Application of hierarchical matrices to large-scale electromagnetic field analyses of coils wound with coated conductors

N. Tominaga, T. Mifune, Y. Sagobe, N. Amemiya (Kyoto University)
A. Ida (The University of Tokyo), T. Washita (Hokkaido University)

1. Background and objective

Background
Electromagnetic field analyses are useful to understand the electromagnetic phenomena in coils wound with coated conductors.
The analyses of large-scale coils require
  • Large amount of memory
  • Long computation time

We introduce
  • Hierarchical matrices (H-matrices)
  • Parallel computation to large-scale analyses

Objective
Verifying the effect of H-matrices and parallelization
Demonstrating an analysis of 3D-shaped coils with over 1.5 million unknowns

2. Governing Equation

Governing equation
\[
\nabla \times \left( \frac{1}{\mu_c} \nabla \times \mathbf{A} \right) + \mathbf{J} = \mathbf{0}
\]

Thin-strip approximation  
\[ \mathbf{E} \cdot \mathbf{J} \text{ characteristic} \]

Discretization spatially and temporally
\[
\nabla \times \left( \frac{1}{\mu_c} \nabla \times \mathbf{A} \right) + \mathbf{J} = \mathbf{0}
\]

This equation is solved iteratively by varying \( \sigma \) by BIGGSTAB.
In BIGGSTAB, matrix-vector multiplication are twice per iteration.

3. H-matrices and parallel computation

The concept of H-matrices

\[
\text{Dense matrix} \quad \rightarrow \quad \text{H-matrix}
\]

Computational cost: \( O(N^3) \rightarrow O(N \log N) \)

We introduce H-matrices by using HACAP library.
In the HACAP library, low-rank approximated submatrices are created by adaptive cross approximation (ACA).

The concept of parallel computation

H-matrix-vector multiplication

Communication between processes

Constructing H-matrix and sparse-matrix vector multiplication are also parallelized.

4. Applications of H-matrices to the analyses

\[
\left( A(\sigma) + \frac{\mu_0 I_0}{4\pi f} B \right) \mathbf{T}[m] \rightarrow \frac{\mu_0 I_0}{4\pi f} B \mathbf{T}[m=1]
\]

H-matrix is constructed from \( B \).
H-matrix is constructed once at the beginning of the iteration.

Concern when applying H-matrices to the analyses

• Influence of mesh shapes on the low-rank approximation

We have to verify that the H-matrices could be applied effectively to the analyses.

5. Effect of H-matrices and parallel computation

Memory consumption

\[ \text{Model A: } 103.3 \text{ MB}, \quad \text{Model B: } 158.5 \text{ MB} \]

Computation time using model A

\[ \text{Model A: } 0.70 \text{ s}, \quad \text{Model B: } 1.96 \text{ s} \]

Memory compression ratio improves as the degrees of freedom increase.

Approximation error

Current distribution and relative error

Parallel scalability

Parallel speed-up ratio

The speed up ratio of whole analyses are 8.8 and 14.3 by 16 and 32 processes, respectively.

6. Demonstration of a large-scale analysis

Coils of a cosine-theta dipole magnet for carbon rotating gantry

\( I = 60 \text{ A (ramped up in 30 s)} \)

Results: the magnitude of current density

A

Isometric view

Viewing from A

The magnitude of current density on the coil surface

Turns of tapes

1.2 \times 10^{-10}

0.8 \times 10^{-10}

0.6 \times 10^{-10}

0.4 \times 10^{-10}

0.2 \times 10^{-10}

1 \times 10^{-10}

Cross section of coils

Viewing from B

Cross section

Tape width

1.0 \times 10^{-10}

8.0 \times 10^{-11}

6.0 \times 10^{-11}

4.0 \times 10^{-11}

2.0 \times 10^{-11}

1 \times 10^{-11}

Results: memory consumption

Model A

Model B

181 GB

106 GB

memory consumed

1.5 million

1.0 million


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