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Fluctuations of the freeze-out temperature in Pb-Pb collisions at the LHC

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CPOD2016, Wrocław, 3 June 2016

Source

Based on:

DP, J. Phys. G**43** (2016) 055101

The Single-Freeze-Out Model

W. Broniowski and W. Florkowski, PRL **87** (2001) 272302; PRC **65** (2002) 064905

1. The freeze-out hypersurface and the Hubble-like expansion

$$\tau_f = \sqrt{t^2 - r_x^2 - r_y^2 - r_z^2} = const, \qquad u^\mu = \frac{x^\mu}{\tau_f}$$

   with condition $r = \sqrt{r_x^2 + r_y^2} < \rho_{max}$.

2. Contributions from resonance decays to the measured particle multiplicities and momentum distributions are taken into account completely.

Parameters of the Single-Freeze-Out Model

- Statistical parameters $T_f,\ \mu_B$
- Geometric parameters $\tau_f,\ \rho_{max}$
- All four parameters T_f, μ_B, ρ_{max} and τ_f are fitted to the spectra simultaneously in this version of the model [for RHIC: DP, APPB **40**, 2825 (2009)].
- For LHC $\mu_B = 0$ (and $\mu_S = \mu_Q = 0$), so there are three parameters: T_f, ρ_{max} and τ_f.

The invariant distribution of particle species i

$$\boxed{\frac{dN_i}{d^2p_T\;dy} = \int p^\mu d\sigma_\mu\; f_i(p\cdot u)}$$

f_i - final distribution of the ith particle, *i.e.* with contributions from resonance decays:

$d\sigma_\mu$ - normal vector to the freeze-out hypersurface

$$f_i = f_i^{prim} + \sum_{decay} f_i^{decay}$$

The distribution describes production from one collision ($\equiv$ one event).

Fit to the spectra I

Fitting the single-freeze-out model invariant distribution

$$\boxed{\frac{dN_i}{d^2p_T\,dy}}$$

to π^+, π^-, K^+, K^-, p and $\bar{p}$ spectra measured in central Pb-Pb collisions at $\sqrt{s_{NN}}$=2.76 TeV [ALICE, Phys. Rev. Lett. **109**, 252301 (2012)] and within whole ranges resulted in χ^2/n_{dof}=1.74 with *p-value* $= 2 \cdot 10^{-11}$ $(n_{dof} = 235)$.

Unacceptable!

What are data "points" really?

$$\frac{1}{N_{ev}} \frac{1}{2\pi p_T} \frac{d^2 N}{dp_T dy} = \frac{1}{N_{ev}} \sum_{k=1}^{N_{ev}} \frac{1}{2\pi p_T} \frac{d^2 N_k}{dp_T dy}$$

N_k - the number of counts from kth event in the sample

$$\boxed{\frac{1}{2\pi p_T} \frac{d^2 N_k}{dp_T dy} \leftarrow k\text{th observation of } \frac{1}{2\pi p_T} \frac{d^2 N}{dp_T dy}}$$

for given particle species and a centrality and p_T bin

Data "points" are sample means!

The theoretical equivalent of data

The correct equivalent of the data "point" is the theoretical average:

$$\boxed{\left\langle \frac{dN_i}{d^2p_T\ dy} \right\rangle_\theta = \int \frac{dN_i}{d^2p_T\ dy} f(\theta) d\theta\ .}$$

$\theta = \beta_f, \tau_f \text{ or } \rho_{max}$ is a random variable now!

Only $\theta = \beta_f (= 1/T_f)$ works!

Probability distributions considered

log-normal p.d.f.

$$f(\beta_f;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\frac{1}{\beta_f}\exp\left\{-\frac{(\ln\beta_f-\mu)^2}{2\sigma^2}\right\}$$

triangular p.d.f.

$$f(\beta_f;\breve{\beta}_f,\Gamma) = \begin{cases} \frac{\Gamma-|\beta_f-\breve{\beta}_f|}{\Gamma^2} & ,\mid \beta_f-\breve{\beta}_f \mid\leq \Gamma \\ 0 & ,\mid \beta_f-\breve{\beta}_f \mid> \Gamma \end{cases}$$

Fit to the spectra II

Fitting the average invariant distributions

$$\boxed{\left\langle \frac{dN_i}{d^2p_T\, dy} \right\rangle_{\beta_f}}$$

to the spectra measured in central Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV (and within whole ranges) resulted in $\chi^2/n_{dof} = 1.49$ with *p-value* $= 2 \cdot 10^{-6}$ $(n_{dof} = 234)$, for both p.d.f.'s

Still unacceptable!

Removal of some resonance states & Fit III

The most weakly bound resonances are removed from the hadron gas, with the full width $\Gamma > 250$ MeV (and masses below 1600 MeV).

Removed resonances: $f_0(500)$, $h_1(1170)$, $a_1(1260)$, $\pi(1300)$, $f_0(1370)$, $\pi_1(1400)$, $a_0(1450)$, $\rho(1450)$, $K_0^*(1430)$ and $N(1440)$.

The removal of $f_0(500)$ state (the lightest one) has found the theoretical justification recently,
W. Broniowski, F. Giacosa and V. Begun, Phys. Rev. C **92**, 034905 (2015).

The removal itself is not enough, without randomization
$\chi^2/n_{dof} = 1.5$, *p-value* $= 10^{-6}$ $(n_{dof} = 235)$,
again unacceptable!

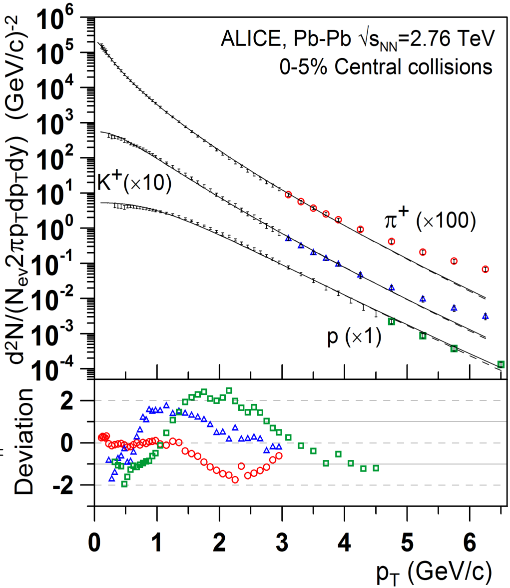
Fit to the spectra IV

Fitting the average invariant distributions with the removal, acceptable!

log-normal p.d.f.			
τ_f (fm)	ρ_{max} (fm)	μ	σ
13.80 $\pm$ 0.40	20.48 $\pm$ 0.60	-4.7439 $\pm$ 0.0235	0.1764 $\pm$ 0.0090
$E[T_f]$ (MeV)	$\sqrt{V[T_f]}$ (MeV)	χ^2/n_{dof}	*p-value* (%)
116.7 $\pm$ 3.0	20.7 $\pm$ 1.6	1.048	29

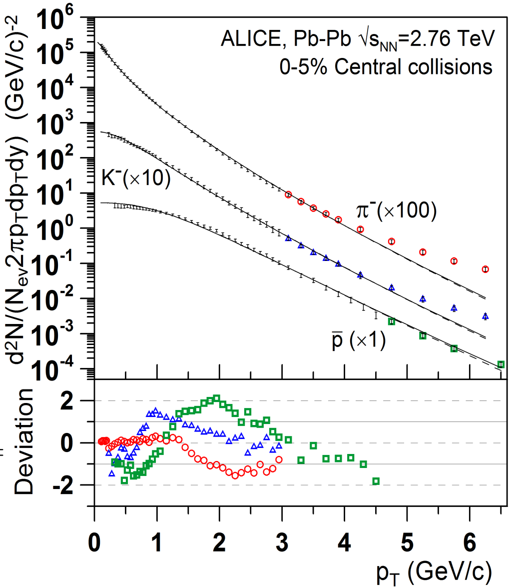
triangular p.d.f.			
τ_f (fm)	ρ_{max} (fm)	$\breve{\beta}_f$ (MeV^{-1})	Γ (MeV^{-1})
14.42	21.45	0.0092482	0.0040906
$E[T_f]$ (MeV)	$\sqrt{V[T_f]}$ (MeV)	χ^2/n_{dof}	*p-value* (%)
111.6	22.6	1.026	38

Spectra of positive pions, kaons and protons



$$\frac{f_{exp}-f_{mod}}{\sigma_{exp}} = \text{Deviation}$$

Spectra of negative pions, kaons and $\overline{\text{protons}}$



$$\frac{f_{exp} - f_{mod}}{\sigma_{exp}} = \text{Deviation}$$

Yields in the Single-Freeze-Out Model

$$\boxed{\frac{dN_i}{dy} = \pi \rho_{max}^2 \tau_f n_i}$$

V. Begun, W. Florkowski and M. Rybczynski,
Phys. Rev. C **90**, 054912 (2014)

The average particle yield per unit rapidity

$$\boxed{\left\langle \frac{dN_i}{dy} \right\rangle_{\beta_f} = \pi \rho_{max}^2 \tau_f \left\langle n_i \right\rangle_{\beta_f}}$$

Midrapidity particle yields and ratios

		Model: $\left\langle \frac{dN_i}{dy} \right\rangle_{\beta_f}$	
Species	Data	triangular p.d.f.	log-normal p.d.f.
π^+	733 $\pm$ 54.0	745.3	739.2
π^-	732 $\pm$ 52.0	745.3	739.2
K^+	109 $\pm$ 9.0	106.9	107.5
K^-	109 $\pm$ 9.0	106.9	107.5
p	34 $\pm$ 3.0	33.0	32.9
$\bar{p}$	33 $\pm$ 3.0	33.0	32.9

	Ratios		
	Data	triangular p.d.f.	log-normal p.d.f.
p/π	0.046 $\pm$ 0.003	0.044	0.045
K/π	0.149 $\pm$ 0.010	0.143	0.145

Centrality dependence

Pb-Pb at $\sqrt{s_{NN}} = 2.76$ TeV, log-normal p.d.f., $n_{dof} = 234$

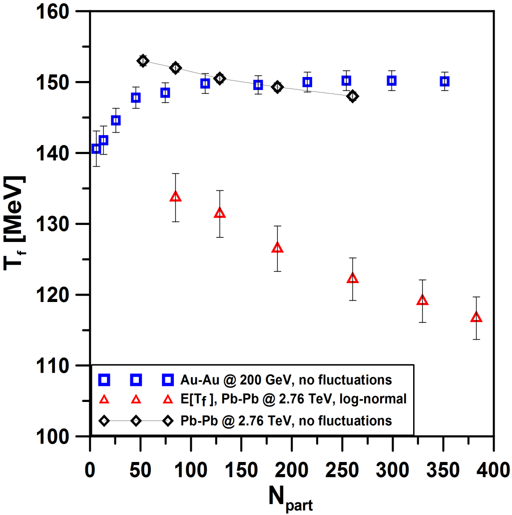
Cent. [%]	τ_f [fm]	ρ_{max} [fm]	$E[T_f]$ [MeV]	$\sqrt{V[T_f]}$ [MeV]	χ^2/n_{dof}	p-v [%]
0-5	13.8 ± 0.4	20.5 ± 0.6	116.7 ± 3.0	20.7 ± 1.6	1.05	29
-10	12.6 ± 0.4	18.7 ± 0.5	119.1 ± 3.0	20.2 ± 1.7	0.84	96
-20	11.1 ± 0.3	16.4 ± 0.5	122.2 ± 3.0	19.6 ± 1.7	0.59	100
-30	9.3 ± 0.3	13.6 ± 0.4	126.5 ± 3.2	18.7 ± 1.9	0.34	100
-40	7.8 ± 0.2	11.0 ± 0.3	131.4 ± 3.3	17.3 ± 2.0	0.30	100
-50	6.6 ± 0.2	9.0 ± 0.3	133.7 ± 3.4	16.7 ± 2.2	0.61	100
-60	5.5 ± 0.2	7.2 ± 0.2	134.7 ± 3.6	16.7 ± 2.3	1.37	0.01
-70	4.6 ± 0.2	5.8 ± 0.2	133.4 ± 3.6	17.4 ± 2.2	2.82	0
-80	3.6 ± 0.1	4.4 ± 0.2	132.5 ± 3.6	17.6 ± 2.2	4.36	0

Centrality dependence, no fluctuations

Pb-Pb at $\sqrt{s_{NN}} = 2.76$ TeV, $n_{dof} = 235$

Cent. [%]	τ_f [fm]	ρ_{max} [fm]	T_f [MeV]	χ^2/n_{dof}	p-v [%]
0-5	10.0 $\pm$ 0.1	14.7 $\pm$ 0.9	147.0 $\pm$ 0.5	1.74	$1.6 \cdot 10^{-9}$
-10	9.3 $\pm$ 0.1	13.7 $\pm$ 0.8	147.3 $\pm$ 0.5	1.45	$9.2 \cdot 10^{-4}$
-20	8.4 $\pm$ 0.1	12.3 $\pm$ 0.8	148.0 $\pm$ 0.6	1.09	17.2
-30	7.3 $\pm$ 0.1	10.6 $\pm$ 0.7	149.3 $\pm$ 0.6	0.69	99.99
-40	6.3 $\pm$ 0.1	8.9 $\pm$ 0.6	150.5 $\pm$ 0.6	0.49	100
-50	5.4 $\pm$ 0.1	7.3 $\pm$ 0.5	152.0 $\pm$ 0.6	0.66	99.999
-60	4.5 $\pm$ 0.1	5.9 $\pm$ 0.5	153.0 $\pm$ 0.7	1.24	0.65
-70	3.7 $\pm$ 0.1	4.6 $\pm$ 0.4	153.5 $\pm$ 0.7	2.47	0
-80	2.9 $\pm$ 0.1	3.4 $\pm$ 0.3	154.1 $\pm$ 0.8	3.86	0

Freeze-out temperature vs centrality



Au-Au: DP, APPB **40**, 2825 (2009)

Conclusions

- The 2 most central bins of Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV are inhomogeneous, during each event the thermal system is created indeed and with approximately the same size at its end, however with different temperature. And the final shape of the spectra is the consequence of summing emissions from many different sources.

- The centrality bins of Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV can be divided into 3 groups: the first, the 2 most central bins where the freeze-out temperature fluctuates significantly; the second, the mid central bins where the situation looks similar to that at the RHIC, the same freeze-out temperature, $T_f \sim 150$ MeV, only ρ_{max} factor ~ 1.5 greater (τ_f approx. the same) what causes that the volume is greater ~ 2.5 times; the third, the peripheral bins where nothing helps.

Conclusions, *cont.*

- The distribution of the freeze-out temperature means the distribution within a bin here. But the significant part of the freeze-out temperature fluctuations might be of *non-thermal* origin, so this would represent the possible variation of the freeze-out conditions event-by-event within the bin.
- A great deal of data in high energy physics are averages, so in any theoretical modeling (of these data) one should be aware of possible misinterpretations when an average is compared with a prediction for a single event.

What are data "points" really?

$$\frac{1}{N_{ev}}\frac{1}{2\pi p_T}\frac{d^2N}{dp_Tdy} \Longrightarrow \frac{1}{N_{ev}(i)}\frac{1}{2\pi p_T}C_{ij}(p_T)\frac{N^j(i,p_T)}{\Delta p_T\Delta y}$$

$N_{ev}(i)$ - the number of events in the ith centrality bin
p_T - the value in the middle of the p_T bin Δp_T
$N^j(i,p_T)$ - the number of counts of particle species j in the centrality bin i and the p_T bin Δp_T
$C_{ij}(p_T)$ - the total correction factor
N_k^j - the number of counts of j's from kth event $\in (i,\Delta p_T)$

$$N^j = \sum_{k=1}^{N_{ev}} N_k^j \Rightarrow \frac{1}{2\pi p_T}C_{ij}\frac{N^j}{\Delta p_T\Delta y} = \sum_{k=1}^{N_{ev}}\frac{1}{2\pi p_T}C_{ij}\frac{N_k^j}{\Delta p_T\Delta y}$$

$$\frac{1}{N_{ev}}\frac{1}{2\pi p_T}\frac{d^2N}{dp_Tdy} = \frac{1}{N_{ev}}\sum_{k=1}^{N_{ev}}\frac{1}{2\pi p_T}\frac{d^2N_k}{dp_Tdy}$$

The least-squares (LS) test statistic

$$\chi^2_{LS}(\vec{Y};\vec{\theta}) = \sum_{i,j=1}^{N} (Y_i - \Lambda(X_i;\vec{\theta}))[V^{-1}]_{ij}(Y_j - \Lambda(X_j;\vec{\theta}))$$

$$\chi^2_{LS}(\vec{Y};\vec{\theta}) = \sum_{i=1}^{N} \frac{(Y_i - \Lambda(X_i;\vec{\theta}))^2}{\sigma_i^2}$$

$\Lambda(X;\vec{\theta})$ - the true value function

$\vec{\theta} = (\theta_1, ..., \theta_m)$ - unknown parameters

V - covariance matrix

$n_{dof} = N - m$ - the number of degrees of freedom

p-value of the test statistic

The probability of obtaining the value of the test statistic equal to or greater then the value just obtained for the present data set (*i.e.* χ^2_{min}), when repeating the whole experiment many times:

$$\boxed{p = P(\chi^2 \geq \chi^2_{min}; n_{dof}) = \int_{\chi^2_{min}}^{\infty} f(z; n_{dof}) dz \ ,}$$

$f(z; n_{dof})$ - the χ^2 p.d.f.

χ^2 (chi-square) distribution

$0 \le z \le +\infty$,

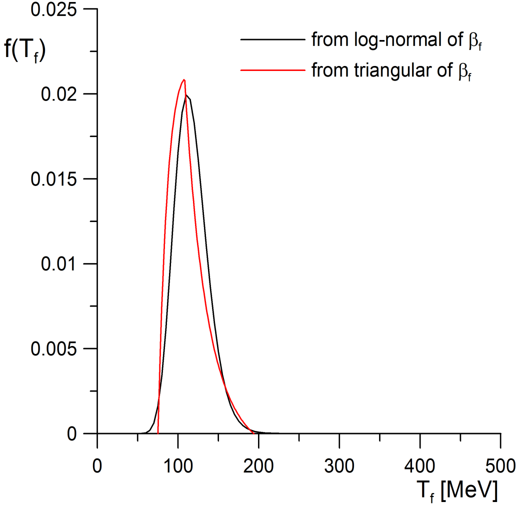
$n = 1, 2, ...$ - the number of degrees of freedom

$$\boxed{f(z;n) = \frac{1}{2^{n/2}\Gamma(n/2)} z^{n/2-1} \cdot e^{-z/2}}$$

$$\Gamma(n) = (n-1)!\ , \quad \Gamma(x+1) = x\Gamma(x), \quad \Gamma(1/2) = \sqrt{\pi}$$

$$E[z] = n, \qquad V[z] = 2n$$

Distributions of the freeze-out temperature



Central Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV

PHENIX @ $\sqrt{s_{NN}} = 200$ GeV

DP, APPB **40**, 2825 (2009), $n_{dof} = 122$

Centr. [%]	T_f [MeV]	μ_B [MeV]	ρ_{max} [fm]	τ_f [fm]	χ^2/n_{dof}
0-5	150.1±1.3	24.1±3.7	9.3±0.2	9.5±0.2	0.69
5-10	150.2±1.4	23.5±3.7	8.8±0.2	8.8±0.2	0.50
10-15	150.2±1.4	22.8±3.7	8.3±0.2	8.2±0.2	0.37
15-20	150.0±1.4	22.4±3.7	7.8±0.2	7.7±0.2	0.37
20-30	149.6±1.3	24.0±3.5	7.1±0.2	7.0±0.1	0.45
30-40	149.8±1.4	23.8±3.6	6.1±0.1	6.0±0.1	0.66
40-50	148.5±1.4	22.5±3.7	5.3±0.1	5.3±0.1	0.89
50-60	147.8±1.5	22.0±4.0	4.4±0.1	4.6±0.1	0.96
60-70	144.6±1.7	21.6±4.6	3.6±0.1	3.9±0.1	1.12
70-80	141.8±2.0	24.1±5.7	2.8±0.1	3.2±0.1	1.23
80-92	140.6±2.5	14.3±7.1	2.2±0.1	2.8±0.1	1.13

What does SHM mean?

For the boost invariant system:

$$\boxed{\frac{(dN_i/dy)_{y=0}}{(dN_j/dy)_{y=0}} = \frac{N_i}{N_j} = \frac{n_i}{n_j}}, \qquad y = \frac{1}{2}\ln\frac{E+p_L}{E-p_L}$$

$$n_i(T,\mu_B) = n_i^{prim}(T,\mu_B) + \sum_a \varrho(i,a)\, n_a^{prim}(T,\mu_B)\ ,$$

$n_i^{prim}(T,\mu_B)$ - the thermal density of particle species i at the freeze-out

$\varrho(i,a)$ - the final fraction of particle species i which can be received from all possible decays (cascades) of particle a, the sum is over all kinds of resonances in the hadron gas

Primordial distributions

At the freeze-out the momentum distributions are frozen and these are primordial distributions:

$$\boxed{f_i^{prim} = \frac{(2s_i+1)}{(2\pi\hbar c)^3} \frac{1}{\exp\left\{\frac{E_i-\mu_i}{T}\right\} \pm 1}}$$

$$\mu_i = B_i\mu_B + S_i\mu_S + Q_i\mu_Q$$

$$\boxed{n_i^{prim} = \int d\vec{p}\; f_i^{prim}(\vec{p})}$$

$$\sum S_i n_i = 0\;, \qquad \frac{\sum Q_i n_i}{\sum B_i n_i} = \frac{Z}{A}$$

The distribution in the presence of the flow

$$\boxed{f_i(\vec{r}, \vec{q}, t) = \frac{(2s_i + 1)}{(2\pi\hbar c)^3} \frac{1}{\exp\left\{\frac{q_\nu u^\nu(\vec{r},t) - \mu_i(\vec{r},t)}{T(\vec{r},t)}\right\} \pm 1}}$$