Muon System Performance

FCC Hadron Detector Meeting
Dec., 9th 2015

W. Riegler

Dates for Next Meetings

Next hadron detector meetings:

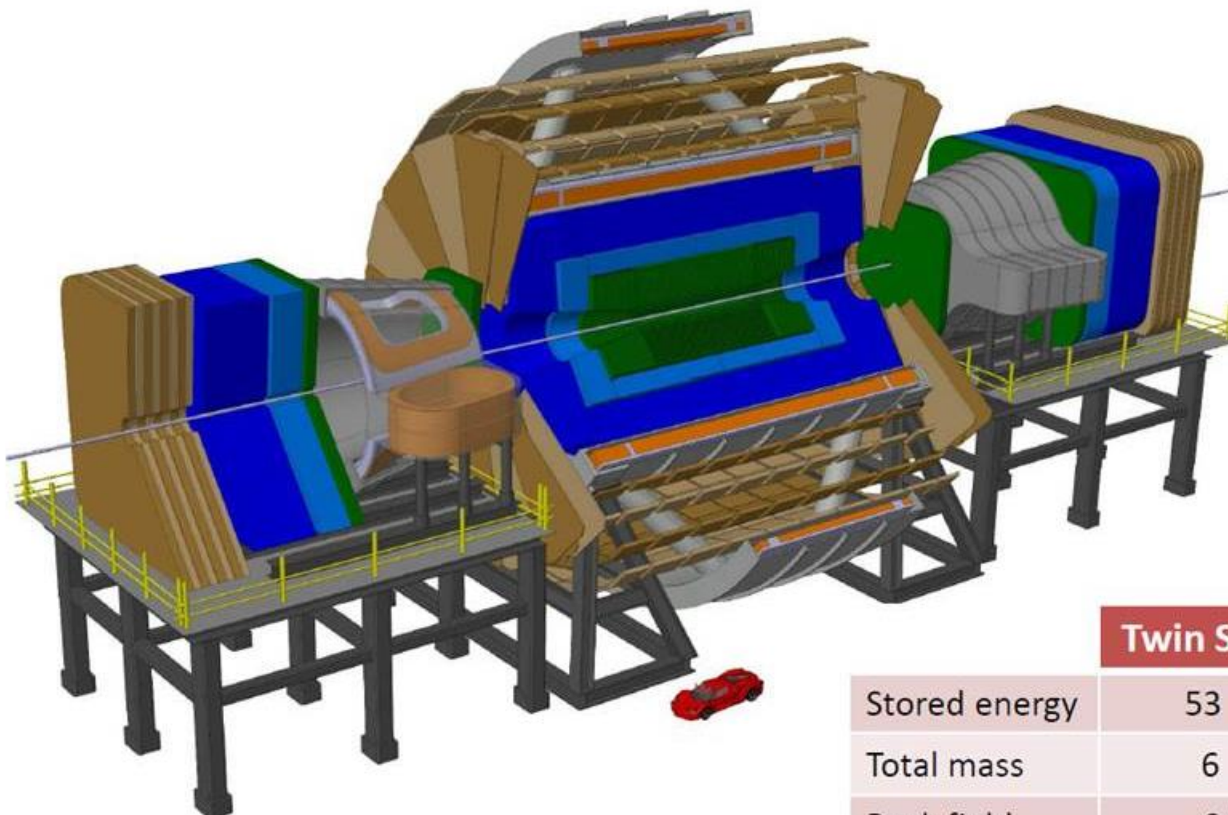
Jan. 21, 2016
Mar. 03, 2016
Apr. 06, 2016

FCC week Rome
Apr. 11-15 2016

https://indico.cern.ch/category/6069/

Twin Solenoid + Dipole Magnet System

Matthias Mentink, Alexey Dudarev, Helder Filipe Pais Da Silva, Christophe Paul Berriaud, Gabriella Rolando, Rosalinde Pots, Benoit Cure, Andrea Gaddi, Vyacheslav Klyukhin, Hubert Gerwig, Udo Wagner, and Herman ten Kate

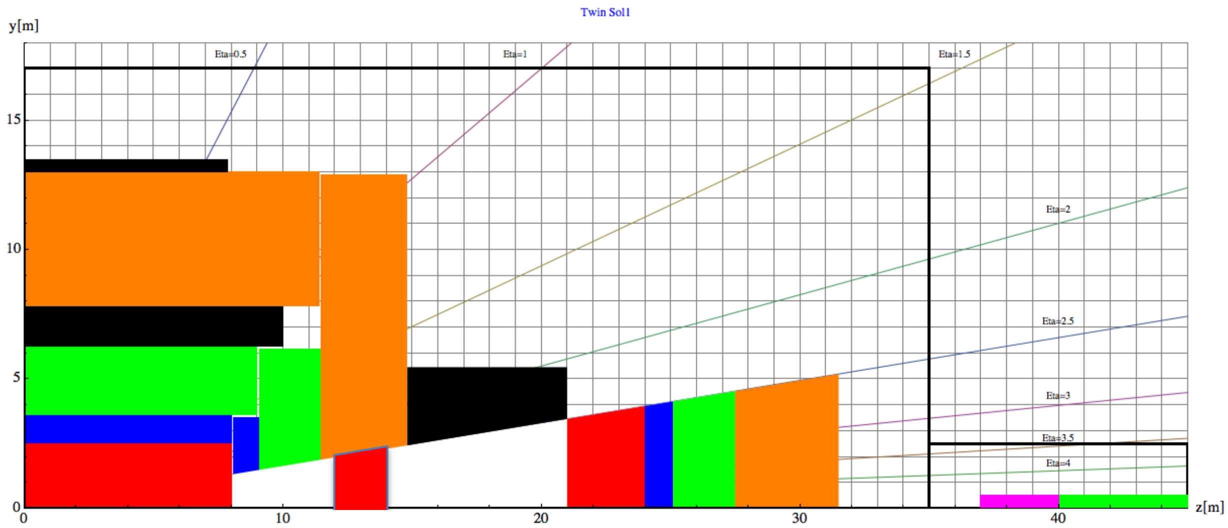


FCC Air core Twin solenoid and Dipoles

State of the art high stress / low mass design.

	Twin Solenoid	Dipole
Stored energy	53 GJ	2 x 1.5 GJ
Total mass	6 kt	0.5 kt
Peak field	6.5 T	6.0 T
Current	80 kA	20 kA
Conductor	102 km	2 x 37 km
Bore x Length	12 m x 20 m	6 m x 6 m

Baseline Geometry, Twin Solenoid



Barrel:

Tracker available space:
R=2.1cm to R=2.5m, L=8m

EMCAL available space:
R=2.5m to R= 3.6m → dR= 1.1m

HCAL available space:
R= 3.6m to R=6.0m → dR=2.4m

Coil+Cryostat:
R= 6m to R= 7.825 → dR = 1.575m, L=10.1m

Muon available space:
R= 7.825m to R= 13m → dR = 5.175m

Coil2:
R=13m to R=13.47m → dR=0.475m, L=7.6m

Endcap:

EMCAL available space:
z=8m to z= 9.1m → dz= 1.1m

HCAL available space:
z= 9.1m to z=11.5m → dz=2.4m

Muon available space:
z= 11.5m to z= 14.8m → dz = 3.3m

Forward:

Dipole:
z= 14.8m to z= 21m → dz=6.2m

FTracker available space:
z=21m to R=24m, L=3m

FEMCAL available space:
Z=24m to z= 25.1m → dz= 1.1m

FHCAL available space:
z= 25.1m to z=27.5m → dz=2.4m

FMuon available space:
z= 27.5m to z=31.5m → dz=4m

CMS Muon Performance

e the first muon station, the offline
ibout 9% for small values of η and p
dalone momentum resolution varies
tum fit using also the inner tracker

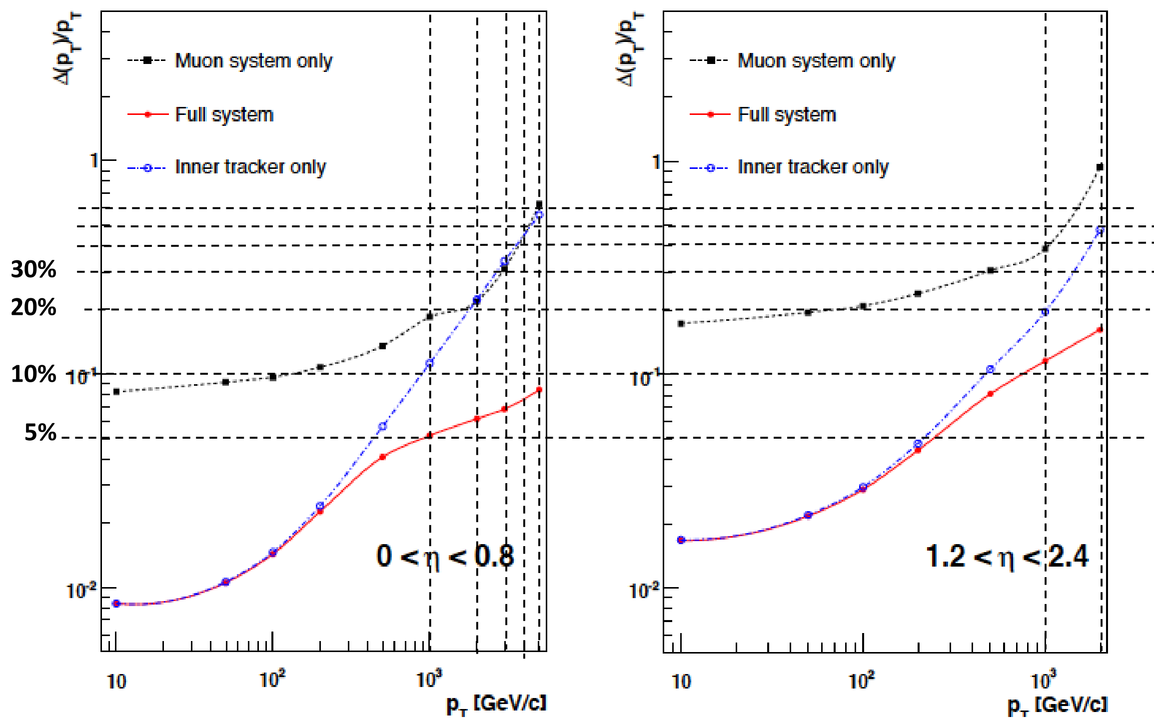


Figure 1.2: The muon transverse-momentum resolution as a function of the transverse-momentum (p_T) using the muon system only, the inner tracking only, and both. Left panel: $|\eta| < 0.8$, right panel: $1.2 < |\eta| < 2.4$.

P_T=1TeV/c, 0<eta < 0.8:
- 20% muon standalone (angle)
- 10% inner tracker only
- 5% combined

P_T=1TeV/c, eta 0<eta<2.4:
- 40% muon standalone (angle)
- 20% inner tracker only
- 10% combined

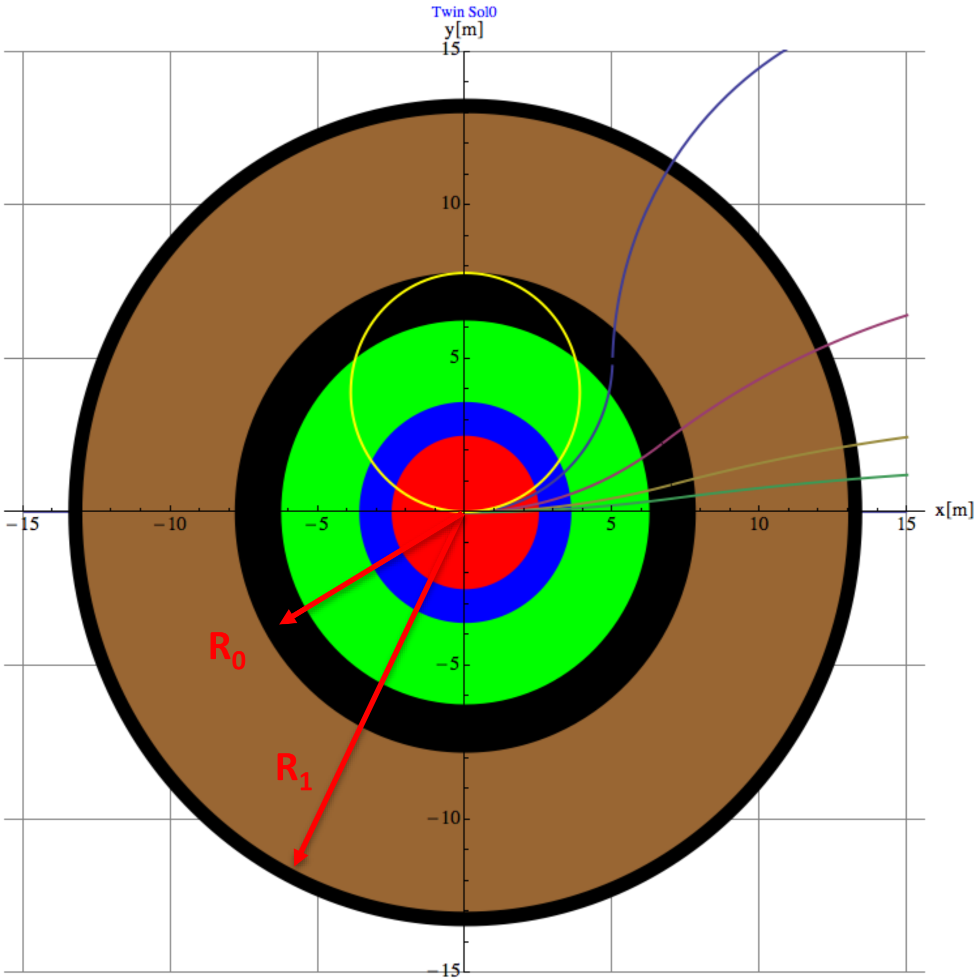
Muon Momentum can be measured by

1) The inner tracker

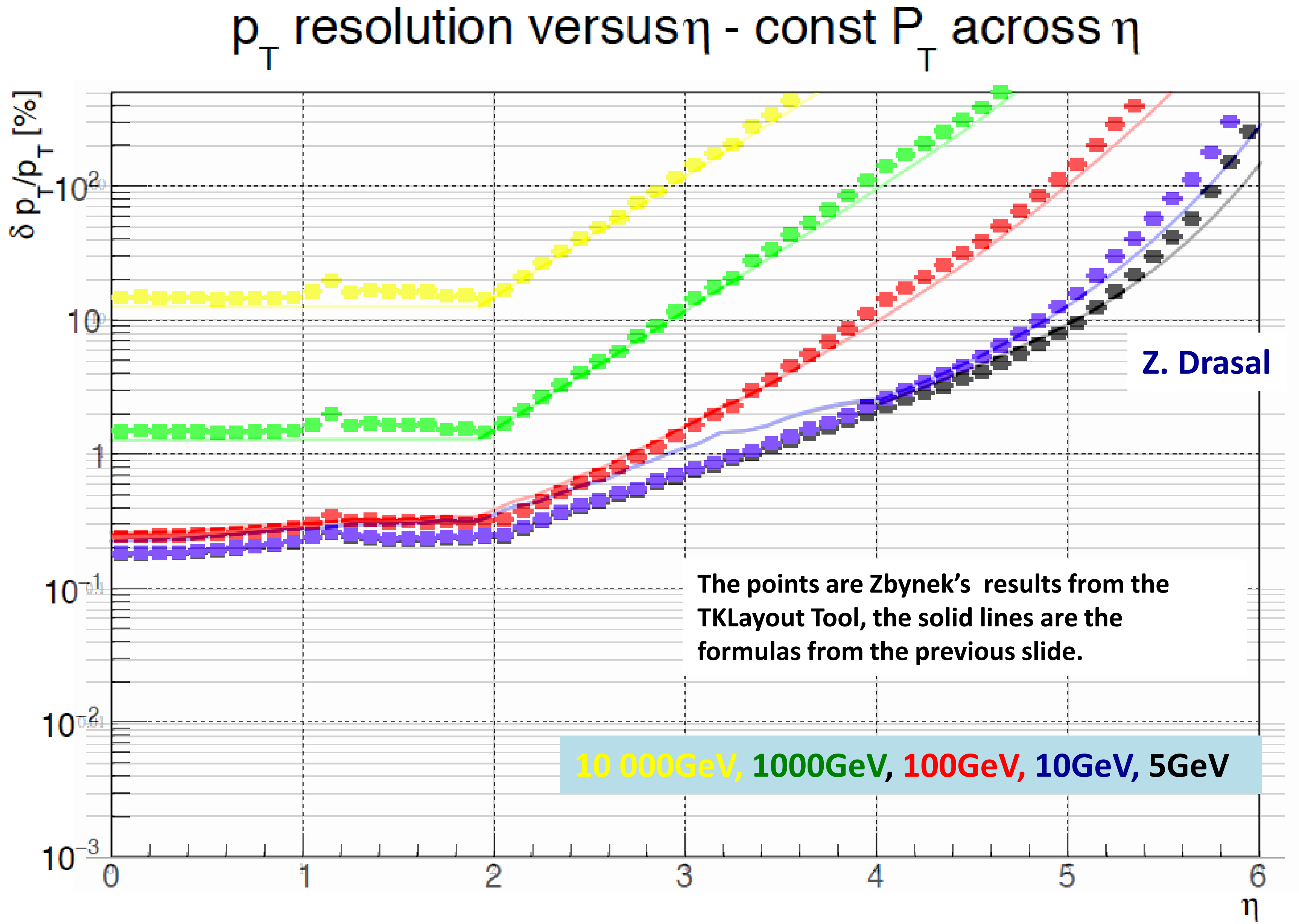
2) The track angle at the entrance of the muon system → Trigger

3) The combined fit of inner tracker and outer layers of the muon system.

4) A sagitta measurement in the muon system

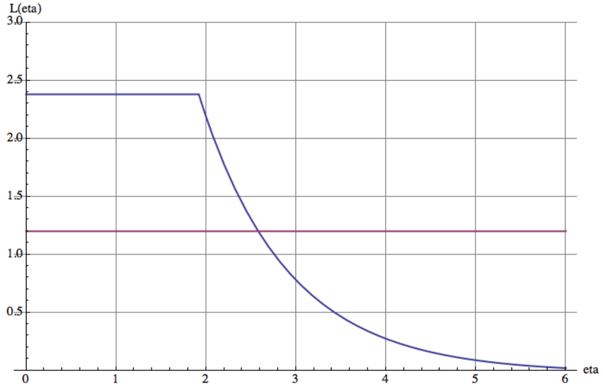


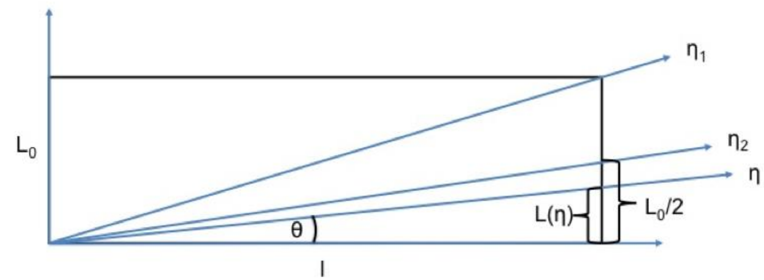
Tracker



p_T resolution versus η - const P_T across η
$\delta p_T/p_T$ [%]
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
0
1
2
3
4
5
6
η
Z. Drasal
The points are Zbynek's results from the TKLayout Tool, the solid lines are the formulas from the previous slide.
10 000GeV, 1000GeV, 100GeV, 10GeV, 5GeV

Tracker

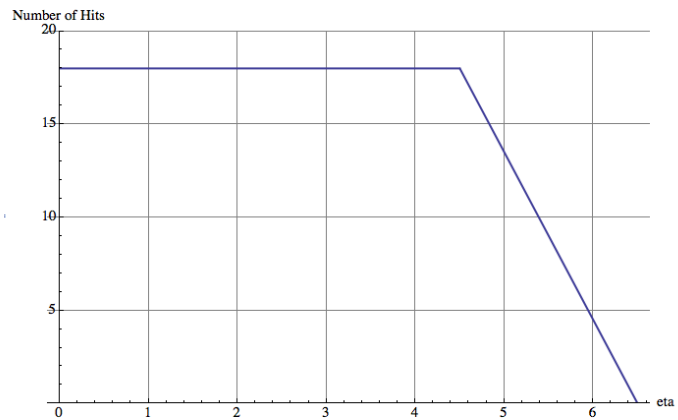




$$\eta_1 = -\ln\tan\left(\frac{1}{2}\arctan\frac{L_0}{l}\right) \qquad \eta_2 = -\ln\tan\left(\frac{1}{2}\arctan\frac{L_0}{2l}\right)$$

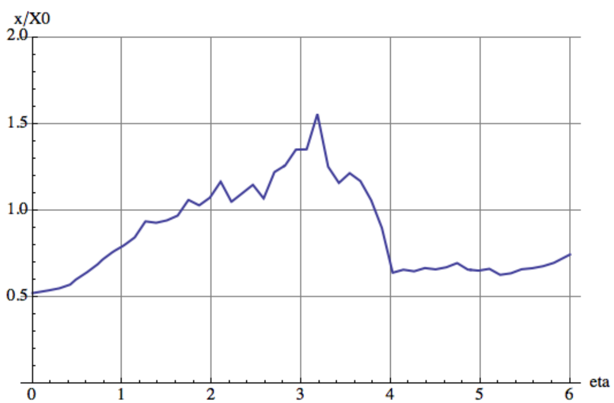
For a geometry with $L_0 = 2.4m$ and $l = 8m$ we have $\eta_1 = 1.9$ and $\eta_2 = 2.6$

$$L(\eta) = L_0 \quad \eta < \eta_1 \qquad\qquad L(\eta) = L_0\frac{\sinh\eta_1}{\sinh\eta} \quad \eta > \eta_1$$



$$\frac{\Delta p_T}{p_T}\Big|_{reso.} = \frac{\sigma\, p_T}{0.3BL(\eta)^2}\sqrt{\frac{720}{N(\eta)+4}} \qquad \frac{\Delta p_T}{p_T}\Big|_{m.s.} = \frac{0.0136}{0.3\,BL(\eta)}\sqrt{\frac{x}{X_0}(\eta)}$$

$$\frac{\Delta p_T}{p_T} = \sqrt{\left(\frac{\Delta p_T}{p_T}\Big|_{reso.}\right)^2 + \left(\frac{\Delta p_T}{p_T}\Big|_{m.s.}\right)^2}$$

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In[60]:= L0 = 2.4;

In[61]:= l = 8;

In[62]:= B = 6;

In[63]:= sig = 23 * 10 ^ (-6);
```

2) Muon measurement by track angle at the entrance of the muon system

$$\frac{\Delta p_T}{p_T} = \Delta\theta \sqrt{\left(\frac{2p_T}{0.3B_0R_0}\right)^2 - 1} \approx \frac{2p_T}{0.3B_0R_0}\Delta\theta \quad \text{for a large } p_T$$

If we want 10% at 10TeV for B_0=6T, R_0=6m
We need $\Delta\theta$=50μRad

→ 2 stations at 1.5m distance with 50um position resolution

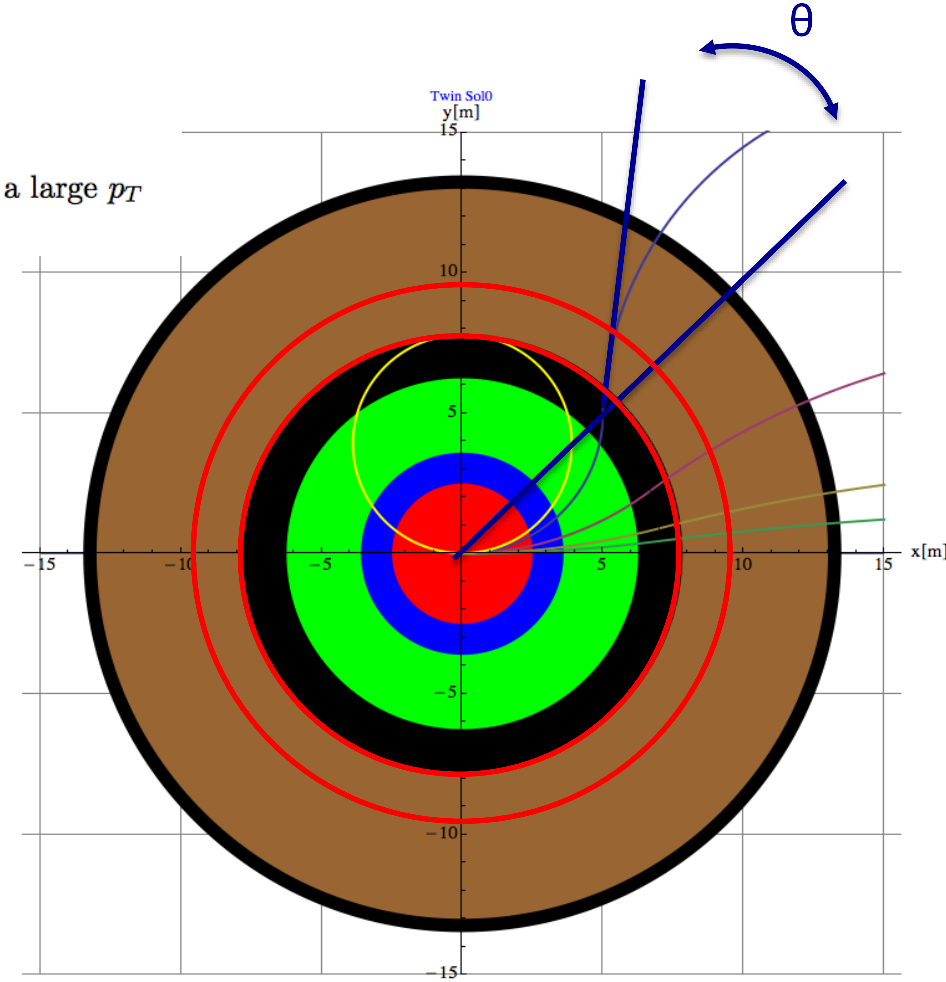
For low momentum, limit due to multiple scattering in the calorimeters and coil:

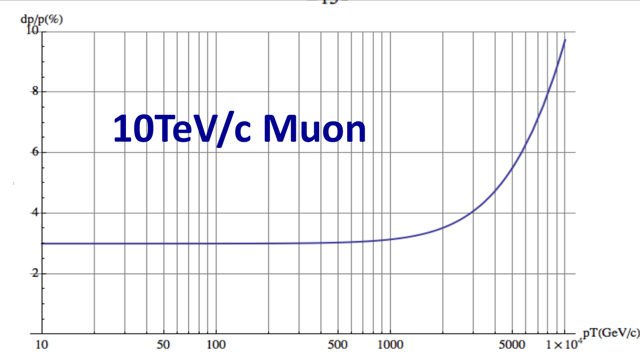
Calorimeter+Cryostat: $35X_0$
HCAL: $110X_0$
Coil: $5X_0$
→ $x_{tot}/X_0 \approx 150$

$\Delta\theta = 0.0136/p_T * \mathrm{Sqrt}(x_{tot}/X_0)$

$$\frac{\Delta p_T}{p_T} = \frac{2 \times 0.0136}{0.3B_0R_0}\sqrt{\frac{x_{tot}}{X_0}}$$

B_0=6T, R_0=6m → $dp_T/p_T \approx 3\%$
This is approximate for an infinitely thin coil and for η=0.





3) Muon Measurement by combined measurement of inner tracker and muon system

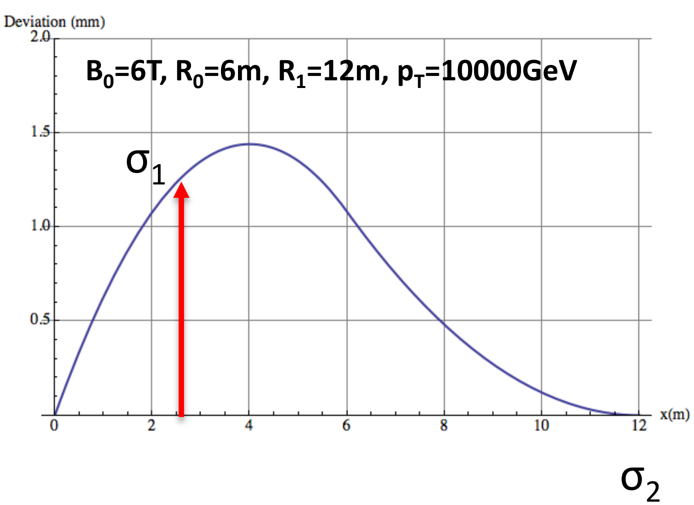
If the full flux is returned trough the muon system, the muon trajectory at the exit of the system points exactly to the IP !

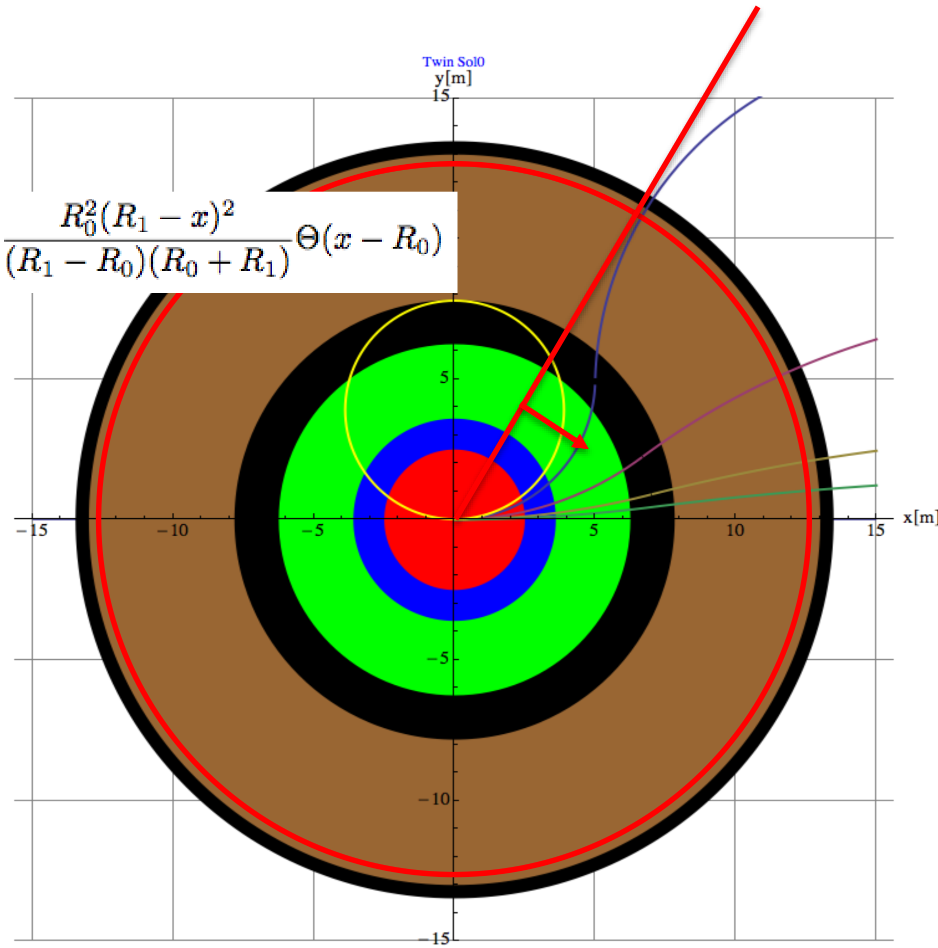
$$y_t(x) = \frac{0.3B_0}{2p_T}\left(x^2 - \frac{2R_0R_1}{R_0+R_1}x\right)\Theta(R_0-x) - \frac{0.3B_0}{2p_T}\frac{R_0^2(R_1-x)^2}{(R_1-R_0)(R_0+R_1)}\Theta(x-R_0)$$

The maximum excursion $y_t(x_0)$ is always at the same radial distance of x_0

$$x_0 = \frac{R_0R_1}{R_0+R_1} \qquad y_t(x_0) = -\frac{0.3B_0}{2p_T}x_0^2 = -\frac{0.3B_0}{2p_T}\left(\frac{R_0R_1}{R_0+R_1}\right)^2$$

For values below: x_0=4m, y_t(x0)=1.44mm
Ideal measurement point is at the peak, but y_t(2.4m)= 1.24mm still good !





$\sigma^2=\sigma_1^2+(x/R_1\sigma_2)^2$

x=2.4m, R_1=12m, σ_1=50µm, σ_1=250µm,
σ=64µm, dp_T/p_T=5% at 10TeV !

Measuring just in the last tracker layer and in the outermost muon station already beats the full inner tracker performance (14 layers, 23um).

4) Muon measurement by sagitta measurement in the muon system

The return field is 2.45T

Measuring over the 5m lever arm with stations of sig=50um resolution we have

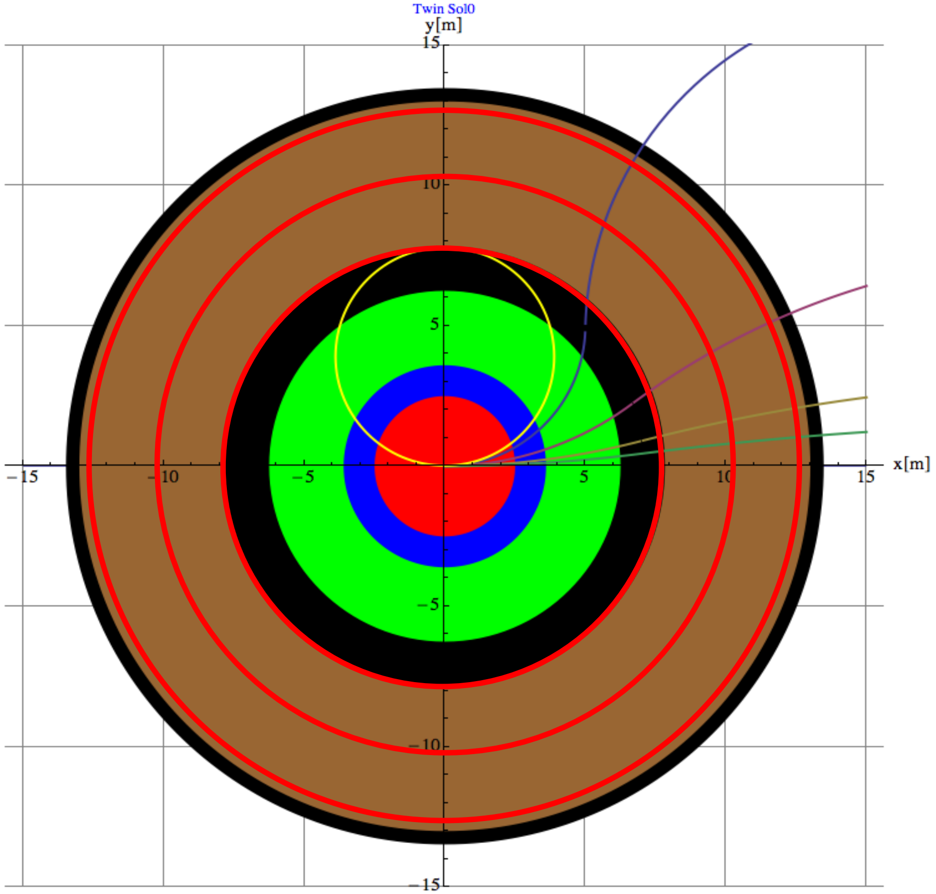
dp_T/p_T= sig*p_T/(0.3*B*L^2)*8
= 20% @ 10TeV

with possibly excellent performance at low p_T due to the absence of iron (vs. CMS) .

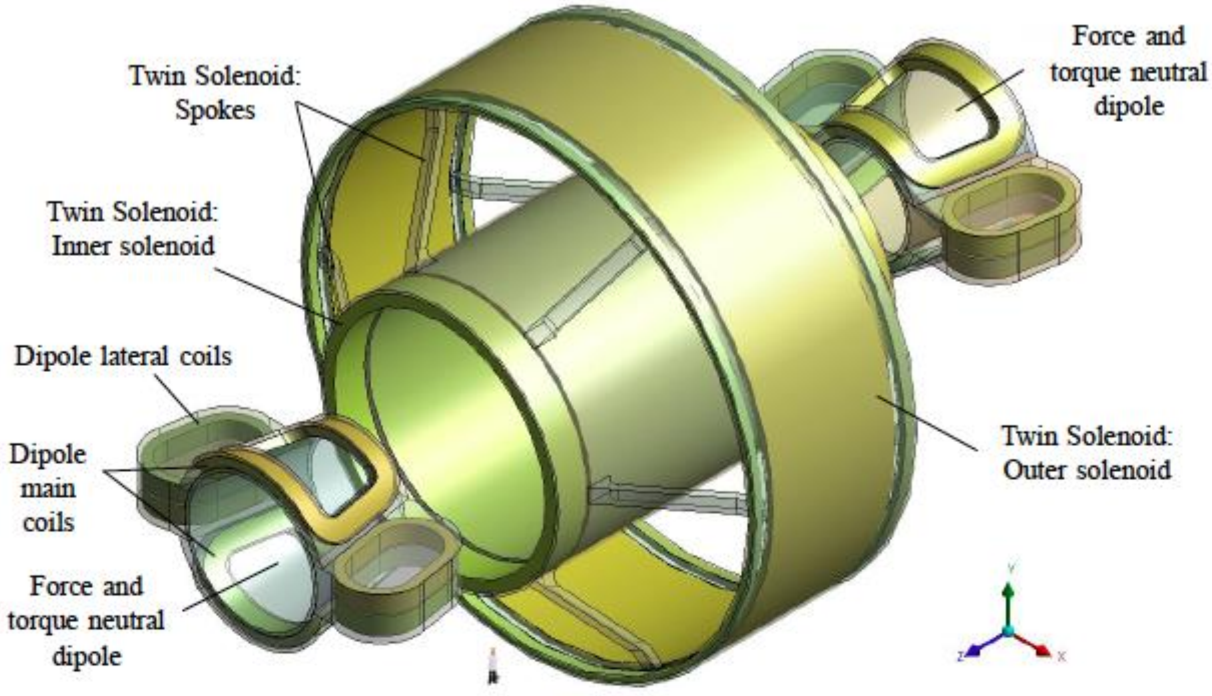
but very hard to beat the angular measurement at high p_T and the inner tracker at low p_T.

Surface > 5000 m^2

→ Seems not competitive



Muon performance versus η using the twin solenoid fieldmap and the baseline detector layout described in the previous meetings.



Muon Trajectories in B-fields

If we use the units $[x] = [s] = m, [B] = T, [p] = GeV/c, q = e_0$ we have

$$\frac{d^2\vec{x}(s)}{ds^2} = \frac{0.3}{p}\frac{d\vec{x}(s)}{ds} \times \vec{B}(\vec{x}(s)) \tag{157}$$

or explicitly

$$x''(s) = \frac{0.3}{p}\left[y'(s)B_z[x(s),y(s),z(s)] - z'(s)B_y[x(s),y(s),z(s)]\right] \tag{158}$$

$$y''(s) = \frac{0.3}{p}\left[z'(s)B_x[x(s),y(s),z(s)] - x'(s)B_z[x(s),y(s),z(s)]\right] \tag{159}$$

$$z''(s) = \frac{0.3}{p}\left[x'(s)B_y[x(s),y(s),z(s)] - y'(s)B_x[x(s),y(s),z(s)]\right] \tag{160}$$

The curvature is then given by

$$k = \frac{0.3}{p}\left|\frac{d\vec{x}(s)}{ds} \times \vec{B}(\vec{x}(s))\right| = \frac{0.3}{p}B_T(\vec{x}(s)) \tag{161}$$

Let's assume a straight line along a given η direction and a transverse B-field $B_T(s)$ along this line. Assuming rotational symmetry of the magnetic field, B_T is inside one plane. A high momentum particle of momentum p starting from the origin i.e. $y(0) = 0$. The trajectory $y(s)$ is defined by

$$k = \frac{y''(s)}{[1 + y'(s)^2]^{3/2}} = \frac{0.3}{p}B_T(s) \tag{162}$$

For small deflections, $y'(s) \ll 1$ and the equation becomes

$$y''(s) = \frac{0.3}{p}B_T(s) \tag{163}$$

Since it holds that

$$\oint \vec{B}(\vec{x})d\vec{A} = 0 \tag{164}$$

and in case the magnetic field is zero outside the shielding coil and at large values of z, it holds with $s = z/\cos\theta$

$$\int_0^a sB_T(s)ds = 0 \tag{165}$$

With this consition we have

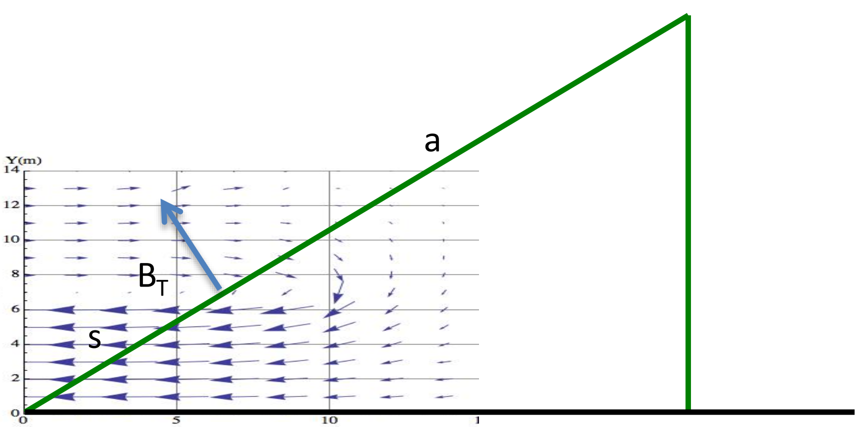
$$\int_0^a sy''(s)ds = 0 \tag{166}$$

$$sy'(s)|_0^a - \int_0^a y'(s)ds = 0 \tag{167}$$

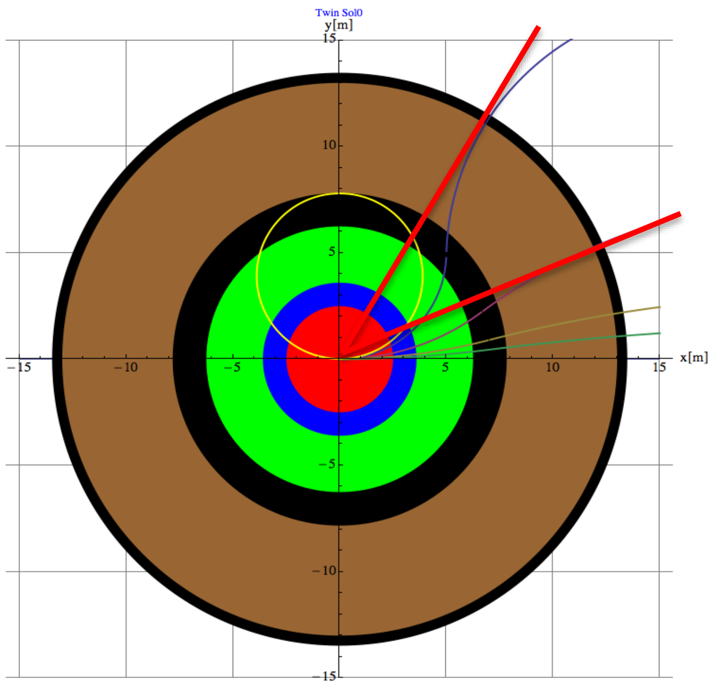
$$ay'(a) - y(a) + y(0) = 0 \tag{168}$$

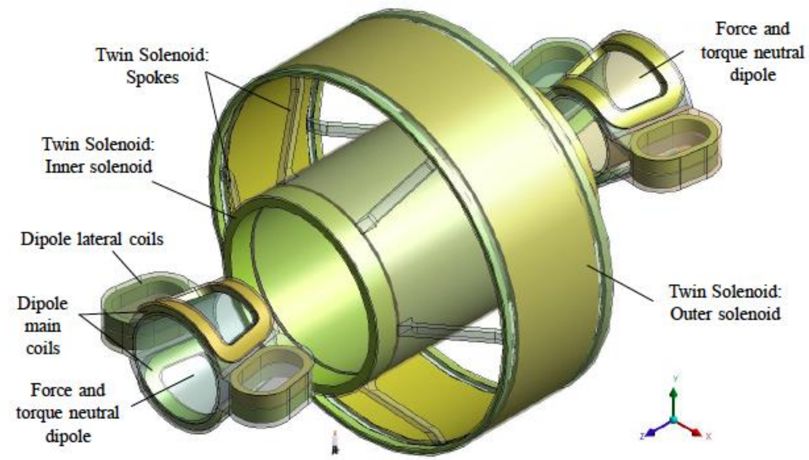
$$y'(a) = \frac{y(a)}{a} \tag{169}$$

so at the point $x = a$ the particle trajectory points back to the origin.



Flux Return and Muon Trajectories






```
In[156]:= tab = Import["/Users/riegler/Documents/X/FCC/Bfield/Bfield_may_2015.txt", "table"];

In[157]:= (*tab=Import["/Users/riegler/Documents/X/FCC/Bfield/Bfield_test.txt","table"];*)

In[158]:= n = Length[tab]

Out[158]= 624777

In[1756]:= eq1 = x''[s] == 0.3 / p * (y'[s] * Bz[x[s], y[s], z[s]] - z'[s] * By[x[s], y[s], z[s]]);

In[1757]:= eq2 = y''[s] == 0.3 / p * (z'[s] * Bx[x[s], y[s], z[s]] - x'[s] * Bz[x[s], y[s], z[s]]);

In[1758]:= eq3 = z''[s] == 0.3 / p * (x'[s] * By[x[s], y[s], z[s]] - y'[s] * Bx[x[s], y[s], z[s]]);

In[1759]:= (* X[s], Y[s], Z[s] --> parametrization of the muon trajectory *)

In[1760]:= {X, Y, Z} = NDSolveValue[{eq1, eq2, eq3, x'[0] == Sin[theta0] * Cos[phi], x[0] == 0, y'[0] == Sin[theta0] * Sin[phi], y[0] == 0, z'[0] == Cos[theta0], z[0] == 0},
    {x, y, z}, {s, sd[5, theta0]}]

Out[1760]= {InterpolatingFunction[{{0., 26.4379}}, <>], InterpolatingFunction[{{0., 26.4379}}, <>], InterpolatingFunction[{{0., 26.4379}}, <>]}
```

We assume a high momentum particle originating from the IP in a given θ, ϕ direction. The particle will deviate only by a small amount from a straight line along this direction and the deviation is given by the transverse magnetic field $\vec{B_T}(s)$ along this direction. We have $g''(s) = 0.3/pB_T(s) = kB_T(s)$. We let the particle start from the origin and therefore have $g(0) = 0$ and also assume that this point is measured with infinite precision. Since we don't know the original direction of the particle we have to allow a small rotation and we have a template function $f(s)$ given by

$$f(s) = k\, g(s) + \theta\, s \tag{170}$$

and we have to find the parameters k and θ that minimize the difference to the measurement points

$$\sum_{n=1}^{N} w_n \left[y_n - f(s_n)\right]^2 \quad \to \min \tag{171}$$

Defining $g_n = g(s_n)$ we have

$$F(k,\theta) = \sum_{n=1}^{N} w_n \left[y_n - k g_n - \theta s_n\right]^2 \qquad \frac{\partial F}{\partial k} = 0 \qquad \frac{\partial F}{\partial \theta} = 0 \tag{172}$$

$$k\, F_{gg} + \theta\, F_{gs} = F_{gy} \qquad k\, F_{gs} + \theta\, F_{ss} = F_{sy} \tag{173}$$

$$F_{gg} = \sum_{n=1}^{N} w_n g_n^2 \quad F_{gs} = \sum_{n=1}^{N} w_n s_n g_n \quad F_{ss} = \sum_{n=1}^{N} w_n s_n^2 \tag{174}$$

$$F_{sy} = \sum_{n=1}^{N} w_n s_n y_n \qquad F_{gy} = \sum_{n=1}^{N} w_n g_n y_n \tag{175}$$

$$k = \frac{F_{gs}F_{sy} - F_{gy}F_{ss}}{F_{gs}^2 - F_{gg}F_{ss}} \qquad \theta = \frac{F_{gs}F_{gy} - F_{gg}F_{sy}}{F_{gs}^2 - F_{gg}F_{ss}} \tag{176}$$

$$\Delta_k^2 = \sum_{n=1}^{N} \left(\frac{\partial k}{\partial y_n}\right)^2 \sigma_n^2 \tag{177}$$

$$\frac{\partial k}{\partial y_n} = \frac{w_n[F_{gs}s_n - F_{ss}g_n]}{F_{gs}^2 - F_{gg}F_{ss}} \tag{178}$$

and we have the general solution

$$\Delta_k^2 = \frac{F_{ss}^2 H_{gg} - 2F_{ss}F_{gs}H_{gs} + F_{sg}^2 H_{ss}}{(F_{gg}F_{ss} - F_{sg})^2} \tag{179}$$

with

$$H_{gg} = \sum_{n=1}^{N} \sigma_n^2 w_n^2 g_n^2 \qquad H_{gs} = \sum_{n=1}^{N} \sigma_n^2 w_n^2 g_n s_n \qquad H_{ss} = \sum_{n=1}^{N} \sigma_n^2 w_n^2 s_n^2 \tag{180}$$

Knowing the position of the detection layers s_n, the resolution of these layers σ_n and the value of the template function at the detection layer $g_n = g(s_n)$, the variance Δ_k^2 is only a function of the relative weights of the measurement points and has to be minimized with respect to these weights.

$$\Delta_k(w_2, w_3, ..., w_N) \to \min \tag{181}$$

For the momentum resolution we then have

$$k = \frac{0.3}{p} \qquad \to \qquad \frac{\sigma_p}{p} = \frac{0.3}{p}\Delta_k \tag{182}$$

For $N = 2$ we have

$$\Delta_k^2 = \frac{s_1^2\sigma_2^2 + s_2^2\sigma_1^2}{[s_1 g_2 - s_2 g_1]^2} \tag{183}$$

Analytic, back of the envelope, pseudo-precise determination of the muon momentum resolution using the tracker+muon system measurement.

Template function g(s) is calculated with MATHEMATICA based on the correct field map.

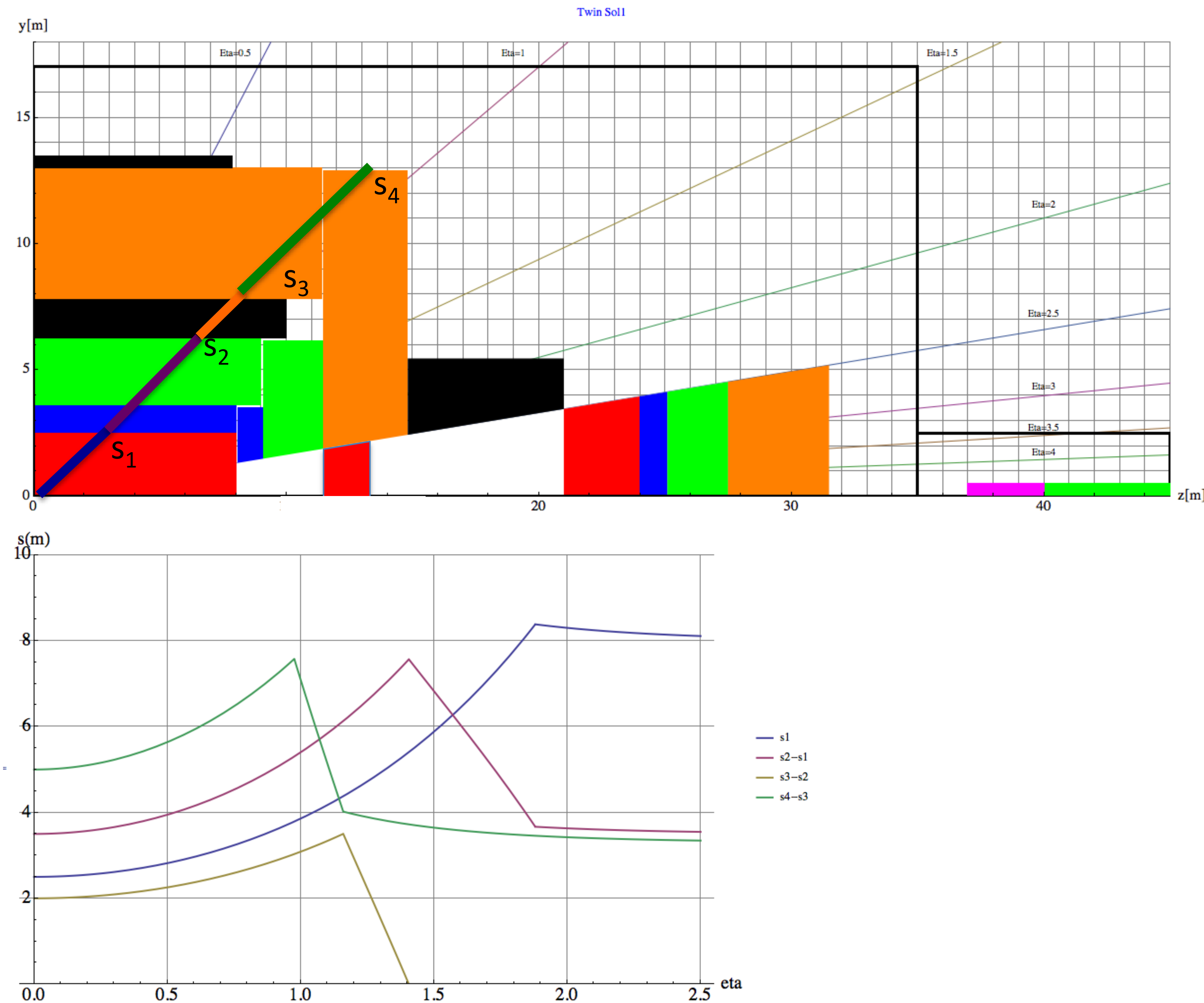
14 equally spaced layers of central tracker are assumed with a resolution of 23um per layer.

The layers in the muon system are then added with their respective resolution.

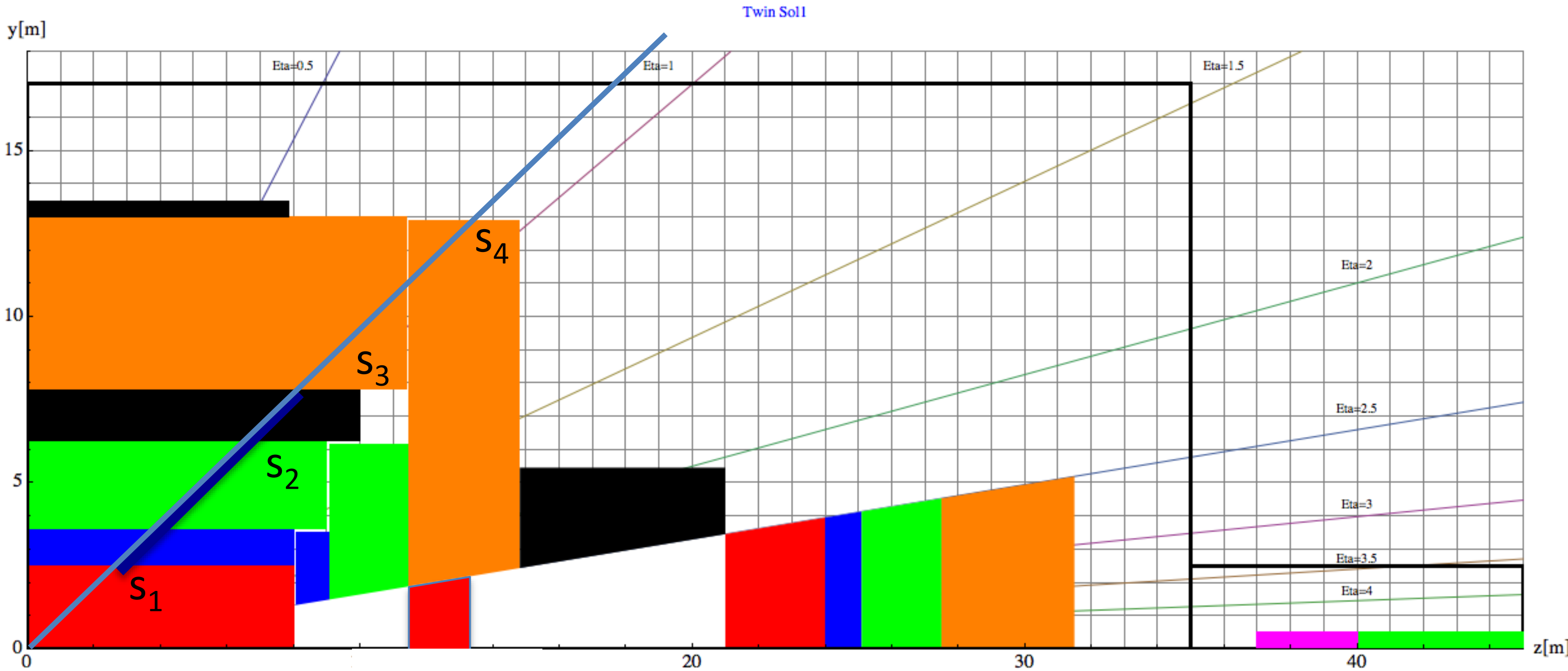
A global minimization of Δ_k for the weights w_n is performed.

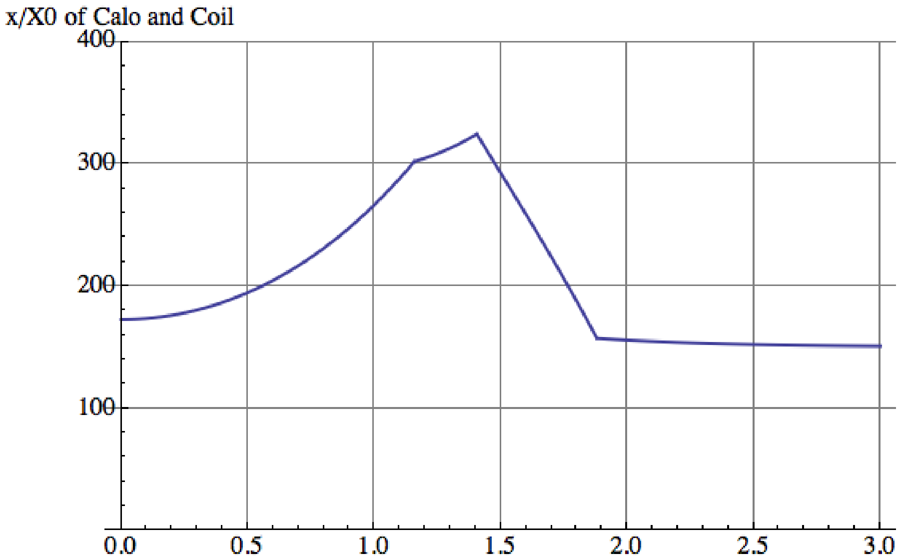
Before assuming specific resolution number for the muon system, a key question is: How is the position measurement in the muon system affected by the multiple scattering the in calorimeters ?

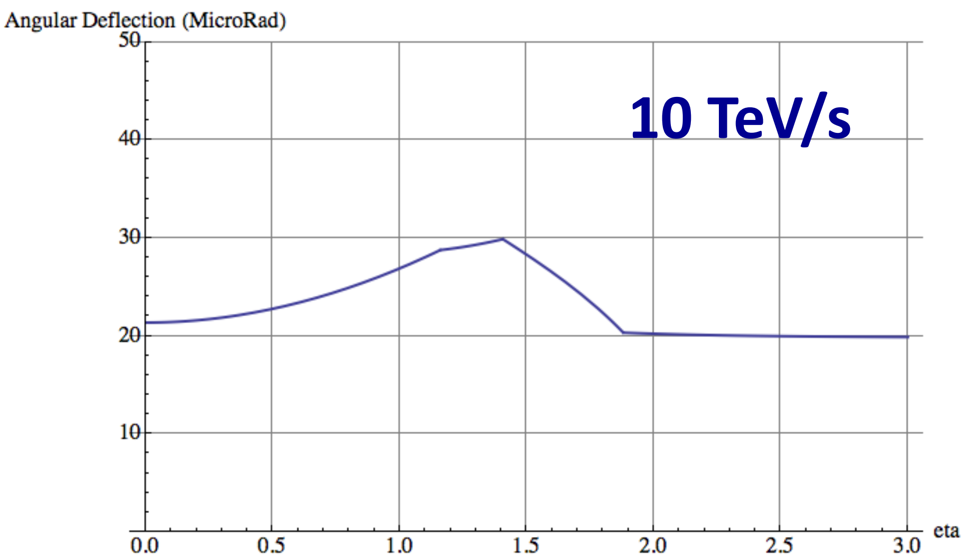
Thickness of different subsystems seen by a high energy muon



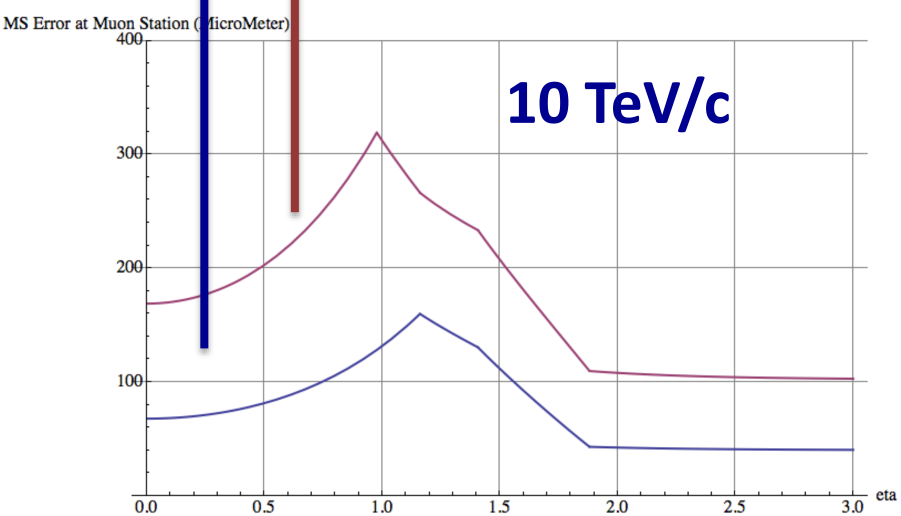
Radiation length and angular deflection of muons in calo+coil







Measurement error in the muon system due to multiple scatering



The r.m.s. (in plane) angular deflection θ_0 and spatial displacement σ_y of a particle passing through material of thickness x are given by

$$\theta_0 = \frac{0.0136}{\beta p[GeV/c]} \sqrt{\frac{x}{X_0}} \left(1 + 0.038 \log \frac{x}{X_0}\right) \tag{184}$$

$$\sigma_y = \frac{1}{\sqrt{3}} x \, \theta_0 \tag{185}$$

For the proper correlated M.C. generation of these two quantities, one uses two independent Gaussian random variables z_1, z_2 with mean of zero and sigma of one, and calculates

$$y = z_1 \, x \frac{\theta_0}{\sqrt{12}} + z_2 \, x \frac{\theta_0}{2} \tag{186}$$

$$\theta = z_2 \theta_0 \tag{187}$$

In case a muon passes the calorimeter of thickness L_1 and then propagates through the (material less) muons system of length L_2 the spatial deflection is given by

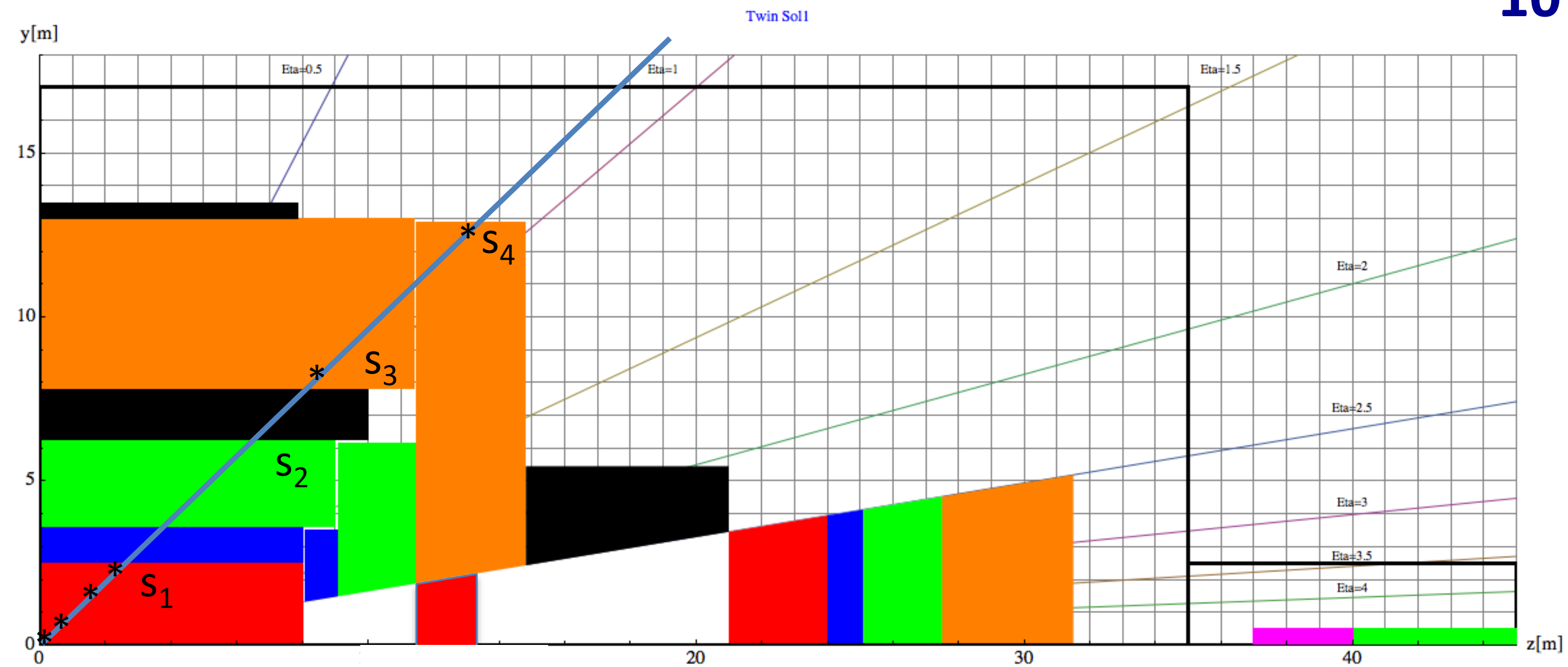
$$s = z_1 \, L_1 \frac{\theta_0}{\sqrt{12}} + z_2 \, L_1 \frac{\theta_0}{2} + z_2 \theta_0 L_2 \tag{188}$$

and therefore a position error of

$$\sigma_s^2 = \theta_0^2 \left[\frac{L_1^2}{12} + \left(\frac{L_1}{2} + L_2 \right)^2 \right] \tag{189}$$

Radiation length and angular deflection of muons in calo+coil

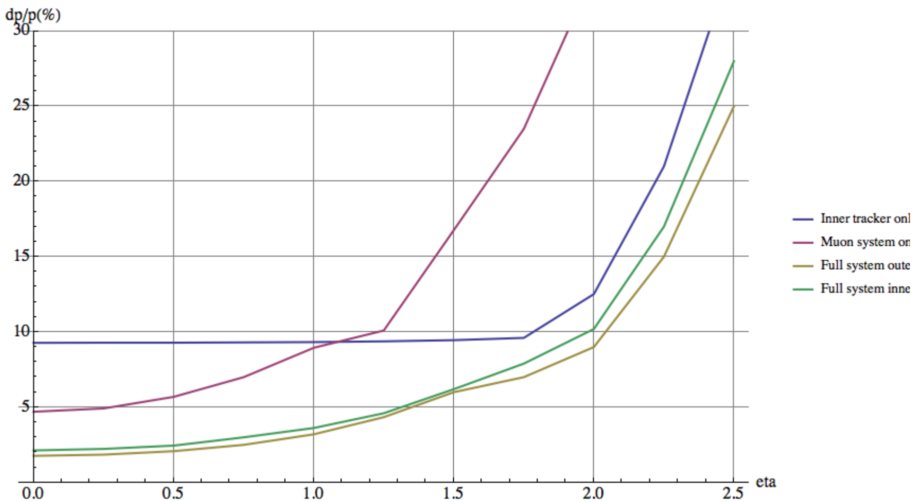
10 TeV/s

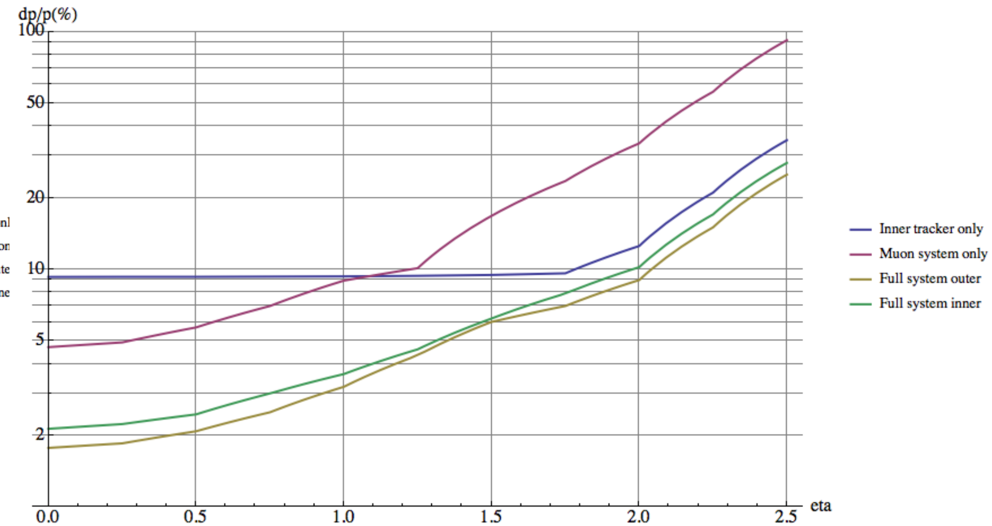


Fit of the template function to 14 measurement points of 23um in the inner tracker and 1 layer in the muon system with a resolution that is much better than the multiple scattering limit.

i.e. assuming the resolution at s4 to be better than 100um and at s3 to be better than 50um and the resolution of the angular measurement better than 20uRad.

Momentum resolution for a 10 TeV/s muon vs. eta





Twin Solenoid assuminginner tracker with baseline resolution curves and multiple scattering limit in the muons system.

P_T=10TeV/c eta = 0: 5% muon standalone (angle)
10% inner tracker only
2% combined

P_T=10TeV/c eta=2.: 35% muon standalone (angle)
12.5% inner tracker only
8% combined

Compare to the CMS numbers:

P_T=1TeV/c, 0<eta < 0.8: 20% muon standalone (angle)
10% inner tracker only
5% combined

P_T=1TeV/c, eta 0<eta<2.4: 40% muon standalone (angle)
20% inner tracker only
10% combined

Summary

First shot at the global muon performance.

What CMS does for 1TeV/c Muons seems feasible for 10TeV/c at FCC.

For now, the 'global fit' assumed just one measurement point in the muon system in addition to the 14 points of the inner tracker.

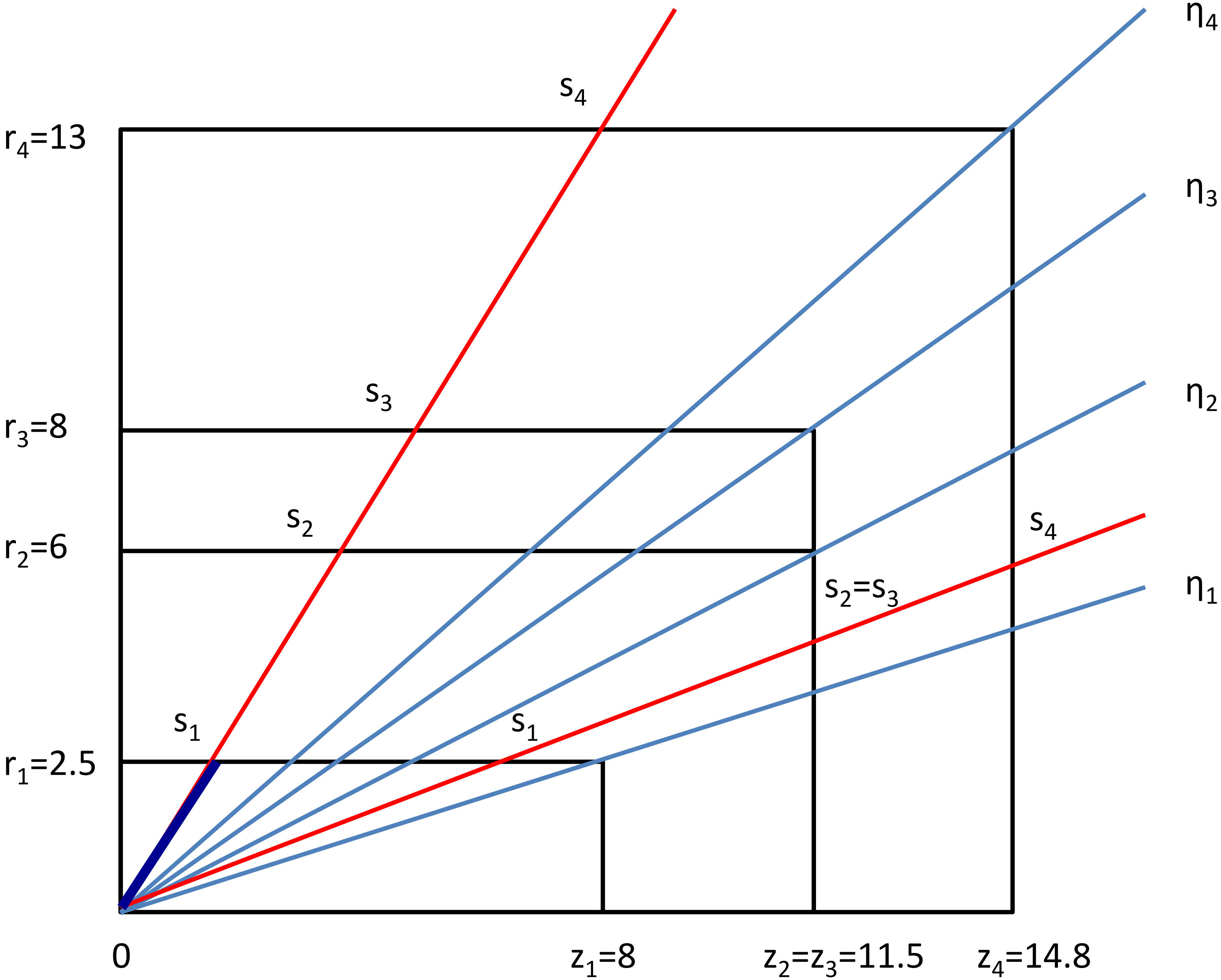
We have to do proper evaluation of the radiation length of the Calo using FLUKA/GEANT.

We have to understand whether the scattering of high energy muons is really approximated well by the given formulas (see todays presentation on Muon scattering in GEANT).

We have to include energy loss.

Another attempt to include all this in the semi-analytical calculation will be done.

A proper track fit in a proper MC simulation will be needed soon to have a solid result.



η4
η3
η2
η1
s4
s3
s2
s1
s4
s2=s3
s1
r4=13
r3=8
r2=6
r1=2.5
0
z1=8
z2=z3=11.5
z4=14.8