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Hadron Thermodynamics defines its own Limits

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Hagedorn's Legacy **CERN, November 13, 2015**

Before 1950, the world of hadrons was simple:

nucleons & π-mesons

– one species of massive Fermionic matter particle (p,n)
– one species of massive Bosonic force particle $\pi^{\pm,0}$

In the next 30 years, explosion in abundance of species:
– excited states of nucleons & mesons ("resonances"), of integer & half-integer spin values, ever increasing masses
– flavor species: strangeness, charm, beauty, top

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Two theoretical lines of thought:

- **classical reductionism: the many species must have infra-structure of fewer simpler species → quark model, QCD**
- **novel question: is the number of different species unbounded? if so, what effect on thermodynamics of hadronic matter?**

Hagedorn (1965): **unlimited number of hadron species; assume their composition & decay of species to be self-similar:**

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poetic formulation by Augustus de Morgan (1801-1871):

Great flees have little fleas upon their backs to bite'em,
and little fleas have lesser fleas, and so ad infinitum.
And the great fleas themselves, in turn, have greater fleas to go on,
while these again have greater still, and greater still, and so on.

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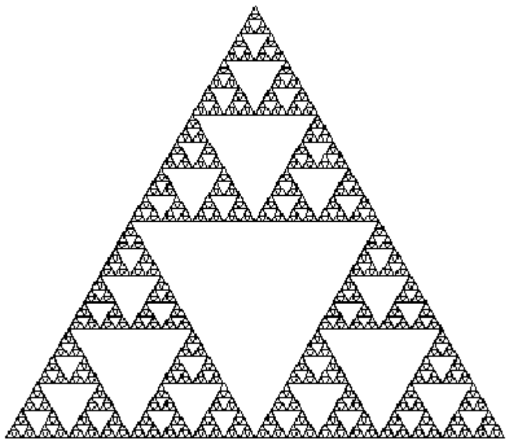
artistic formulation by
by Wacław Sierpinski (1915):



mathematical partition problem: how many ways to partition an integer into integers?

Euler (1753), Schröder (1870), Hardy & Ramanujan (1918):

simple version: count all orders

1=1	$p(1) = 1 = 2^{n-1}$
2=2 ,1+1	$p(2) = 2 = 2^{n-1}$
3=3, 2+1, 1+2, 1+1+1	$p(3) = 4 = 2^{n-1}$
4=4, 3+1, 1+3, 2+2, 2+1+1, 1+2+1, 1+1+2, 1+1+1+1	$p(4) = 8 = 2^{n-1}$

and so on: $$p(n) = 2^{n-1} = \frac{1}{2}\, e^{n \ln 2}$$

number of partitions of n grows exponentially with n

Hagedorn: partition hadrons into more hadrons into still more hadrons. Not only masses, but also kinetic energies.

Result: The Statistical Bootstrap Equation

Hagedorn had in mind
the legendary Baron von Münchhausen;
but he pulled himself and his horse out of
a swamp by his hair, not by his bootstraps.
The bootstraps came into the picture only a
hundred years later in American versions.

Münchhausen O. Herrfurth pinx

The point remains the same:
the mechanism is self-induced.

partition problem:

"large integers consist of smaller integers which consist of still smaller integers, and so on..." – formulate that as an equation!

The number of partitions $\rho(n)$ of integer n is determined by

$$\rho(n) = \delta(n-1) + \sum_{k=2}^{n} \frac{1}{k!} \prod_{i=1}^{k} \rho(n_i)\ \delta(\Sigma_i n_i - n).$$

Convolution of many "similar" partitions of smaller n; result $\rho(n) \sim \exp\{n \log 2\}$; number at large n is fixed by that for smaller n: self-similarity.

Hagedorn's problem was more complex: the heavy resonance is not just sum of the lighter ones, but lighter ones in motion; total energy must add up to the mass of the heavy one. Result: the statistical bootstrap equation

$$\rho(m, V_0) = \delta(m - m_0)\ + \sum_N \frac{1}{N!} \left[\frac{V_0}{(2\pi)^3}\right]^{N-1} \int \prod_{i=1}^{N}\ [dm_i\ \rho(m_i)\ d^3 p_i]\ \delta^4(\Sigma_i p_i - p)$$

m_0: lightest hadron ("pion") $\quad V_0 = 4\pi R_0^3/3$: fireball volume

basic hadronic scale $R_0 \simeq 1/m_0$

Solution [W. Nahm, 1972]

$$\rho(m, V_0) = \text{const. } m^{-3} \exp\{m/T_H\}.$$

with

$$\frac{V_0 T_H^3}{2\pi^2}(m_0/T_H)^2 K_2(m_0/T_H) = 2\ln 2 - 1,$$

Hagedorn:

basic hadronic scale $R_0 \simeq 1/m_\pi$ gives $T_H \simeq 150$ MeV

Chiral limit $m_0 \to 0$ with $R_0 \simeq 1$ fm gives $T_H \simeq 200$ MeV

crucial: hadronic range, not lowest hadron mass

- What is the thermodynamics of an ideal gas of resonances whose degeneracy is determined by the bootstrap $\rho(m, V_0)$?

⋆ Ideal gas of identical particles, grand canonical partition function

$$\mathcal{Z}(T,V) = \sum_N \frac{1}{N!}\left[\frac{V}{(2\pi)^3}\int d^3p \;\; \exp\{-\sqrt{p^2+m_0^2}\;/T\}\right]^N$$

gives

$$\ln \mathcal{Z}(T,V) = \frac{VTm_0^2}{2\pi^2}\; K_2(\frac{m_0}{T}).$$

and hence

$$\epsilon(T) = -\frac{1}{V}\;\frac{\partial\; \ln\; \mathcal{Z}(T,V)}{\partial\,(1/T)} \simeq \frac{3}{\pi^2}\; T^4 \qquad n(T) = \frac{\partial\; \ln\; \mathcal{Z}(T,V)}{\partial\, V} \simeq \frac{1}{\pi^2}\; T^3,$$

for energy density and particle density.

Therefore average energy per particle: $\omega \simeq 3\ T$.

Increasing energy of an ideal gas of identical particles leads to

- **a higher temperature,**
- **more constituents, and**
- **more energetic constituents.**

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Interacting gas with resonance formation as dominant interaction: equivalent to an ideal gas of all possible resonances

[Beth & Uhlenbeck 1937; Dashen, Ma & Bernstein 1969]

⋆ Ideal resonance gas with bootstrap spectrum

$$\ln \mathcal{Z}(T,V) = \sum_i \frac{VTm_i^2}{2\pi^2}\,\rho(m_i)\ K_2(\frac{m_i}{T}) \simeq \frac{VT}{2\pi^2}\int dm\ m^2\rho(m_i)\ K_2(\frac{m_i}{T})$$

leads to a singular form

$$\ln \mathcal{Z}(T,V) \sim V \left[\frac{T}{2\pi}\right]^{3/2} \int dm\; m^{-3/2} \exp\{-m \left[\frac{1}{T} - \frac{1}{T_H}\right]\}.$$

diverging for $T > T_H$:

T_H is the ultimate temperature of (hadronic) matter.

Further energy input does not increase temperature, instead more and more massive resonance species are formed.

A new, non-kinetic use of energy: increase number and mass of different hadron species, not their momenta.

Compare pion gas and resonance gas:

pion gas	resonance gas
$n_\pi \sim \epsilon^{3/4}$	$n_{res} \sim \epsilon$
$\omega_\pi \sim \epsilon^{1/4}$	$\omega_{res} \sim m_{res}$

General solution of bootstrap type equation

$$\rho(m, V_0) \sim m^{-a} \exp\{m/T_H\},$$

NB: also result of dual resonance model [Huang & Weinberg 1970]

Nahm's solution of Hagedorn eq'n: $a = 3$; for $T \to T_H$, partition function exists, energy density diverges.

Hagedorn (1965): T_H is highest temperature of matter

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partition function exists, energy density diverges.

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For $4 > a > 3$, $T \to T_H$ **leads to finite energy density,**
divergent specific heat

Cabbibo & Parisi (1975): T_H **is critical temperature**
signalling transition to a new state of matter:
deconfinement → Quark-Gluon Plasma;
the end of the world we know.

What happens at the transition point?

resonance degeneracy form of
Hagedorn's bootstrap, dual resonance model:

singularity in higher derivatives of partition function,
continuous critical behavior, critical exponents

so far, pointlike hadrons; intrinsic hadron size?

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Pomeranchuk (1951): hadron size limits hadron density

$$n_h(T) \leq \frac{3}{4\pi R_h^3} \quad \Rightarrow \quad \left(\frac{T_c}{m_0}\right) K_2(m_0/T_c) = \frac{3\pi}{2}$$

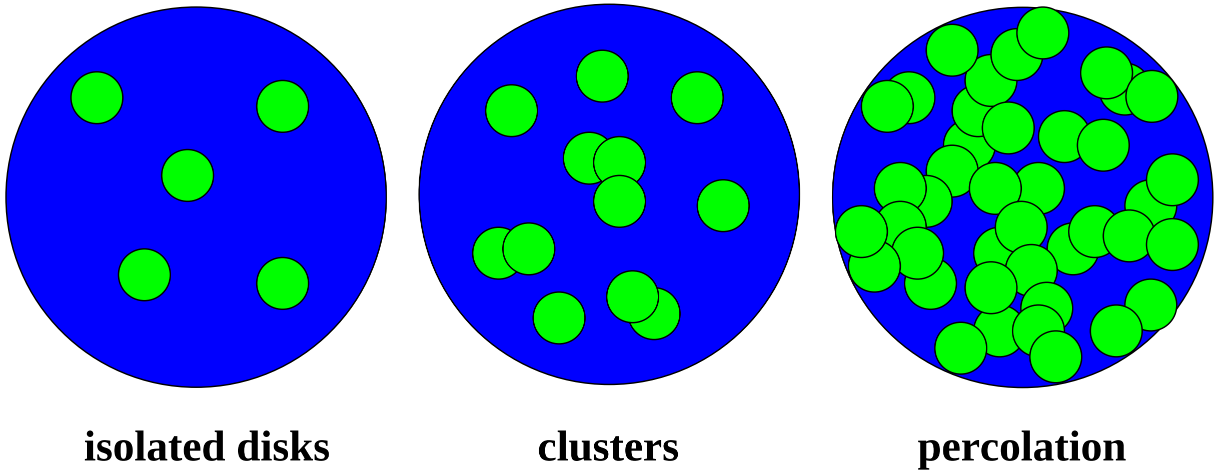
ideal gas of pions has temperature limit $T \leq T_c = 190$ MeV

what happens after that?

increasing density of spatial objects → cluster formation,

percolation transition

percolation: transition from isolated objects to connected medium



once ∃ connected medium, quarks can move around freely:
onset of color conductivity = deconfinement

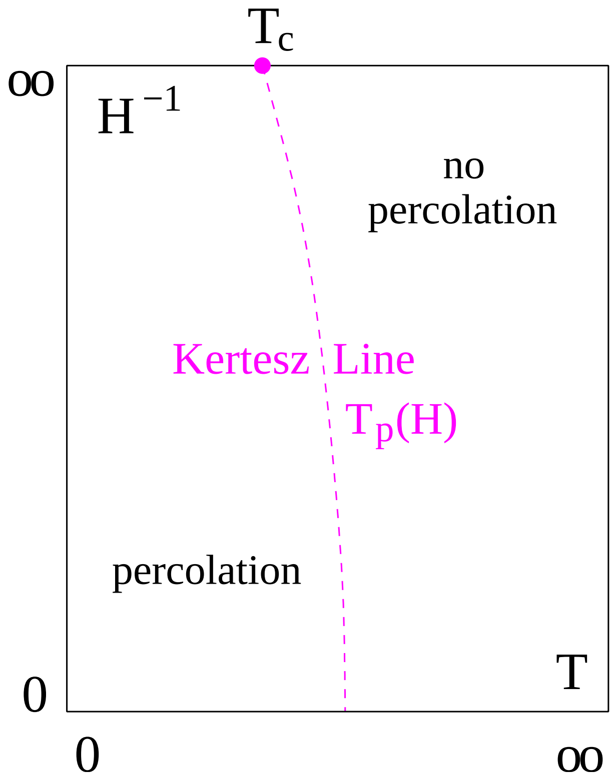
two transition points:
– hadrons percolate, vacuum also still connected
– hadrons percolate, vacuum no longer connected
first order transition?

percolation transition = geometric critical phenomenon

critical expt's of percolation etc.,
can but need not lead to singular partition function

example: 2-d Ising model, $H = 0$,
percolation transition = magnetization transition

$H \neq 0$:
no magnetization transition
percolation transition remains
deconfinement = hadron percolation?



Hadron thermodynamics defines its own limits

dynamics:

temperature limit because hadronic resonance spectrum leads to resonances of increasing degeneracy

geometry:

temperature limit because hadronic size leads to percolation transition, onset of color connectivity

Rolf Hagedorn:

the temperature of our world has an upper limit