Towards real time computation of 3D magnetic field in parametrized Polyhelix magnets using a reduced basis Biot-Savart model

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Background

The Laboratoire Nationale des Champs Magnétiques Intenses (LNCMI) is the French facility enabling researchers to perform experiments in the highest possible magnetic fields. The numerical modeling jointly developed with the Center of Modeling and Simulation of Strasbourg (Cemosis) plays an essential role in the understanding and the optimization of magnets made of set of Copper Alloy tubes cut by EDM technics connected in series. In this context, it is important to be able to compute the different fields of the model efficiently, in order to reach a real time evaluation. It is done by using the Reduced Basis Method.

Objectives

- Design a Reduced Basis method for Biot & Savart law, allowing to change experimental (conductivity, heat transfer coefficient,...) or geometric (helices shape, radius,...) parameters.
- Compare Finite Element and Reduced Basis (RB) results, and illustrate the use of the method with different applications.

Conclusion & Perspectives

- A Finite Element, full 3D, parallel, multi-physics model, coupling thermoelectric, magneto static (Maxwell and Biot & Savart) and mechanic.
- A Reduced Basis method for Biot & Savart allowing a real time computation for the magnetic field in the region of interest.
- Use of the Discrete Empirical Interpolation Method (DEIM) to deal with geometric parameters, giving the opportunity to do geometrical optimization.
- In the future, use Simultaneous EIM and RB (SER) to deal with non linearities and to accelerate the DEIM offline phase.
- Integrate the RB models into a tool for magnet designers without numerical simulation knowledge to visualize the 3D magnetic field maps in real time while changing parameters.

Methods

Equations

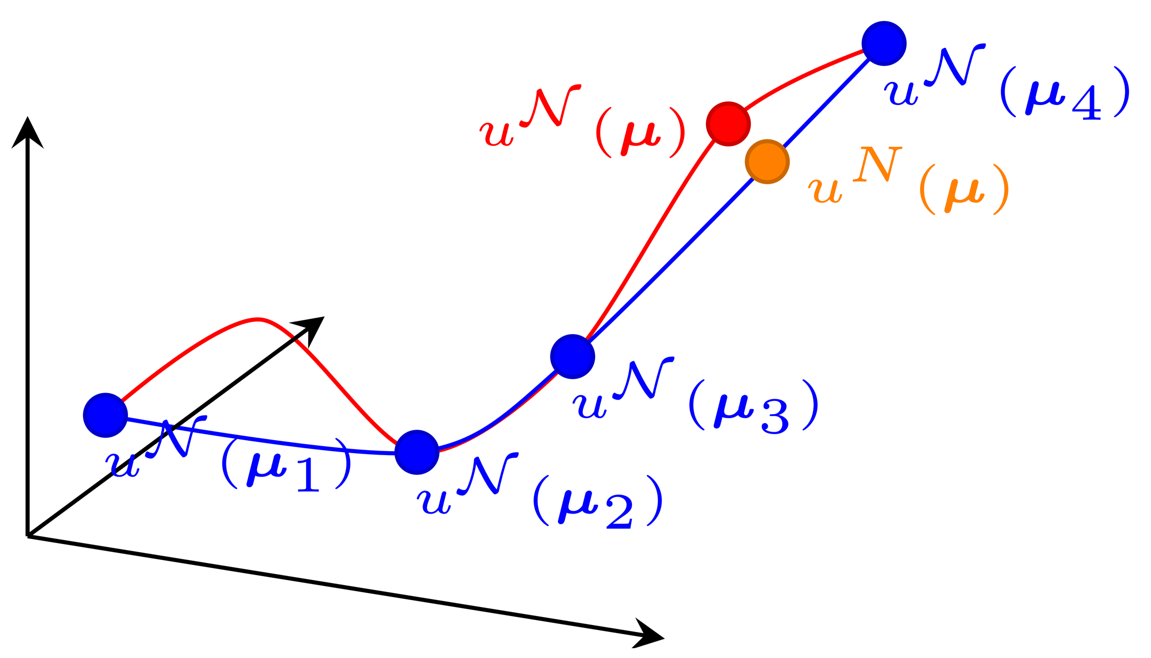
ThermoElectric:

$$\begin{cases} -\nabla\cdot\sigma(T)\nabla V = 0 & \text{in } \Omega_C \\ -\sigma(T)\nabla V\cdot \boldsymbol{n} = 0 & \text{on } \Gamma_N \\ V = V_D & \text{on } \Gamma_I \\ V = 0 & \text{on } \Gamma_O \end{cases} \qquad \begin{cases} -\nabla\cdot k(T)\nabla T = \sigma(T)\nabla V\cdot\nabla V & \text{in } \Omega_C \\ -k(T)\nabla T\cdot\boldsymbol{n} = h(T-T_w) & \text{on } \Gamma_C \\ -k(T)\nabla T\cdot\boldsymbol{n} = 0 & \text{on } \Gamma_E \end{cases}$$

Biot & Savart:

$$\boldsymbol{B}(\mathbf{x};\boldsymbol{\mu}) = \frac{\mu_0}{4\pi}\int_{\Omega_C}\frac{-\sigma(T)\nabla V(\mathbf{r})\times(\mathbf{x}-\mathbf{r})}{|\mathbf{x}-\mathbf{r}|^3}d\mathbf{r} \qquad \forall \mathbf{x}\in\Omega_M$$

Reduced Basis Method



The goal is to project the equations on a space more appropriate.

- Rewrite the (bi)linear forms to separate the parameters from the forms to have an **affine decomposition**: $a(u,v;\boldsymbol{\mu}) = \sum_{q=0}^{Q^a}\Theta_q^a(\boldsymbol{\mu})a_q(u,v)$
- Find the best parameters to approach the problem, via a **POD** or a **greedy algorithm**
- Compute the FE solution for those parameters and precompute the matrices a_q for those basis during the **offline phase**
- During the **online phase**, we assemble the matrix with the coefficient and solve the problem of reduced size

Reduced Basis for Biot & Savart

Reduced Basis for V and EIM for $\sigma(T)$:

$$V = \sum_{n=1}^{N}\Theta_n(\boldsymbol{\mu})\xi_n^V \qquad \sigma(T) = \sum_{m_\sigma=1}^{M_\sigma}\beta(\boldsymbol{\mu})q_{m_\sigma}$$

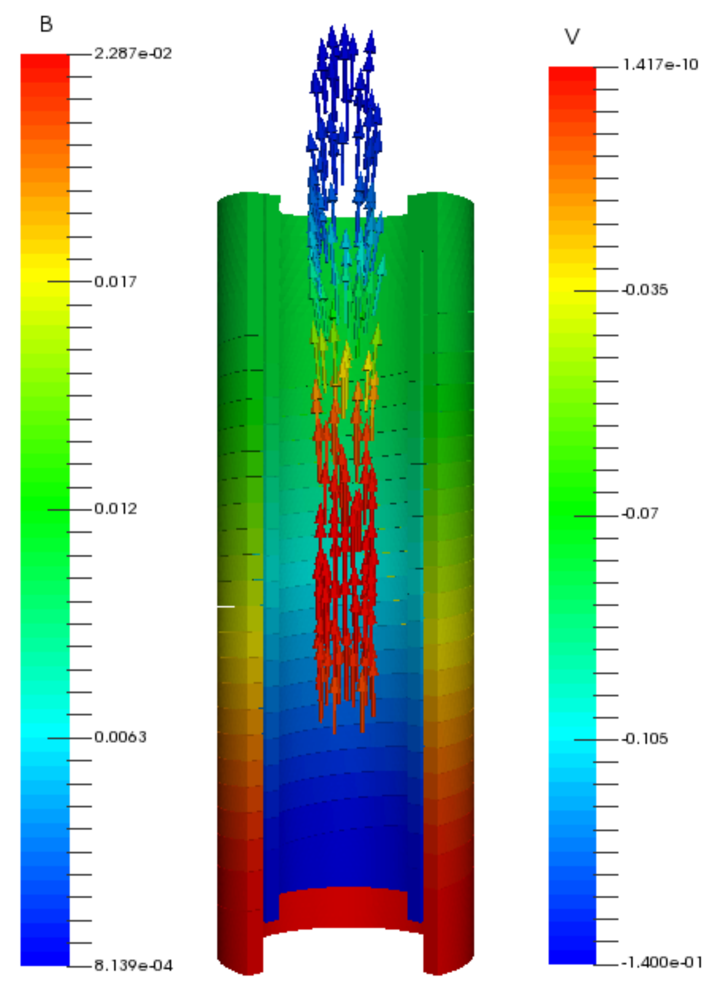
Reduced Basis for Biot & Savart:

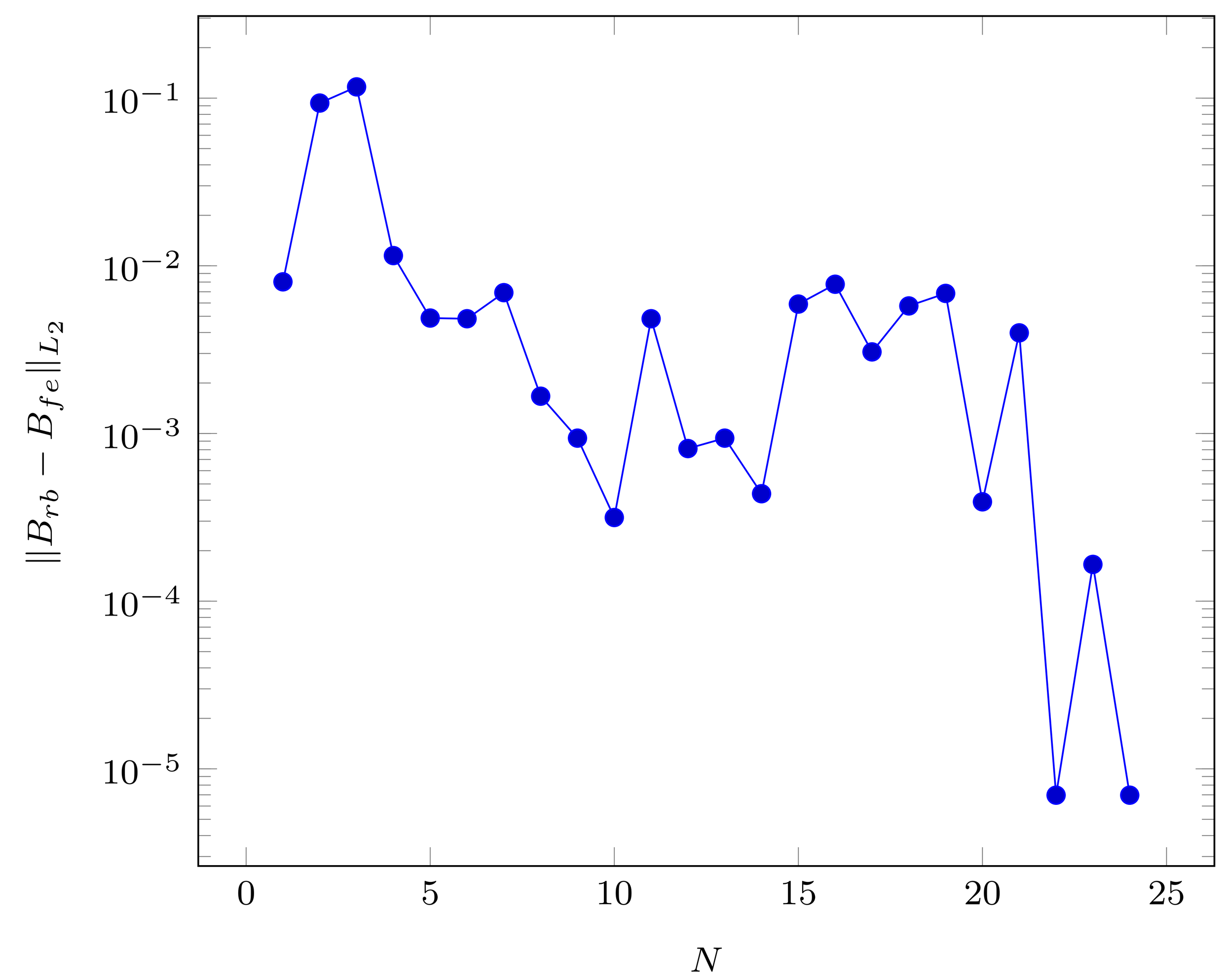
$$\mathbf{B}(\mathbf{x};\boldsymbol{\mu}) = \sum_{n=1}^{N}\sum_{m_\sigma=1}^{M_\sigma}\Theta_n(\boldsymbol{\mu})\beta_{m_\sigma}^{\sigma}(\boldsymbol{\mu})\frac{\mu_0}{4\pi}\int_{\Omega_C}\frac{-q_{m_\sigma}(\mathbf{r})\nabla\xi_n^V(\mathbf{r})\times(\mathbf{x}-\mathbf{r})}{|\mathbf{x}-\mathbf{r}|^3}d\mathbf{r} \qquad \forall\mathbf{x}\in\Omega_M$$

In case of geometric parameter, the distance $|\mathbf{x}-\mathbf{r}|$ can not be easily decomposed, we then use the Discrete Empirical Interpolation Method to approximate the magnetic field: $\boldsymbol{B}(\mathbf{x};\boldsymbol{\mu}) \approx \sum_{m=1}^{M}\beta_m(\boldsymbol{\mu})\boldsymbol{B}_m(\mathbf{x})$

Results

Comparison between Finite Element and Reduced Basis





Linear case:

$$\sigma(T) = \sigma(T_0) \quad k(T) = k(T_0)$$

Dimensions of the problem:

Ω_C: 1 350 000 dofs Ω_M: 3000 dofs

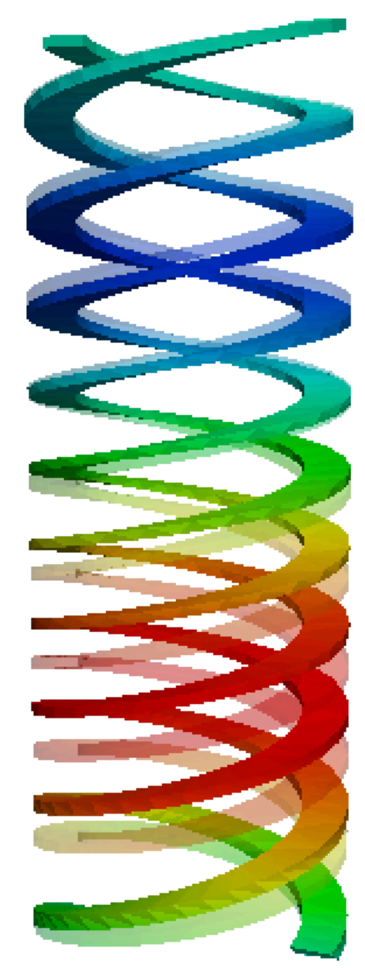
Parameters:

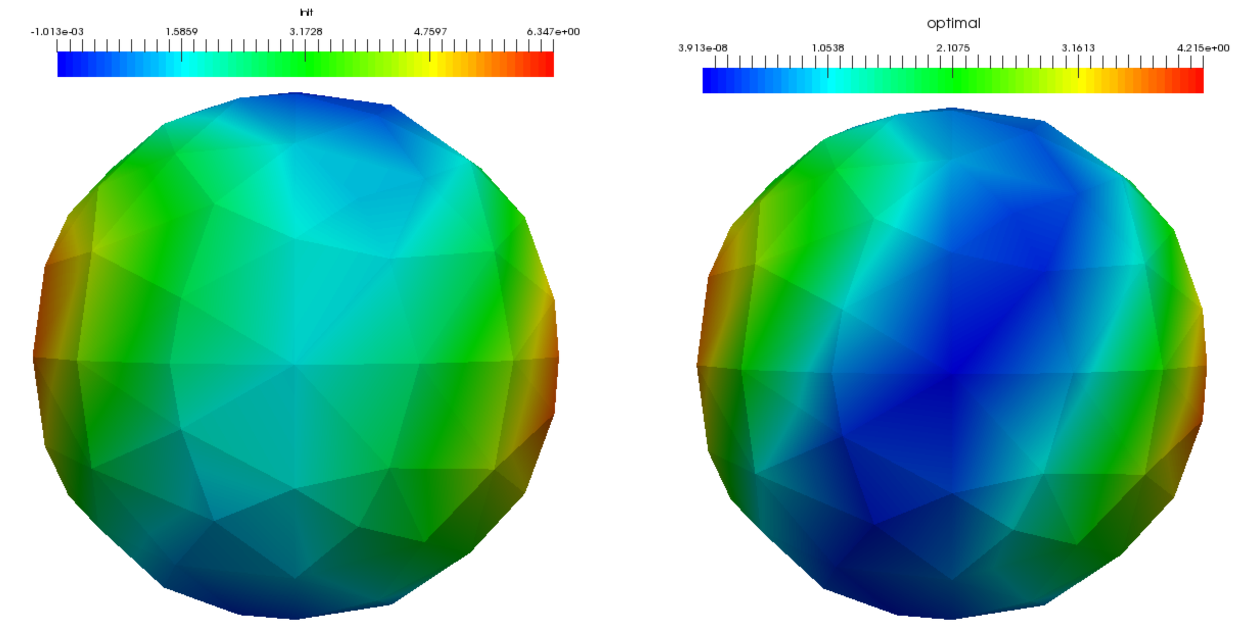
$$\sigma(T),\ k(T),\ V_D,\ h,\ T_w$$

Computation time:

	FE	RB
V	45s	0.05s
B	110s	0.0005s

Optimization of the homogeneity by controlling the shape of the helices





On the left: optimized shape of the helices with the initial shape in transparency.
On the top left: ppm in the initial shape.
On the top right: ppm in the optimized shape.

We apply Φ, a geometric transformation to modify the shapes of the helices to attain a better homogeneity

$$\Phi = \begin{pmatrix} x\cos(\alpha)+y\sin(\alpha) \\ -x\sin(\alpha)+y\cos(\alpha) \\ z \end{pmatrix}$$

where α is a **Bezier curve** which **control points** are the **parameters** of the **optimization**.

	init	optimal
$\max\|B_z\|$	0.41337	0.4051
$\min\|B_z\|$	0.41076	0.4034
$\frac{\max\|B_z\|}{\min\|B_z\|}-1$	5381.9	3544.9

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