

# *Life Above the Hagedorn Temperature*

Quark-Gluon Plasma at SPS, RHIC & LHC

**Berndt Mueller**

Brookhaven National Laboratory  
& Duke University

Hagedorn Symposium

CERN

13 November 2015

**BROOKHAVEN**  
NATIONAL LABORATORY  
*a passion for discovery*

 Office of  
Science  
U.S. DEPARTMENT OF ENERGY





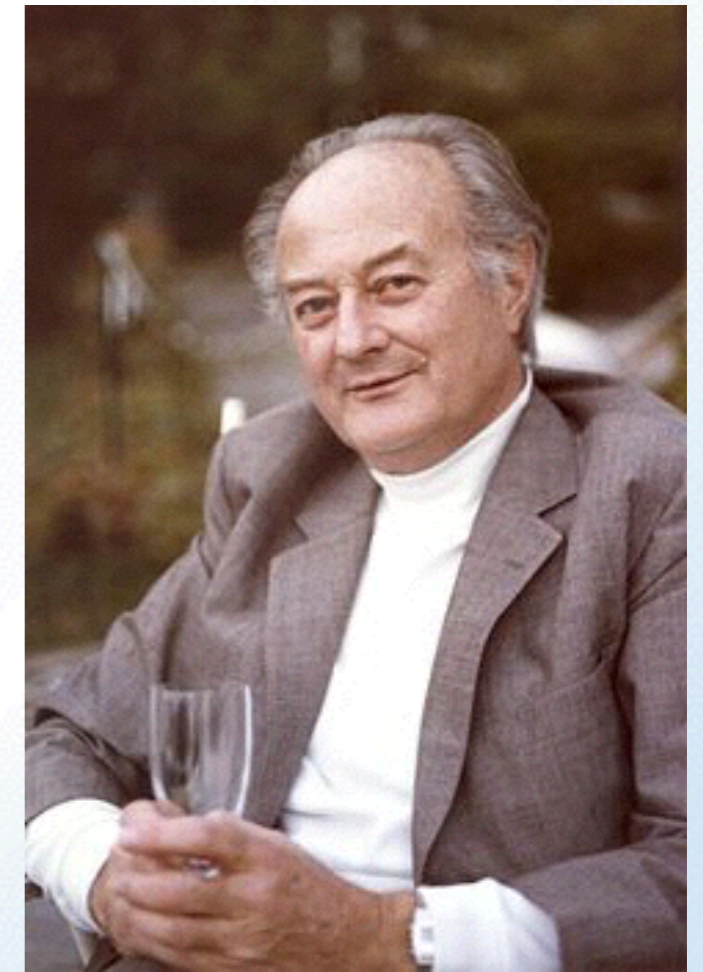
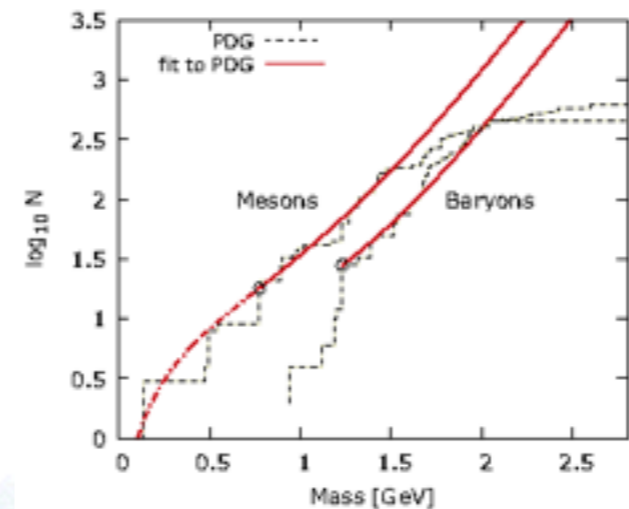
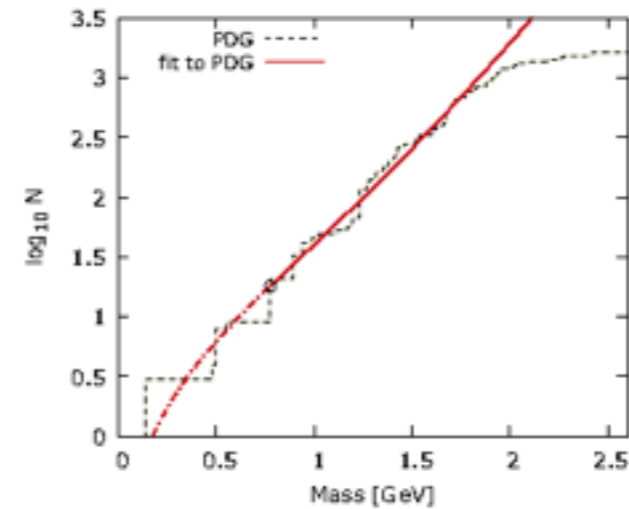
# 1965

was a momentous year



© 2004 Thomson - Brooks/Cole

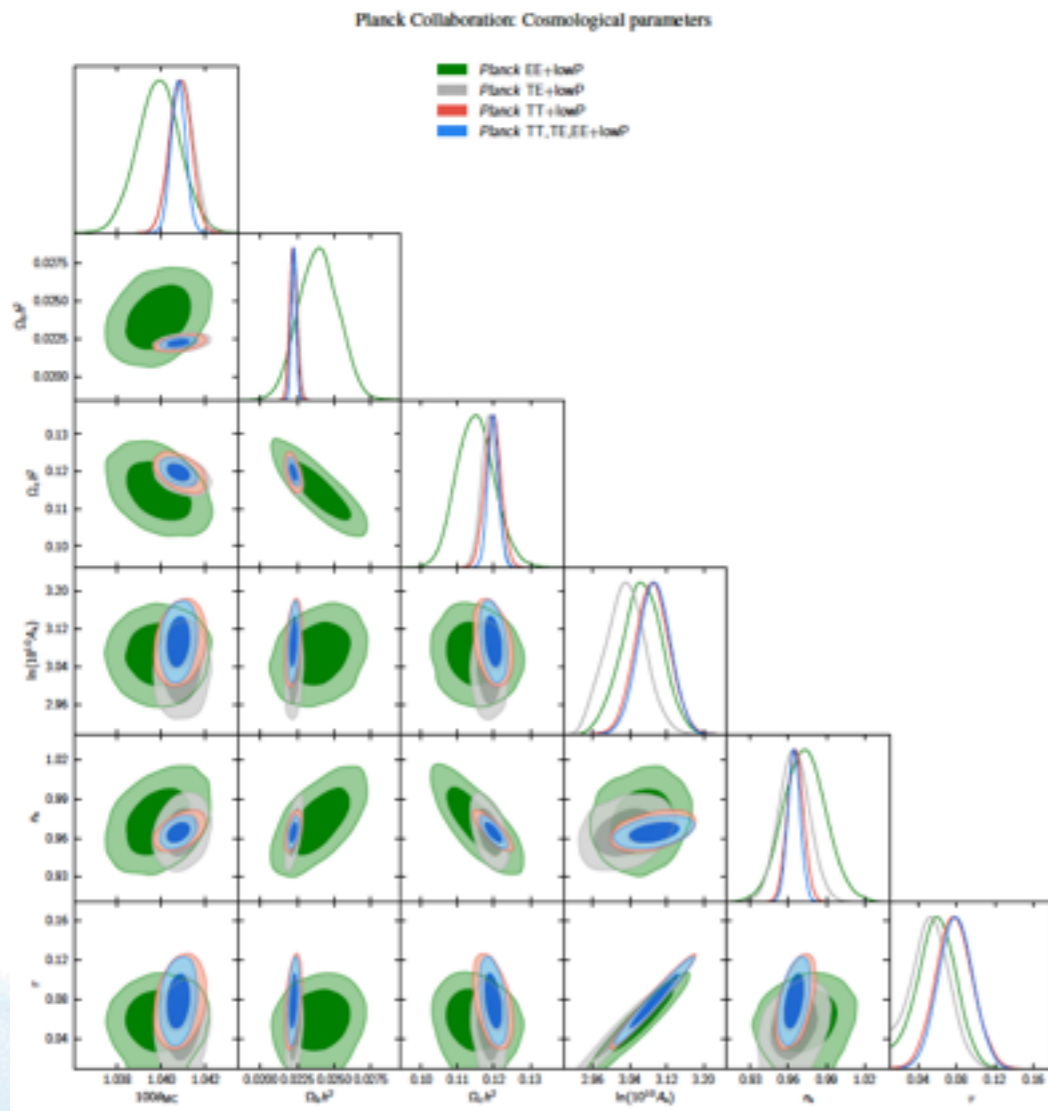
cosmic microwave background



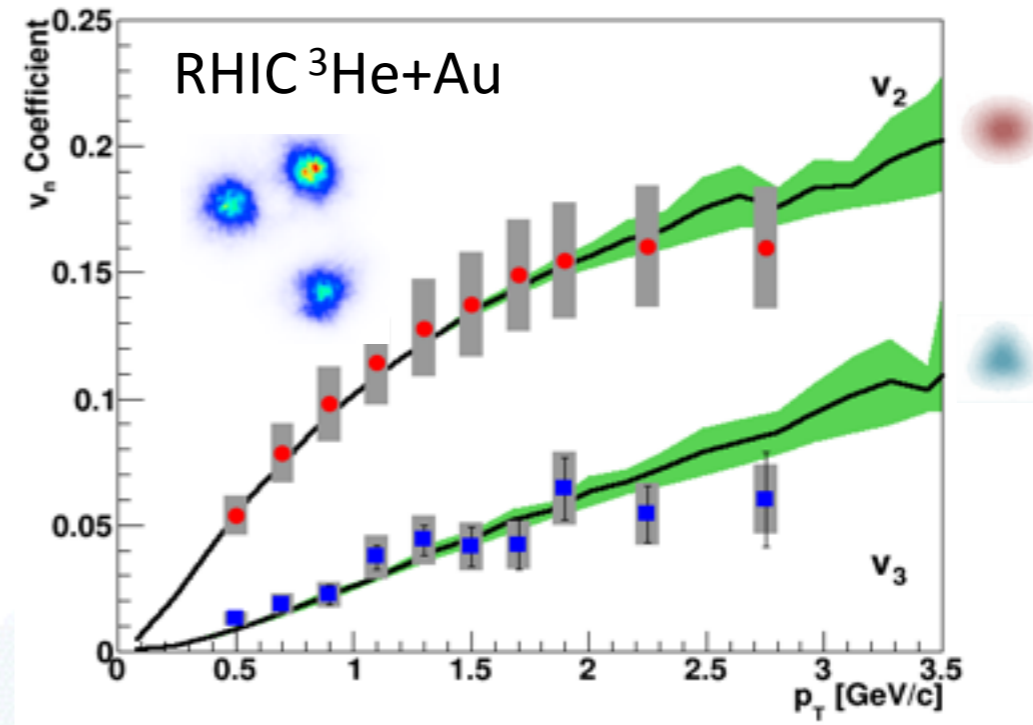
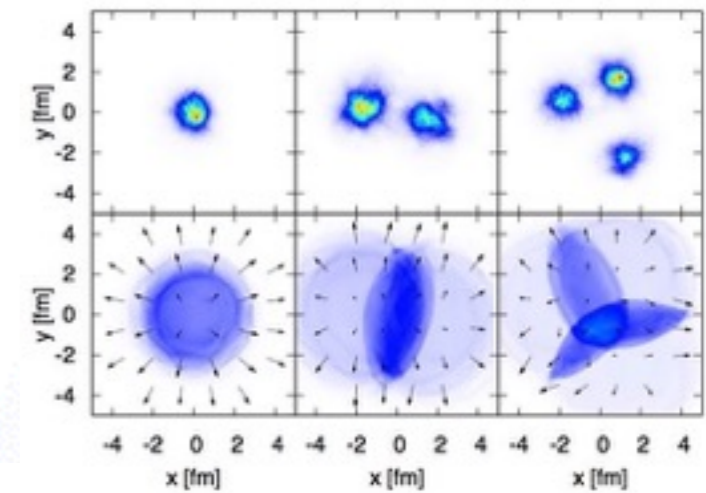
Hagedorn mass spectrum

# 2015

almost unimaginable progress



Planck: cosmological parameters



PHENIX: QGP in p/d/<sup>3</sup>He+Au



# Three things are needed...

...to reach the summit: **good equipment**, **good strategy**, **determination**.

In the study of hot, dense QCD matter this means:



- High luminosity colliders
- Large acceptance, high DAQ rate detectors with good particle ID
- Realistic lattice QCD for thermodynamic quantities
- Realistic transport codes
- Weak (pQCD) and strong (AdS/CFT) coupling dynamical models
- Multivariate model-data comparison

After several decades of experimental and theoretical development, the necessary tools are now all in place.



# The Relativistic Heavy Ion Collider

*...is hexagonal and 3.8 km long*

**RHIC-AGS Complex at BNL**

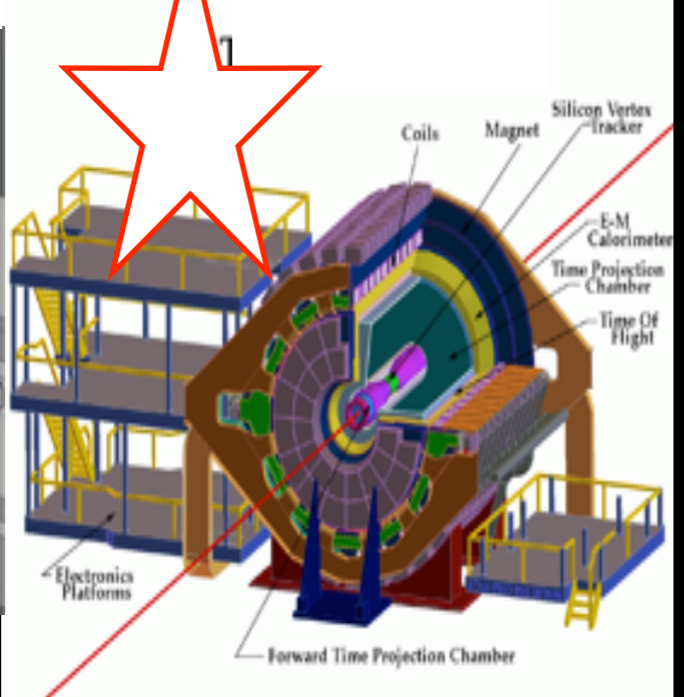
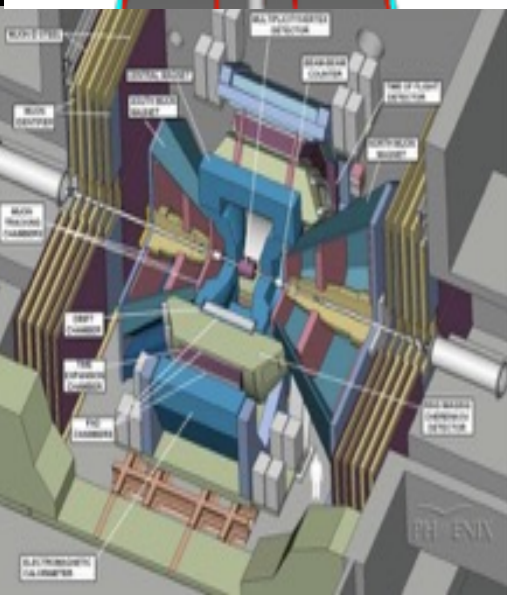
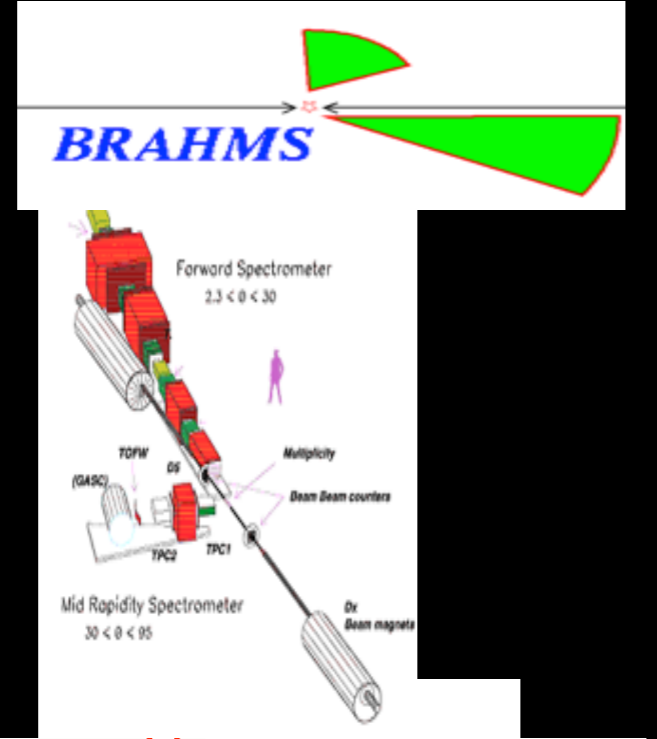
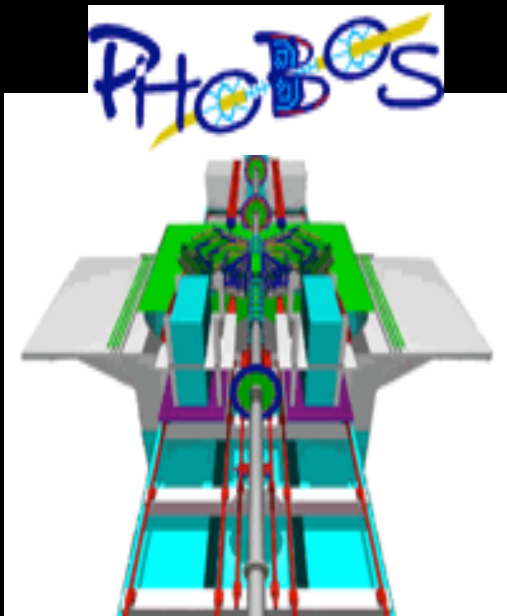




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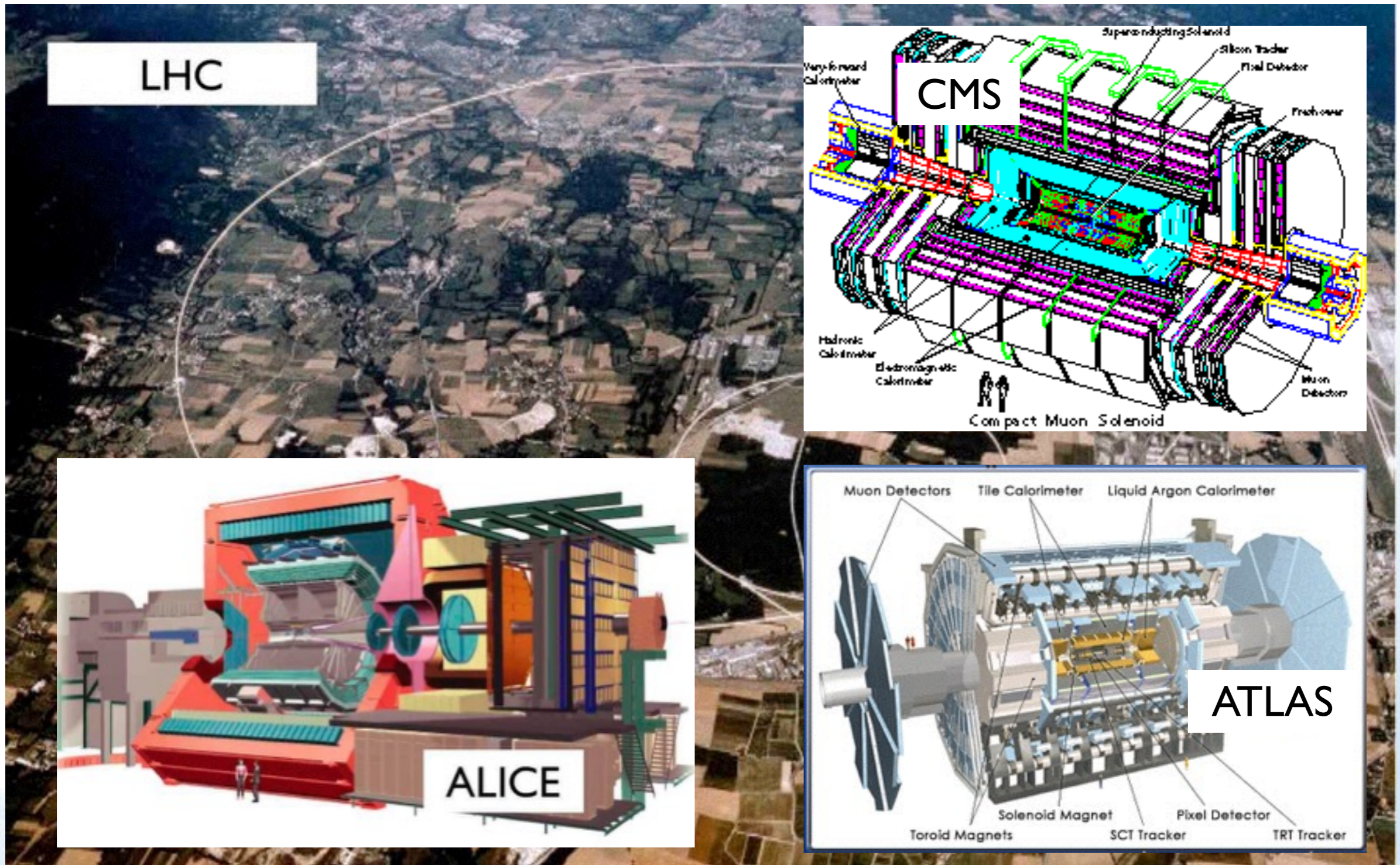


# The highest energies...





# The highest energies...





# Equation of State of QCD Matter



# Equation of State

EOS of flowing matter has conservative and dissipative contributions:

$$\begin{aligned} T_{\mu\nu} &= T_{\mu\nu}^{(\text{cons})} + T_{\mu\nu}^{(\text{diss})} \\ &= (\varepsilon + p)u_\mu u_\nu - p g_{\mu\nu} \\ &\quad + \eta \left( \partial_\mu u_\nu + \partial_\nu u_\mu - \frac{2}{3} g_{\mu\nu} \partial_\alpha u^\alpha \right) + \zeta g_{\mu\nu} \partial_\alpha u^\alpha \end{aligned}$$

When  $\zeta(\partial_\alpha u^\alpha) > p$ , the matter becomes unstable and cavitates.

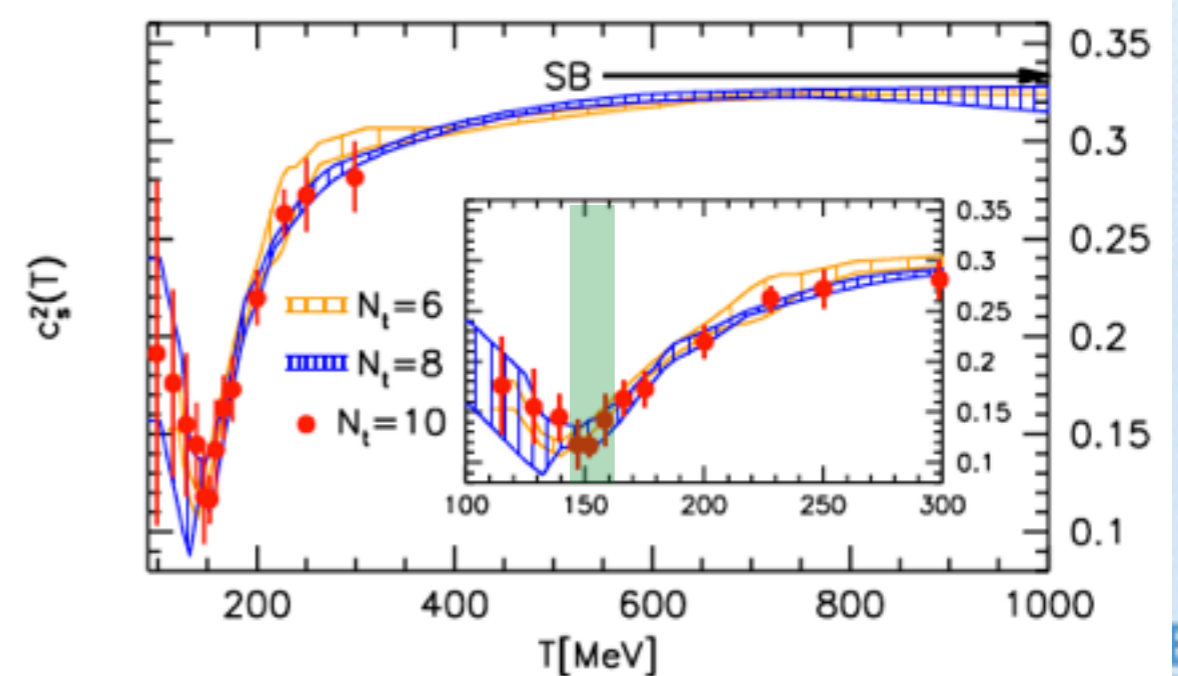
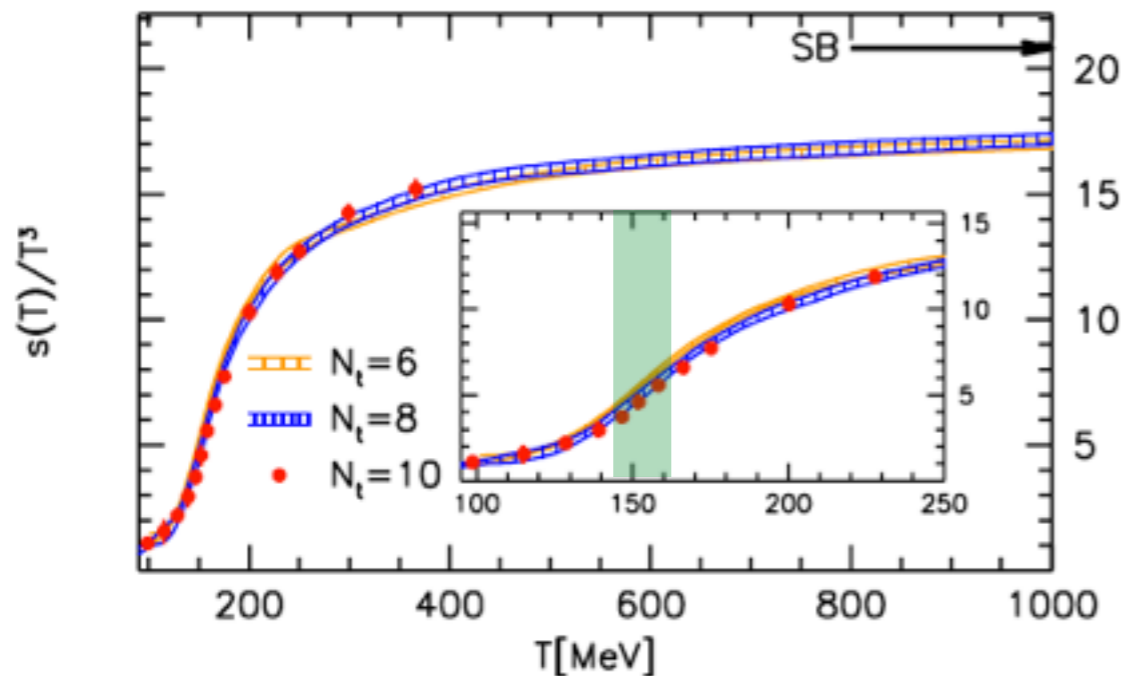
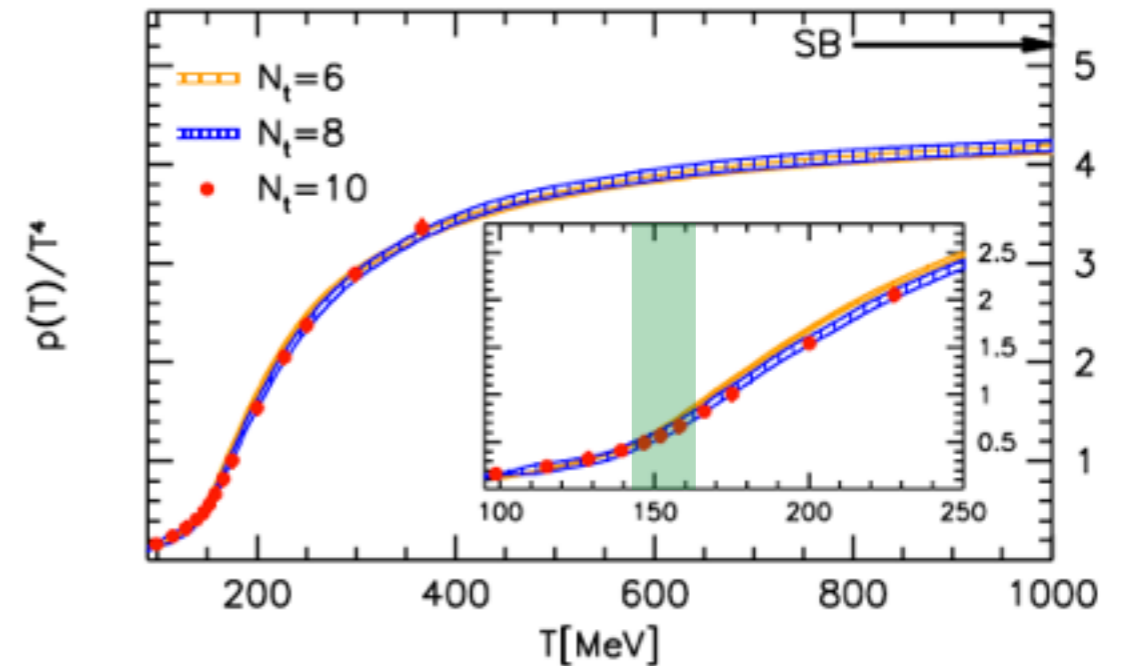
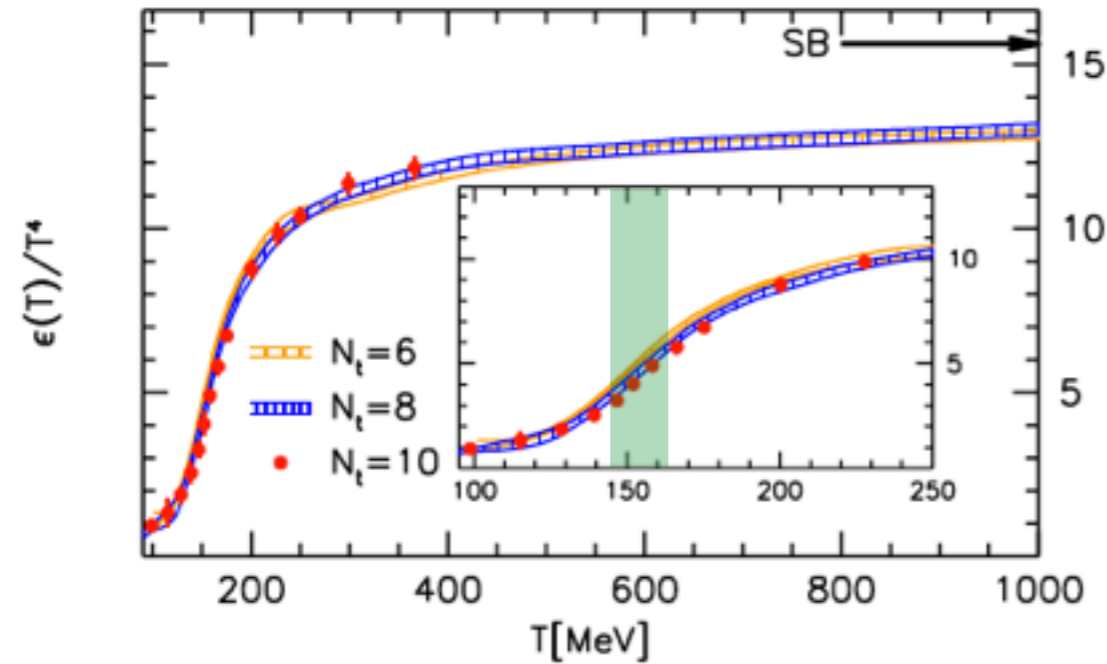
In general,  $T_{\mu\nu}$  is a dynamical quantity that relaxes to its equilibrium value on a time scale  $\tau_\pi$  that itself is related to the viscosity.

While the shear viscosity  $\eta$  has a lower quantum bound, the bulk viscosity  $\zeta$  vanishes for conformally invariant matter.



# QCD EOS at $\mu_B = 0$

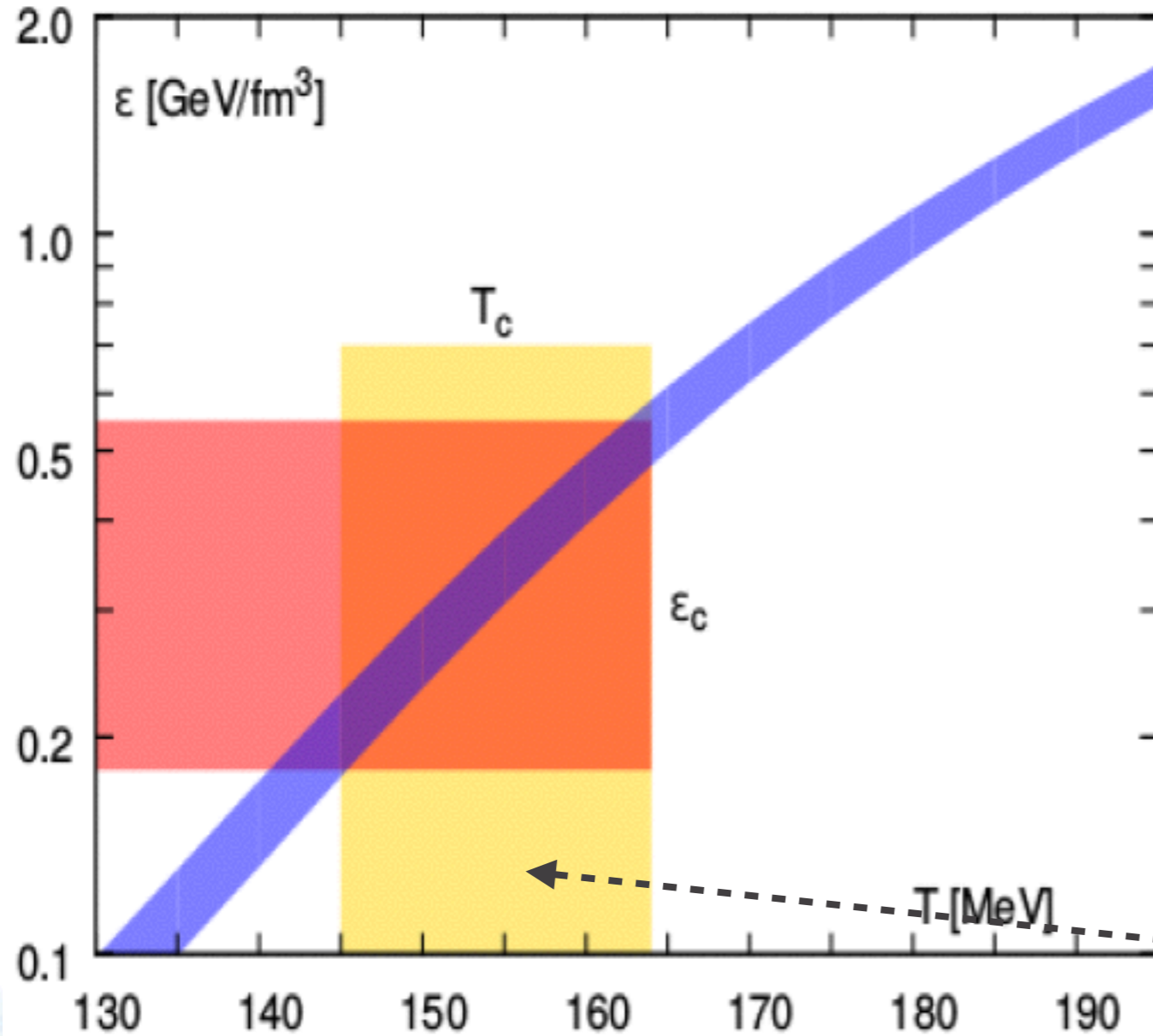
Results (true quark masses, continuum extrapolated) have converged; full agreement found between groups (HotQCD, Wuppertal-Budapest) using different quark actions.





# (Pseudo-) Critical temperature

Transition between hadron gas and quark-gluon plasma is a **cross-over** at  $\mu_B = 0$  and for small  $\mu_B$ . Precise value of  $T_c$  depends on the quantity used to define it.



Pseudo-critical temperature from chiral susceptibility peak:

$$T_c = 154 \pm 9 \text{ MeV}$$

critical energy density:

$$\epsilon_c = 0.18 - 0.50 \text{ GeV/fm}^3$$

$$\epsilon_c = (1.2 - 3.3) \rho_{\text{nuclear}}$$

*Uncertainty in  $T_c$ , not width of cross-over region!*

HotQCD: arXiv:1407.6387



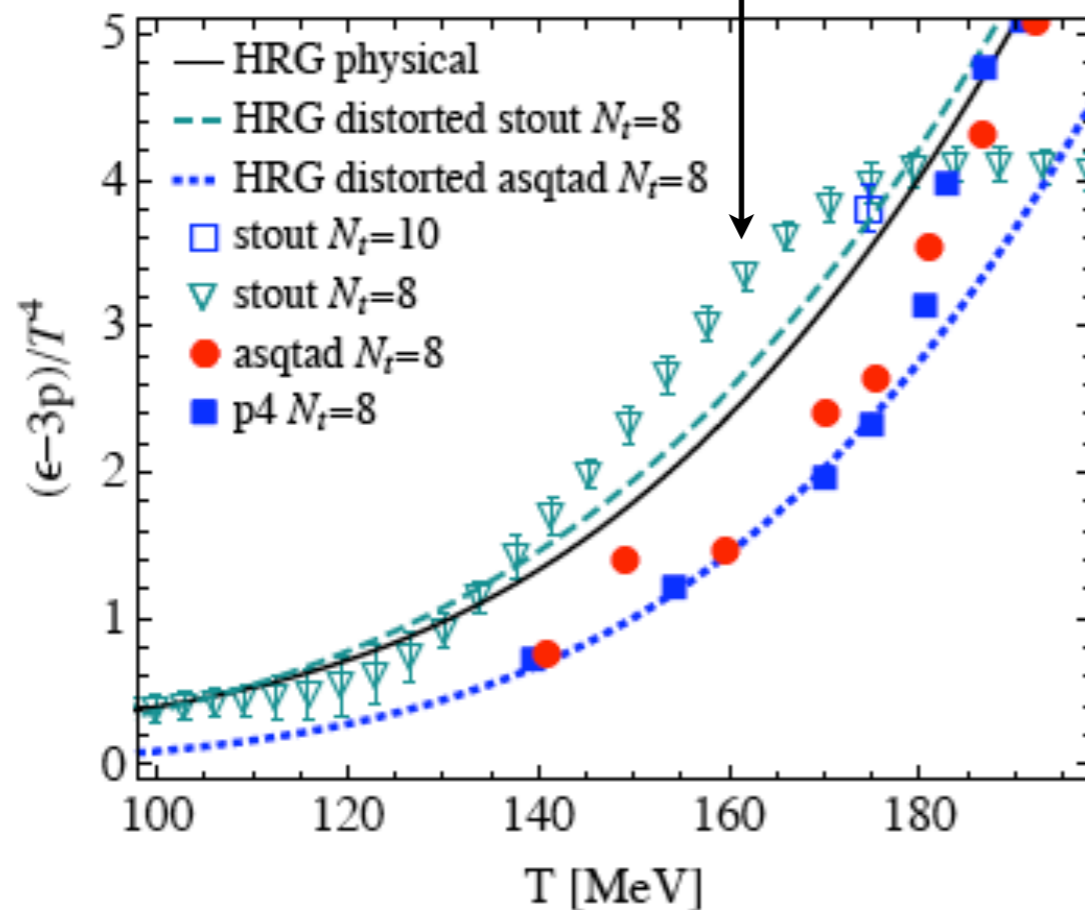
# Hadron mass spectrum

Below  $T_c$ , the quantity  $(\epsilon-3p)/T^4$  measures the level density of massive hadronic excitations of the QCD vacuum.

Hagedorn spectrum ( $T_H \approx 180$  MeV):

$$\rho_H(m) = \frac{A e^{m/T_H}}{(m^2 + m_0^2)^{5/4}}$$

Lines: Hadron resonance gas using only PDG resonances  
 Data points: Lattice QCD  
 LQCD lies **above** HRG for  $T > 140$  MeV  
 Indicates additional hadron resonances

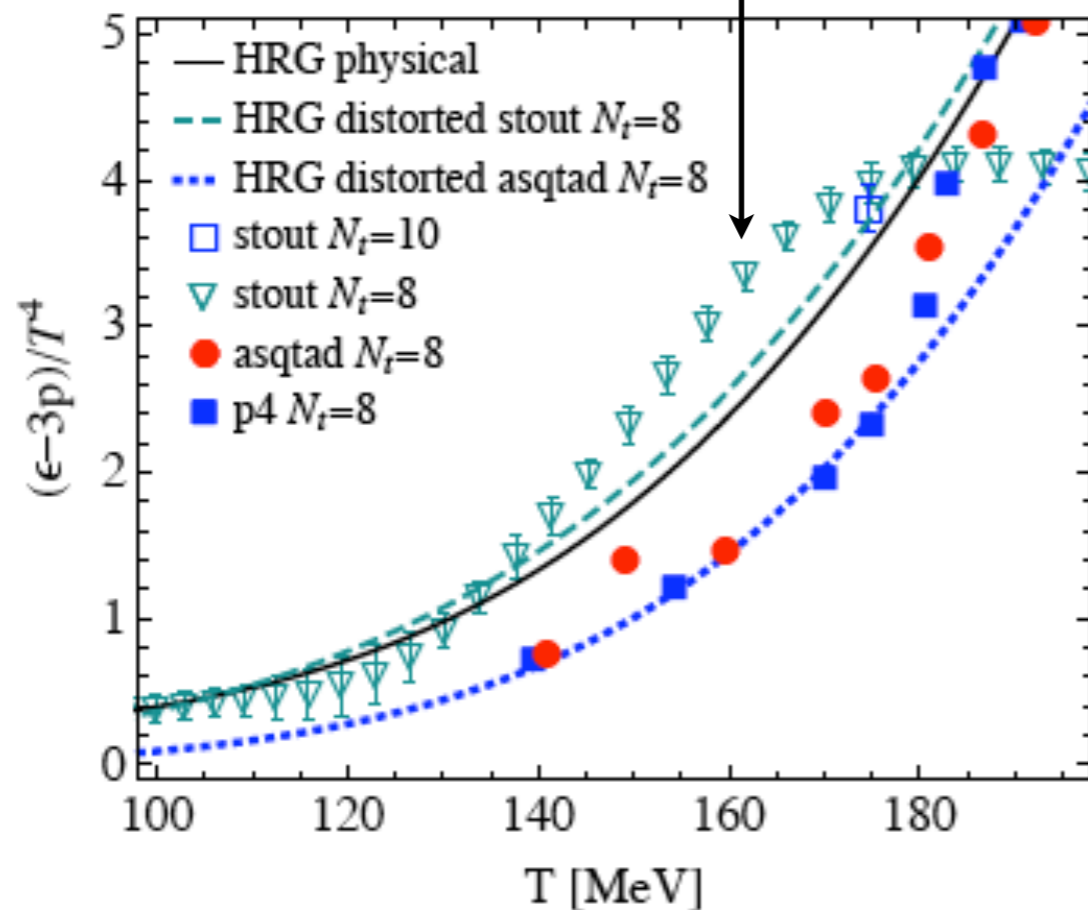




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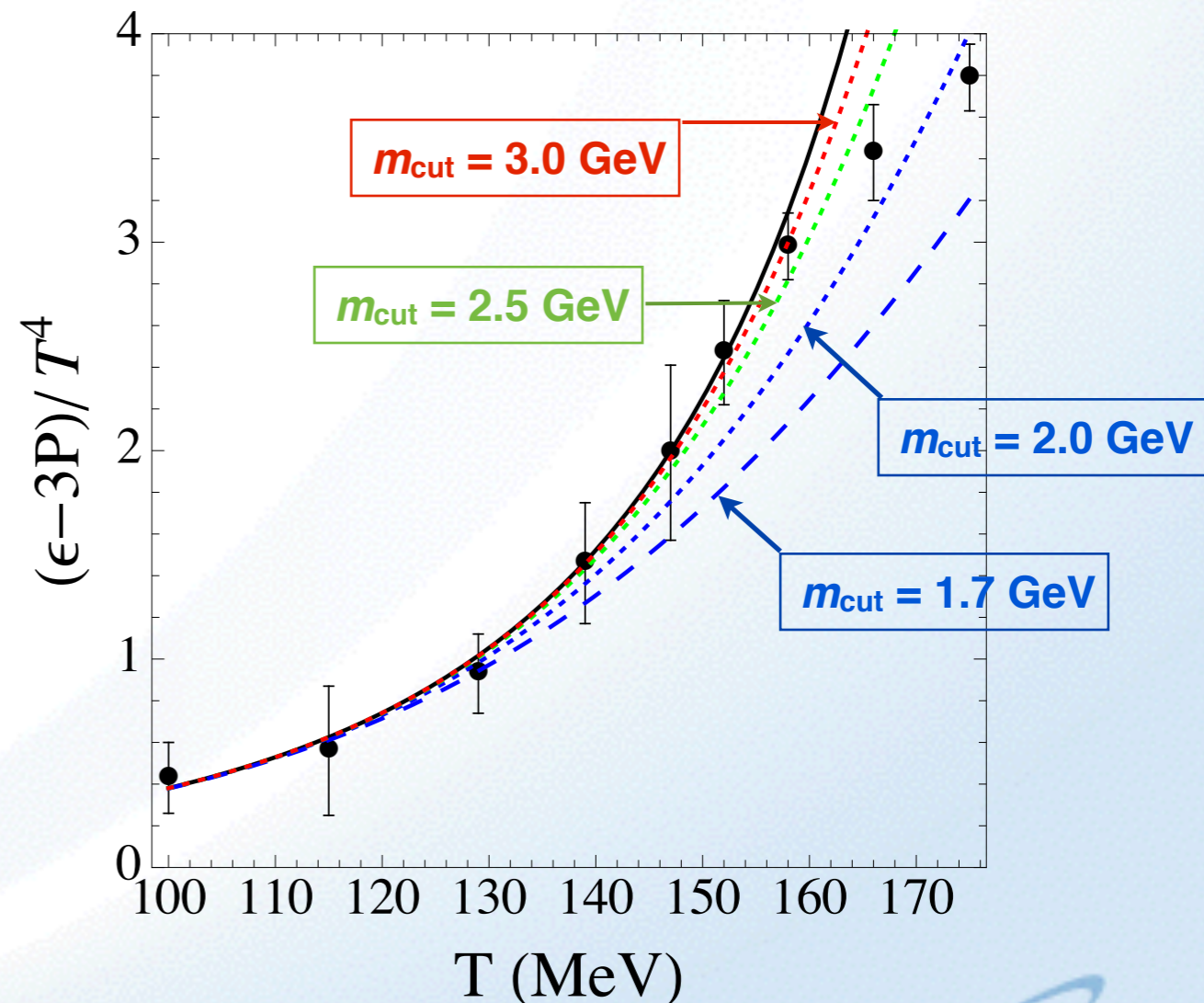
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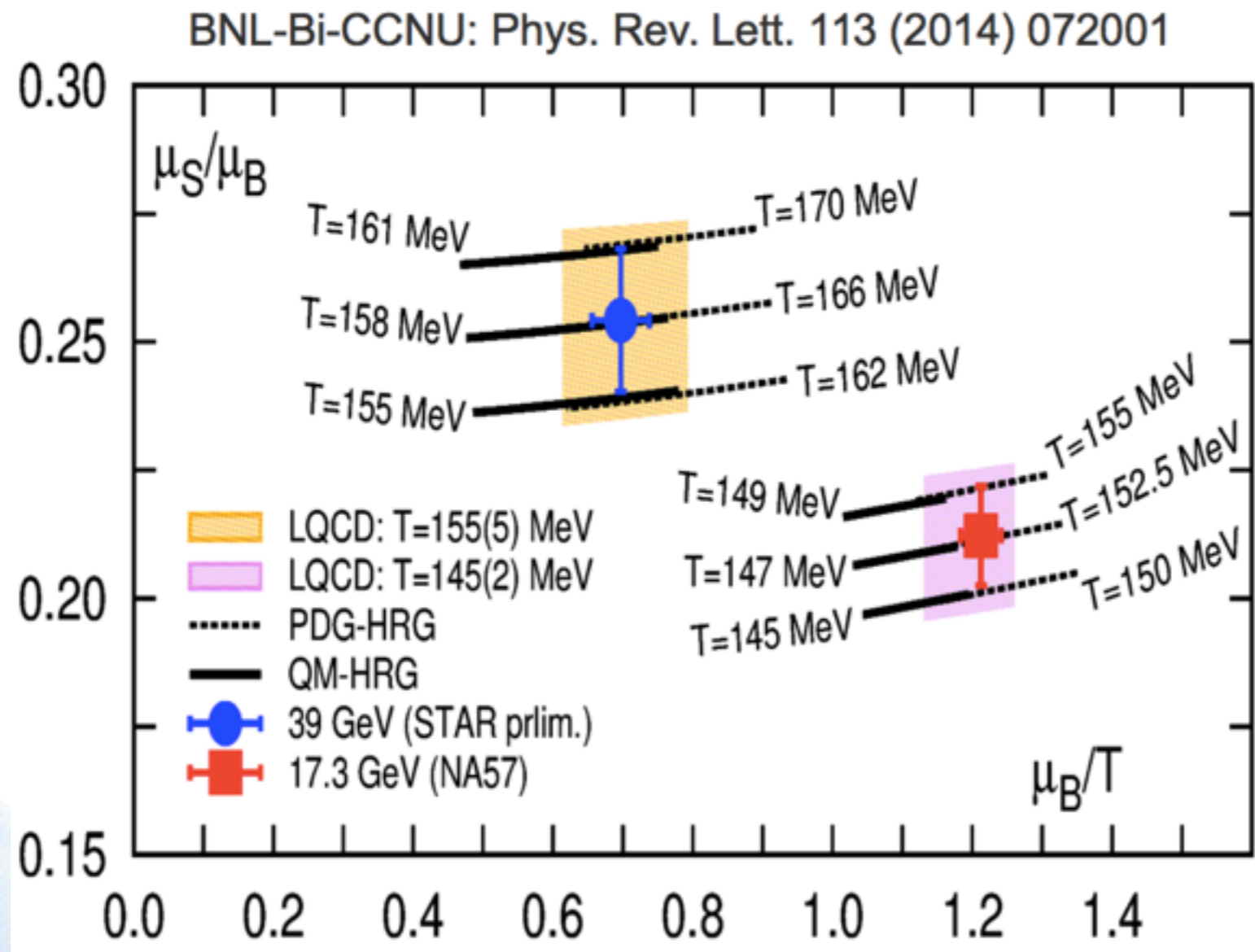
In good agreement with lattice results  
 Hadrons up to 3 GeV mass contribute



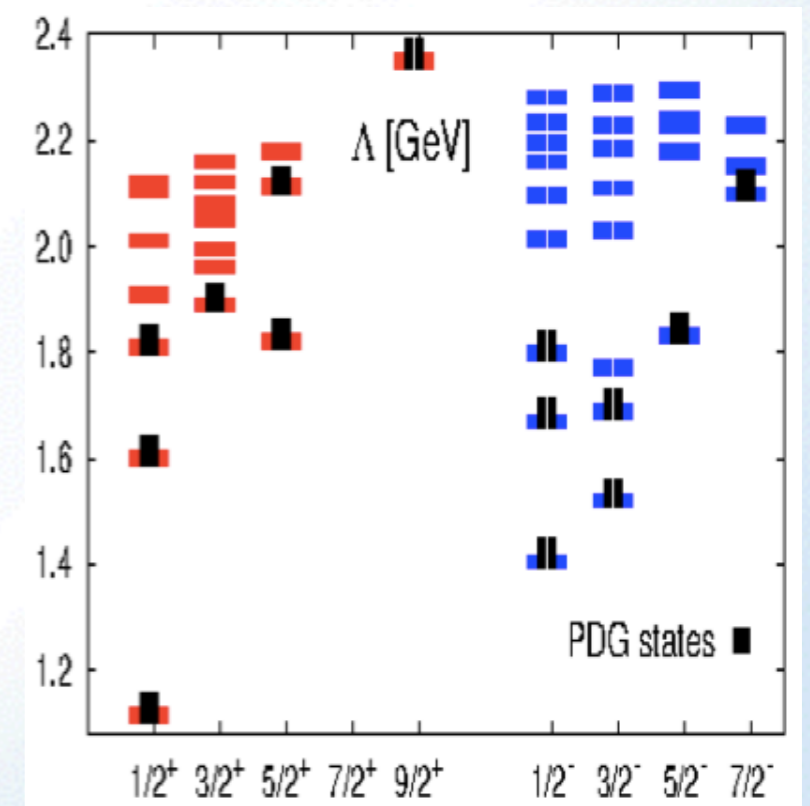


# Probing the baryon spectrum

Consistency of  $\mu_s/\mu_B$  and  $\mu_B/T$  with chemical composition of emitted hadrons and Lattice QCD requires additional strange baryon resonances beyond those in the PDG tables.



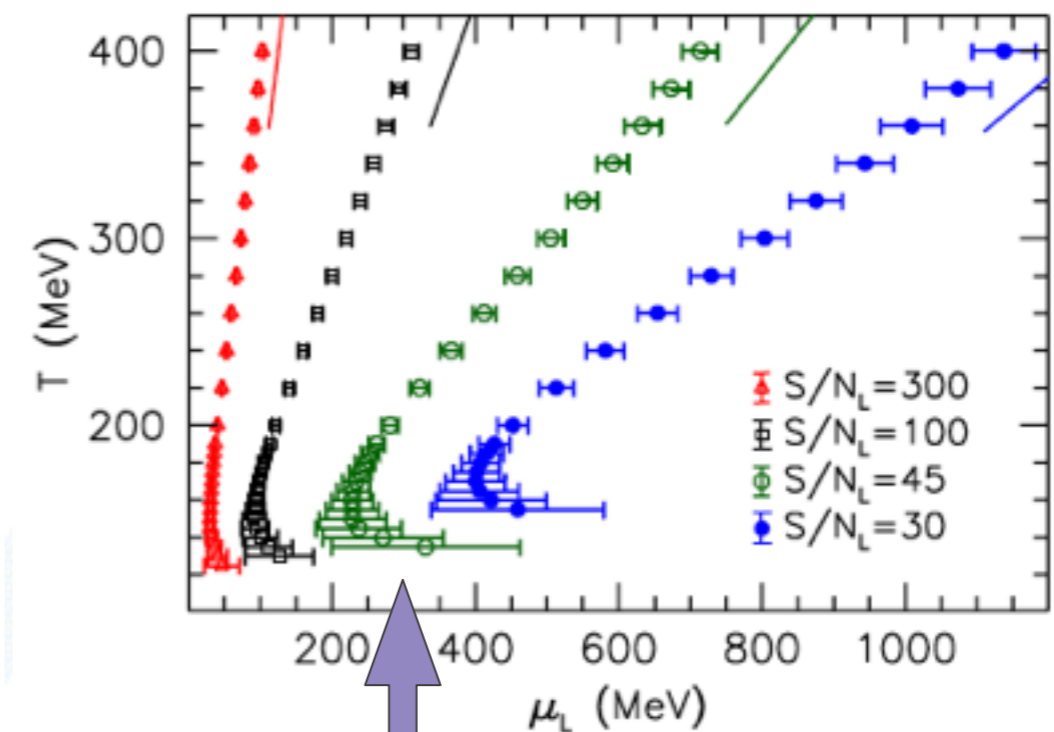
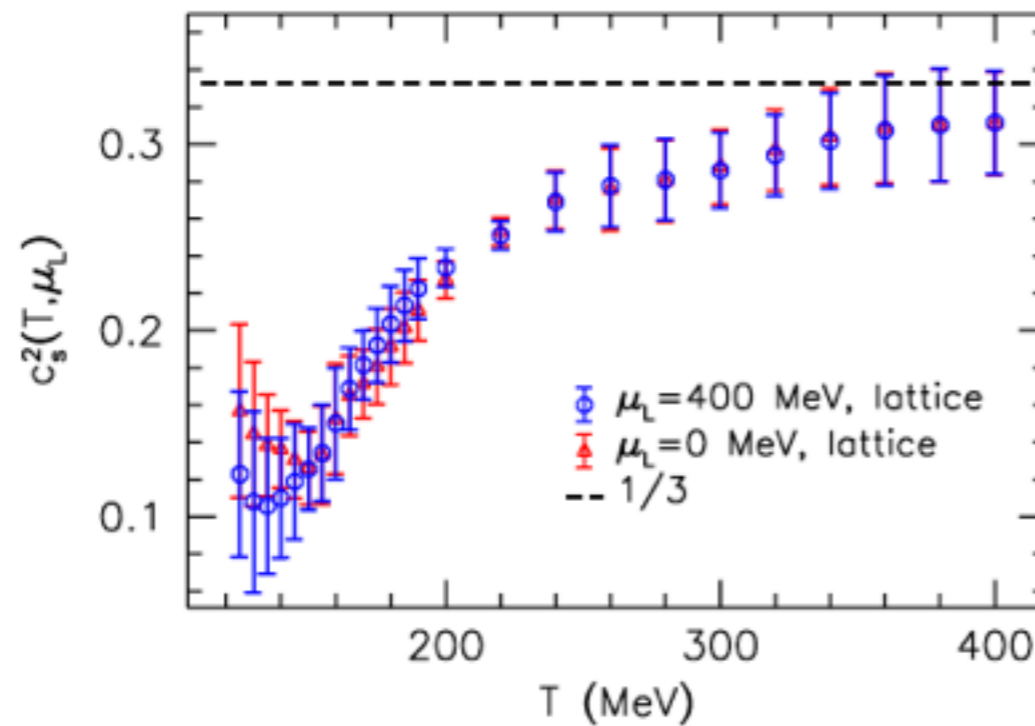
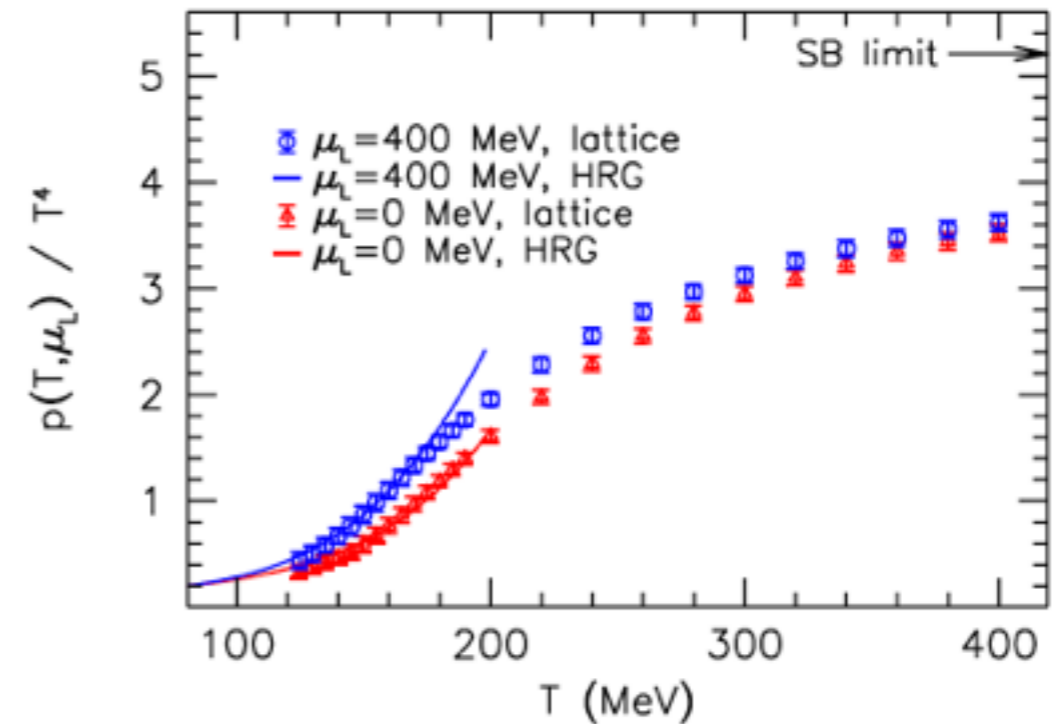
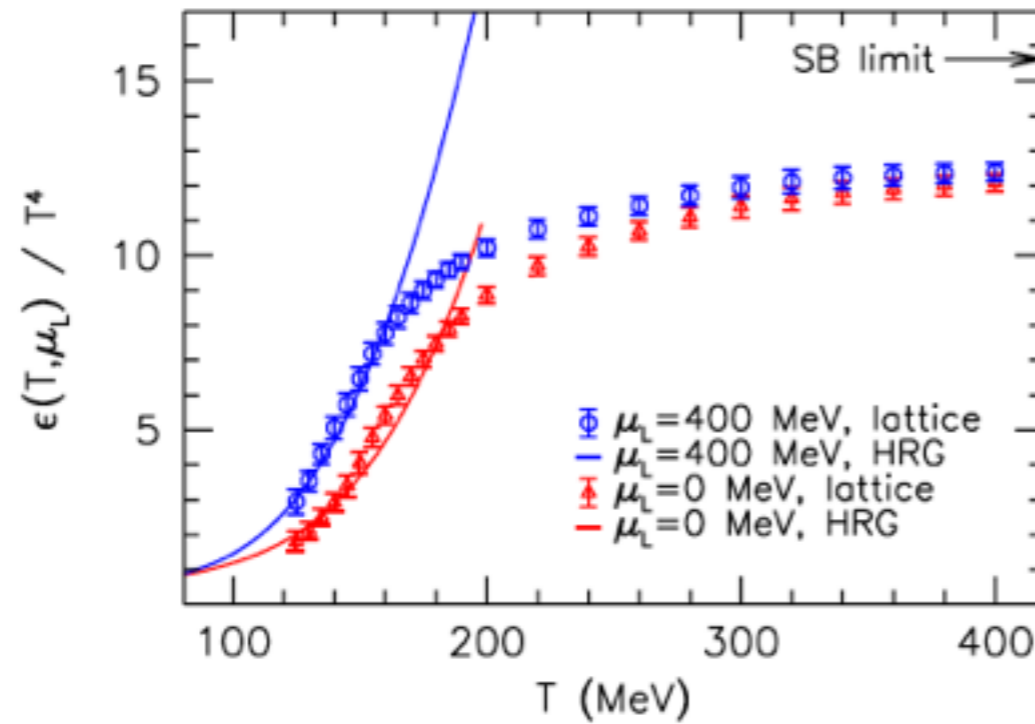
Quark model states of strange baryons





# QCD EOS at $\mu_B \neq 0$

Borzanyi et al., arXiv:1204:6710



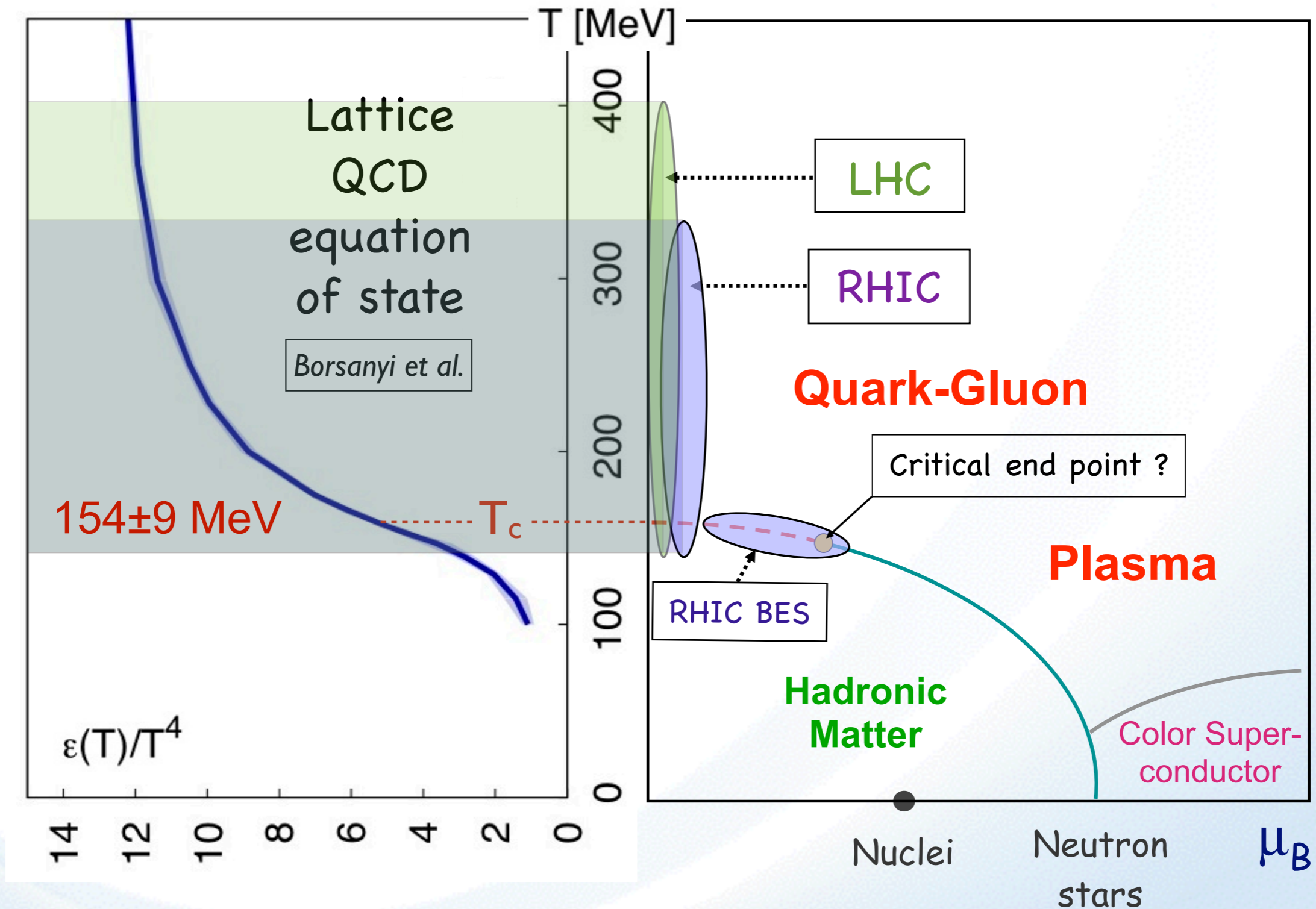
Approximate trajectories in QCD phase diagram



# Probing the QCD Phase Boundary



# QCD Phase Diagram





# Thermodynamic fluctuations

Susceptibilities measure thermodynamic fluctuations.  
Interesting because they exhibit singularities at a critical point.  
Fluctuations of **conserved quantities** (charge  $Q$ , baryon number  $B, \dots$ ) cannot be changed by local final-state processes.

Expt.: mean:  $M_Q$   
variance:  $\sigma_Q^2$   
skewness:  $S_Q$   
kurtosis:  $\kappa_Q$

$$\sqrt{s} \Leftrightarrow (T, \mu_B)$$

Lattice gauge theory:

$$\chi_n^X(T, \mu_X) = \frac{\partial^n (p(T, \mu_X)/T^4)}{\partial (\mu_X/T)^n}$$

Ratios are independent of the (unknown) freeze-out volume:

$$\frac{M_Q(\sqrt{s})}{\sigma_Q^2(\sqrt{s})} = \frac{\chi_1^Q(T, \mu_B)}{\chi_2^Q(T, \mu_B)} \quad \frac{S_Q(\sqrt{s})\sigma_Q^3(\sqrt{s})}{M_Q(\sqrt{s})} = \frac{\chi_3^Q(T, \mu_B)}{\chi_1^Q(T, \mu_B)}$$

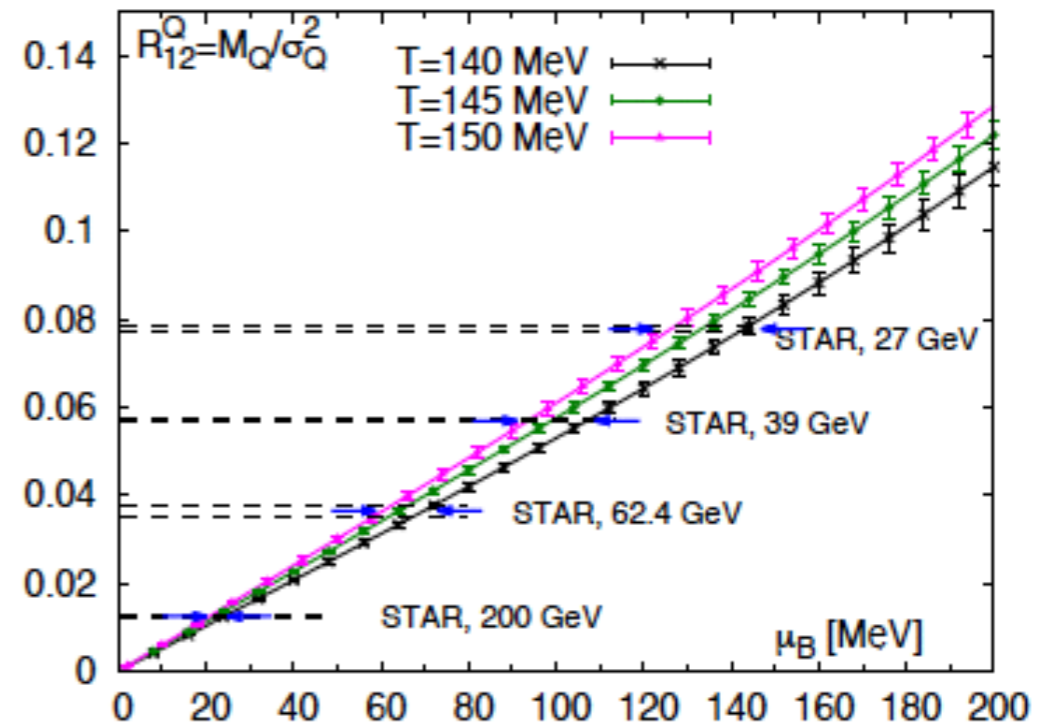
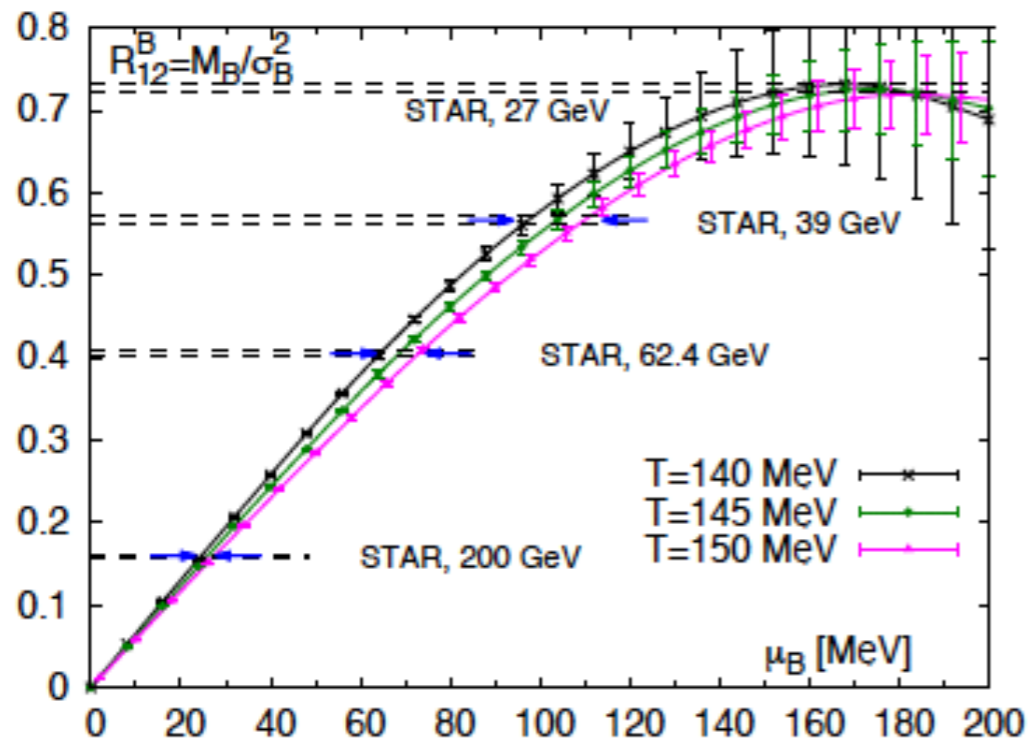


# Chemical freeze-out

... from fluctuations of conserved quantum numbers ( $Q$ ,  $B$ ):

Borsanyi et al. Wuppertal-Budapest Coll. Phys.Rev.Lett. 111, 062005 (2013); Phys.Rev.Lett. 113, 052301 (2014)

use  $M/\sigma^2$  both in the baryon and in the charge sector

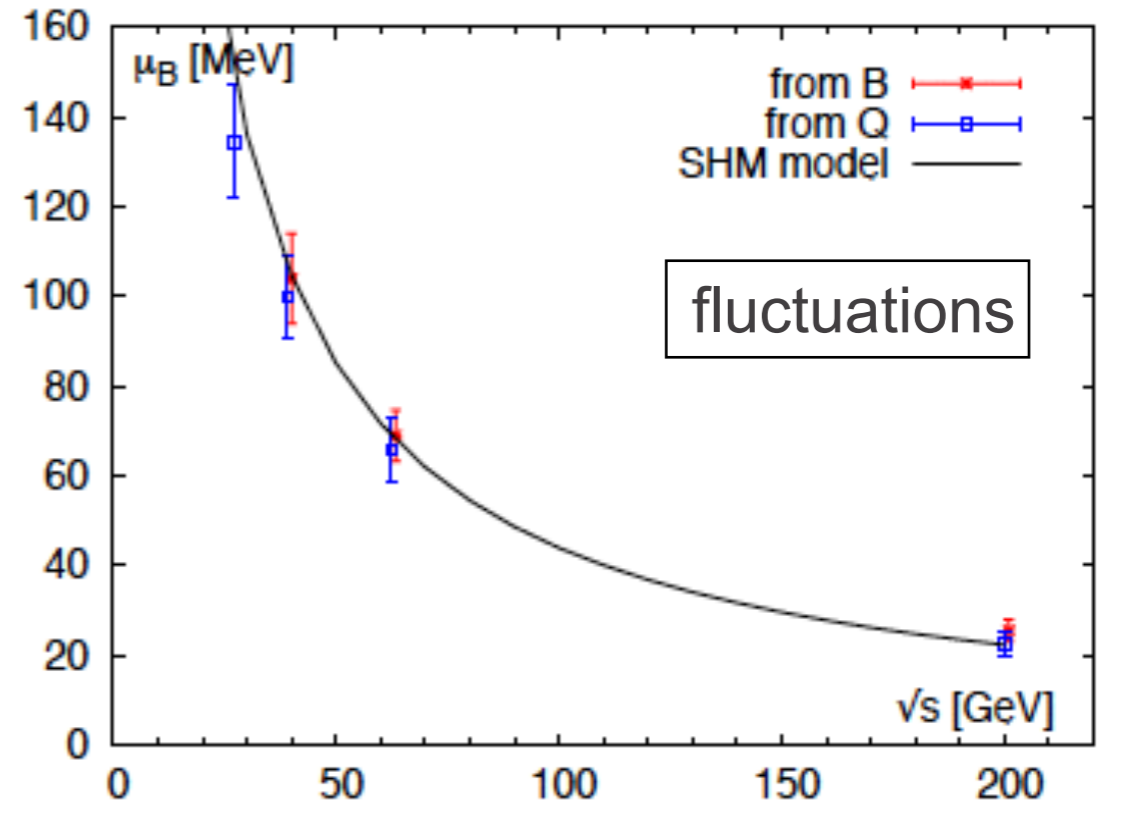
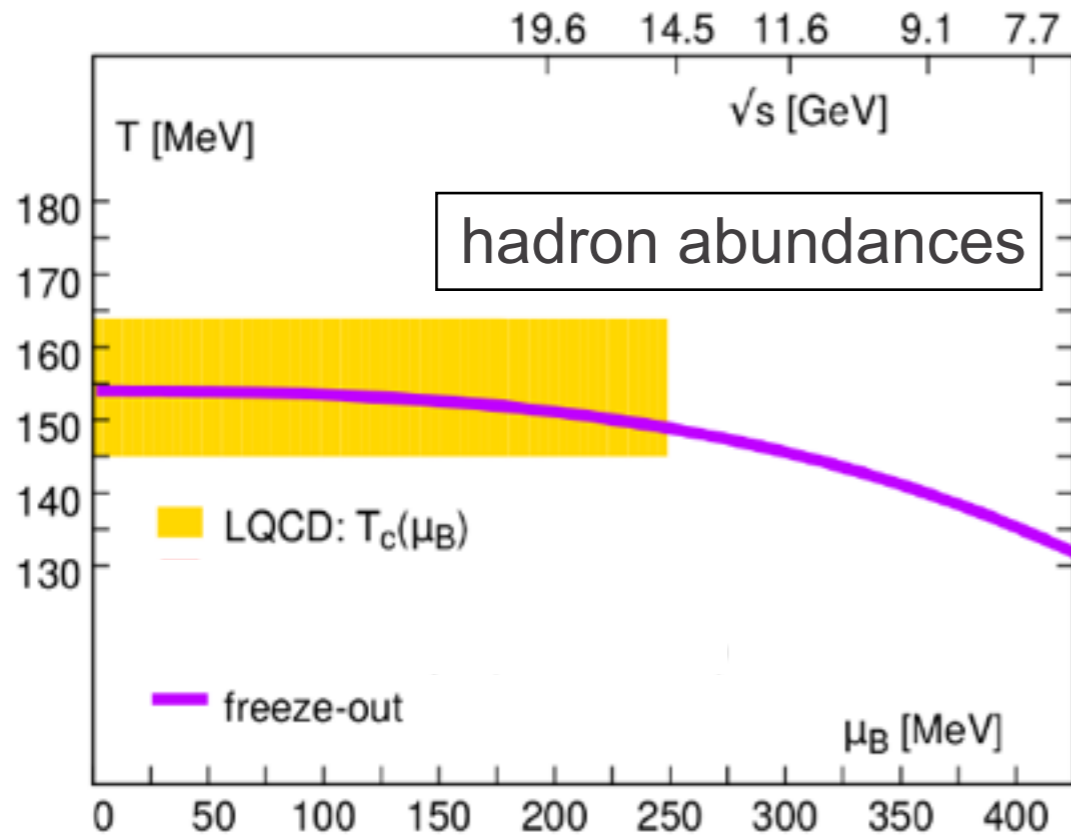


Compare lattice results with the STAR data for the fluctuation ratios in the temperature range 140–150 MeV permits to read off  $\mu_B$ .

Both methods are consistent with each other and with the measured baryon/antibaryon ratios, if additional strange baryon states beyond those in the PDG tables (e.g. in the quark model) are accounted for.



# Chemical freeze-out



Consistency of freeze-out parameters from mean hadron abundances and from fluctuations ( $Q$ ,  $B$ ) opens the door to search for a critical point in the QCD phase diagram by looking for enhanced critical fluctuations as function of beam energy.



# Probing the Quark-Gluon Plasma



# Hot QCD matter properties

Which **properties of hot QCD matter** can we hope to determine and how ?



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Easy  
for  
LQCD

$$T_{\mu\nu} \Leftrightarrow \varepsilon, p, s$$

**Equation of state:** spectra, coll. flow, fluctuations



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$$\eta = \frac{1}{T} \int d^4x \langle T_{xy}(x) T_{xy}(0) \rangle$$

**Shear viscosity:** anisotropic collective flow

Very  
Hard  
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LQCD

$$\hat{q} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle U^\dagger F^{a+i}(y^-) U F_i^{a+}(0) \rangle$$

$$\hat{e} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle iU^\dagger \partial^- A^{a+}(y^-) U A^{a+}(0) \rangle$$

$$\kappa = \frac{4\pi\alpha_s}{3N_c} \int d\tau \langle U^\dagger F^{a0i}(\tau) t^a U F^{b0i}(0) t^b \rangle$$

**Momentum/energy diffusion:**  
parton energy loss, jet fragmentation



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Easy  
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$$m_D = -\lim_{|x| \rightarrow \infty} \frac{1}{|x|} \ln \langle U^\dagger E^a(x) U E^a(0) \rangle$$

**Color screening:** Quarkonium states



# The “perfect” fluid

# Viscous hydrodynamics

Hydrodynamics = effective theory of energy and momentum conservation

**energy-momentum tensor** = **ideal fluid** + **dissipation**

$$\partial_{\mu} T^{\mu\nu} = 0 \quad \text{with} \quad T^{\mu\nu} = (\varepsilon + P)u^{\mu}u^{\nu} - Pg^{\mu\nu} + \Pi^{\mu\nu}$$

$$\tau_{\Pi} \left[ \frac{d\Pi^{\mu\nu}}{d\tau} + \left( u^{\mu}\Pi^{\nu\lambda} + u^{\nu}\Pi^{\mu\lambda} \right) \frac{du^{\lambda}}{d\tau} \right] = \eta \left( \partial^{\mu}u^{\nu} + \partial^{\nu}u^{\mu} - \text{trace} \right) - \Pi^{\mu\nu}$$

**Input:** Equation of state  $P(\varepsilon)$ , shear viscosity, initial conditions  $\varepsilon(x,0)$ ,  $u^{\mu}(x,0)$

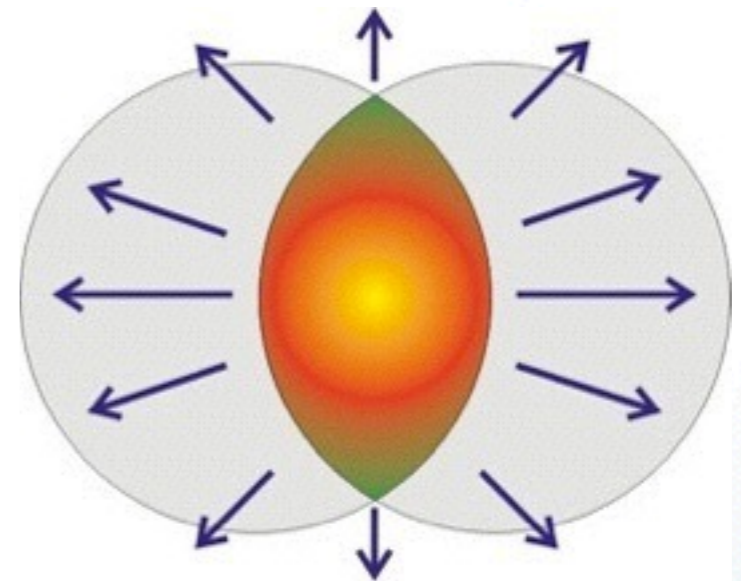
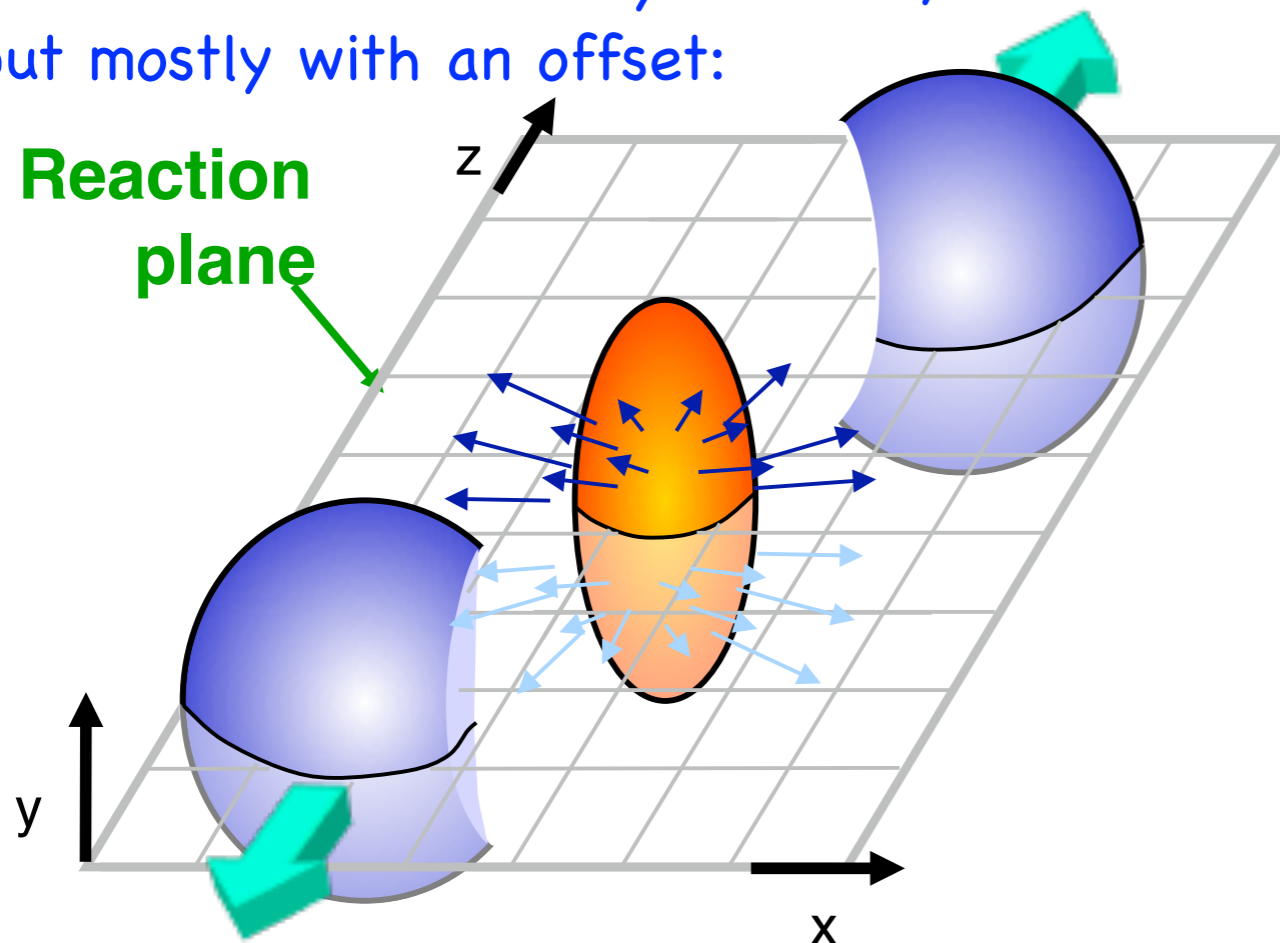
Shear viscosity  $\eta$  is normalized by density: **kinematic viscosity**  $\eta/\rho$ .

Relativistically, the appropriate normalization factor is the **entropy density**  $s = (\varepsilon+P)/T$ , because the particle density is not conserved:  $\eta/s$ .



# Elliptic flow

- two nuclei collide rarely head-on, but mostly with an offset:



only matter in the overlap area gets compressed and heated

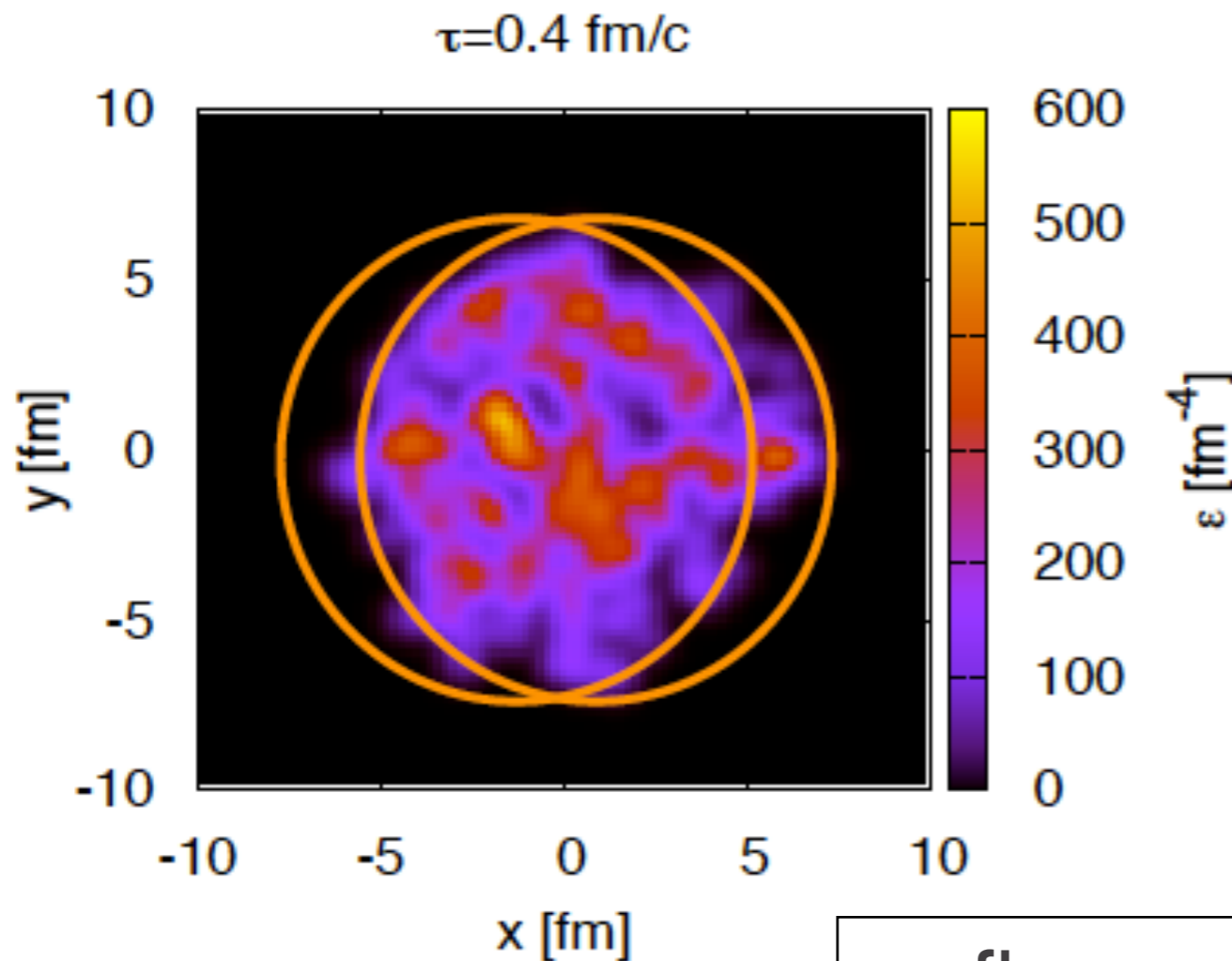
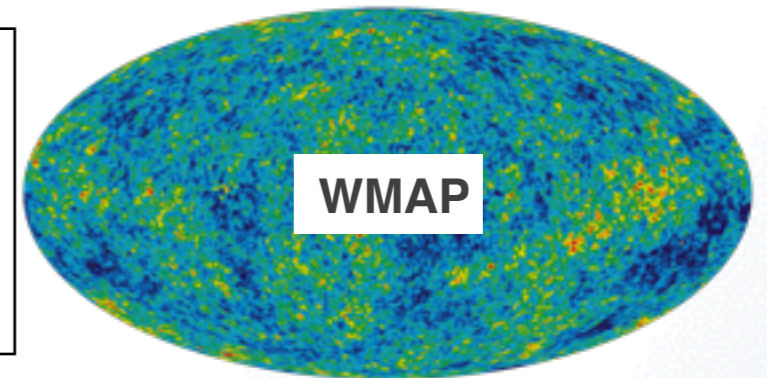
$$2\pi \frac{dN}{d\phi} = N_0 \left( 1 + 2 \sum_n v_n(p_T, \eta) \cos n(\phi - \psi_n(p_T, \eta)) \right)$$

anisotropic flow coefficients

event plane angle

# Event-by-event fluctuations

Initial state generated in A+A collision is grainy  
event plane  $\neq$  reaction plane  
 $\Rightarrow$  eccentricities  $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \text{ etc.} \neq 0$



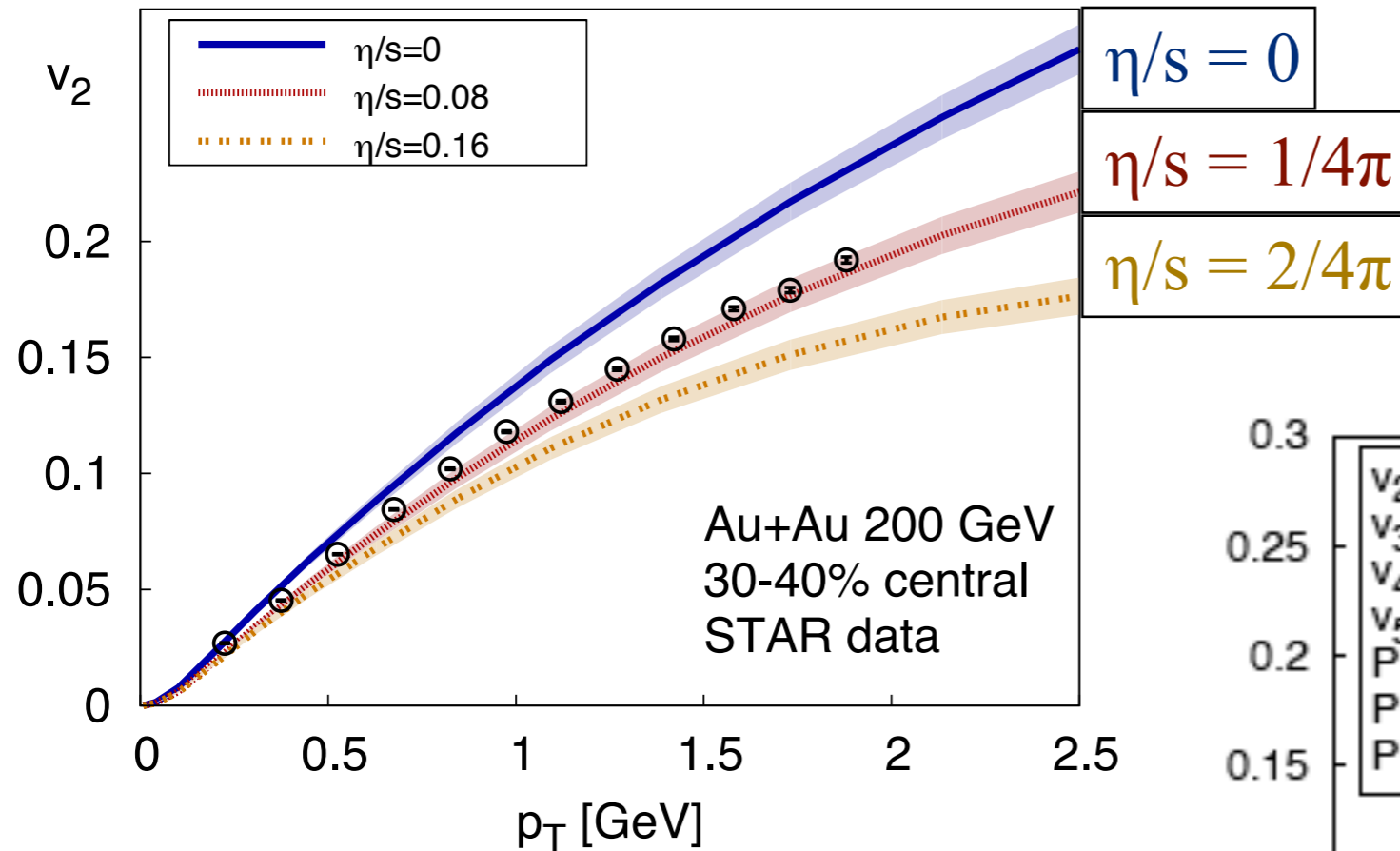
Idea: Energy density fluctuations in transverse plane from initial state quantum fluctuations. These thermalize to different temperatures locally and then propagate hydrodynamically to generate angular flow velocity fluctuations in the final state.

$\Rightarrow$  flows  $v_1, v_2, v_3, v_4, \dots$



# Elliptic flow “measures” $\eta_{\text{QGP}}$

Schenke, Jeon, Gale, PRL 106 (2011) 042301

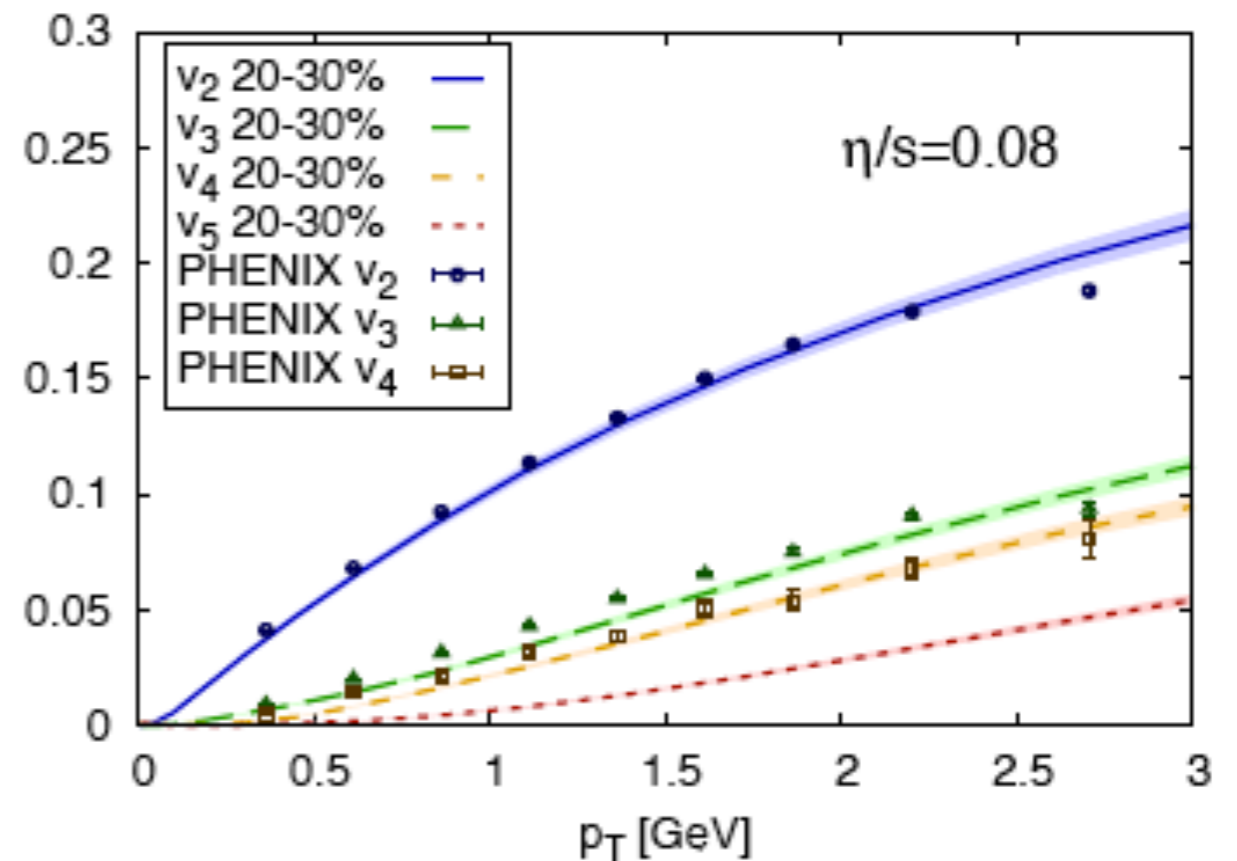


Schenke, Jeon, Gale, PRC 85 (2012) 024901

Universal strong coupling limit of non-abelian gauge theories with a gravity dual:

$$\eta/s \rightarrow 1/4\pi$$

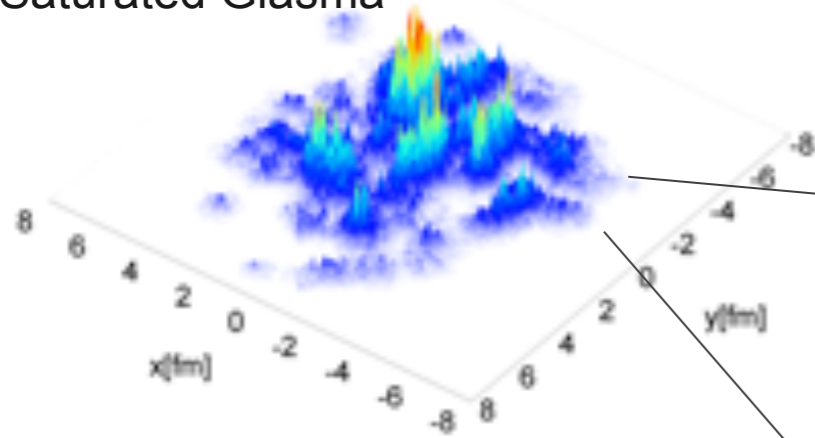
aka: the “perfect” liquid



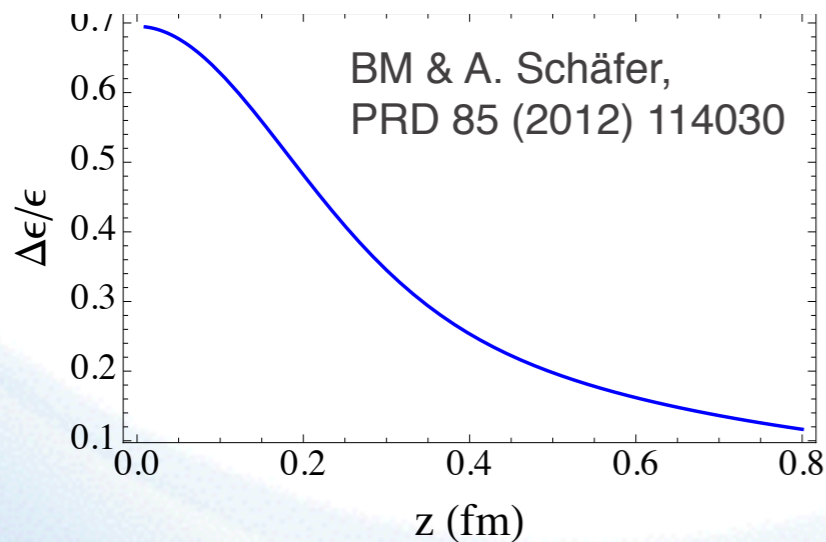
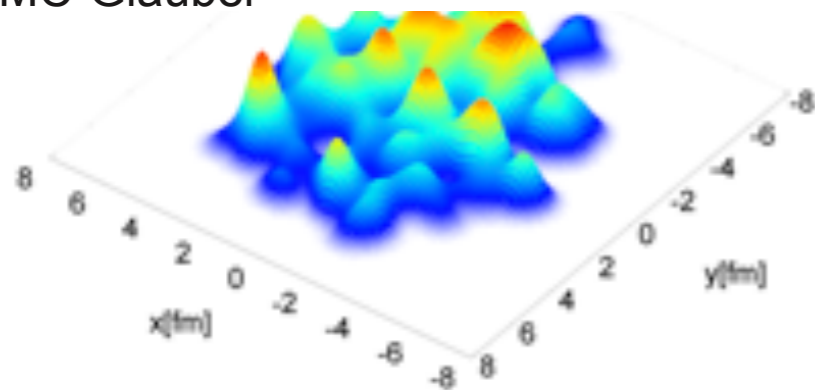
# RHIC vs. LHC

Gale, Jeon, Schenke, Tribedy, Venugopalan, arXiv:1209.6330

Saturated Glasma

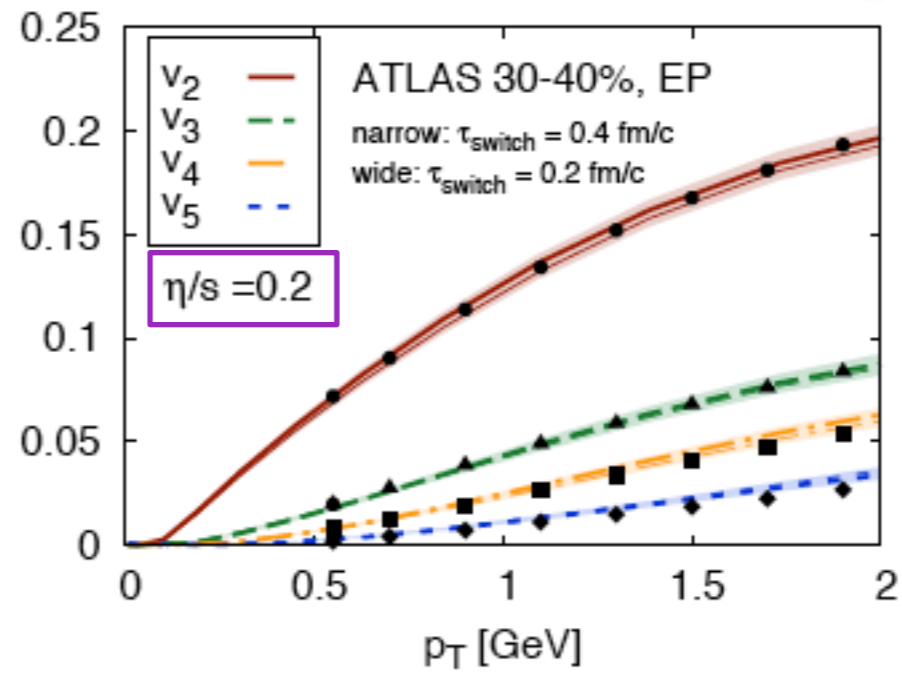


MC-Glauber

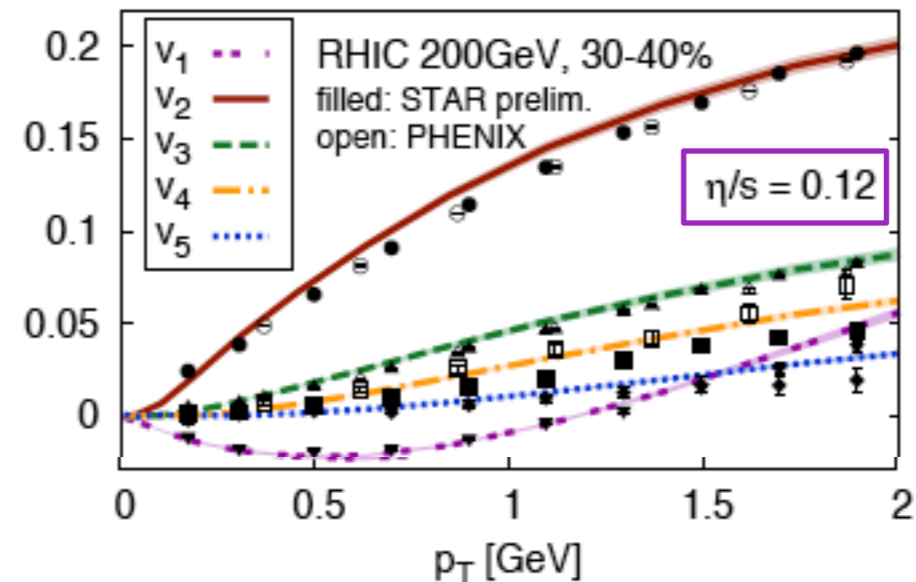


$\langle v_n^2 \rangle^{1/2}$

$\langle v_n^2 \rangle^{1/2}$



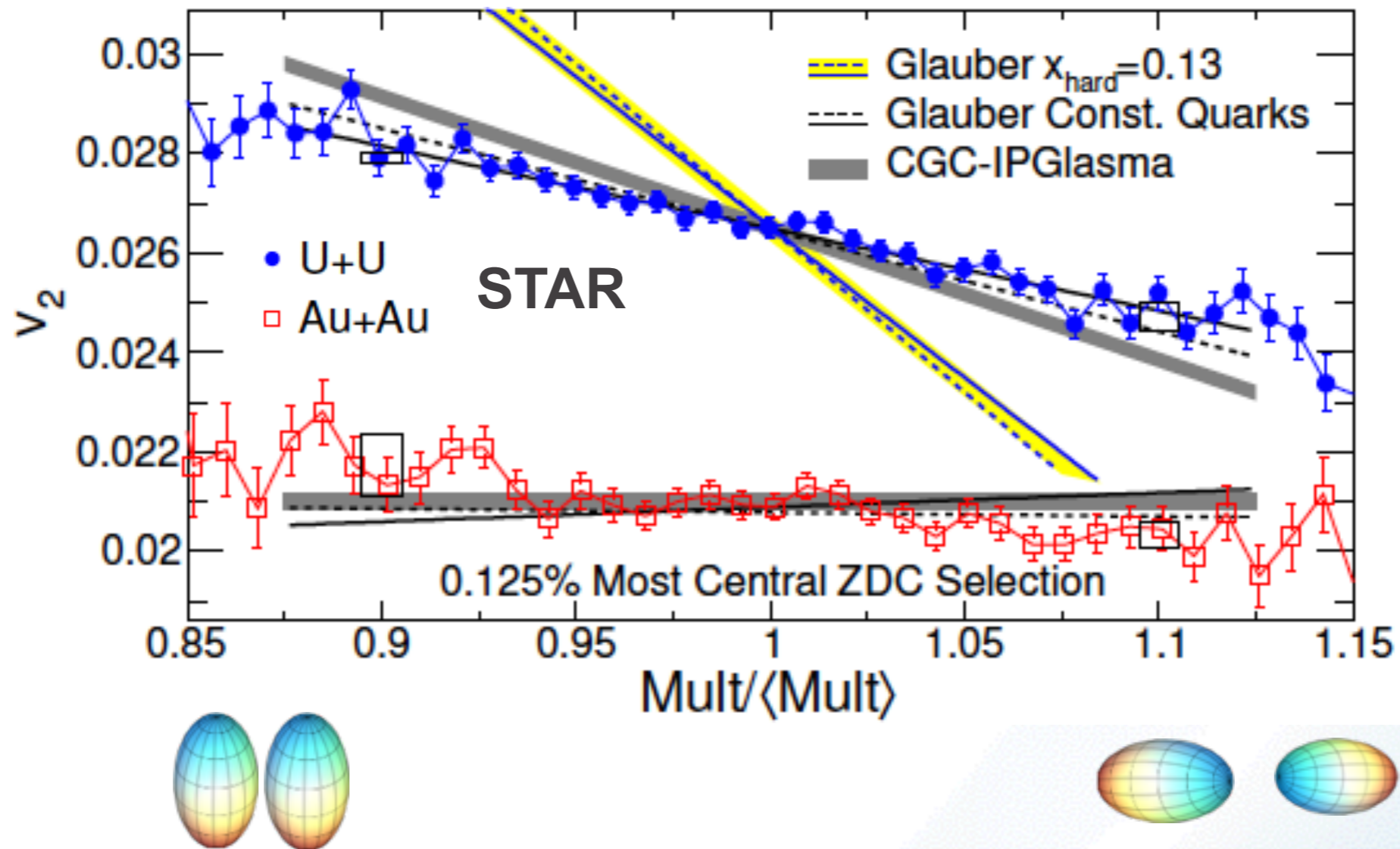
LHC



RHIC



# Shape engineering: U+U collisions



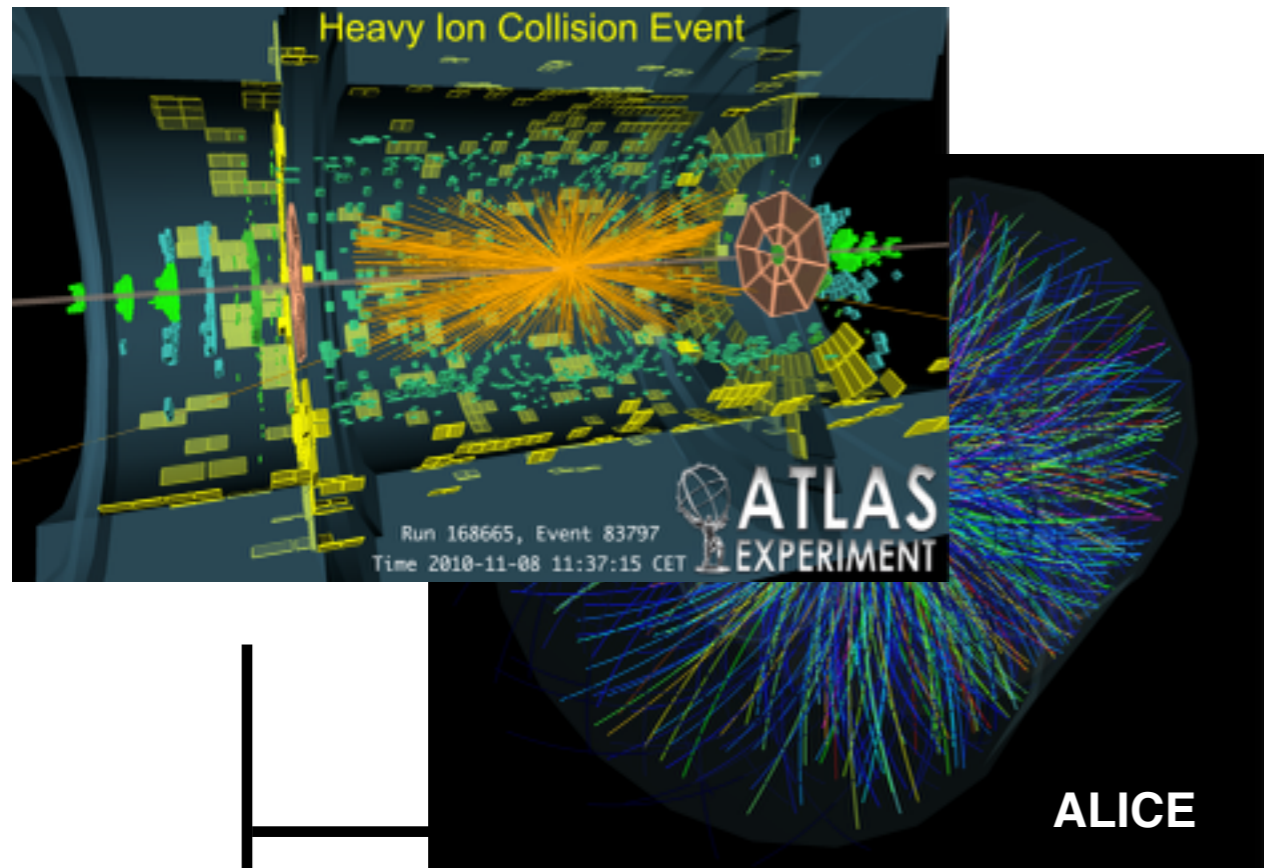
Constituent-quark Glauber model and IP-Glasma model are consistent with the observations, but not the nucleon-nucleon based Glauber model.

→ Initial state fluctuations are driven by interactions at the sub-nucleonic level



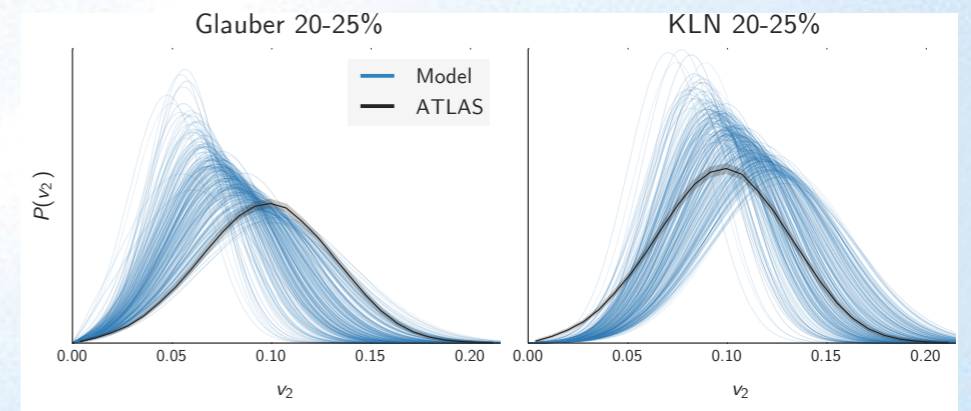
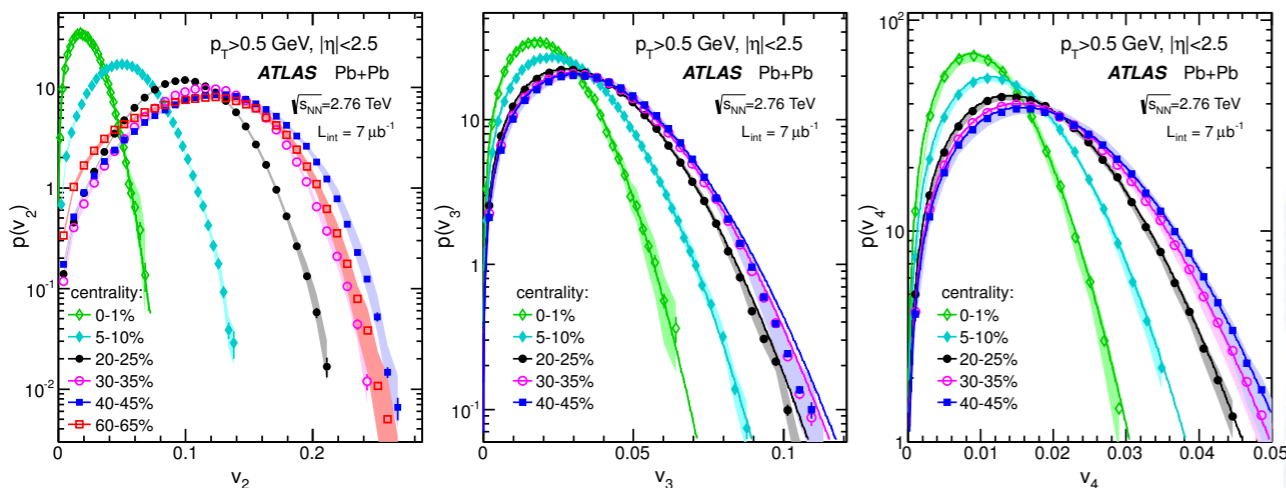
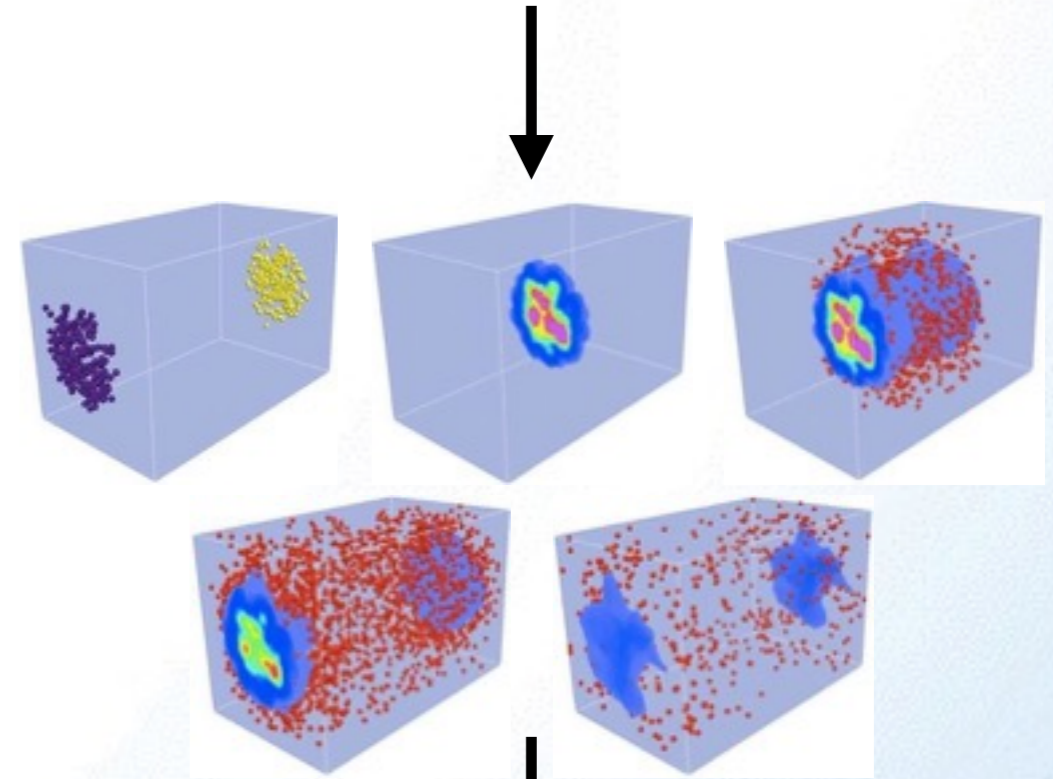
# Discovery by model-data comparison

## Data:



## Model:

initial conditions,  $\tau_0$ ,  $\eta/s$ ,  $\zeta/s$ , ....



extracted QGP properties:  $\eta/s$ , ...



# Flow Analysis of QGP Properties at the LHC

## Data:

- ALICE  $v_2$ ,  $v_3$  &  $v_4$  flow cumulants
- identified particle spectra
- identified particle mean  $p_T$

## Model:

- EbE VISHNU

## Parameter Space:

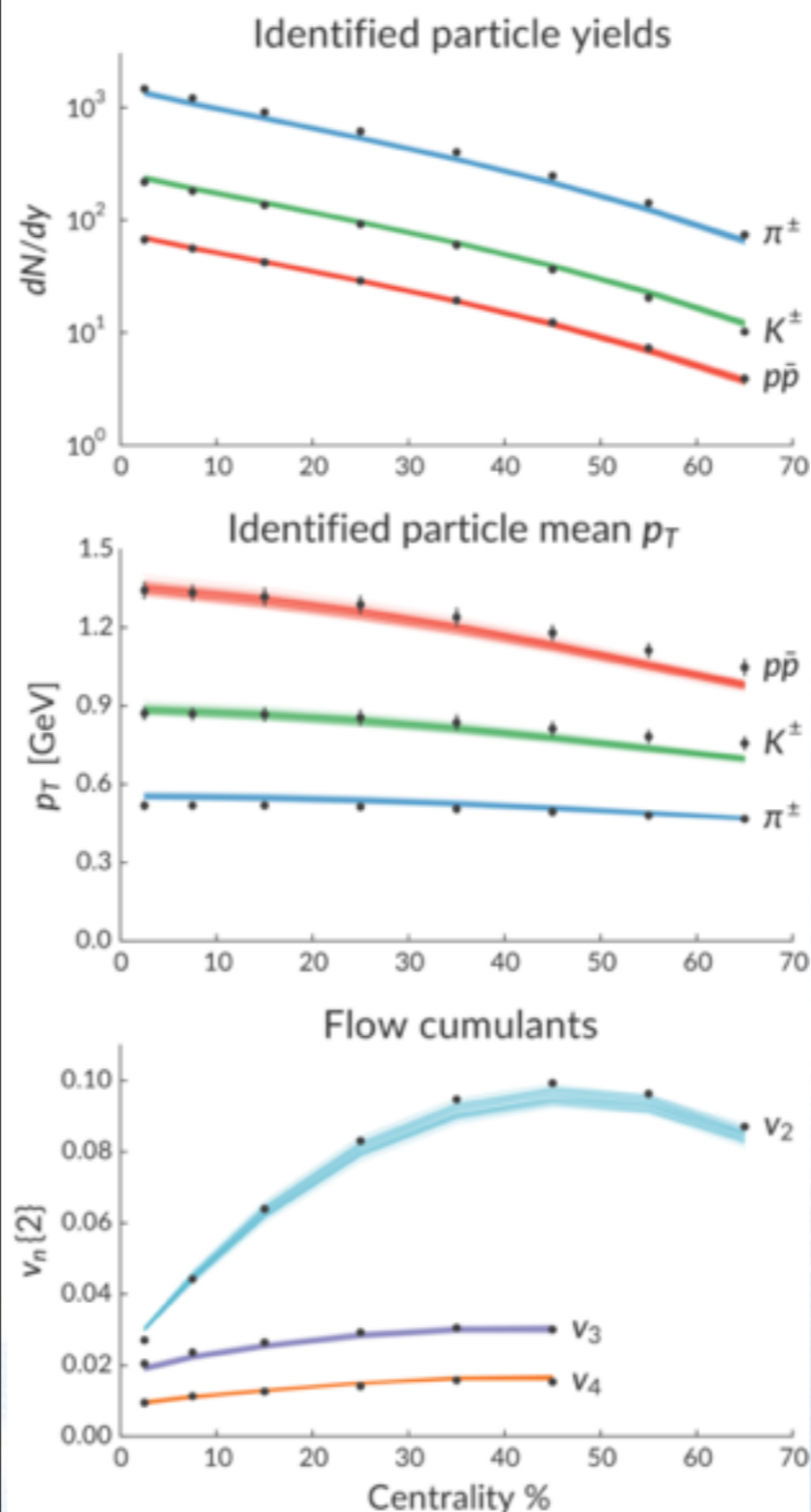
- Trento initial condition:
  - $p$ : entropy deposition
  - $k$ : nucleon fluctuation
  - $w$ : Gaussian nucleon width
- specific shear viscosity  $\eta/s$  slope and intercept at  $T_c$
- normalization scale for  $\zeta/s$
- hydro to micro switching temperature  $T_{sw}$

## Analysis Design:

- 6 centrality bins
- 300 point Latin Hypercube
- total of 10,000,000 events
- Gaussian Process Emulators for interpolation between LH points

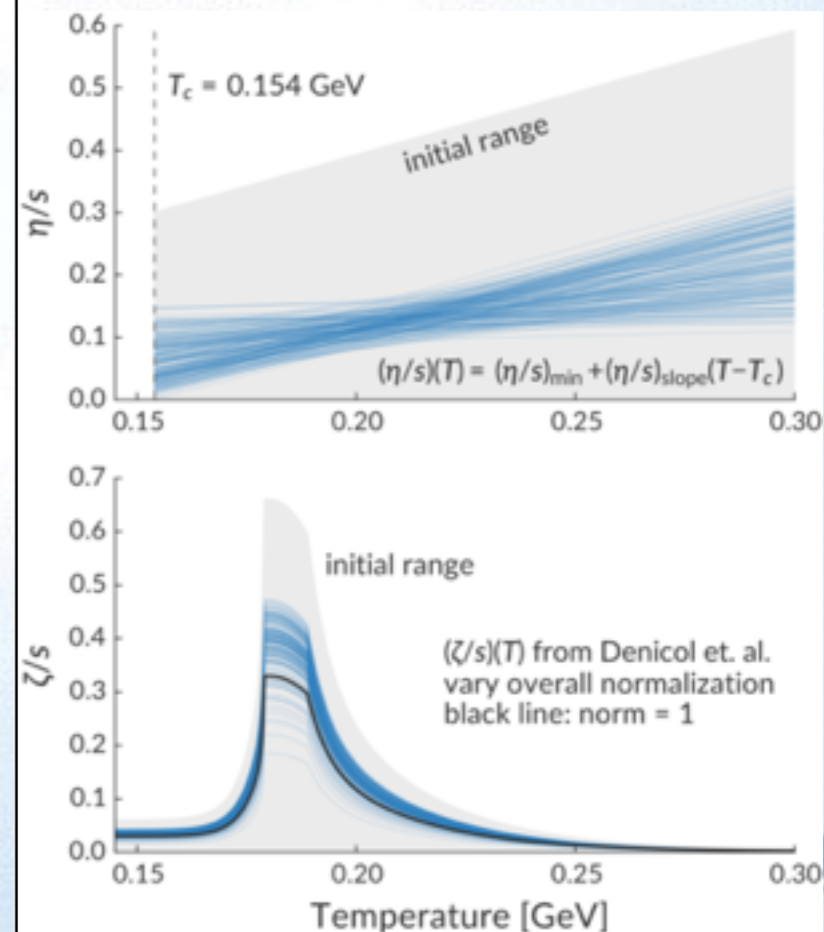
use MCMC for analysis

## Posterior: emulator predictions for highest likelihood parameter values

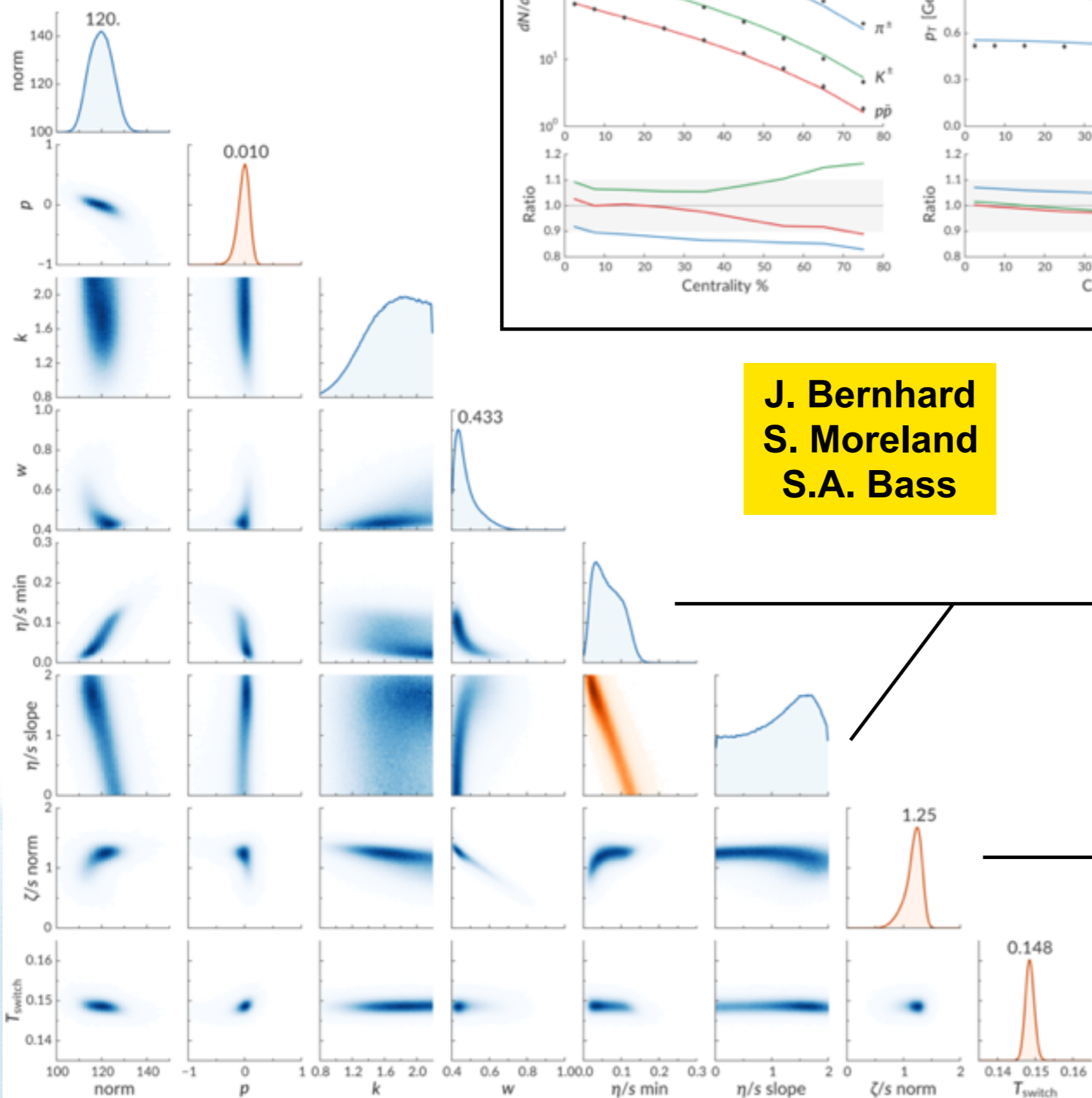


## Key Results:

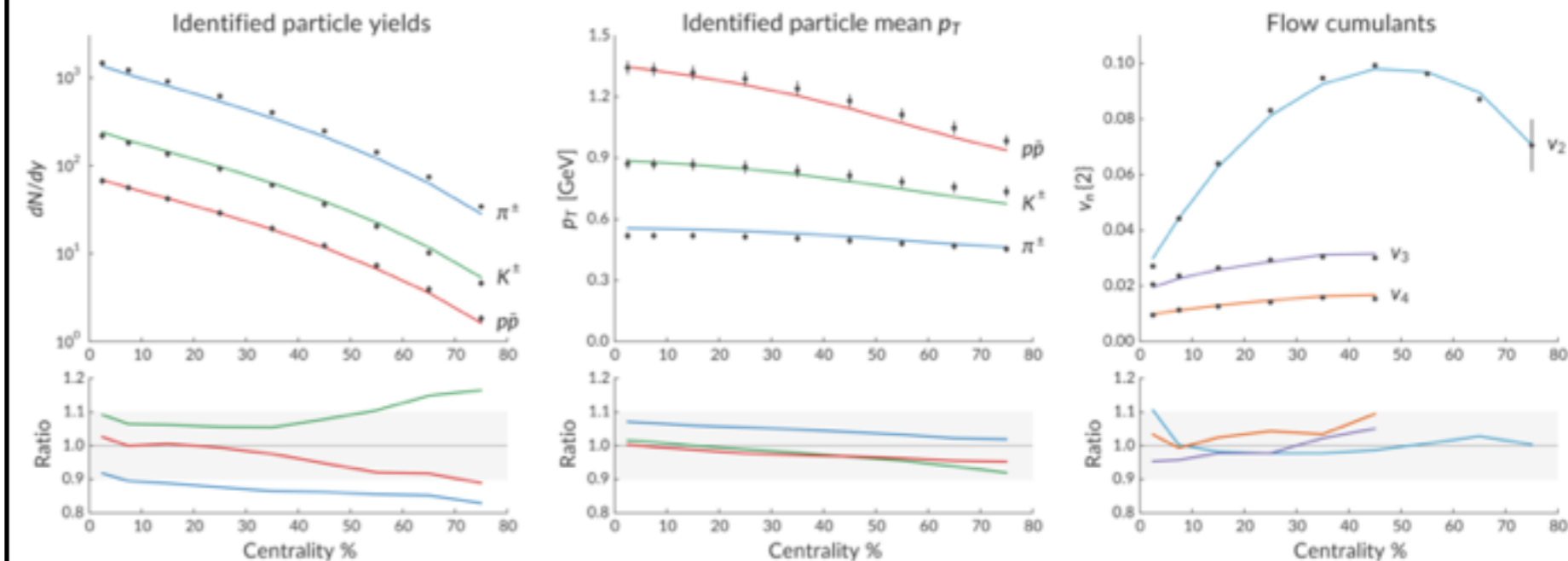
- excellent agreement with data, simultaneous description of  $v_2$ ,  $v_3$  and  $v_4$  data
- initial condition favors scaling properties of IP-Glasma
- non-zero bulk viscosity
- temperature dependence of  $\eta/s$  requires data at several beam energies to pin down



# Calibrated posterior distributions:

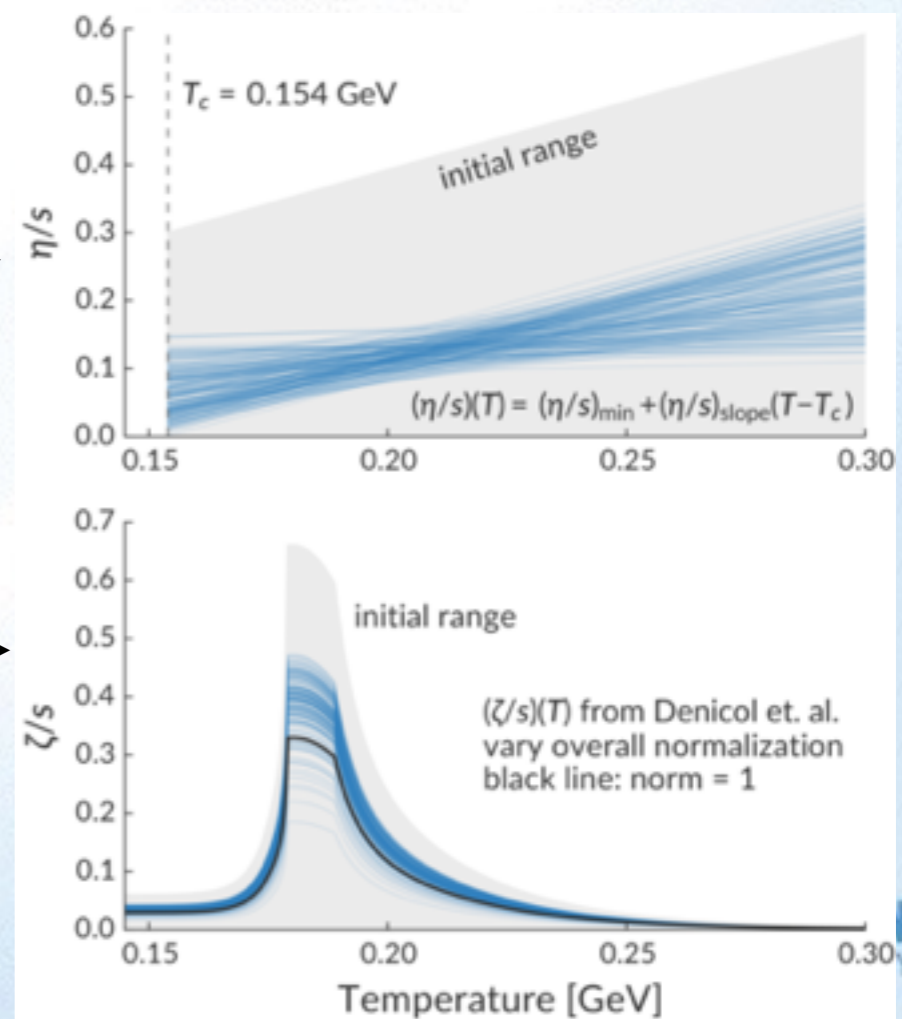


## Explicit model calculations (no emulator):



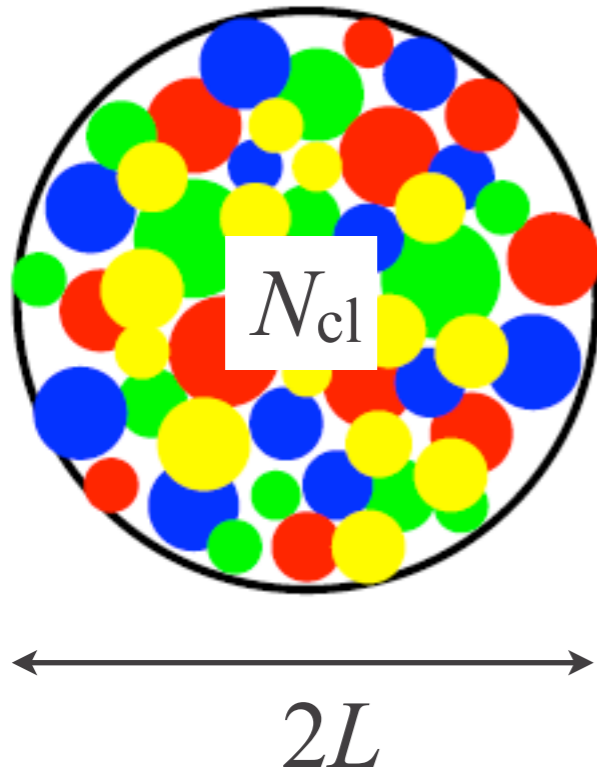
**J. Bernhard  
S. Moreland  
S.A. Bass**

## Temperature-dependent viscosities from the calibrated posterior:





# How small can a QGP be?



saturation scale

$$Q_s^2 \propto \frac{N_{cl}}{\pi L^2}$$

mean free path

$$\ell_{mfp} \propto Q_s^{-1}$$

final multiplicity

$$dN / dy \propto N_{cl}$$

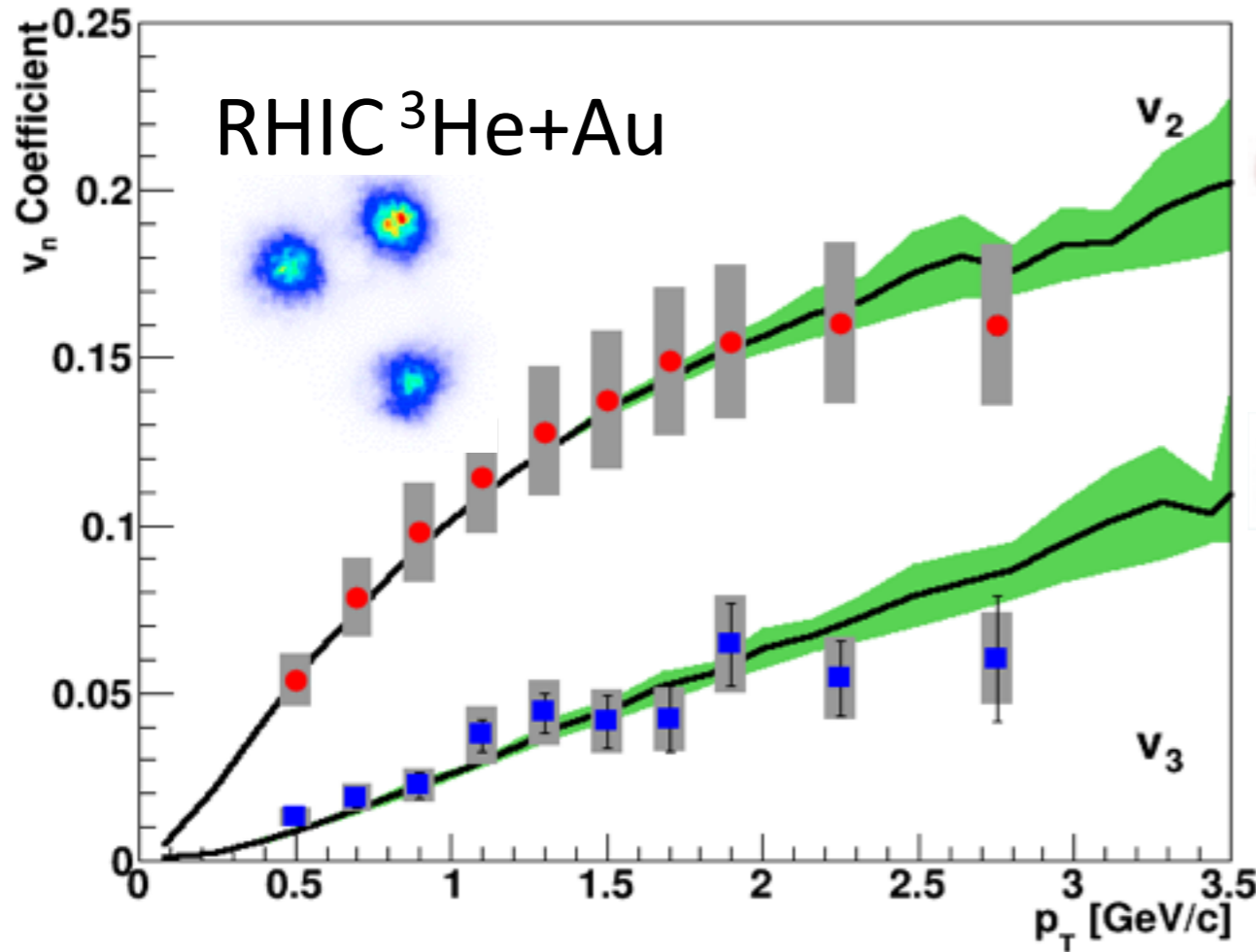
*Basar & Teaney, 1312.6770*

Size scales out of  
Reynolds number:

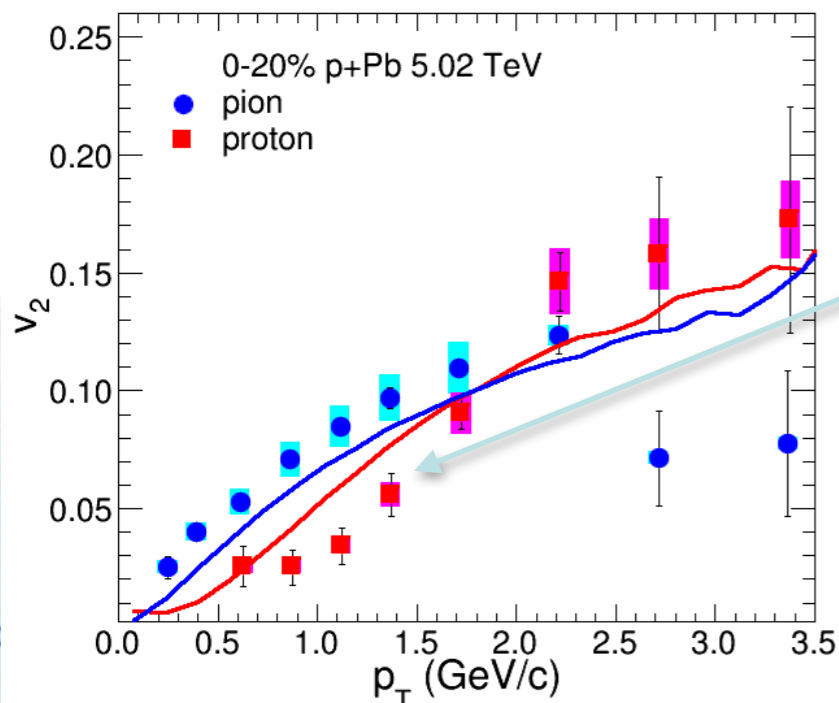
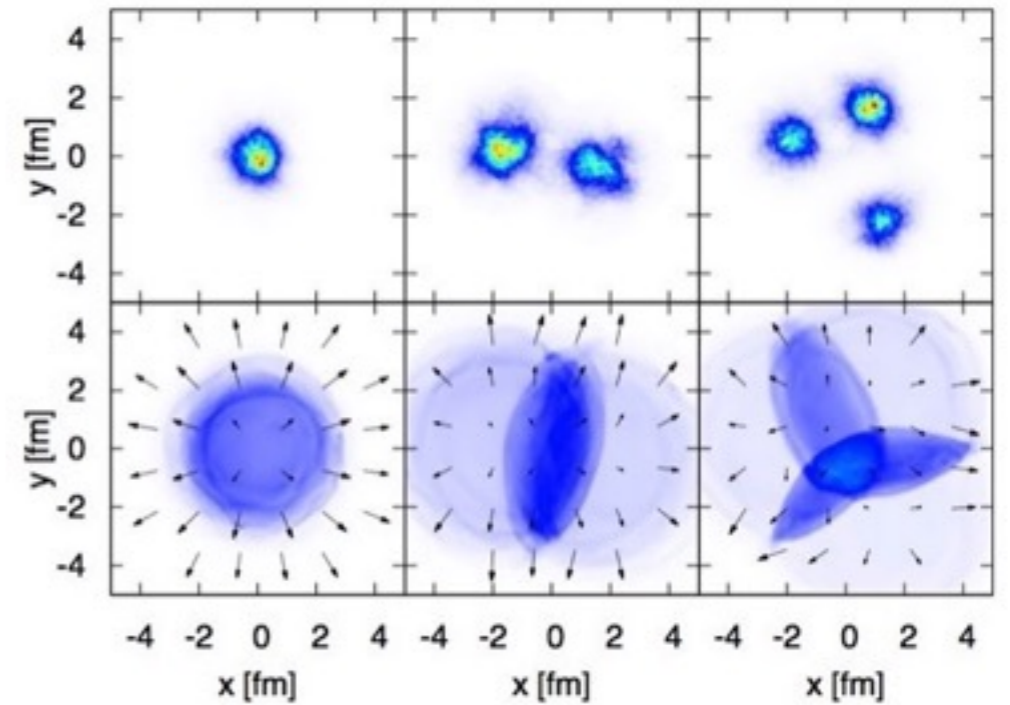
$$\text{Re} = \frac{\ell_{mfp}}{L} \propto \frac{1}{Q_s L} \propto \frac{1}{\sqrt{dN / dy}}$$

This does not mean that hydrodynamics applies for a given  $dN/dy$ , but it suggests that the transport is independent of size.

# How small can a QGP droplet be?

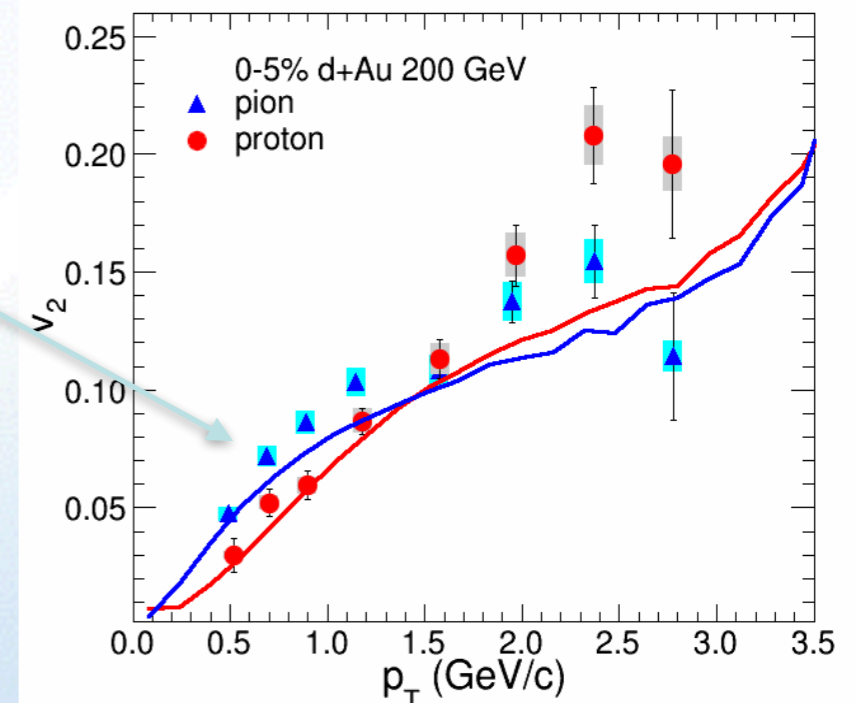


Very successful 3-week run resulted in 2.2 **billion** recorded minimum bias  $^3\text{He} + \text{Au}$  collisions (PHENIX)



Characteristic differential elliptic flow for hadrons of different mass

Evidence for strong coupling?





# QGP Chemistry

# Strangeness enhanced

The original idea (Rafelski):  $\frac{N(\bar{s})}{N(\bar{q})} = \frac{1}{2} \left( \frac{m_s}{T} \right)^2 K_2(m_s / T) e^{\mu_B / (3T)} = 1 \dots 5$

*“We almost always have more  $\bar{s}$  than  $\bar{u}$  or  $\bar{d}$  quarks. When quark matter reassembles into hadrons, some of the numerous  $\bar{s}$  quarks may, instead of being bound into kaons, form multiply strange antibaryons, such as  $\bar{\Lambda}$ ,  $\bar{\Xi}$ ,  $\bar{\Omega}$ .”*

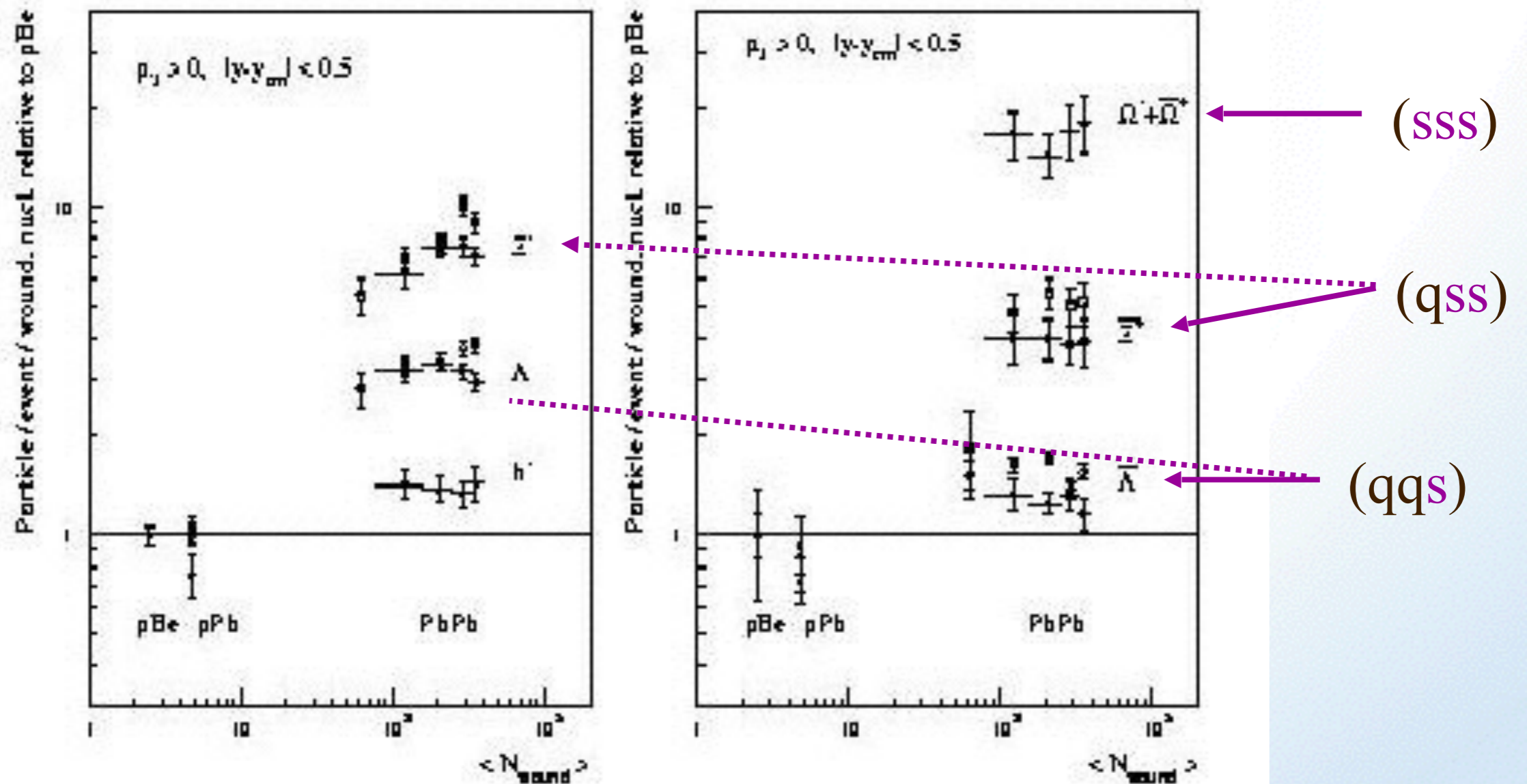
$gg \rightarrow s\bar{s}$  is essential for rapid strangeness equilibration ! (JR & BM)

Almost 30 years of investigation: The liberation of quark and gluon degrees of freedom (not necessarily thermalization) is required for strangeness equilibration.



# s-enhancement at SPS

NA57 measured enhanced strange baryon yields in Pb+Pb collisions at 158 GeV/c

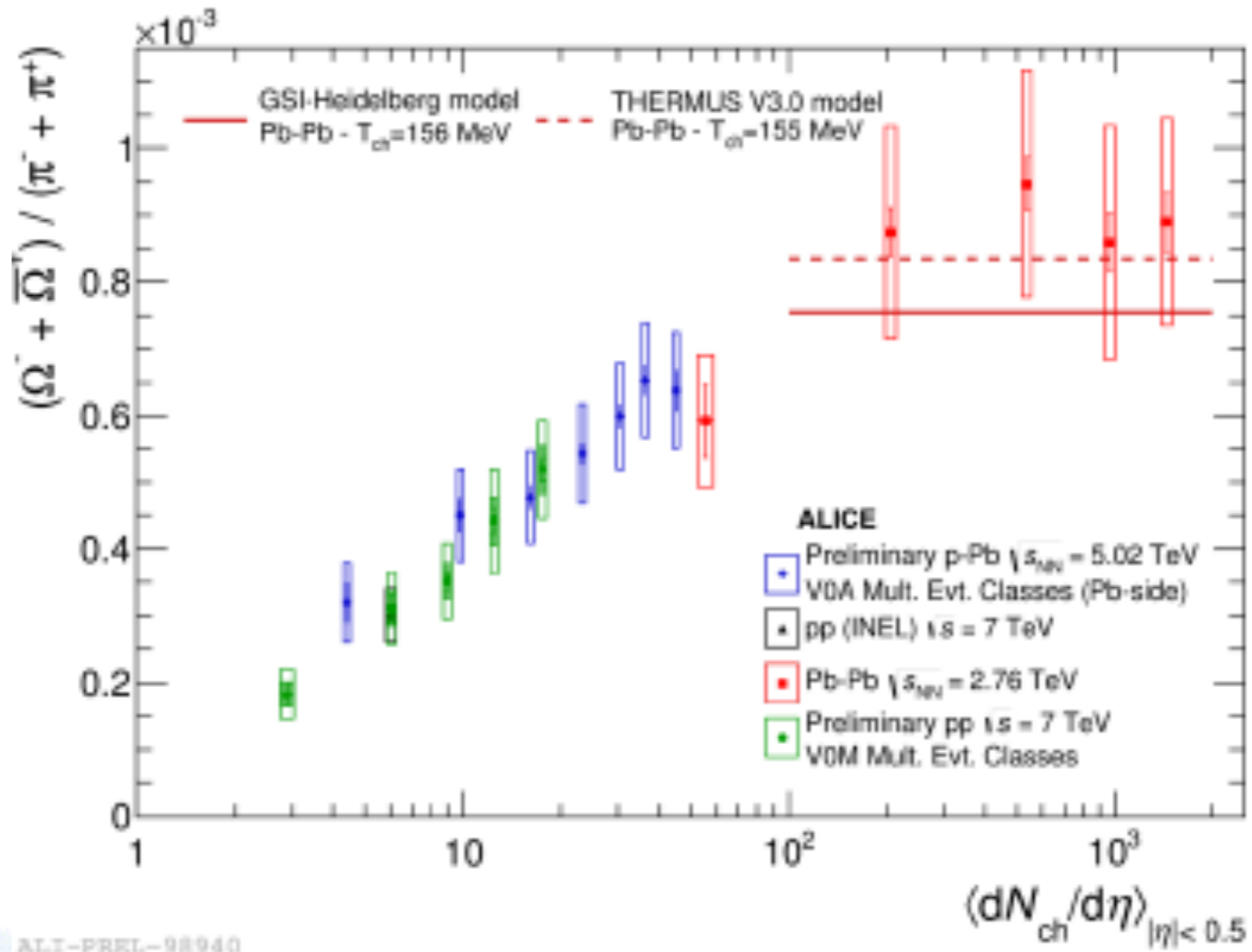




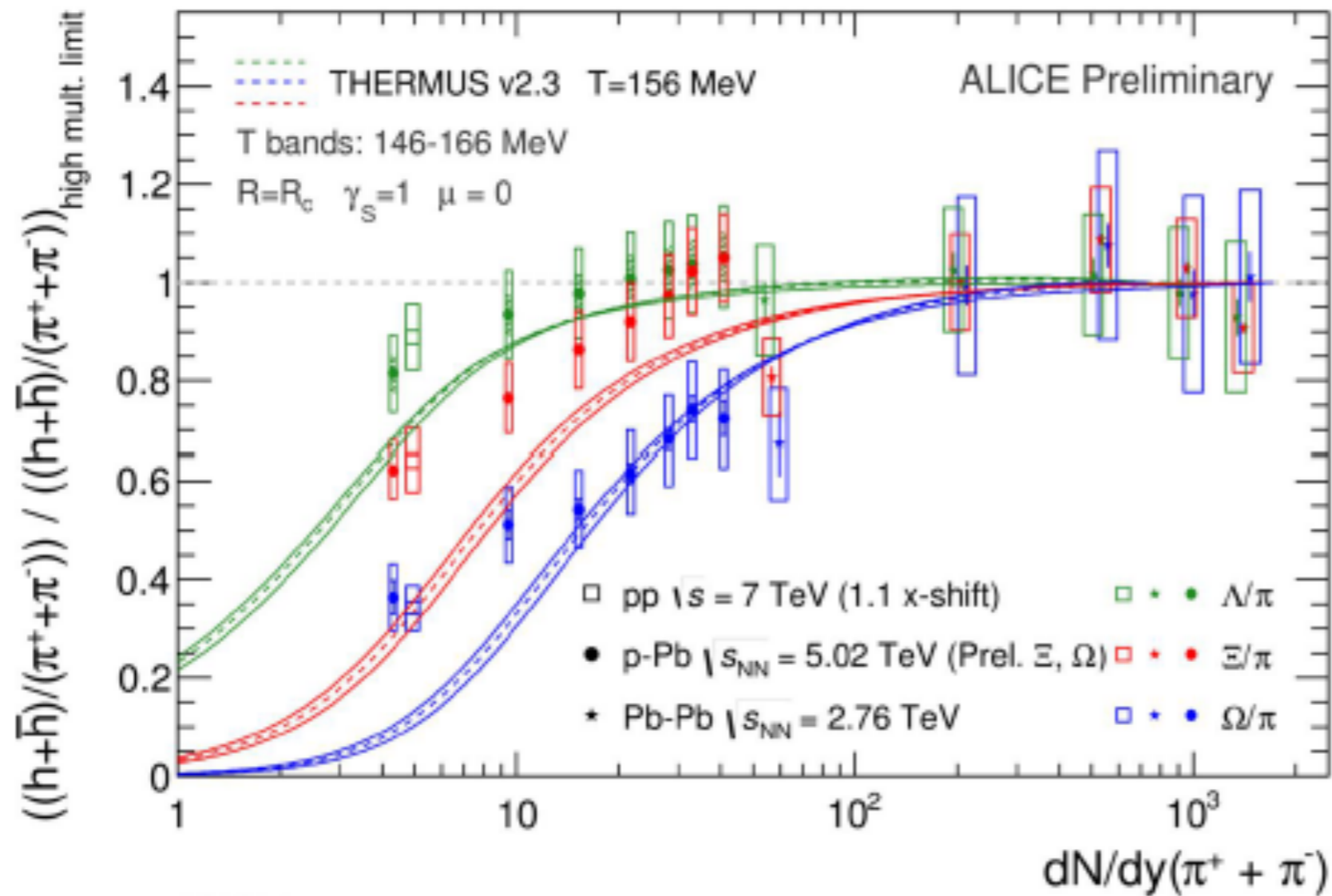


# s-enhancement at LHC

Strangeness enhancement grows with fireball size (or life-time) and saturates at grand canonical equilibrium in Pb+Pb collisions

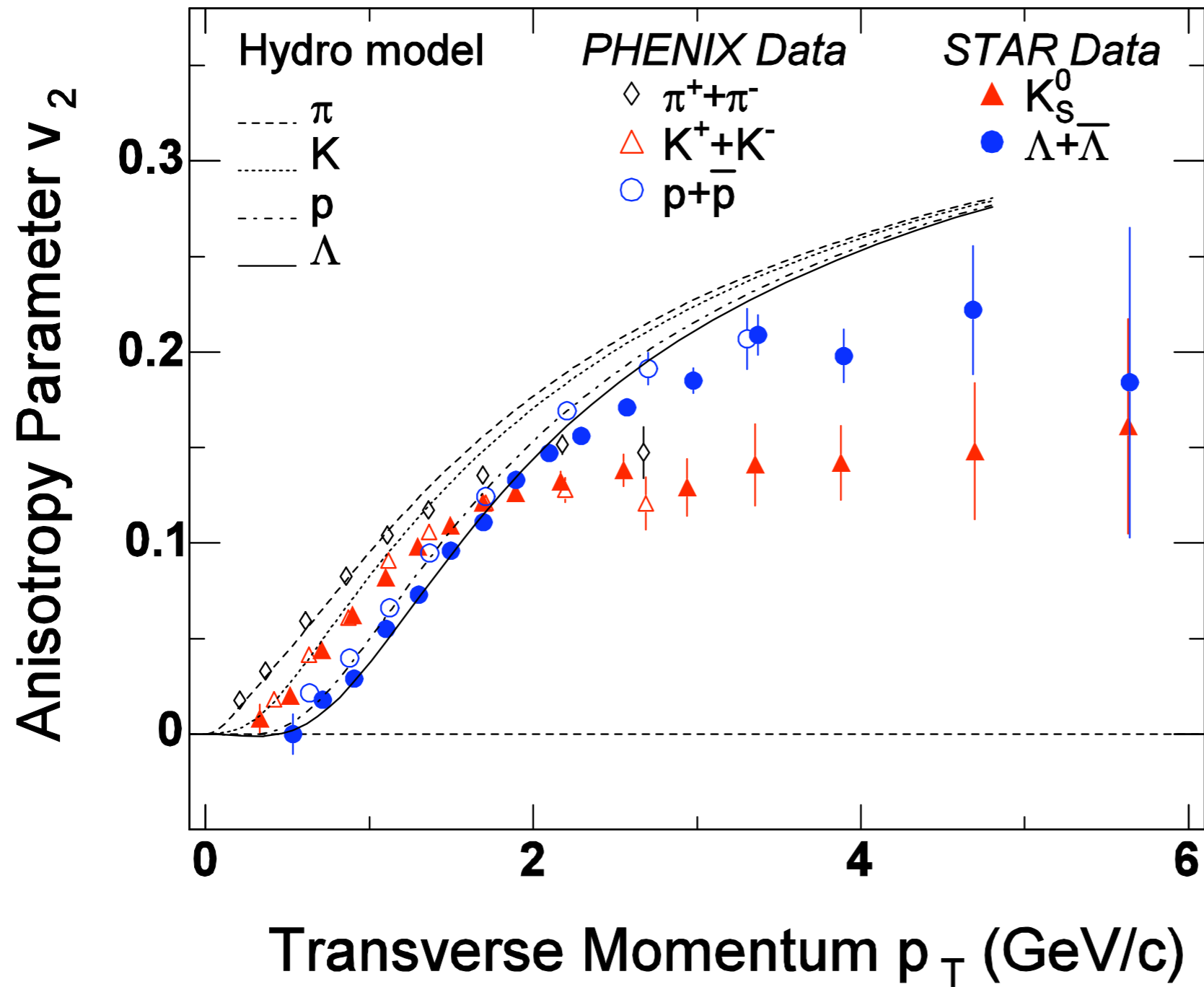


# Size dependence of s-enhancement

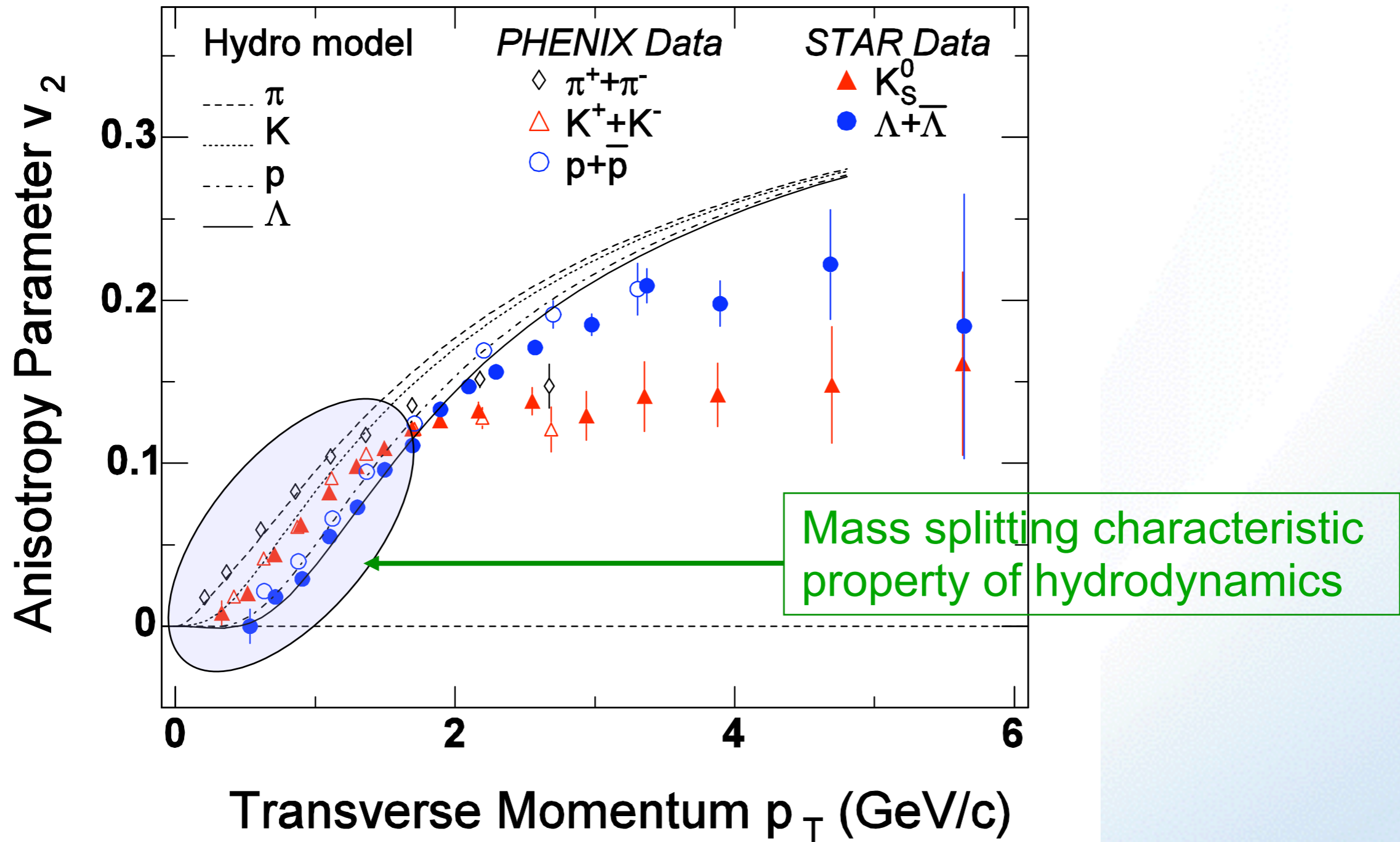




# $v_2(p_T)$ vs. hydrodynamics

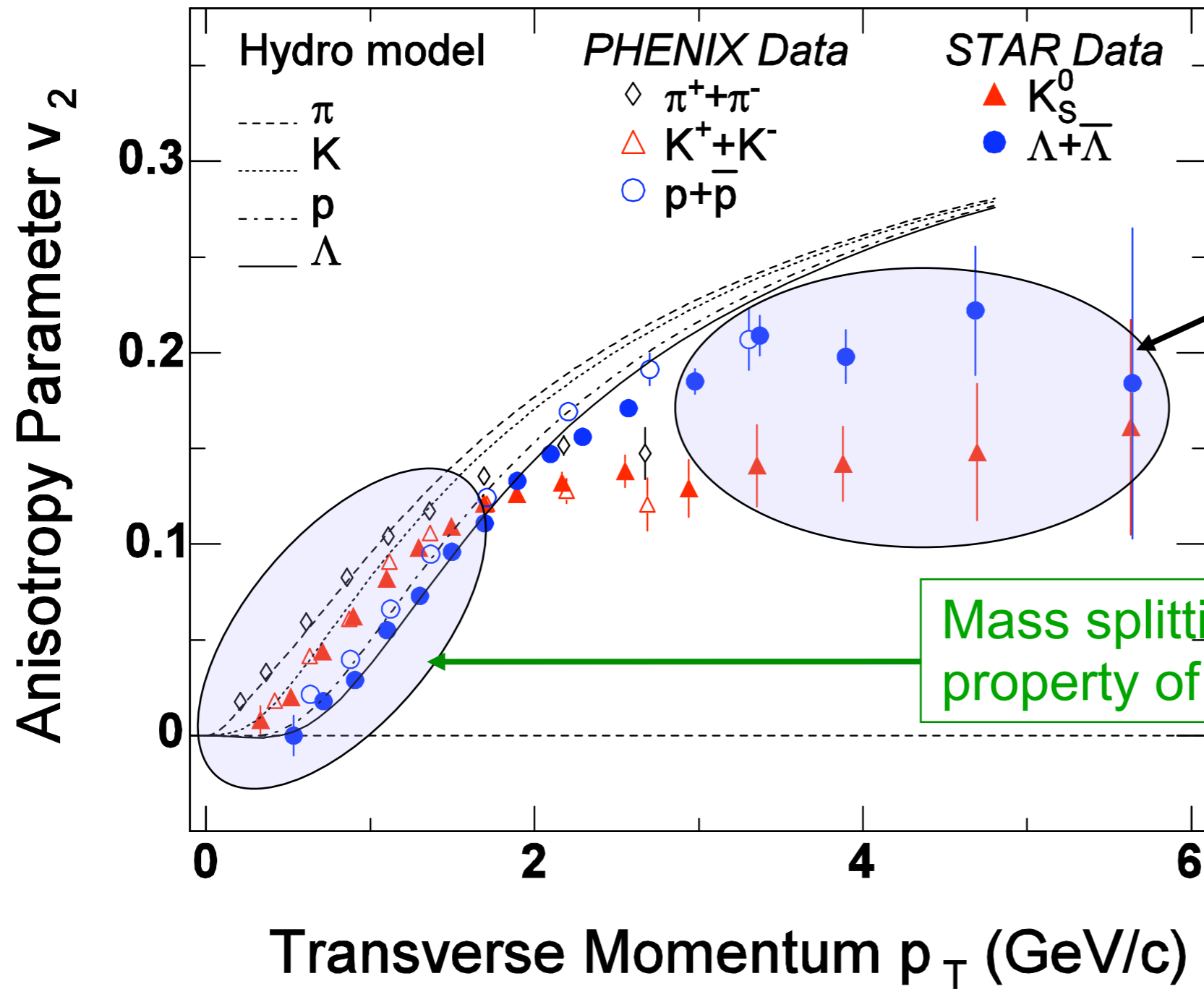


# $v_2(p_T)$ vs. hydrodynamics





# $v_2(p_T)$ vs. hydrodynamics



Failure of ideal hydrodynamics tells us how hadrons form

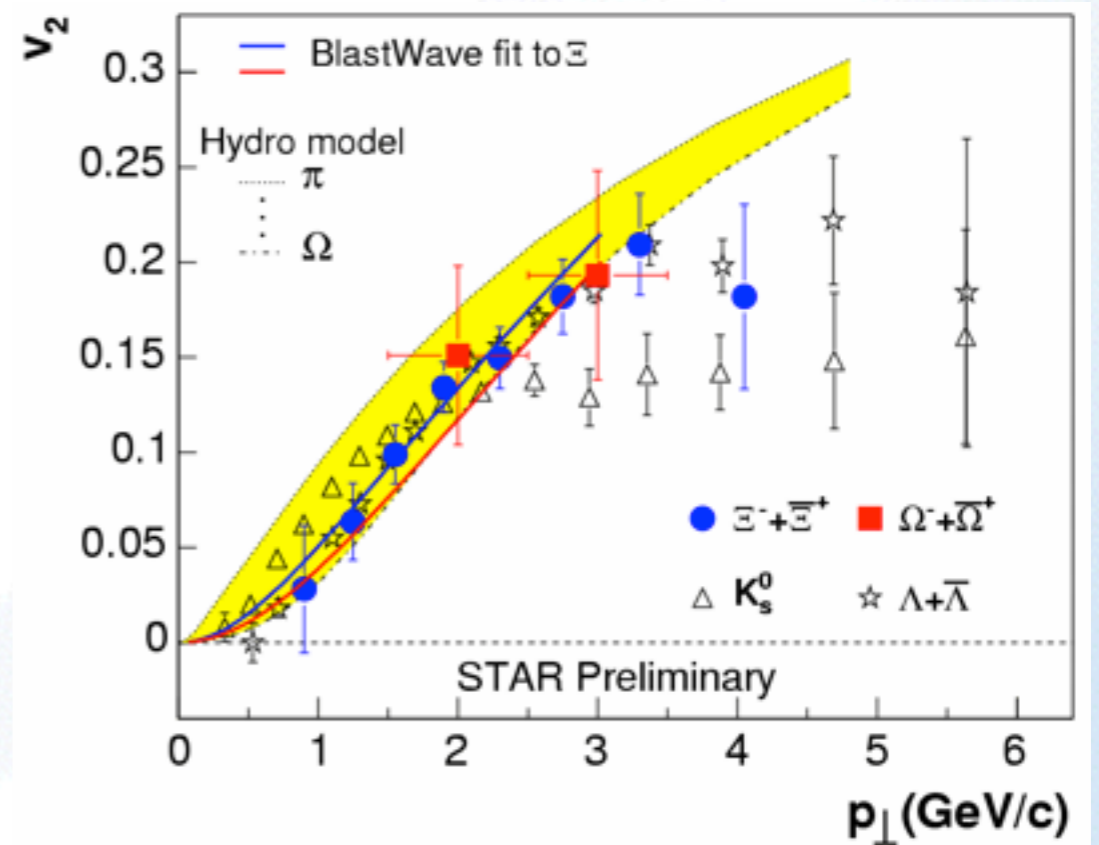
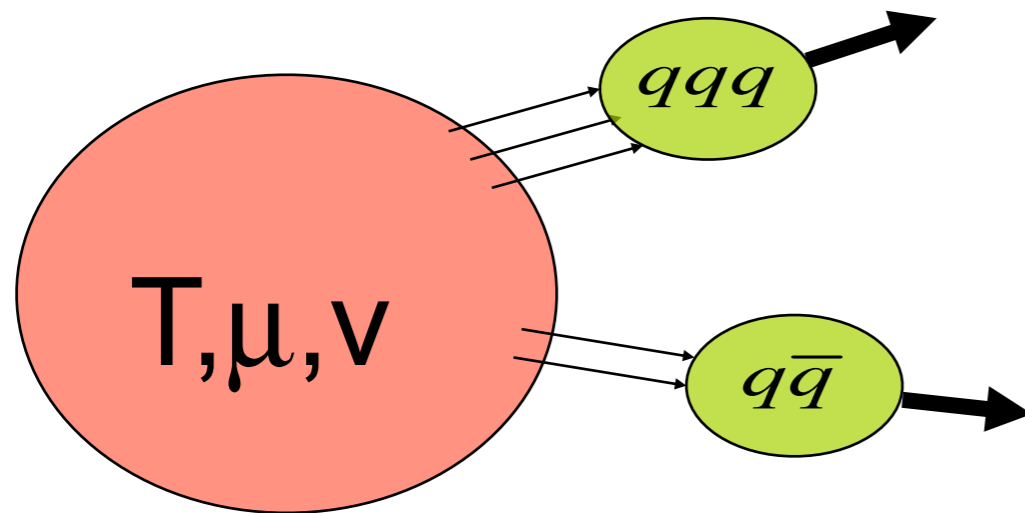
Mass splitting characteristic property of hydrodynamics

# Quark number scaling of $v_2$

In the recombination regime, **meson** and **baryon**  $v_2$  can be obtained from the **quark**  $v_2$  :

$$v_2^M(p_t) = 2v_2^q \left( \frac{p_t}{2} \right)$$

$$v_2^B(p_t) = 3v_2^q \left( \frac{p_t}{3} \right)$$



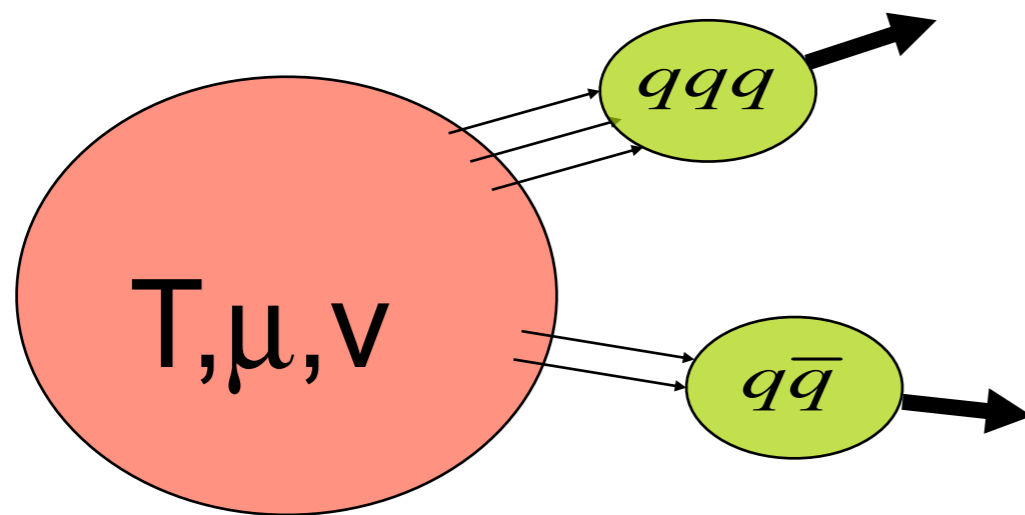


# Quark number scaling of $v_2$

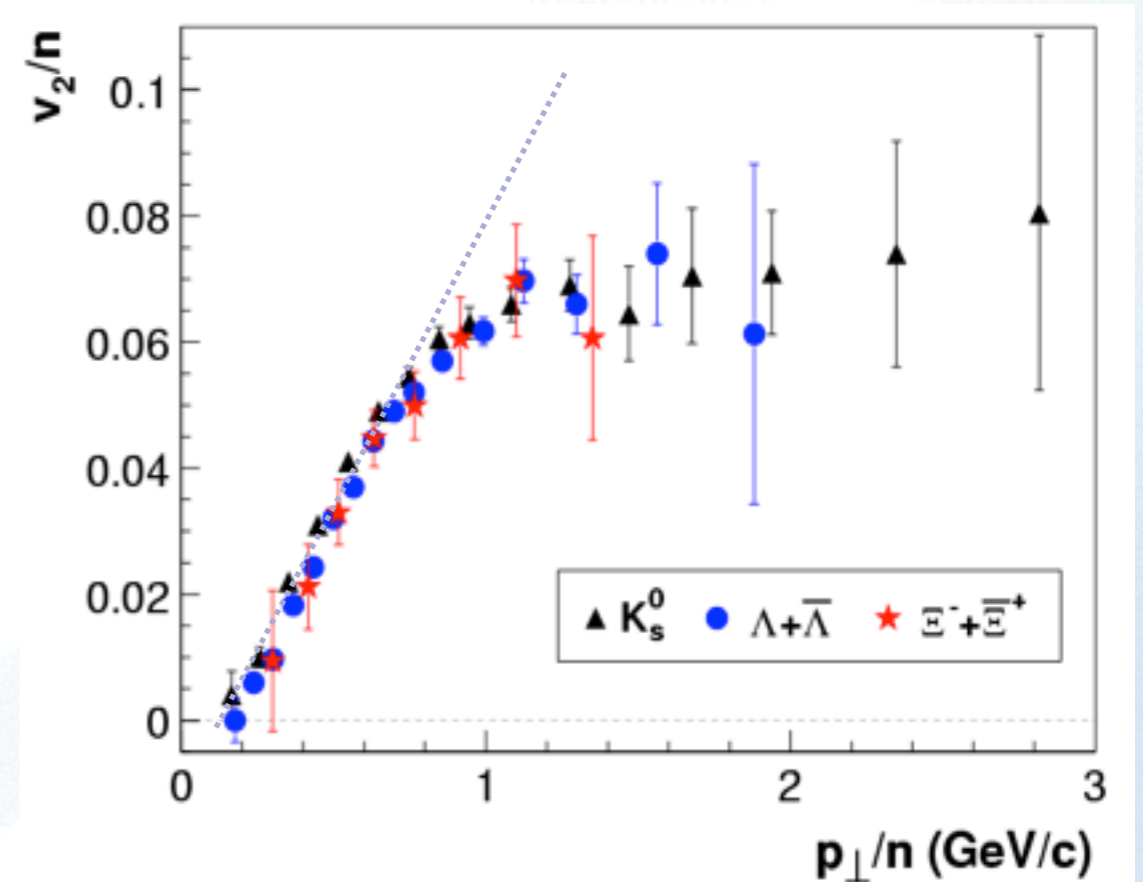
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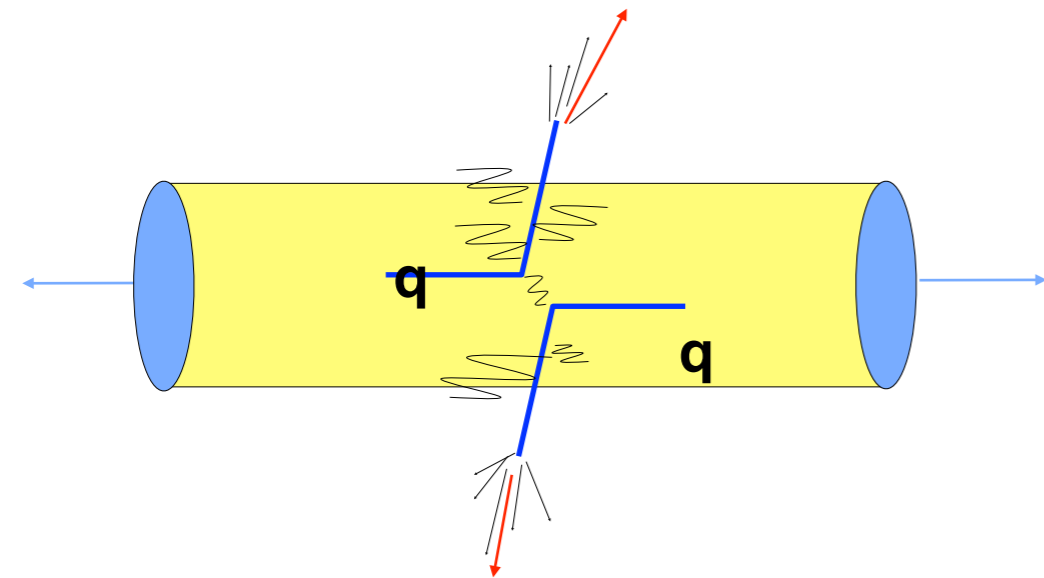
Emitting medium is composed of unconfined, flowing quarks.



# Color screening & color opacity



# Jet quenching



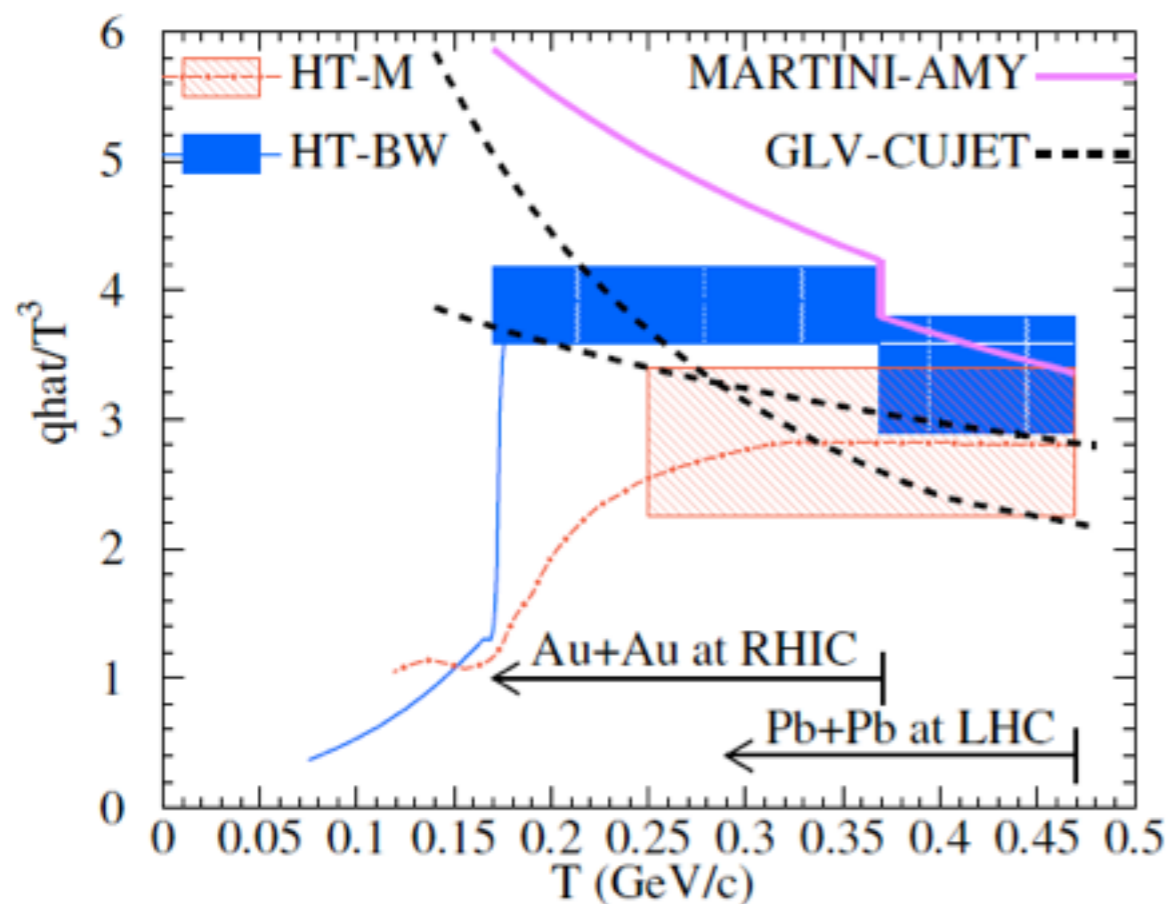
Toward quantitative measurement of basic medium properties:  $\hat{q}$

$$\frac{dE}{dx} = -C_2 \alpha_s \hat{q} L$$

Radiative

$$\frac{dE}{dx} = -C_2 \hat{e}$$

Collisional



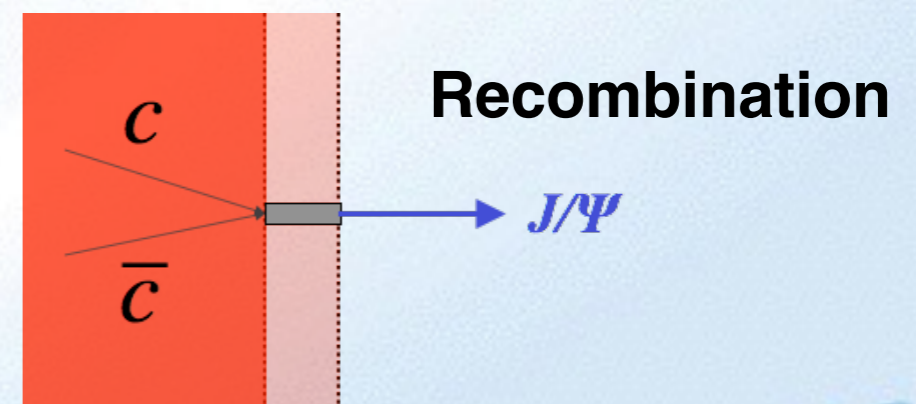
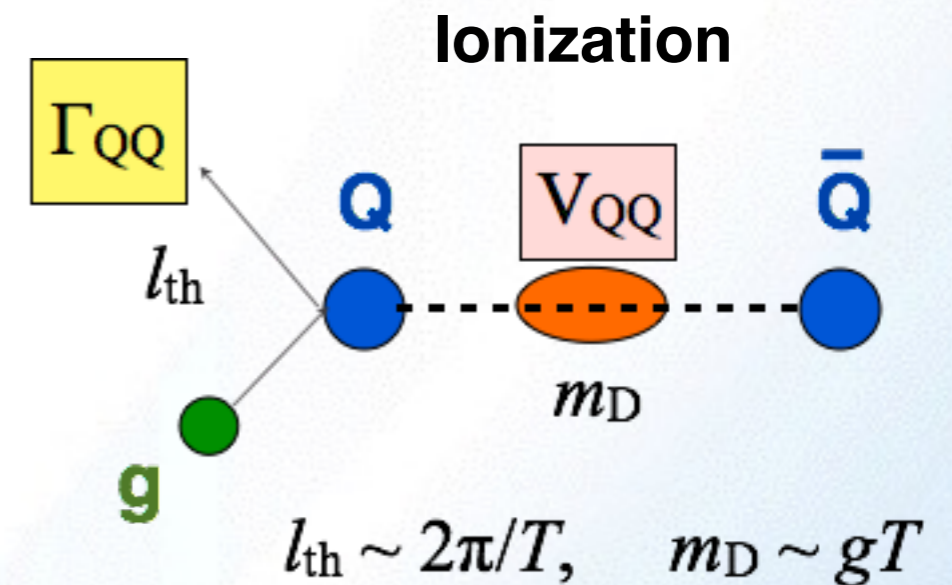
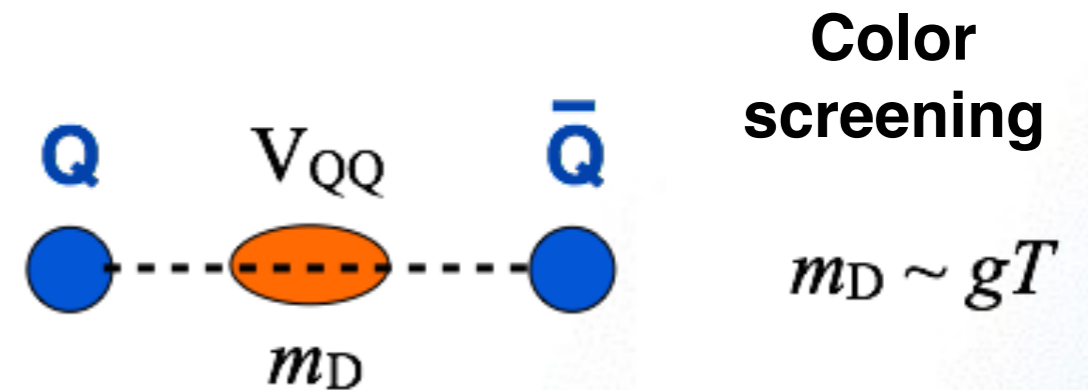
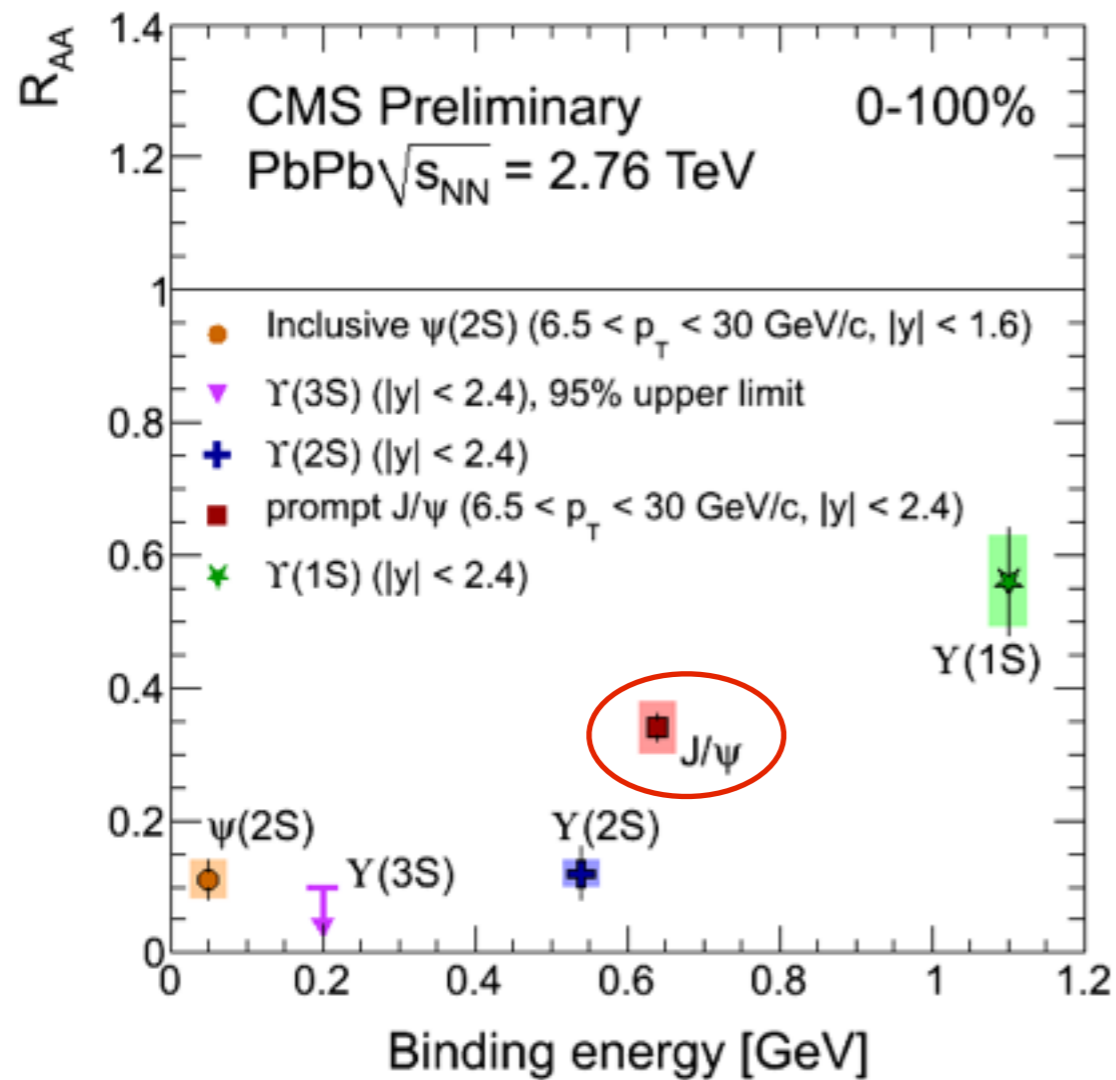
JET Collaboration

$$\frac{\hat{q}}{T^3} = \begin{cases} 4.6 \pm 1.2 & \text{at RHIC} \\ 3.7 \pm 1.4 & \text{at LHC} \end{cases}$$

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**QGP @ RHIC is slightly more strongly coupled than QGP@ LHC.**

# Quarkonium “melting”



Charmonium states “melt” in the QGP but can be regenerated by recombination when the charm quark density is high (at LHC).

Resolved measurement of Upsilon states required at RHIC.



# Future of RHIC

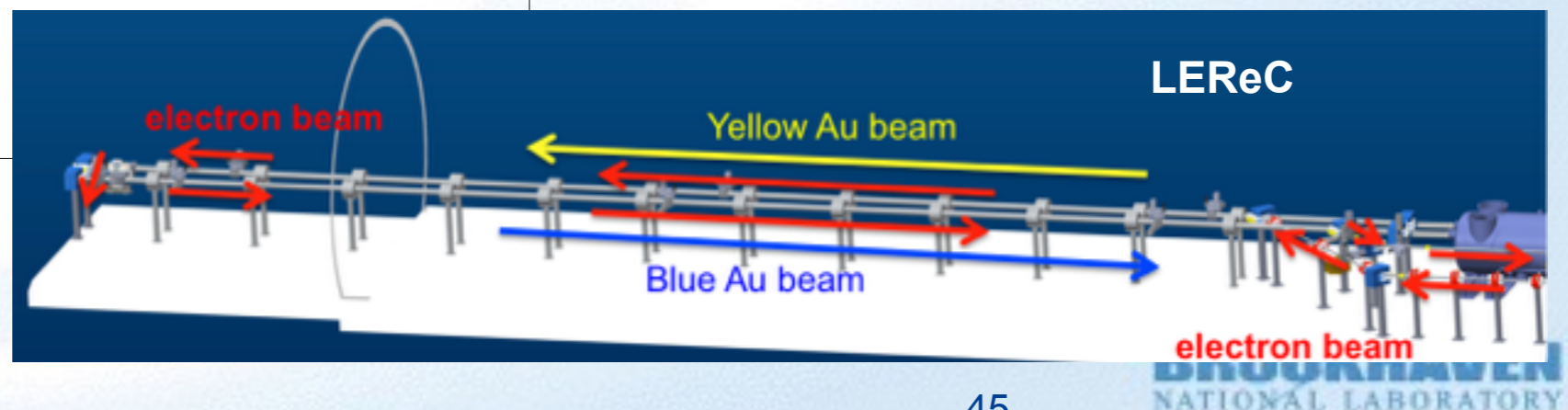
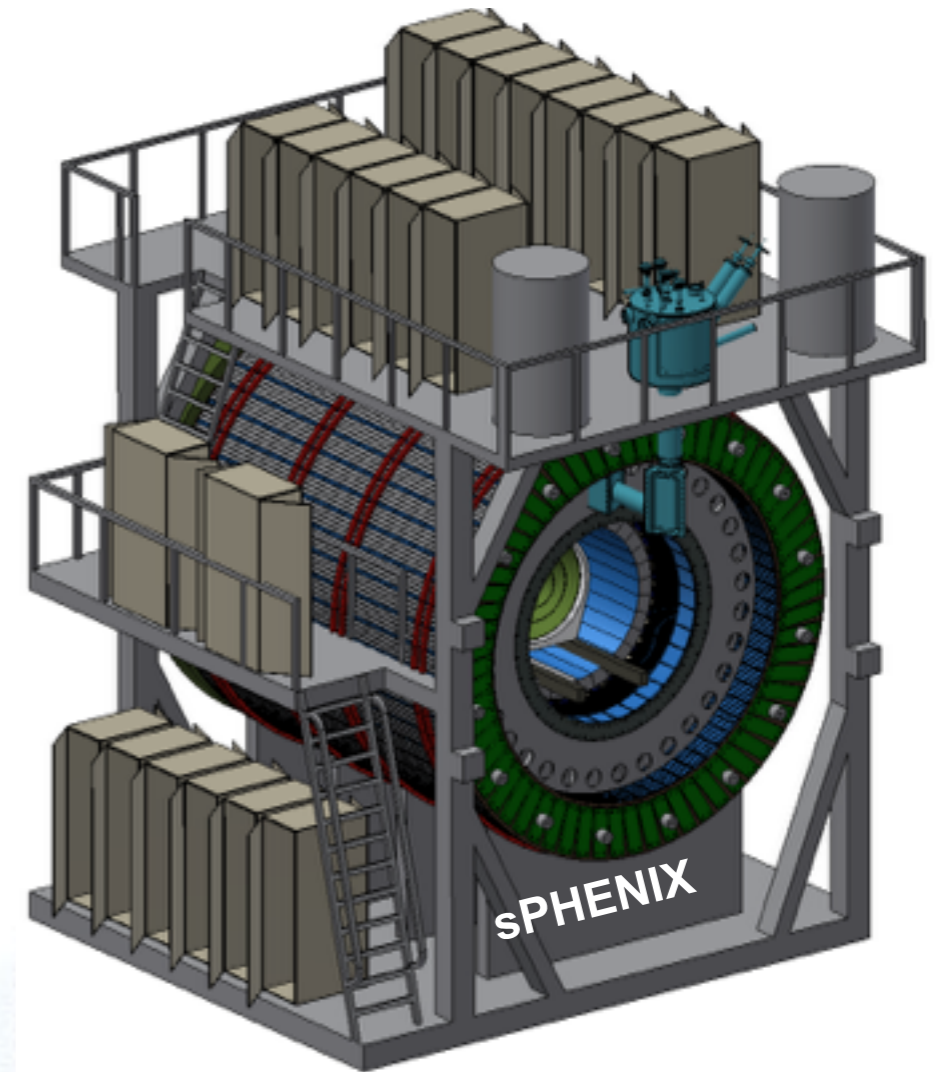
# Completing the RHIC science mission

**Status:** RHIC-II configuration is complete

- Vertex detectors in STAR (HFT) and PHENIX
- Luminosity reaches 25x design luminosity

**Plan:** Complete the RHIC mission in 3 campaigns:

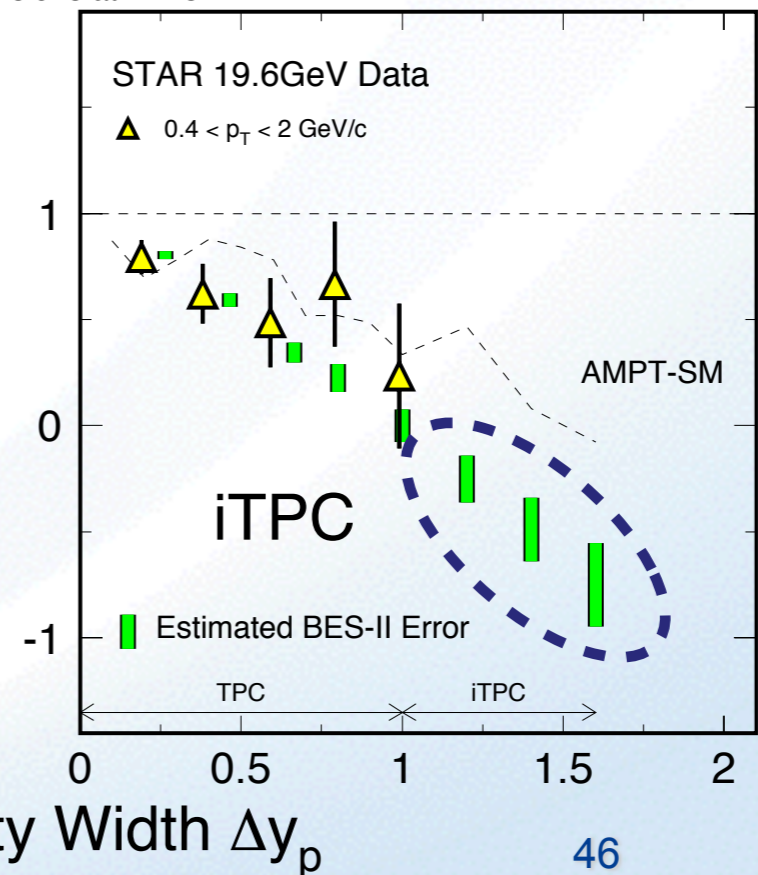
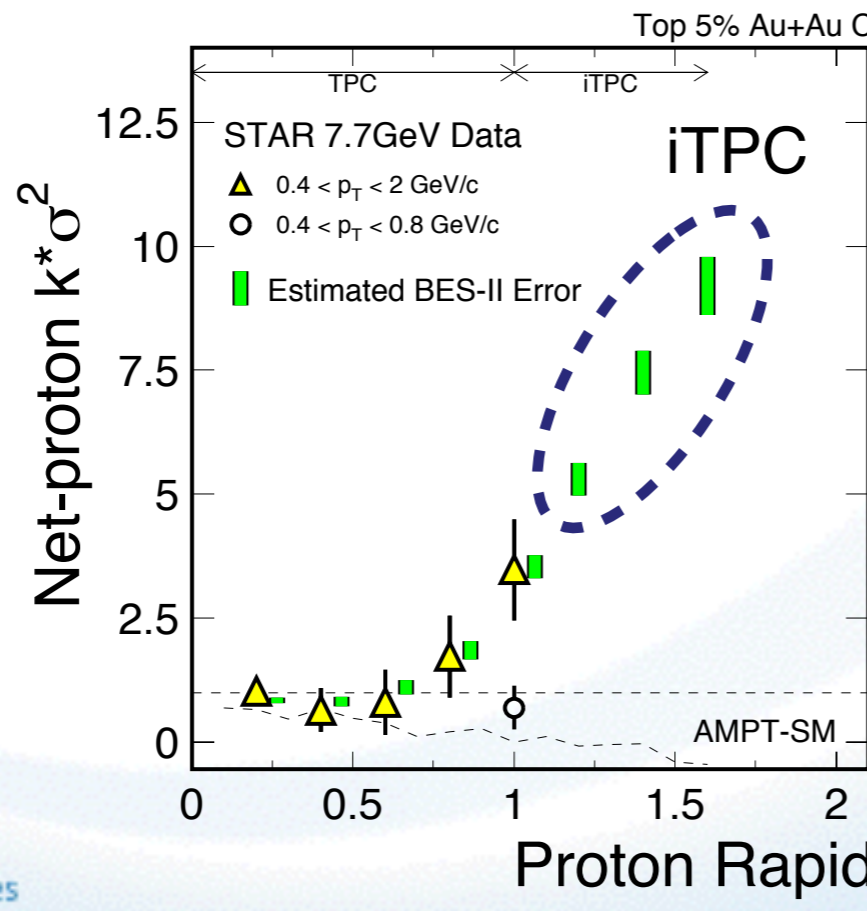
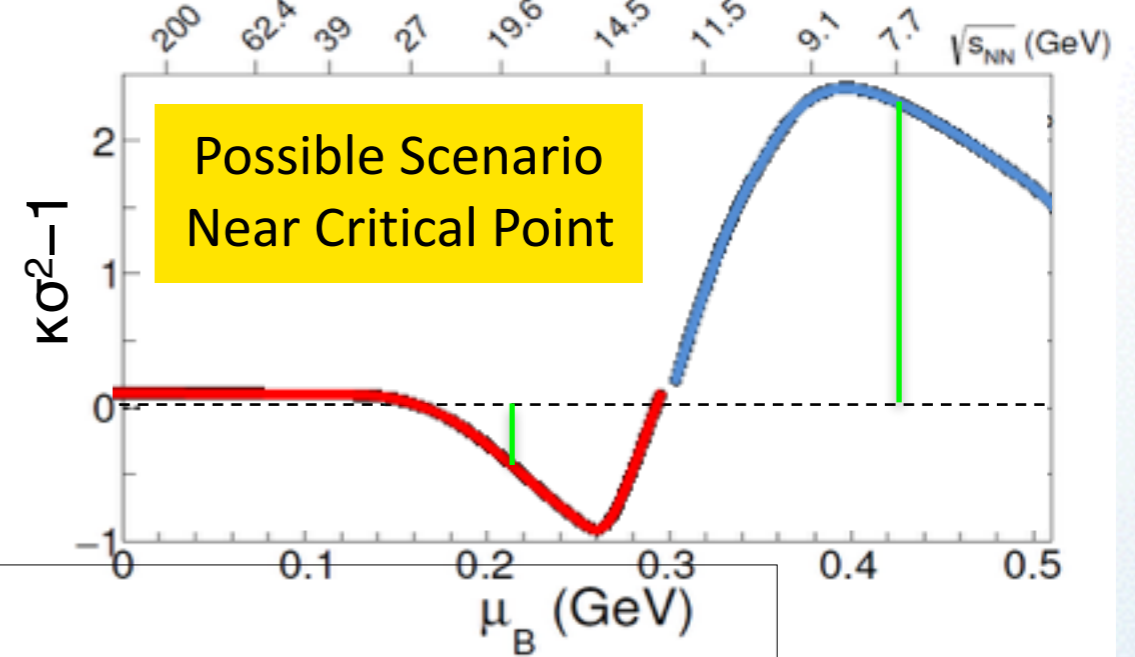
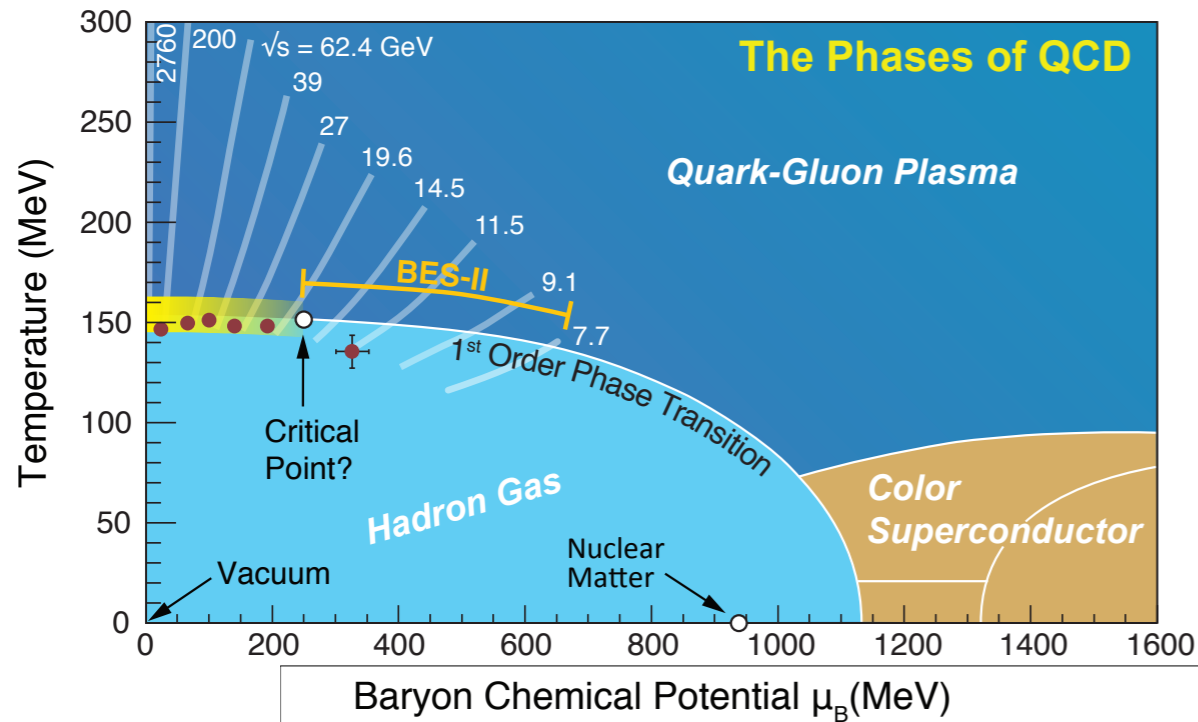
- 2014–17: Heavy flavor probes of the QGP using the micro-vertex detectors;  
Transverse spin physics
- 2018: Install low energy e-cooling (LEReC)
- 2019/20: High precision scan of the QCD phase diagram & search for critical point
- Install sPHENIX
- Probe QGP with precision measurements of jet quenching and Upsilon suppression
- Spin physics and initial conditions at forward rapidities with p+p and p+A collisions ?
- Transition to eRHIC ?



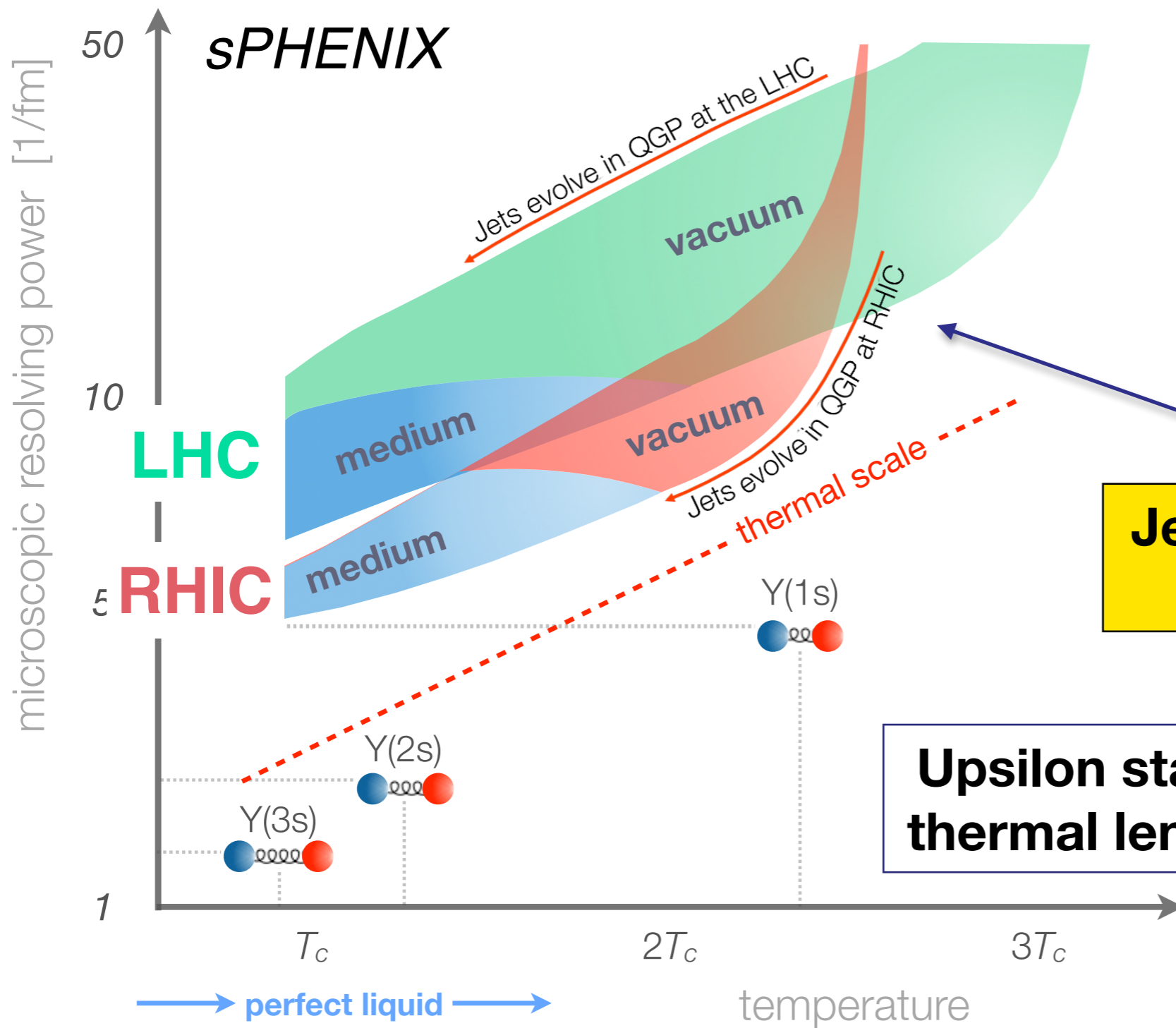


# Critical fluctuations in BES-II

Model independent structure of net baryon number kurtosis



# Probing scales in the medium



*How does the perfect fluidity of the QGP emerge from the asymptotically free theory of QCD?*

**Jets probe sub-thermal length scales**

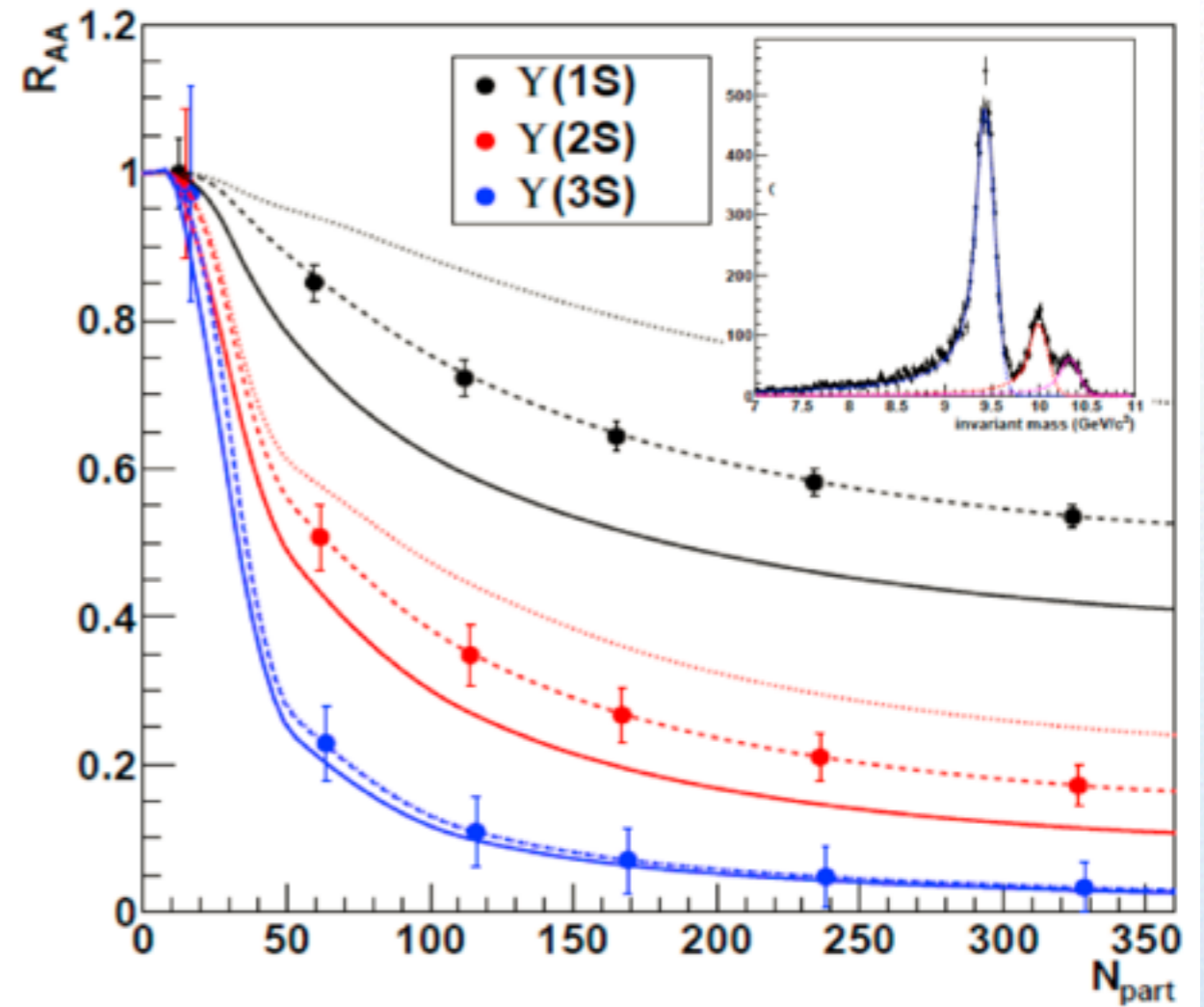
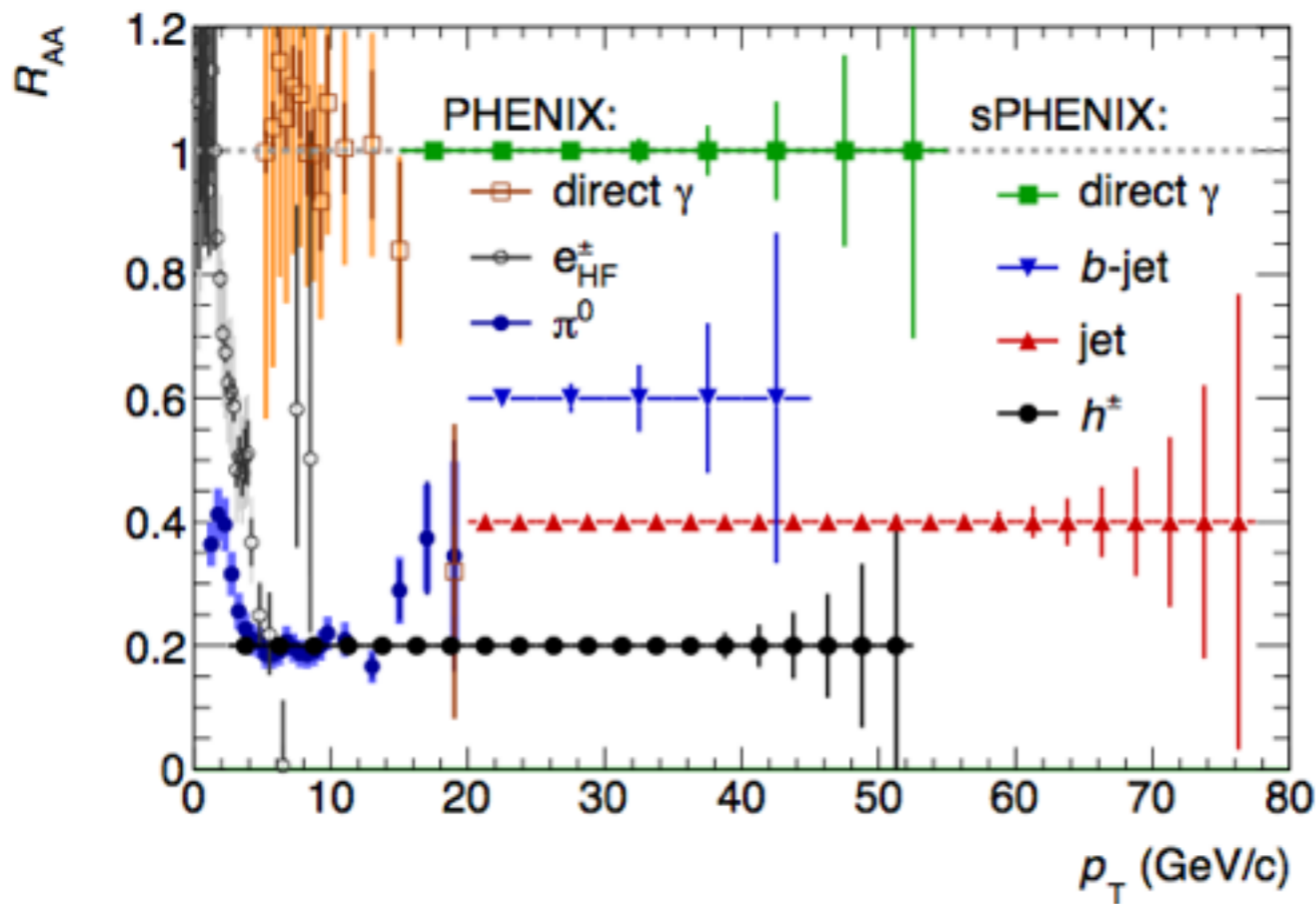
**Upsilon states probe thermal length scales**



# Jets & Upsilon states

**sPHENIX capabilities**

**Complete calorimetric jet spectroscopy**



**Completely resolved Upsilon spectroscopy**

# Beyond the Hagedorn temperature...

... lies the **liquid QGP** - a remarkable discovery by any measure.

*Imagine:* Heating a liquid (nuclear matter) turns it into vapor, i.e. a nucleon/hadron gas, at approximately 100 billion degrees.

But when we heat it to 20 times this temperature (2 trillion degrees) we find that it suddenly turns into a **liquid** again, in fact, into the **most perfect liquid** ever observed.

How is this possible?

It is still a mystery, but precise study of hard probes of various scales at RHIC and LHC, combined with comparative model-data analysis, will resolve the mystery within the next decade.

**Rolf Hagedorn would surely be pleased!**