I. INTRODUCTION

Shielding Current Analysis in High-Temperature Superconducting Film and Its Application

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A schematic view of (a) a pellet injection system used in the SELS, (b) an HTS film for the propulsion and (c) an HTS current loop.

A. FEM Model

Shielding Current Density in HTS Sample

\[ j = \frac{\partial}{\partial t} S \times e_c, \]  
(1) Here, \( S(x, t) \): scalar function, \( b \): thickness.

Intero-Differential Equations

\[ \frac{\partial}{\partial t} \int_0^L \left[ d \Omega(x) \times \frac{e_c(x)}{S(x, t)} \right] + \frac{2}{B} \frac{\partial}{\partial x} \]  
(2) Here, \( \Omega \): applied magnetic field by permanent magnet, \( \frac{\partial}{\partial x} \): average operator over the thickness of the HTS, \( E \): electric field.

J-E Constitutive Equation (Power Law)

\[ E(x, t) = E_c \left( \frac{j}{j_c} \right)^N, \]  
(3b)

Here, \( j_c \): critical current density, \( E_c \): critical electric field, \( N \): index.

Newton's law of motion

\[ m \frac{dv}{dt} = 2 \left( \nabla \times \frac{e_c}{S} \times B \right), \]  
(4) Here, \( m \): mass of container.

Initial and Boundary Conditions

\[ S = 0 \quad \text{at} \quad t = 0, \quad v = v_0 \quad \text{at} \quad t = 0, \quad z = z_0 \quad \text{at} \quad t = 0, \quad \frac{S}{B} = 0 \quad \text{on} \quad \partial \Omega. \]  
(5a)

Ordinary Differential Equations (ODEs)

\[ \frac{dv}{dt} = \left[ \sum \frac{a_j}{c} (S) \right], \]  
(6)

B. Equivalent Circuit Model

Faraday's law

\[ \frac{dI}{dt} = L \left( M(z, t) \frac{dI}{dt} + \frac{dM}{dz} \right) + \epsilon. \]  
(7)

Here, \( I \): self-inductance of HTS current loop; \( M \): mutual inductance between the coil current \( I_c \) and a shielding current \( I \); \( \epsilon \): induced electromotive force of the HTS current loop, \( \epsilon = \epsilon_c (I_c) \epsilon_c \), \( \epsilon_c \): critical current and \( \epsilon_c \): critical voltage.

ODEs

\[ \frac{dv}{dt} = \frac{1}{M(z, t) \frac{dI}{dt} + \frac{dM}{dz} + \epsilon_c (I_c) \epsilon_c}. \]  
(8)

Parameters

\[ R_s = 5 \text{ cm}, \quad H_l = 10 \text{ cm}, \quad m = 10 \text{ g}, \quad v_0 = 0 \text{ m/s}, \quad N = 20, \quad E_c = 1 \text{ mV/m}, \quad j_c = 1 \text{ MA/cm}^2, \quad a = 7 \text{ cm}, \quad b = 1 \text{ mm}, \quad R = 3.5 \text{ cm}, \quad W = 5 \text{ mm}. \]  
(9)

II. GOVERNING EQUATION AND MOTION EQUATION

III. SIMULATION OF PELLETT INJECTION SYSTEM

A. Single Coil for FEM and Equivalent Circuit Models

Dependence of the coil current \( I \) and position \( z \) on the time \( t \) for the case with \( \alpha = 20 \text{ kA/ms} \) and \( z_0 = 1 \text{ mm} \). The coil current:

\[ I_c(t, z) = \begin{cases} \alpha t & (0 \leq z \leq z_{\text{limit}}) \\ 0 & \text{(otherwise)} \end{cases}, \]  
where \( z_{\text{limit}} \) is limit of acceleration region, and its value is fixed as \( z_{\text{limit}} = 20 \text{ cm} \).

B. Multiple Coils for Equivalent Circuit Model

Here, \( \alpha t = \text{mod} (z + p/2, z_b) - z_b/2 \), where \( z_{\text{b}} \) is coil interval and \( z_{\text{b}}\) is the time at \( z_{\text{limit}} = 0 \).

Acceleration Region

Dependence of the velocity \( v \) on the time \( t \) for \( \alpha = 20 \text{ kA/ms} \) and \( z_{\text{limit}} = 5 \).

Dependence of the final velocity \( v_f \) on the initial position \( z_0 \) for the case with \( \alpha = 20 \text{ kA/ms} \) and \( z_{\text{limit}} = 5 \). The inset indicates the velocity \( v \) on the time \( t \) for \( z_0 = 1 \text{ mm} \) and \( z_{\text{limit}} = 5 \).

Dependence of the final velocity \( v_f \) on the initial position \( z_0 \) for the case with \( \alpha = 20 \text{ kA/ms} \) and \( z_{\text{limit}} = 5 \).

Dependence of the velocity \( v \) on the position \( z \) for the case with \( \alpha = 20 \text{ kA/ms} \) and \( z_{\text{limit}} = 5 \).

IV. CONCLUSION

Although the velocity for the FEM model is larger than that for the circuit model, the behavior of the velocity hardly change qualitatively. However, the FEM model is quite time-consuming because requires a large number of FEM nodes.

As the initial position approaches the origin, the acceleration performance improves.

The results of the computations show that the velocity increases with the increasing ratio. As a result, the increasing ratio of the coil is preferably as large as possible.

The pellet injection system using the SELS has the acceleration performance similar to the centrifugal acceleration method.