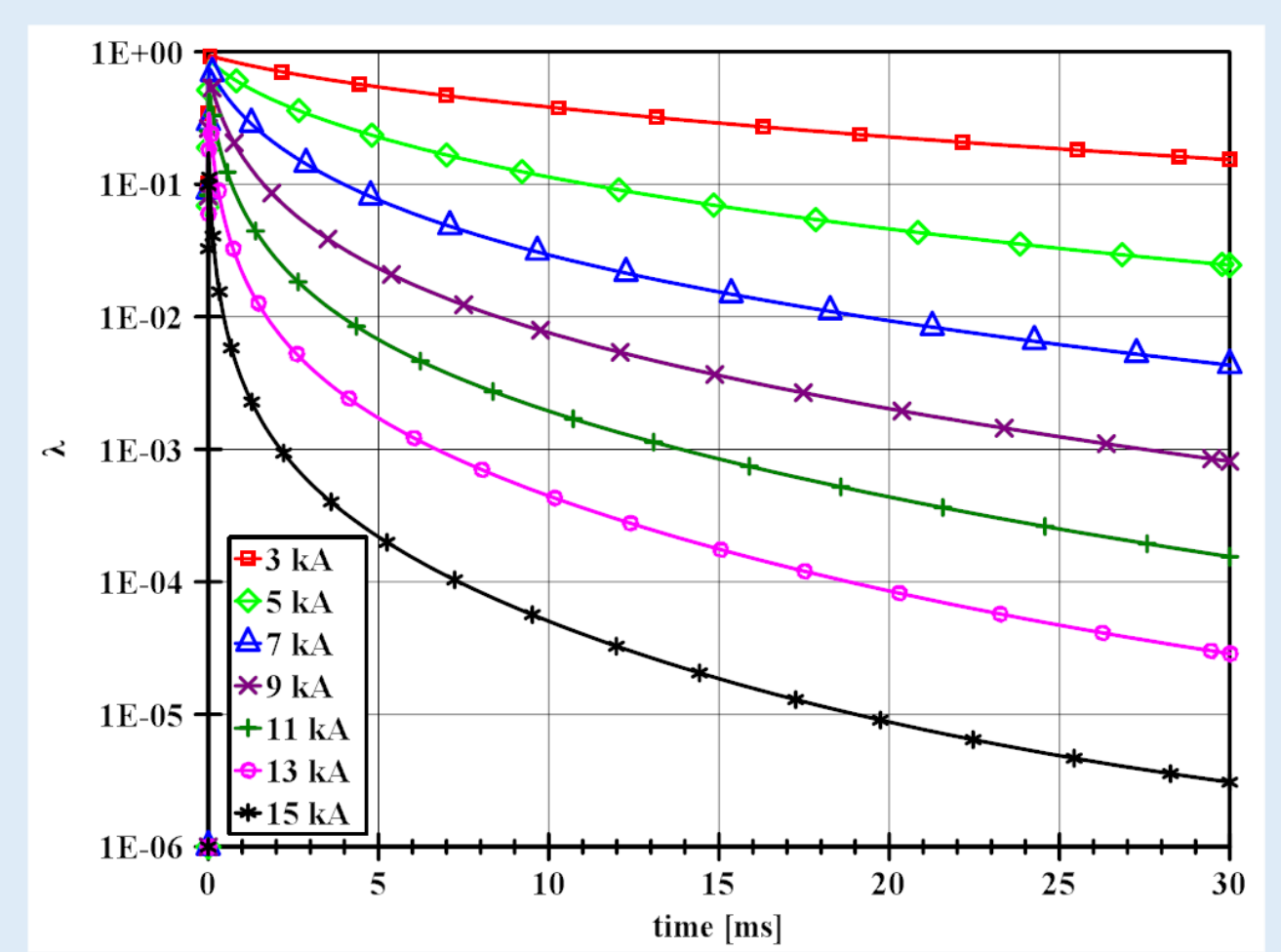
This is usually the case for regular quench currents and conductors.

The distance from the Hot Spot (HS) to the quench front writes as

$$l(t) = l(t_q) + v_q(t - t_q)$$

A QPV formula that includes the effect of the current-sharing region from T_{cs} to T_c is proposed,

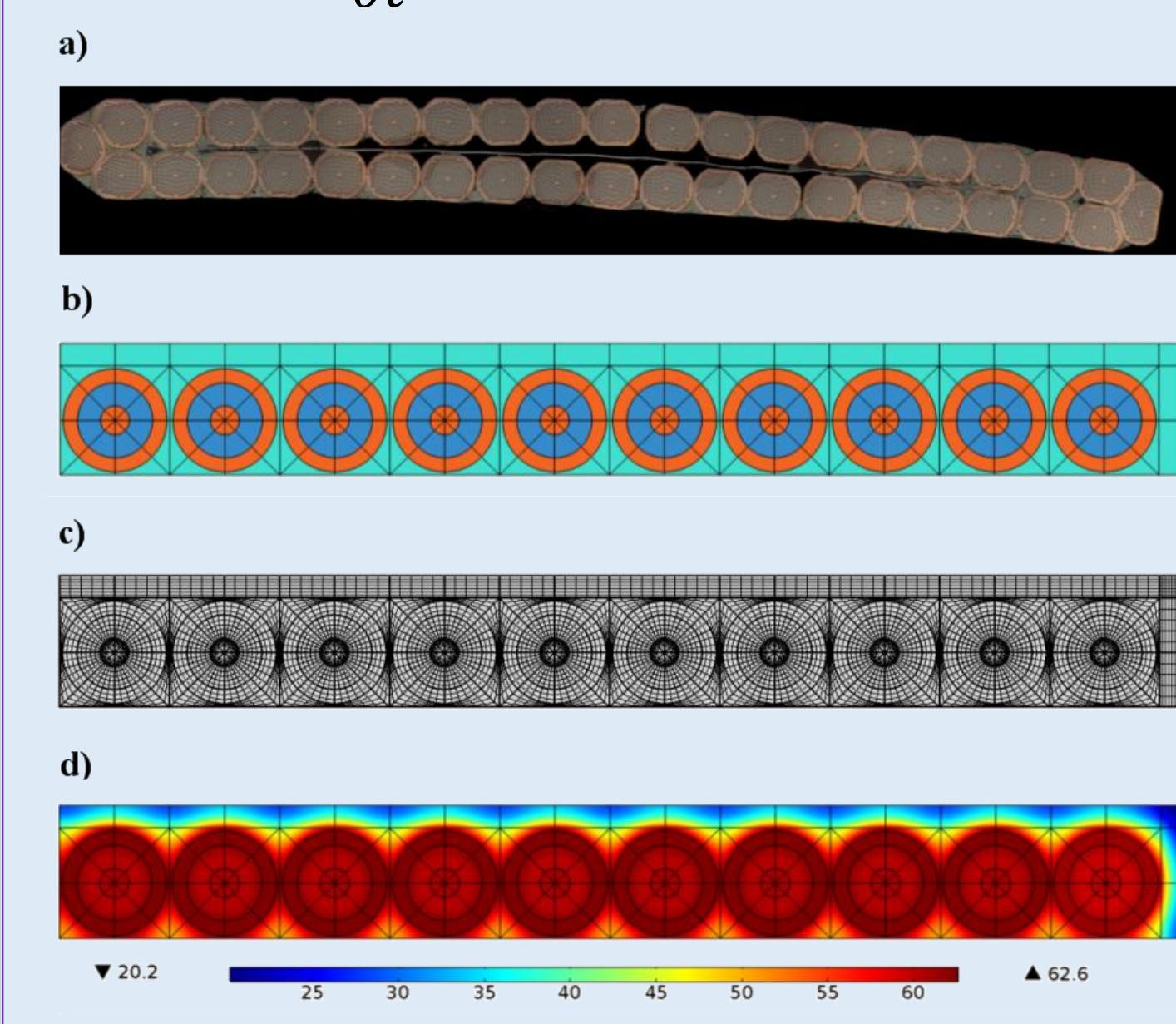
$$v_q = \frac{I}{A} \sqrt{\frac{LT_{cs}}{\int_{T_0}^{T_{cs}} C_{av} dT \left[C_{av}(T_c) - \frac{1}{T_{cs}} \int_{T_0}^{T_{cs}} C_{av} dT \right]}}$$



2-D Model (ANSYS APDL, COMSOL Multiphysics)

2-D Heat Transport:

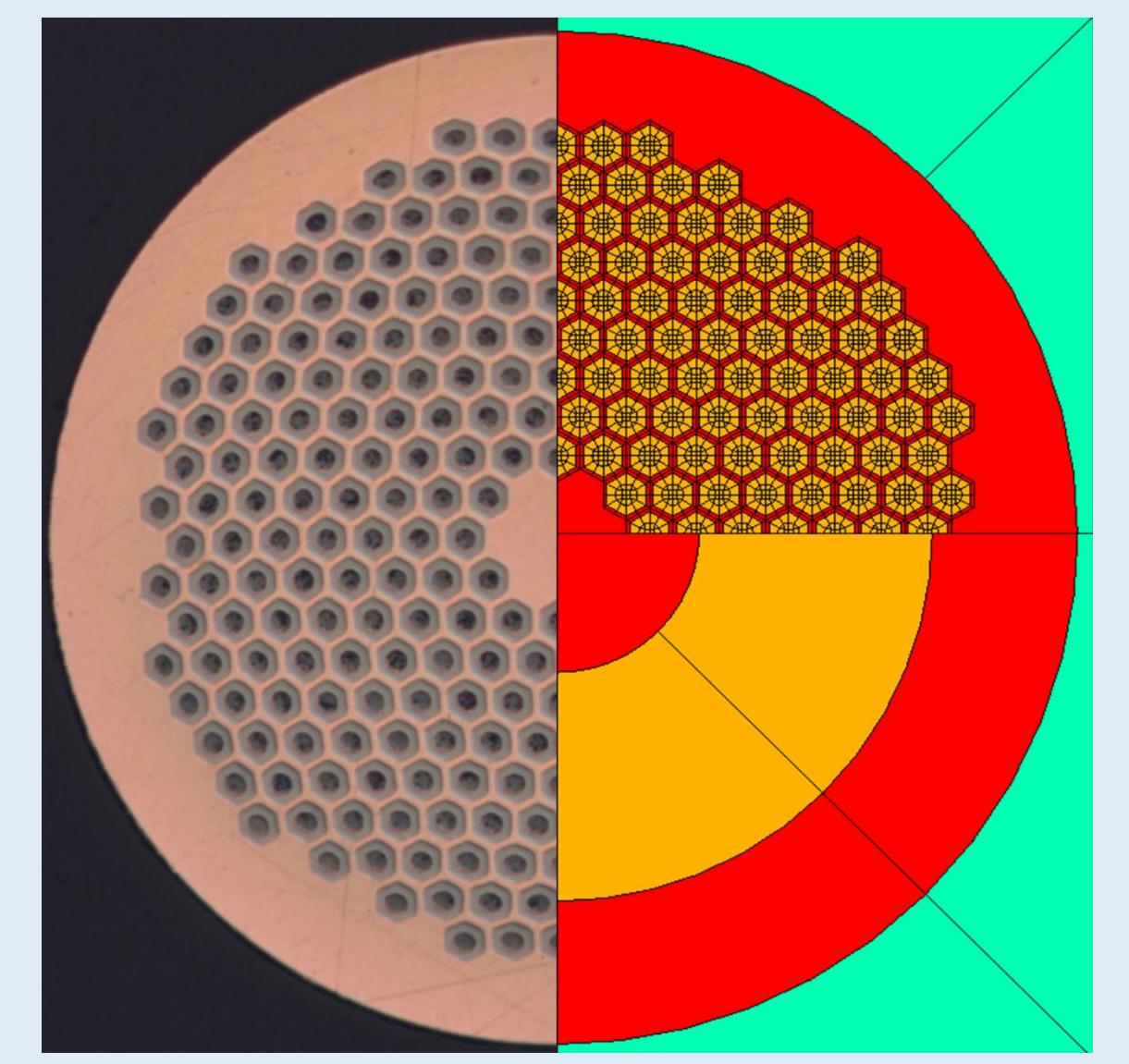
$$C_i \frac{\partial T_i}{\partial t} = \nabla \cdot (\bar{K}_i \cdot \nabla T_i) + \rho_i J_i^2$$



0-D Model (MATLAB, NI DIAdem AQA)

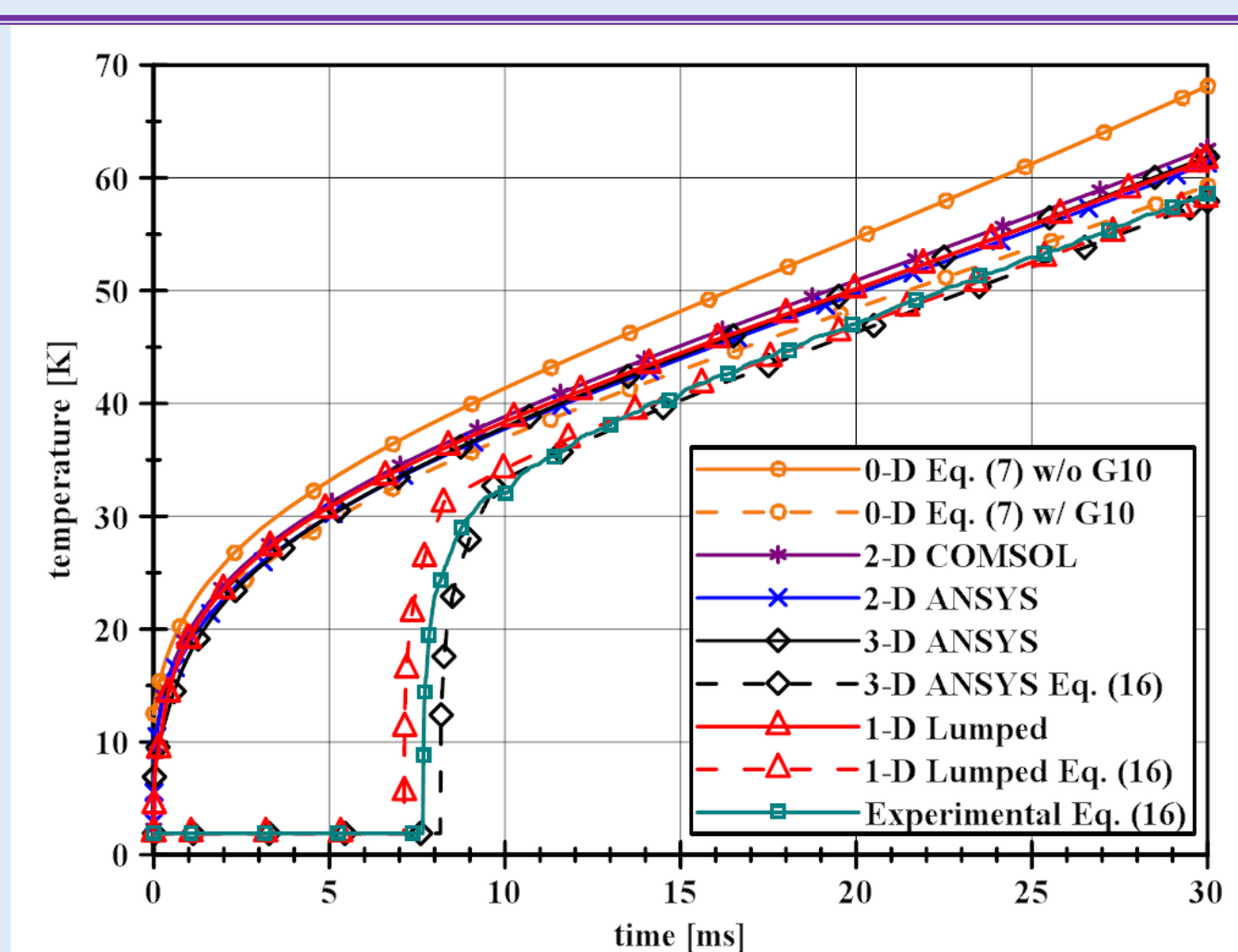
No Heat Transport:

$$C_{av}(T) \frac{dT}{dt} = \frac{\rho_{Cu}(T)}{A_{Cu} \sum_{i=1}^{N_c} A_i} I^2(t)$$



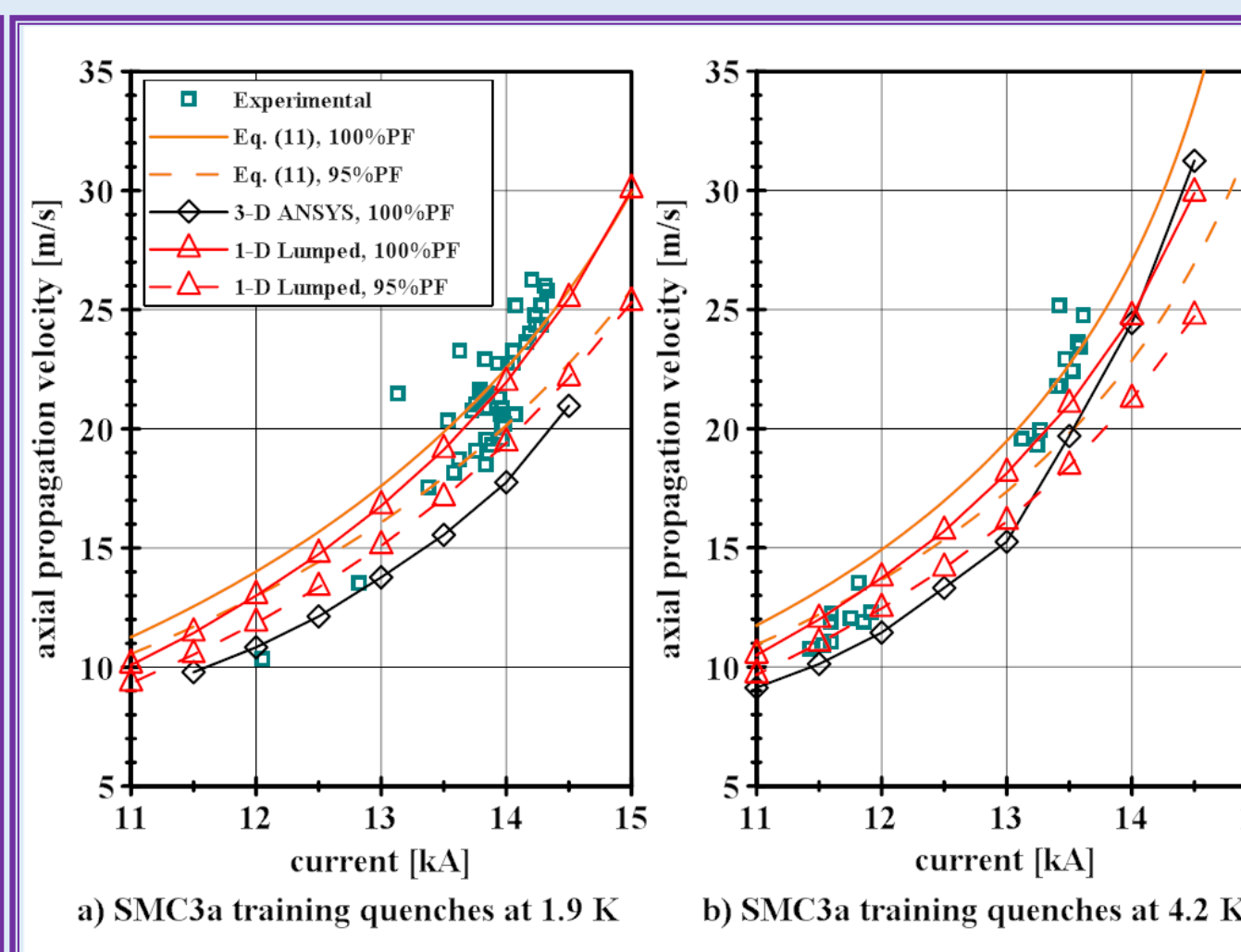
Models Validation

The models are validated with the experimental data sets collected during the tests of Racetrack Coils performed at the CERN Superconducting Magnet Test Facilities SM18. The data has been analysed using SM18 AQA (Automatic Quench Analysis) software, a set of Visual Basic scripts running in National Instruments DIAdem and developed in order to post-process the measured raw data.



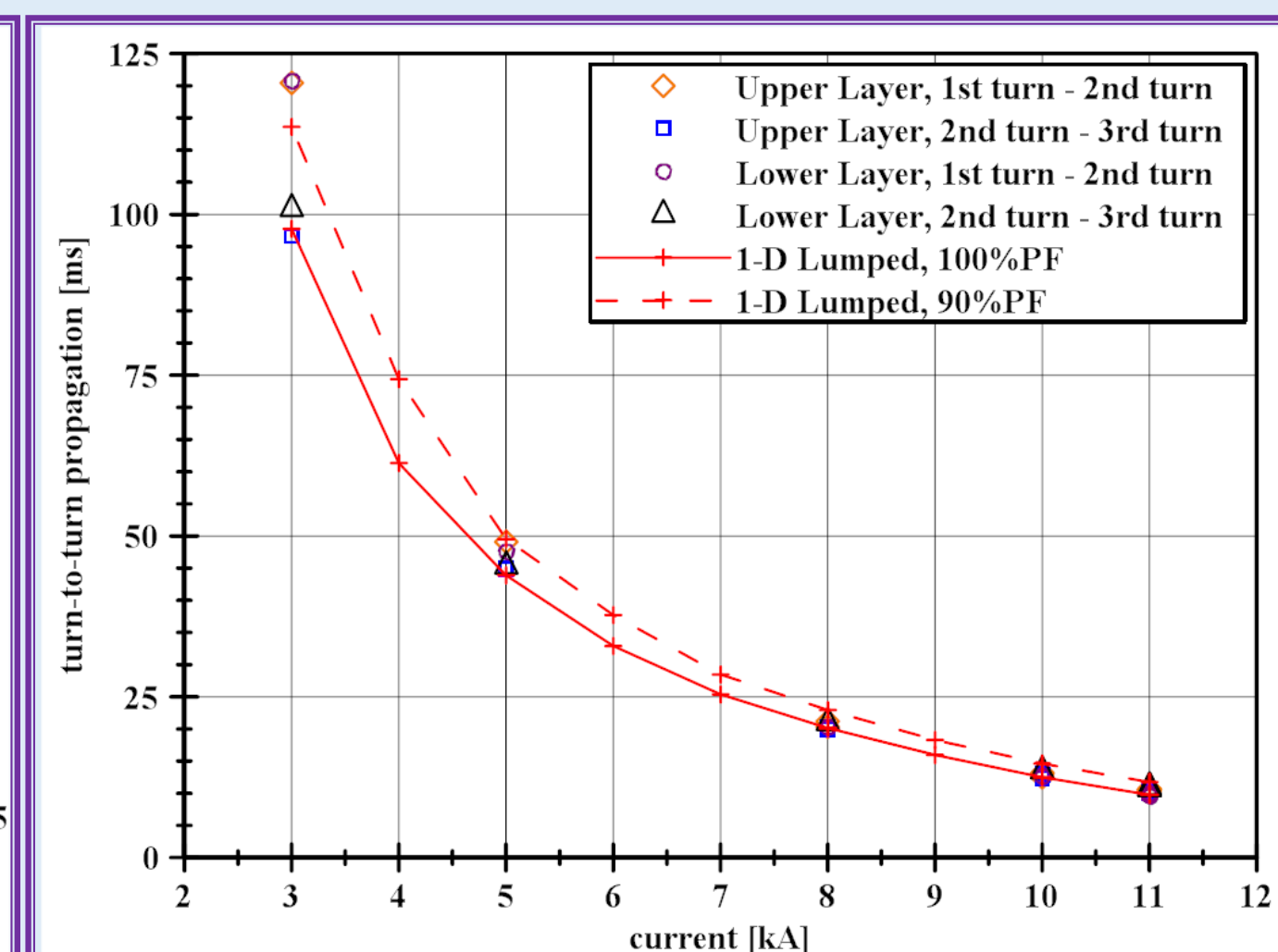
The temperature in the cable is estimated from the registered voltage and current signals with

$$\rho_{Cu}(T) = \frac{A_{Cu} \Delta V(t)}{L I(t)}$$



Experimental axial propagation velocities are obtained with the time of flight method,

$$v_q = \frac{L}{t_1 + t_2 - 2t_0}$$



The time needed for a quench to propagate from turn to turn through the insulation has been measured during spot heater tests in the high field region of the coil.

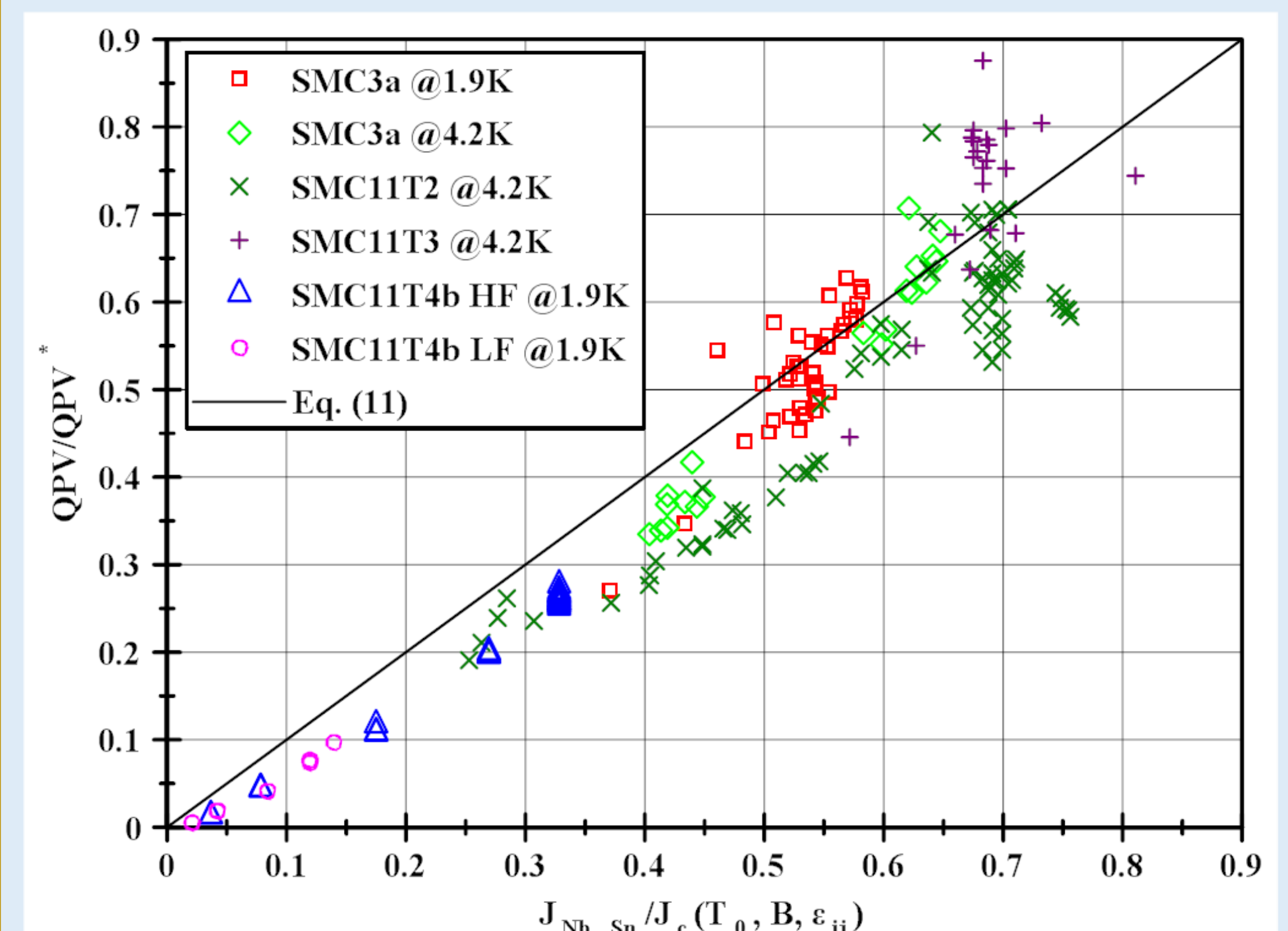
Normalization of QPV Plots

A normalization of QPV plots allows to compare quench velocities for different operation conditions (bath temperature, current, magnetic field) and conductor features (critical surface parameters, PIT-RRP, copper to non-copper ratio).

$$\frac{v_q}{v_q^*} = f \left(\frac{J_{Nb_3Sn}}{J_c(T_0, B, \epsilon_{ij})} \right)$$

$$v_q^* = f_{Nb_3Sn} J_c(T_0, B, \epsilon_{ij}) \sqrt{\frac{LT_{cs}}{\int_{T_0}^{T_{cs}} C_{av} dT \left[C_{av}(T_c) - \frac{1}{T_{cs}} \int_{T_0}^{T_{cs}} C_{av} dT \right]}}$$

T_0 is the initial temperature, B is the magnetic field, and ϵ_{ij} is the strain tensor, all referred to the coil section where the velocity is measured.



Conclusions

The excellent agreement between numerical and test results confirms that adiabatic conditions are an appropriate choice for the boundary conditions during at least a few tens of milliseconds after quench. Furthermore, models results show that taking into account the heat transfer across the cable section due to the presence of insulation is more relevant to the HST than including the heat conduction along the cable. Despite the fact that the insulation has an unquestionable influence in the HST so that it must be considered for a more accurate calculation, it has been shown that a 1-D lumped model is sufficient in order to account for this effect. This results in quite faster simulations as 3-D meshes are not needed. Axial propagation velocities are very well described by the numerical models and an explicit formula is presented that provides with good results without resorting to any free parameter to calculate a transition temperature. Based on this formula, a normalization allowing to compare propagation velocities in different coils regardless of the operation conditions and conductor features is proposed.