Understanding Experimentally-Observed Fluctuations

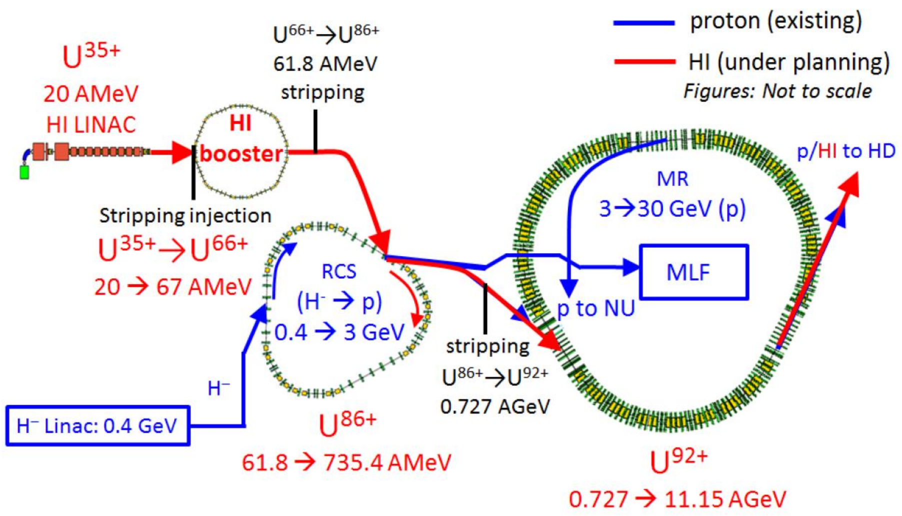
Masakiyo Kitazawa

(Osaka U.)

MK, Asakawa, Ono, Phys. Lett. B728, 386-392 (2014)
Sakaida, Asakawa, MK, PRC90, 064911 (2014)
MK, Nucl. Phys. A942, 65 (2015)
MK, Phys. Rev. C93, 044911 (2016)
Ohnishi, MK, Asakawa, to appear soon.

CPOD2016, Wroclaw, Poland, 2/Jun./2016

J-PARC Heavy-Ion Program

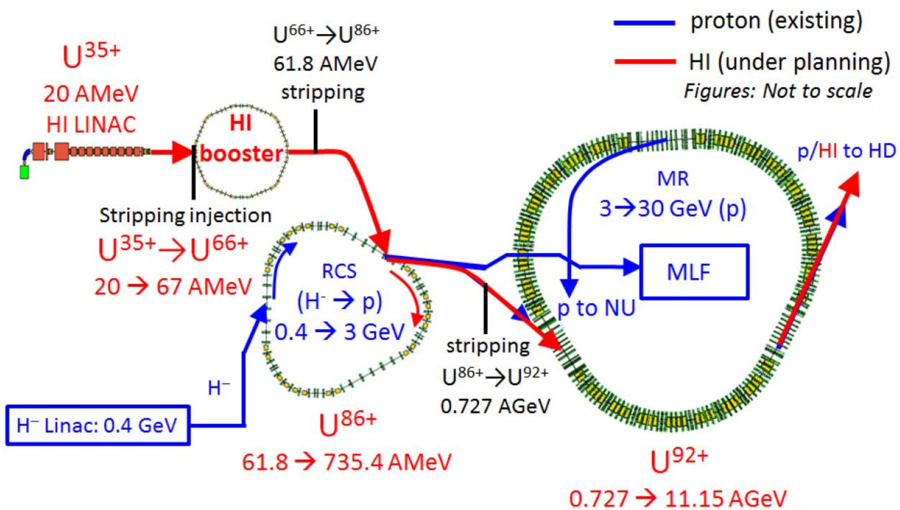


- fixed target HI experiment
- E_{lab}<20GeV/A ($\sqrt{s_{NN}}$<6.2 GeV)
- Exploit Main Ring for p accel.
- High luminosity beam

Earliest possible schedule

Jun 2016	White paper completed
Jun 2016	Submission of LOI
2016-2019	Discussions in J-PARC, KEK, Japanese Nuclear Physics Committee, Science Council of Japan
2020	Funding request to MEXT
2021	Approval of funding
2021-2022	Construction of HI Injector
2021-2023	Construction of HI injection system in RCS
2023-2024	Construction of HI spectrometer
2025	First collision

J-PARC Heavy-Ion Program



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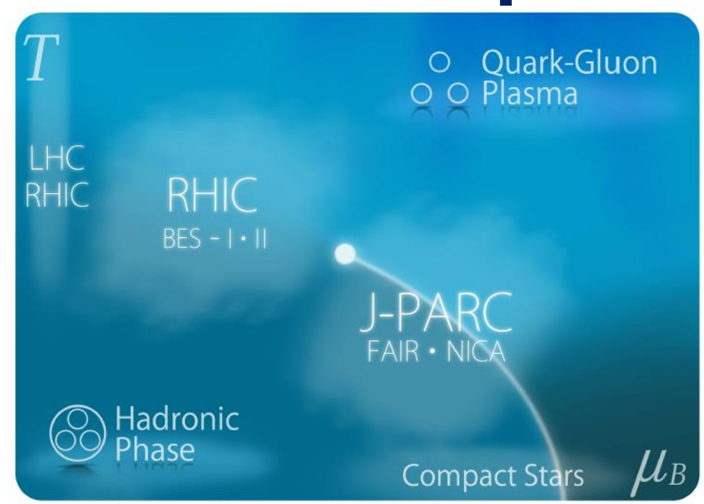
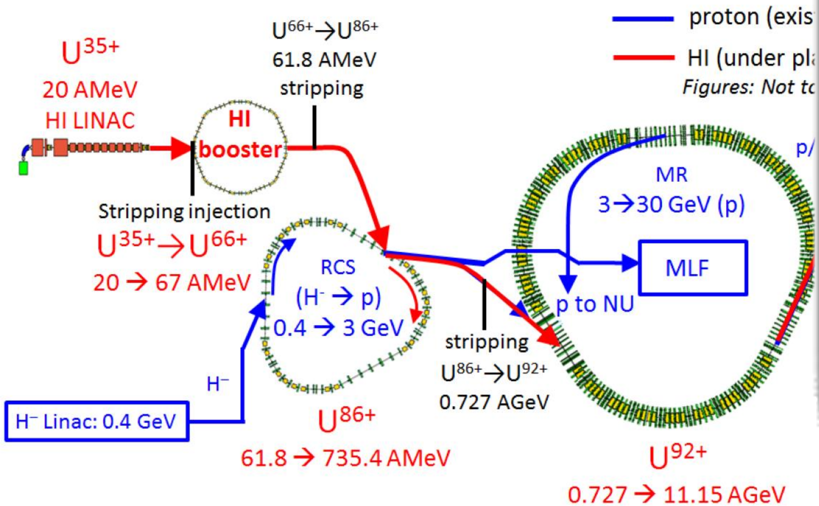
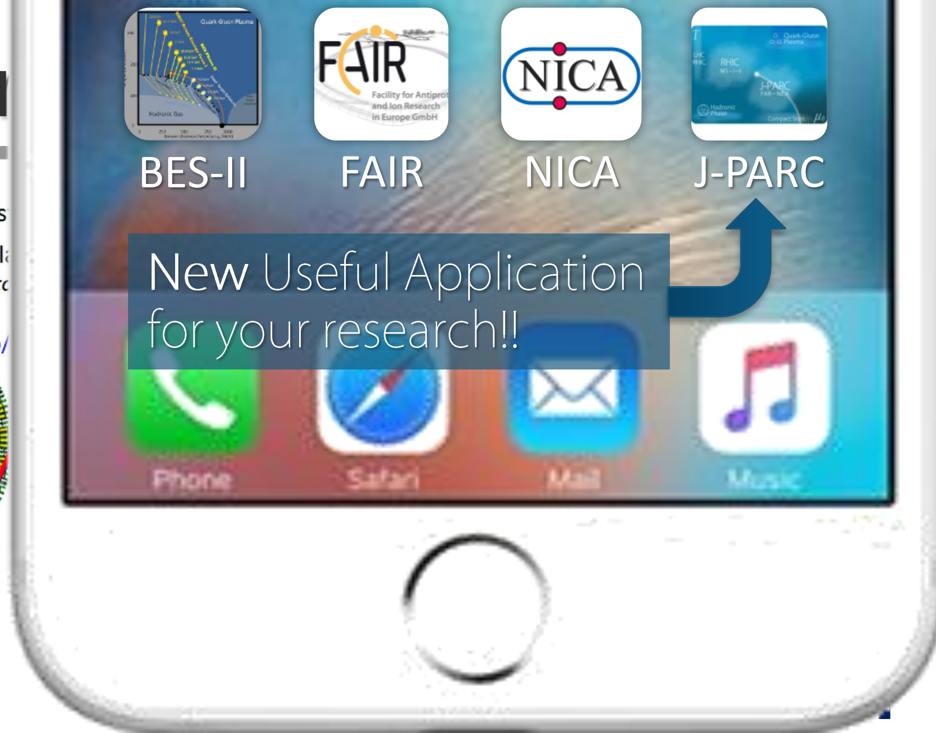
White Paper



Now available on
http://asrc.jaea.go.jp/soshiki/gr/hadron/jparc-hi/

J-PARC Heavy-Ion Pr





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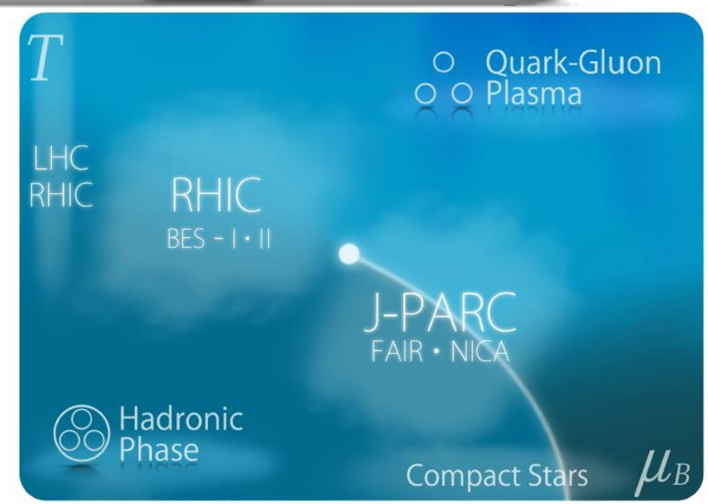

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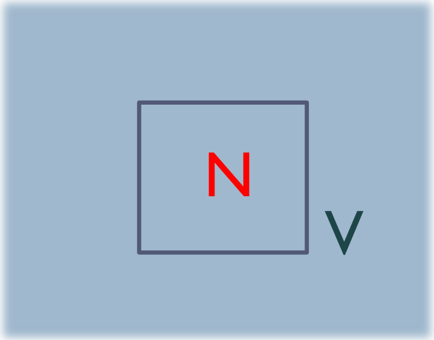
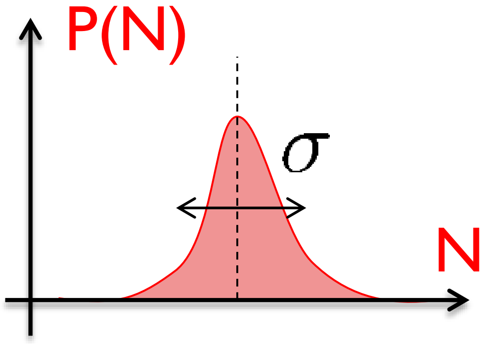
Fluctuations

Thermal Fluctuations

Observables are fluctuating even in an equilibrated medium.





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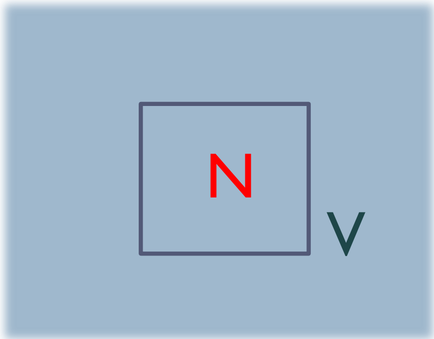
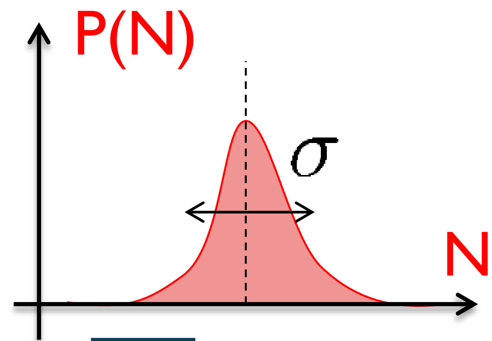




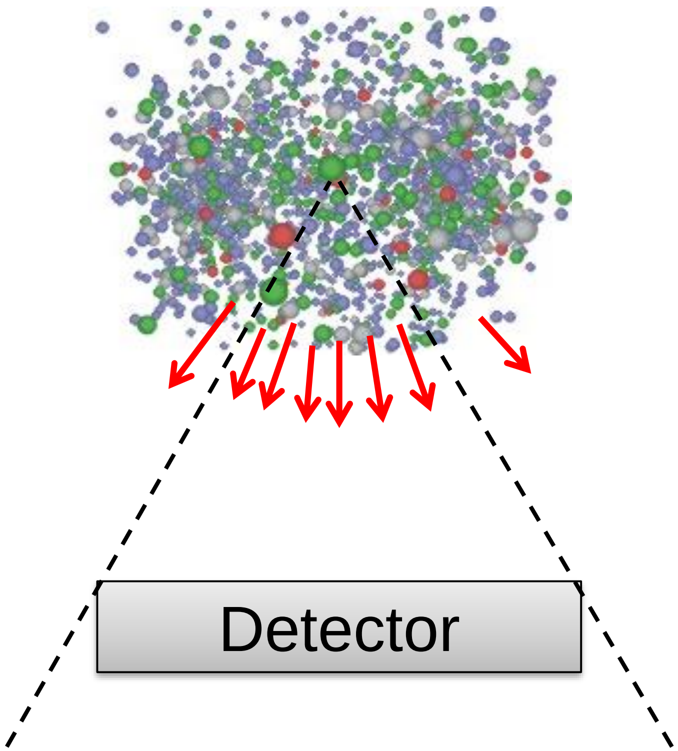
- Variance: $\langle \delta N^2 \rangle = V\chi_2 = \sigma^2$
- Skewness: $S = \frac{\langle \delta N^3 \rangle}{\sigma^3}$
- Kurtosis: $\kappa = \frac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^2}$

$\delta N = N - \langle N \rangle$

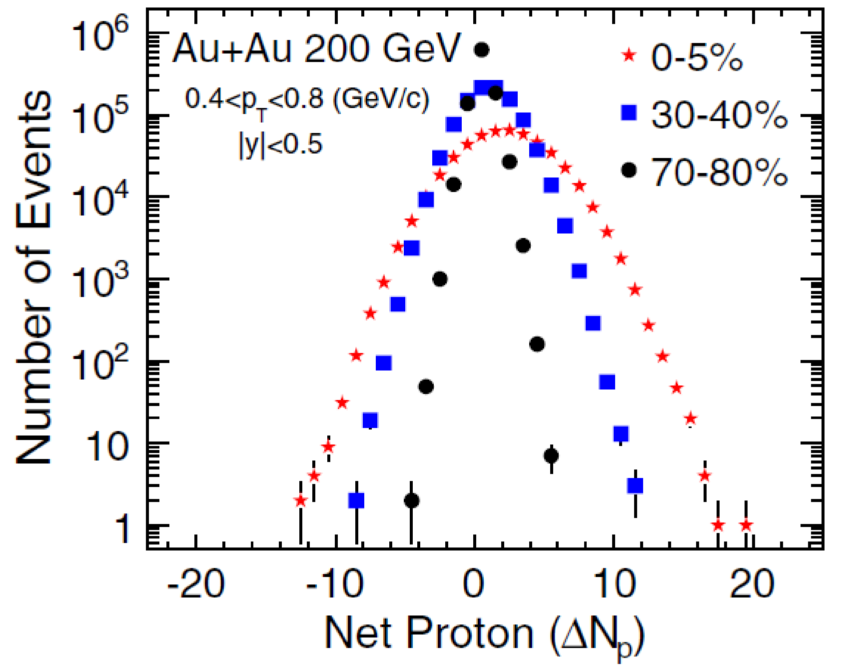
Non-Gaussianity

Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in experiments.



STAR, PRL**105** (2010)



$$\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$$

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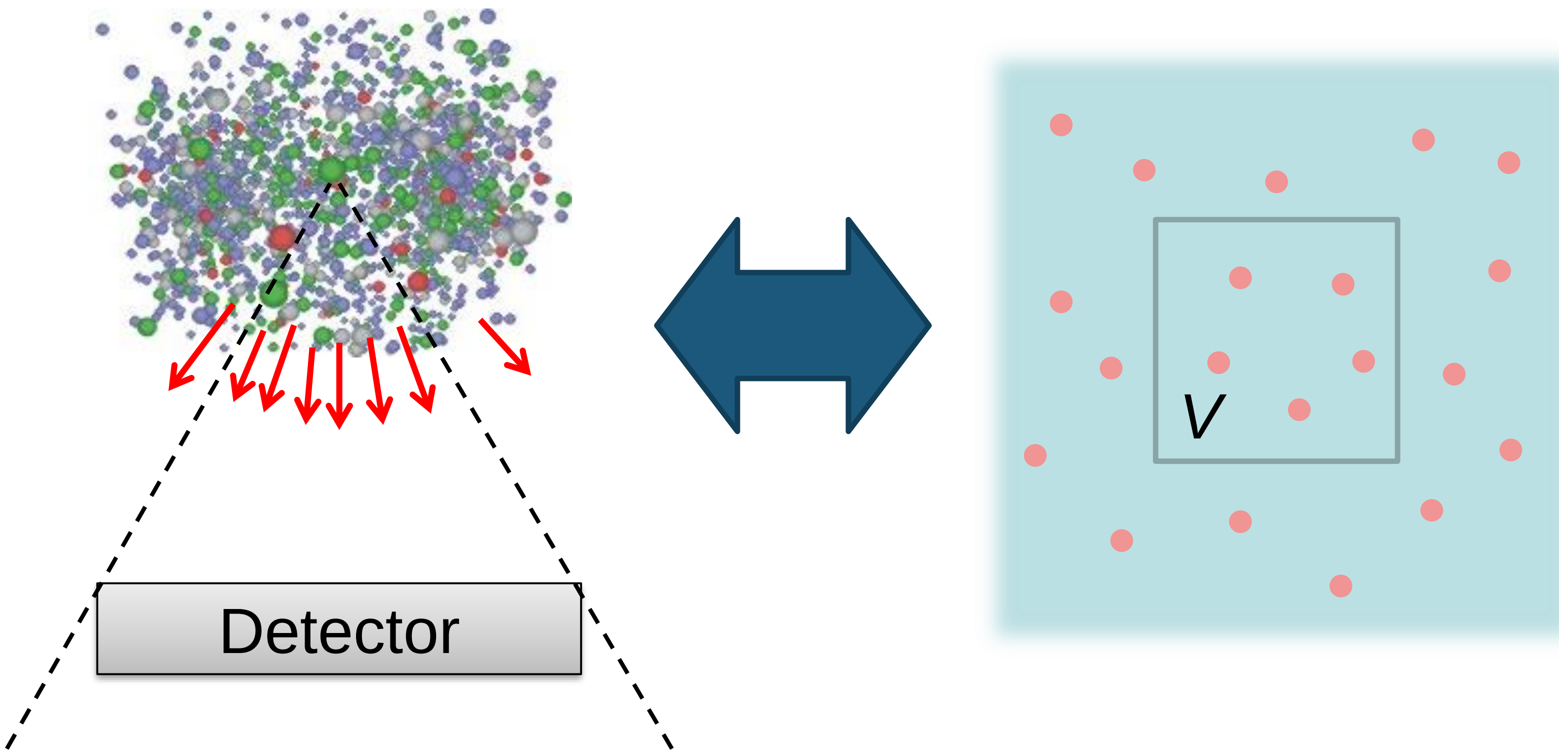


Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

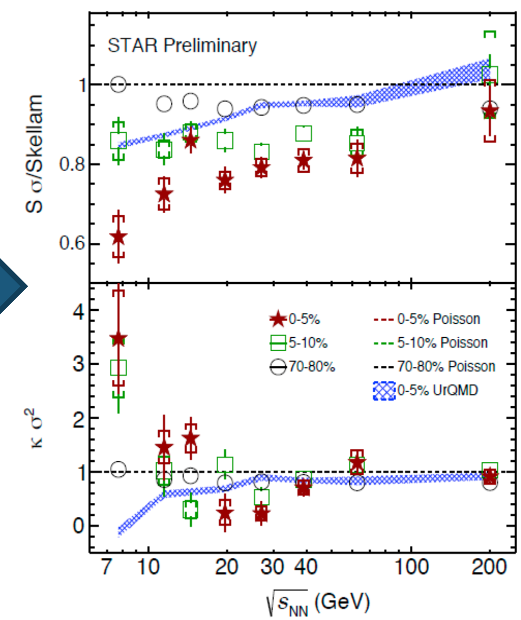
Review

Fluctuations of conserved charges in relativistic heavy ion collisions: An introduction

Masayuki Asakawa, Masakiyo Kitazawa*
Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan

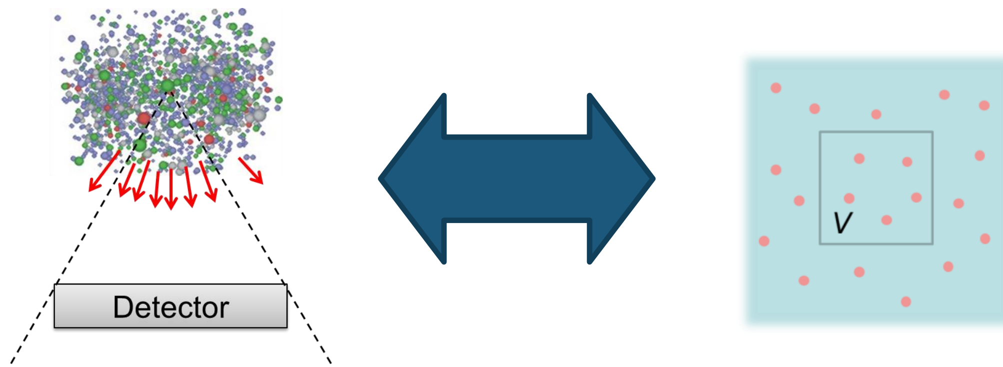
PPNP, in press,
arXiv:1512.05038v2

- What are **cumulants**?
- Why Sσ/(Skellam) and κσ²?
  - what are the "**baselines**"?
- Why **conserved** charges?
- What are **event-by-event** fluctuations?
  - their relation with theoretical analyses?



Two Problems

in connecting experiments with theories



1. Thermal blurring and diffusion of fluctuations

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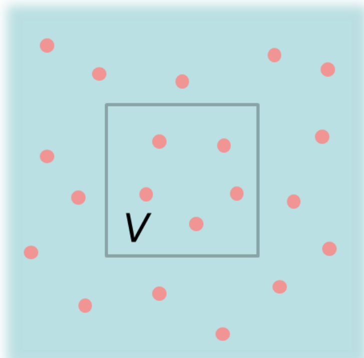
2. Efficiency correction of cumulants

MK, PR**C93**, 044911 (2016).

Fluctuations: Theory vs Experiment

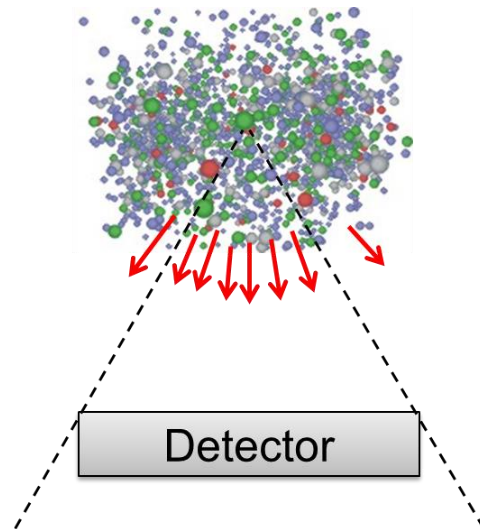
Theoretical analyses
based on statistical mechanics



lattice, critical point,
effective models, …

Fluctuation in a spatial volume

Experiments



Fluctuations in a momentum space

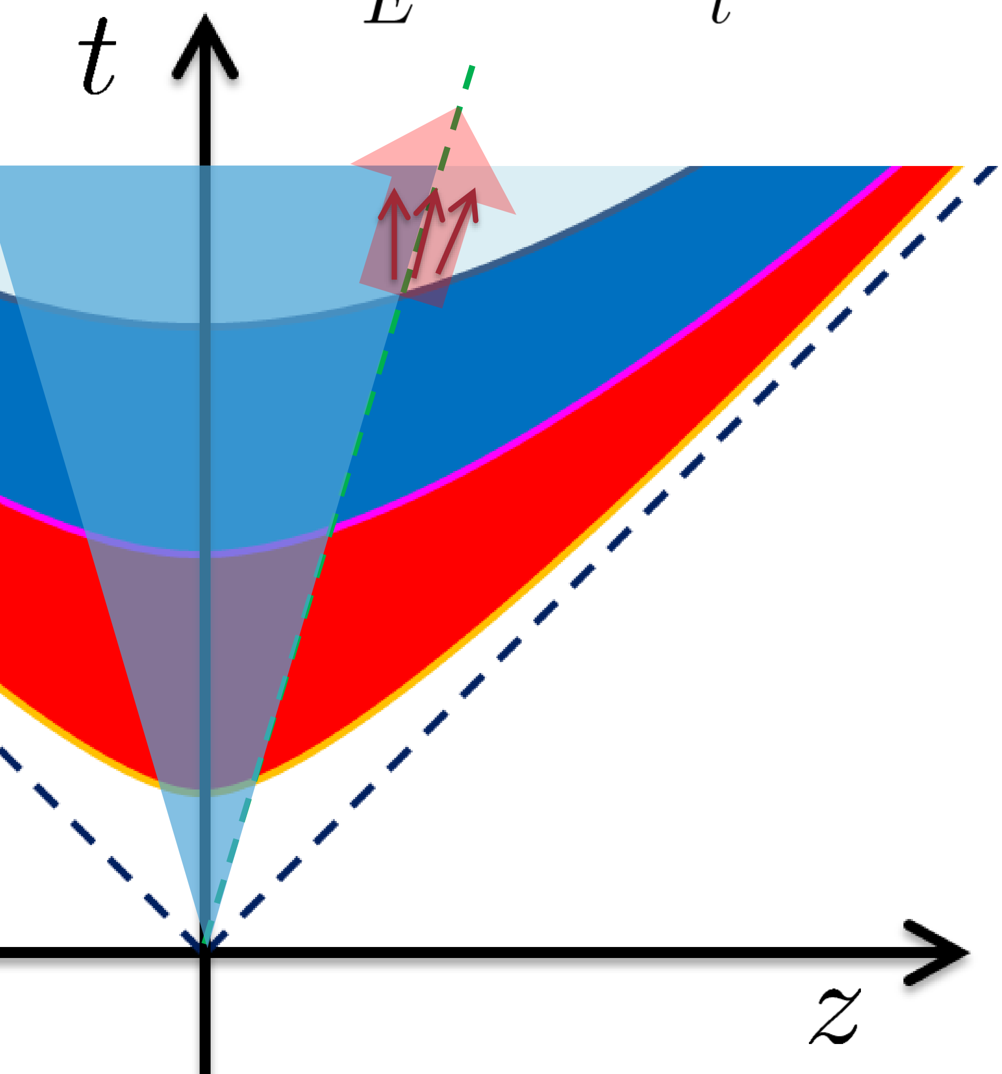
discrepancy in phase spaces

Asakawa, Heinz, Muller, 2000; Jeon, Koch, 2000; Shuryak, Stephanov, 2001

Connecting Phase Spaces

Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000

$$\frac{p_z}{E} = v_z = \frac{z}{t}$$



Under Bjorken picture,

coordinate-space rapidity *Y*

||

momentum-space rapidity *y* of medium

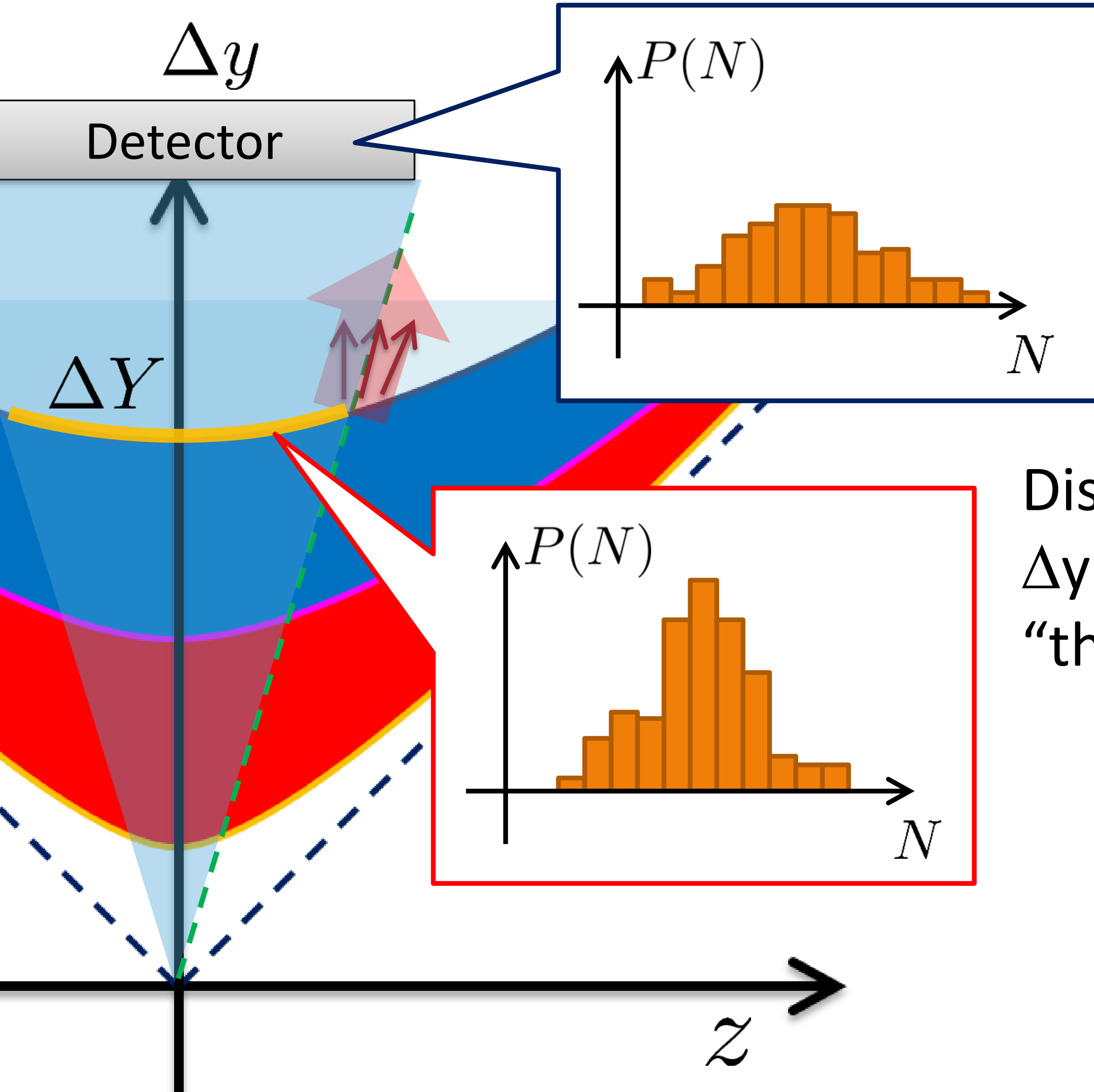
≀|

momentum-space rapidity *y* of individual particles

$$\Delta y \simeq \Delta Y$$

Thermal Blurring

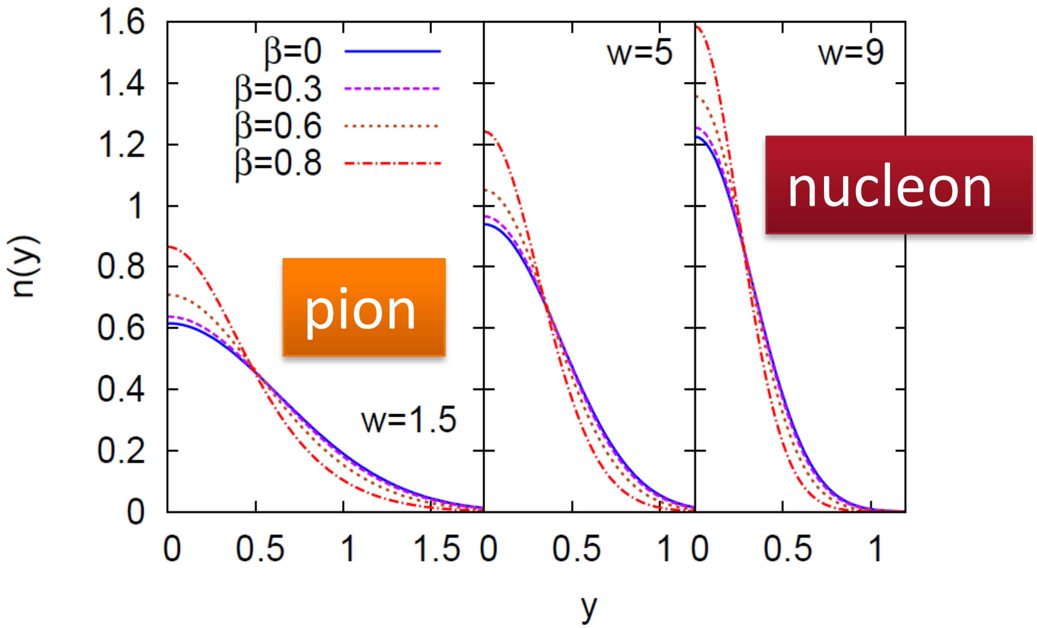
Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000



Distributions in ΔY and Δy are different due to “thermal blurring”.

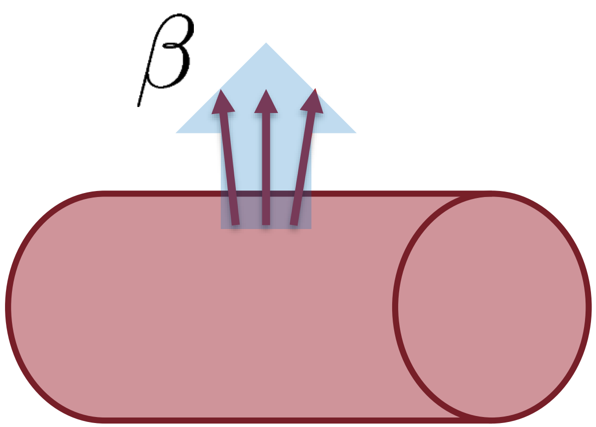
Thermal distribution in y space

Y. Ohnishi+
to appear soon

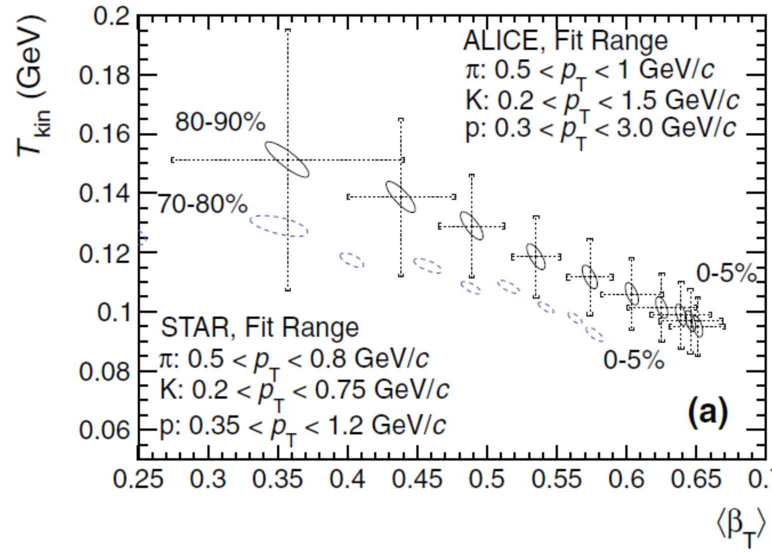


$$w = \frac{m}{T}$$

- pions $w \simeq 1.5$
- nucleons $w \simeq 9$



Blast wave squeezes the distribution in rapidity space



- blast wave
- flat freezeout surface

Thermal distribution in y space

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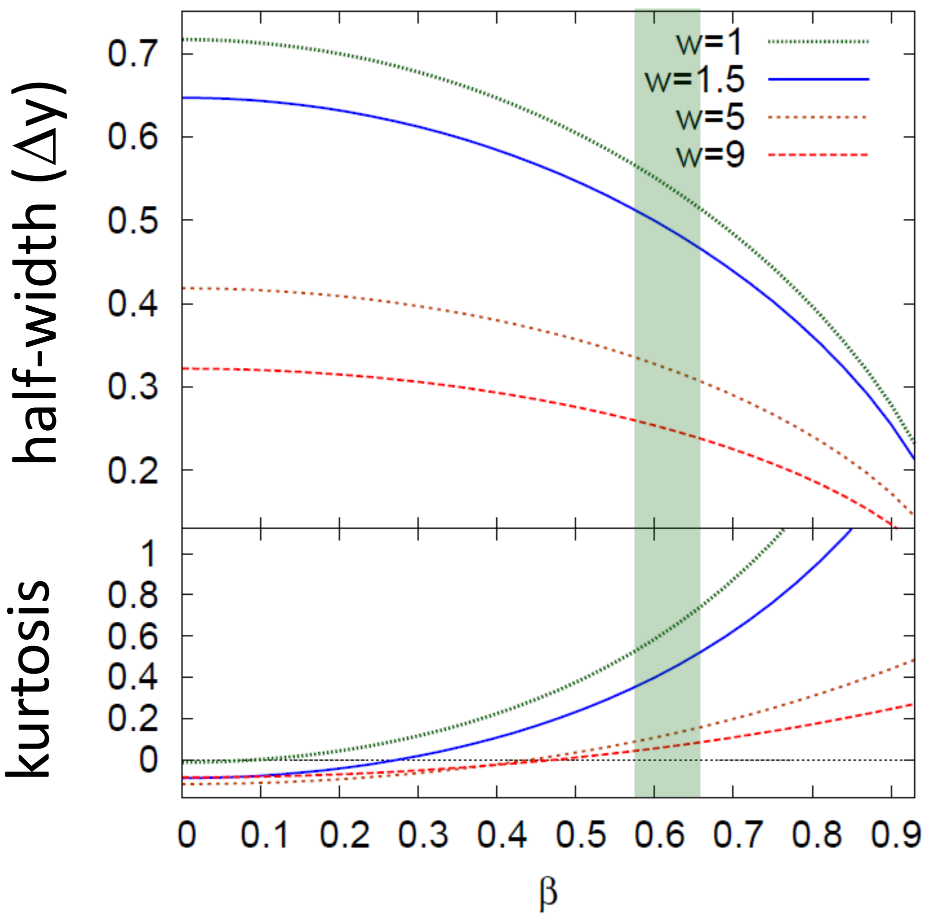

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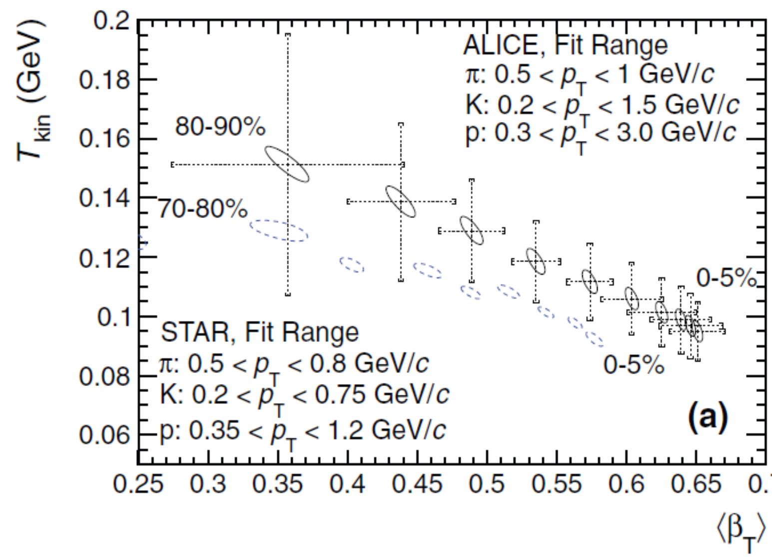
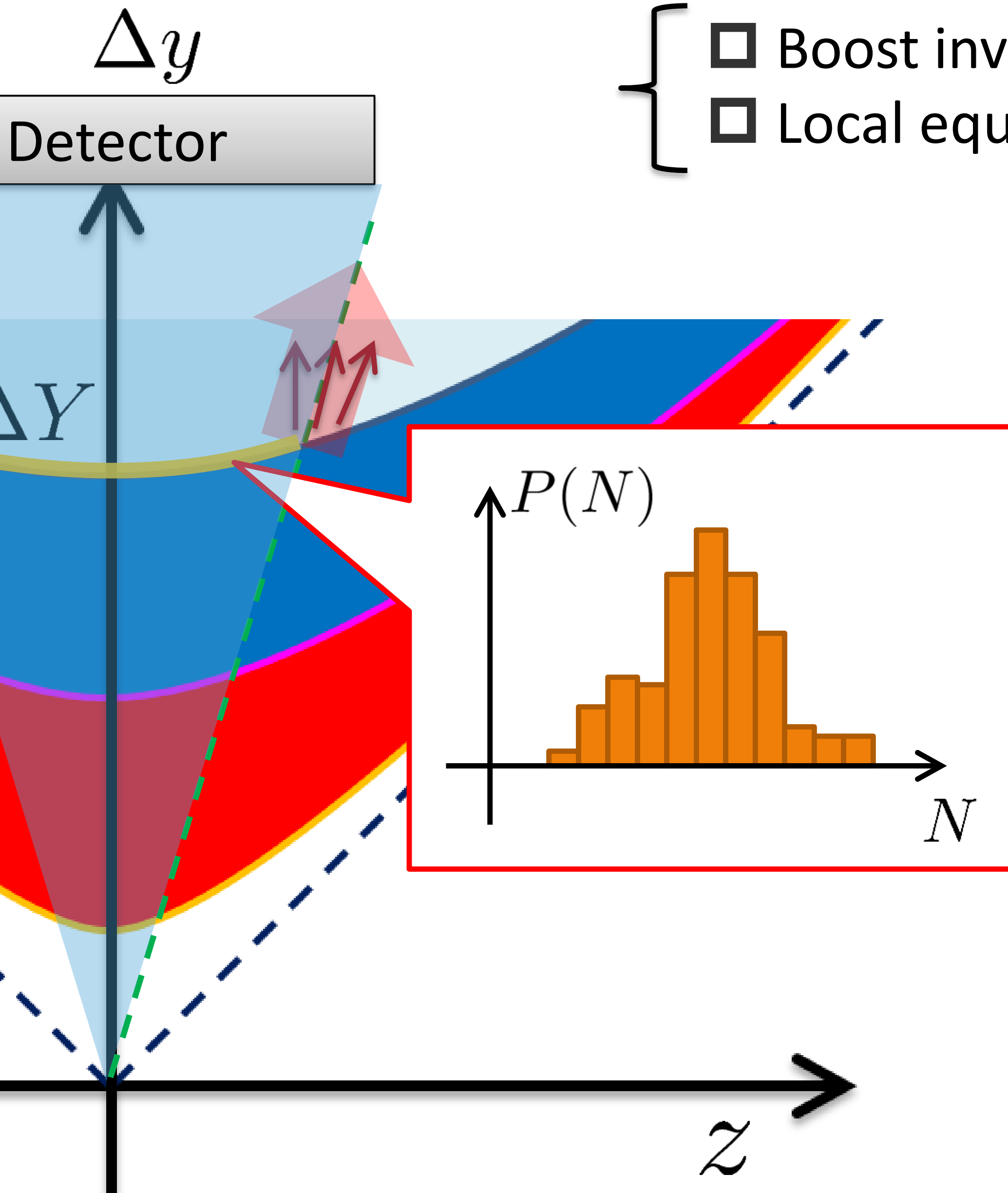


Rapidity distribution can be well approximated by Gaussian.

- blast wave
- flat freezeout surface

Initial Condition

- ☐ Boost invariance / infinitely long system
- ☐ Local equilibration / local correlation



$\langle \bar{Q}^2 \rangle_c, \ \langle \bar{Q}^3 \rangle_c, \ \langle \bar{Q}^4 \rangle_c$

$\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$

$\langle Q^2_{(\mathrm{tot})} \rangle_c, \langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c$

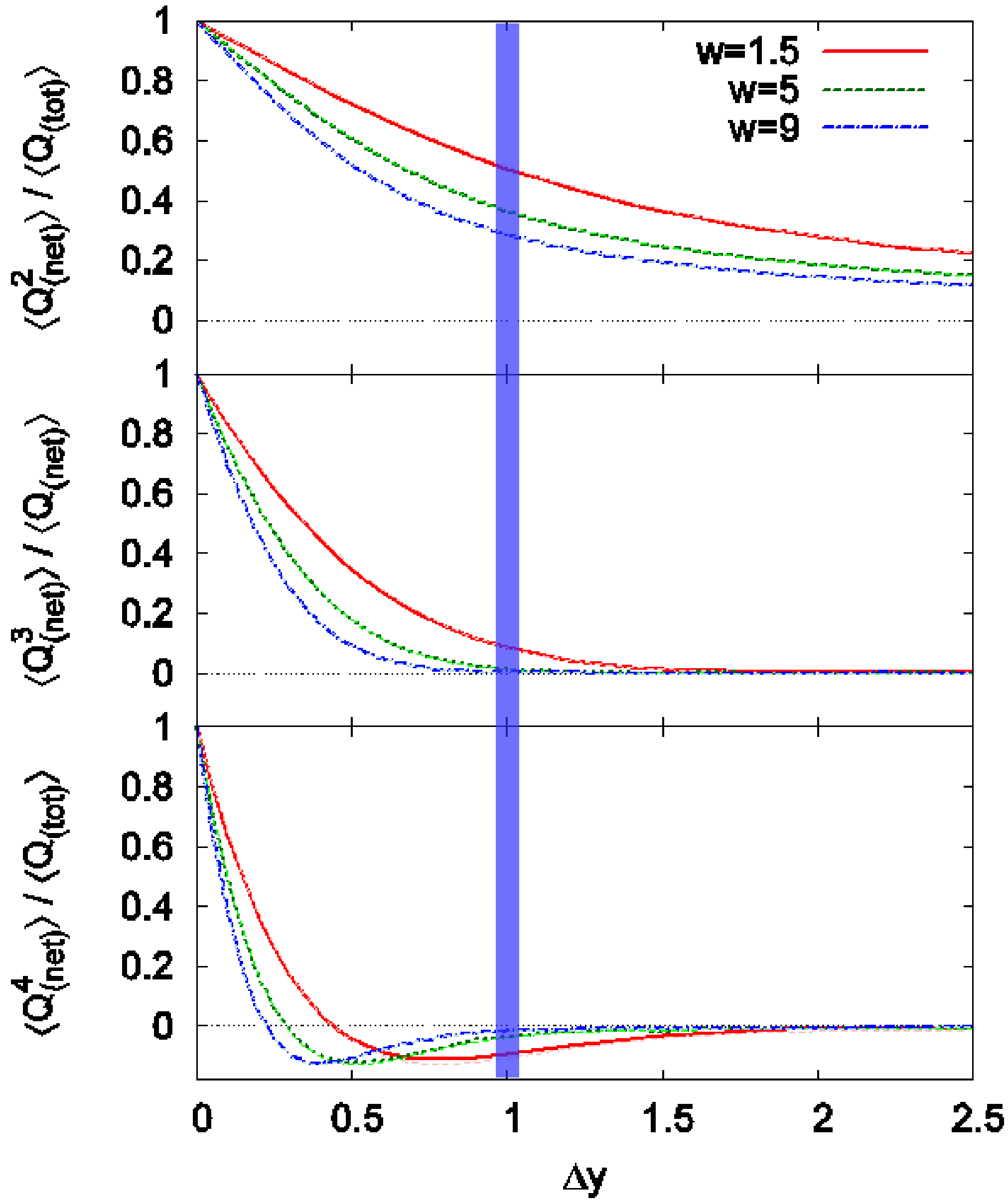
We need 6 parameters
to specify the initial fluctuation

Δη Dependence

Initial condition
(before blurring)
no e-v-e fluctuations

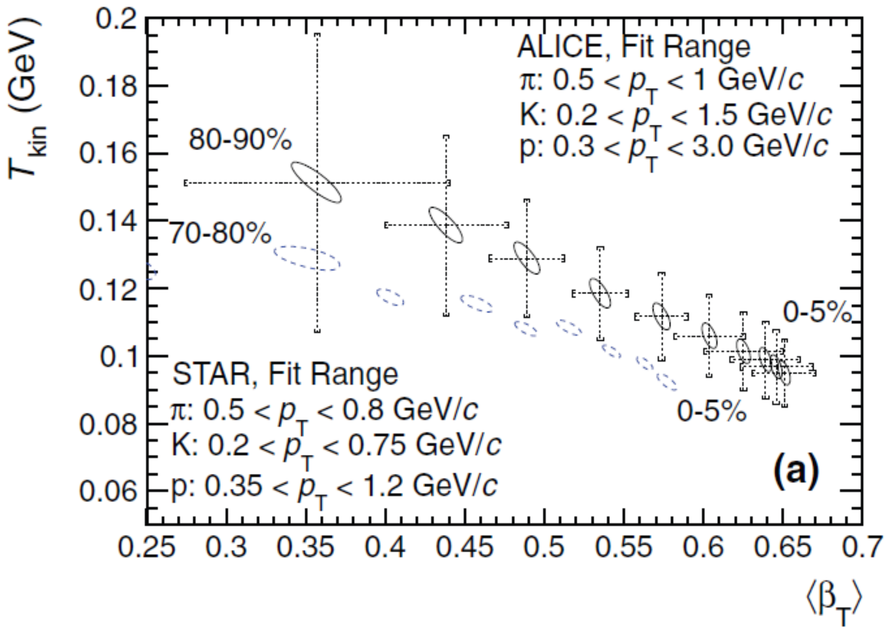
Cumulants after blurring
can take nonzero values

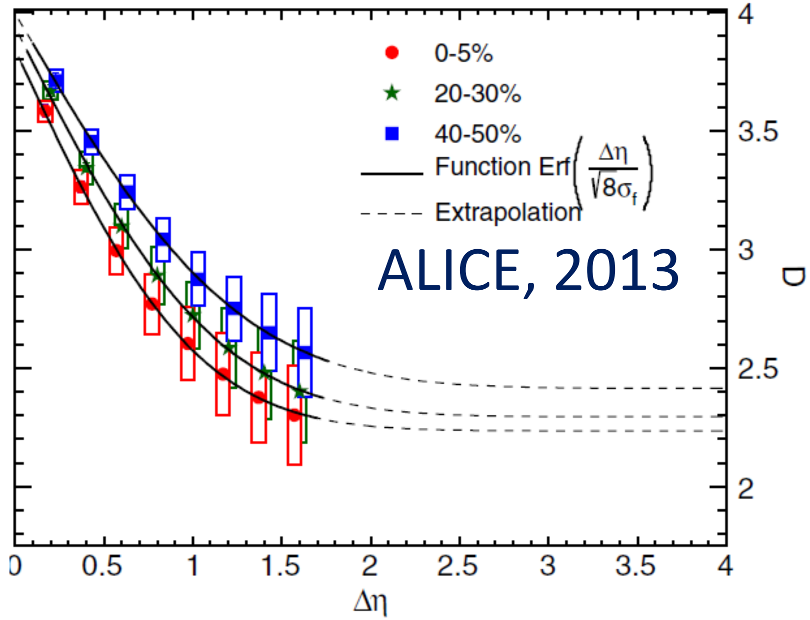
Cumulants after blurring



With Δy=1, the effect is **not** well suppressed

Centrality Dependence





More central → { lower T, larger β } → Weaker blurring

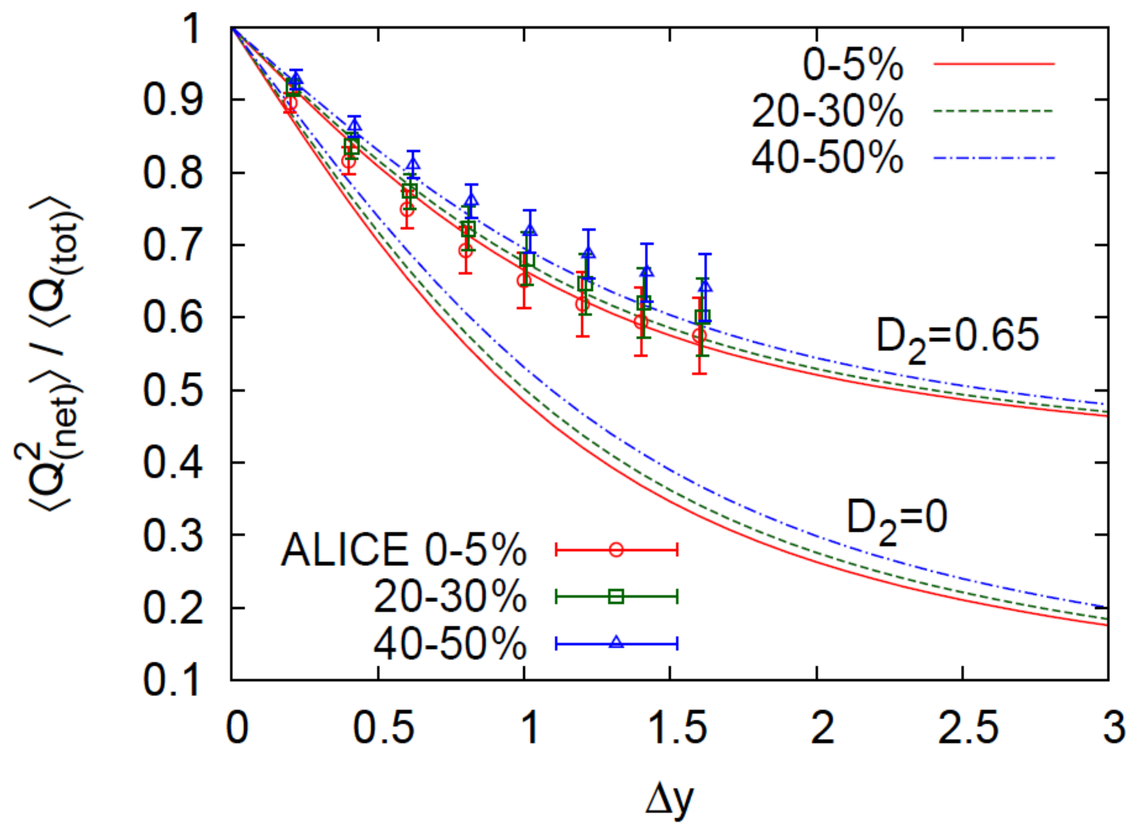
Is the centrality dependence understood solely by the thermal blurring at kinetic f.o.?

Centrality Dependence @ ALICE

$$D_2 = \frac{\langle \delta N_Q^2 \rangle}{\langle \delta N_Q^2 \rangle_{eq.}}$$

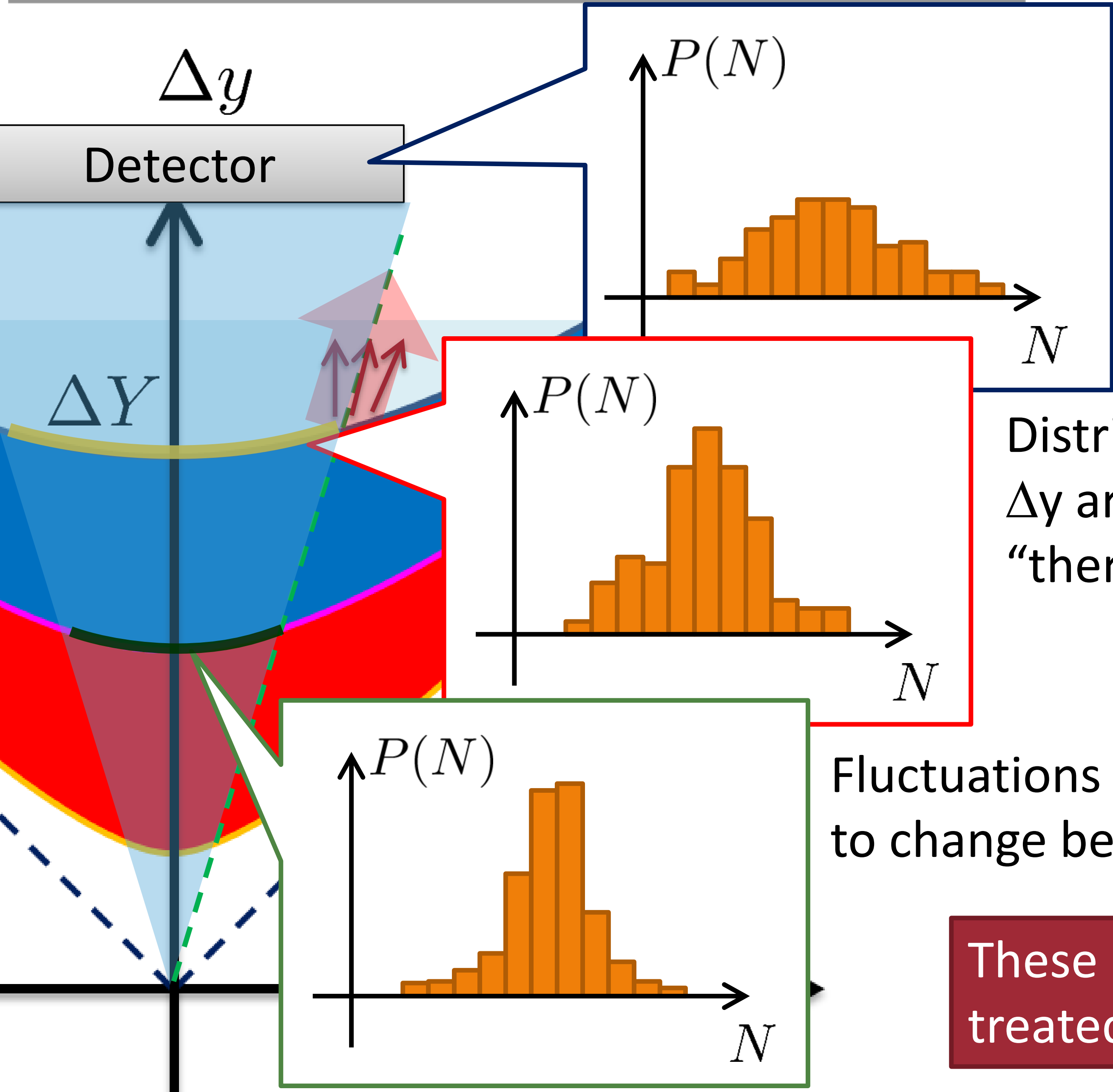
Assumptions:

- Centrality independent cumulant at kinetic f.o.
- Thermal blurring at kinetic f.o.



☐ Centrality dep. of fluctuation can be described by a simple thermal blurring picture.

Diffusion Before Kinetic F.O.



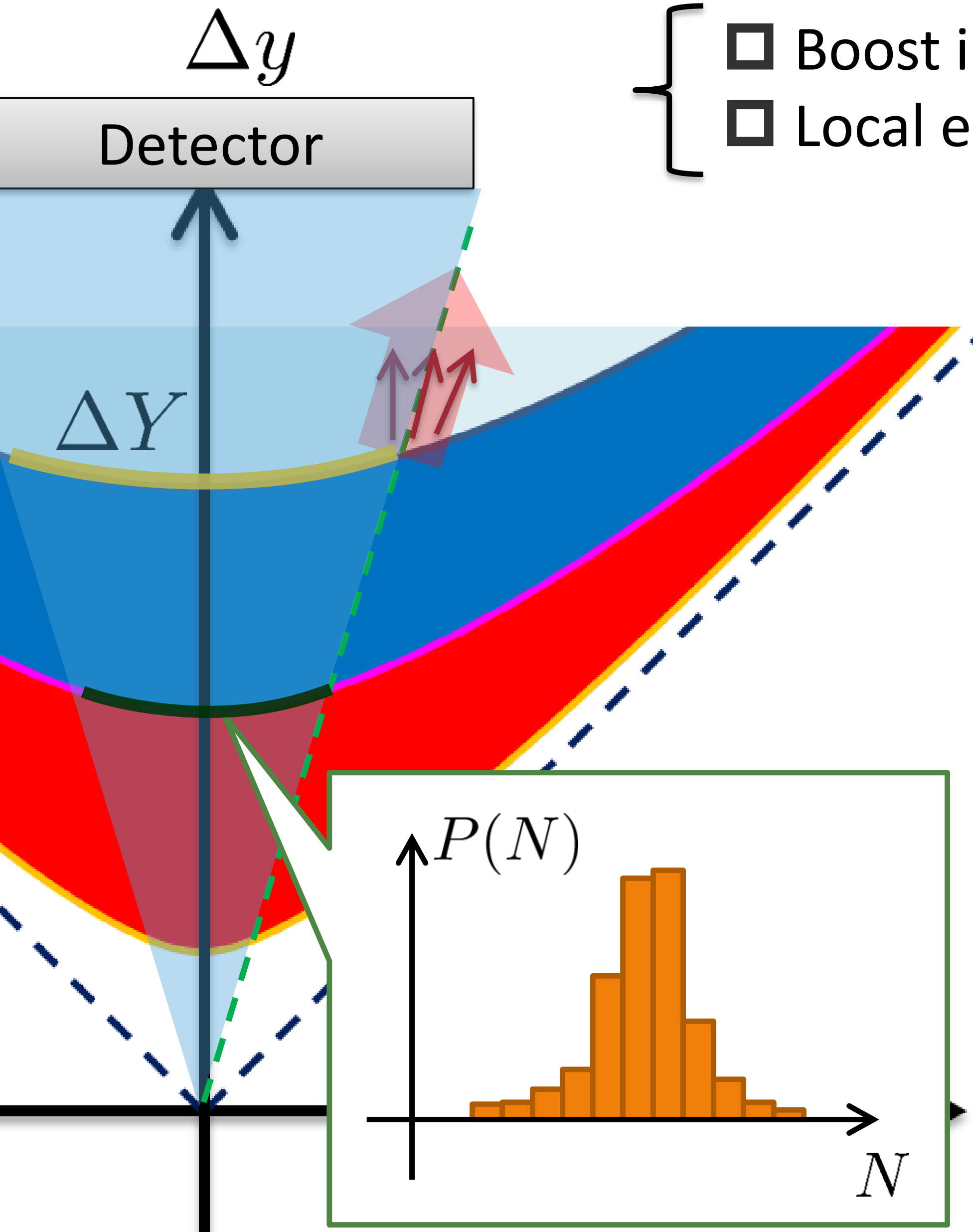
Distributions in ΔY and Δy are different due to "thermal blurring".

Fluctuations in ΔY continue to change before kinetic f.o.

These 2 processes can be treated as a single diffusion.

Diffusion Before Kinetic F.O.

- ☐ Boost invariance / infinitely long system
- ☐ Local equilibration / local correlation



Initial Condition

$$\langle \bar{Q}^2 \rangle_c,\ \langle \bar{Q}^3 \rangle_c,\ \langle \bar{Q}^4 \rangle_c$$

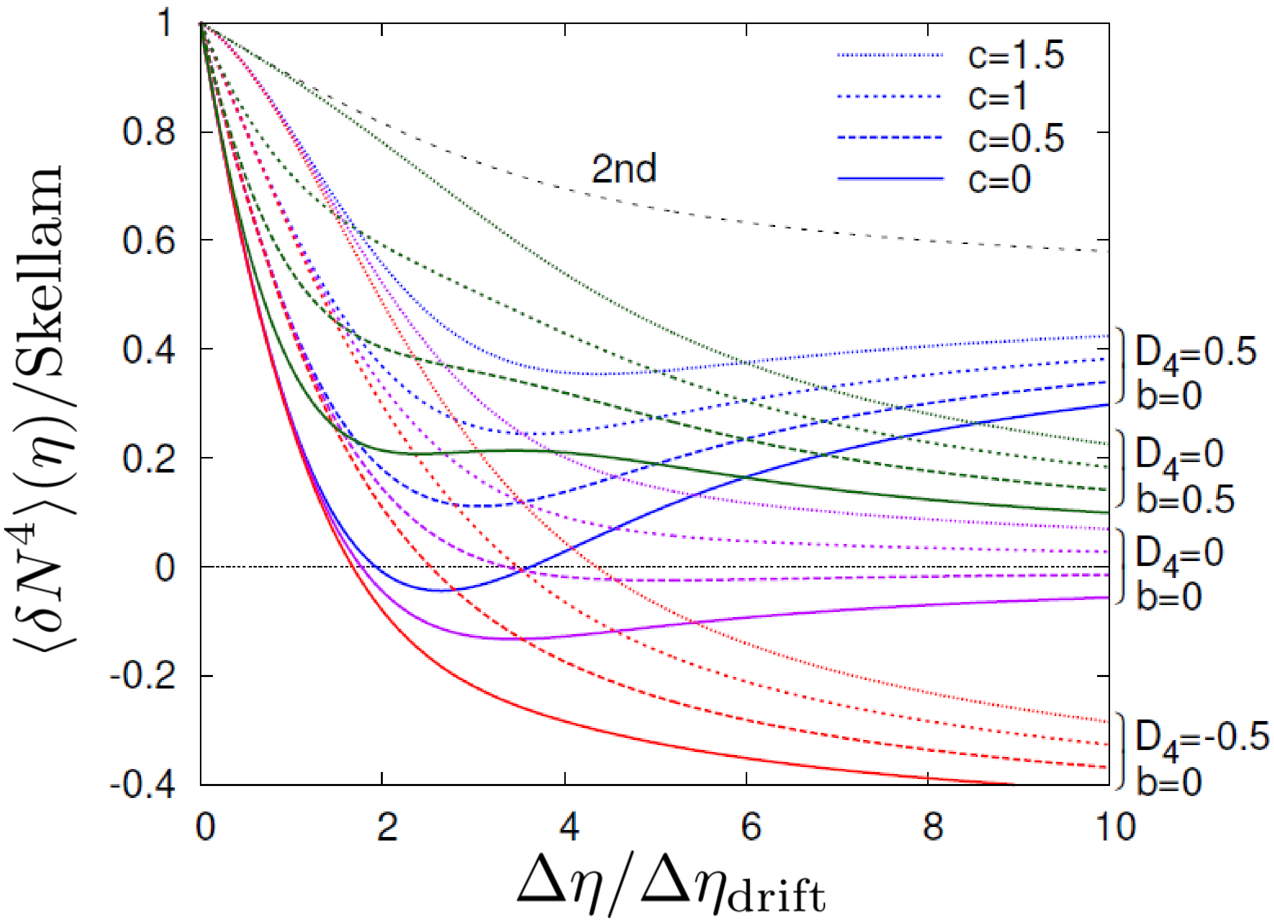
$$\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$$

$$\langle Q_{(\mathrm{tot})}^2 \rangle_c, \langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c$$

We need 6 parameters
to specify the initial fluctuation

Δη Dependence: 4th order

MK, Asakawa, PLB(2014)
MK, NPA(2015)



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\rm net)} \rangle_c}{\langle Q_{(\rm tot)} \rangle}$$

$$b = \frac{\langle Q^2_{(\rm net)} Q_{(\rm tot)} \rangle_c}{\langle Q_{(\rm net)} \rangle}$$

$$c = \frac{\langle Q^2_{(\rm tot)} \rangle_c}{\langle Q_{(\rm tot)} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\rm net)} \rangle_c}{\langle Q_{(\rm tot)} \rangle} = 0.5$$

Characteristic Δη dependences!

Δη Dependence: 4th order

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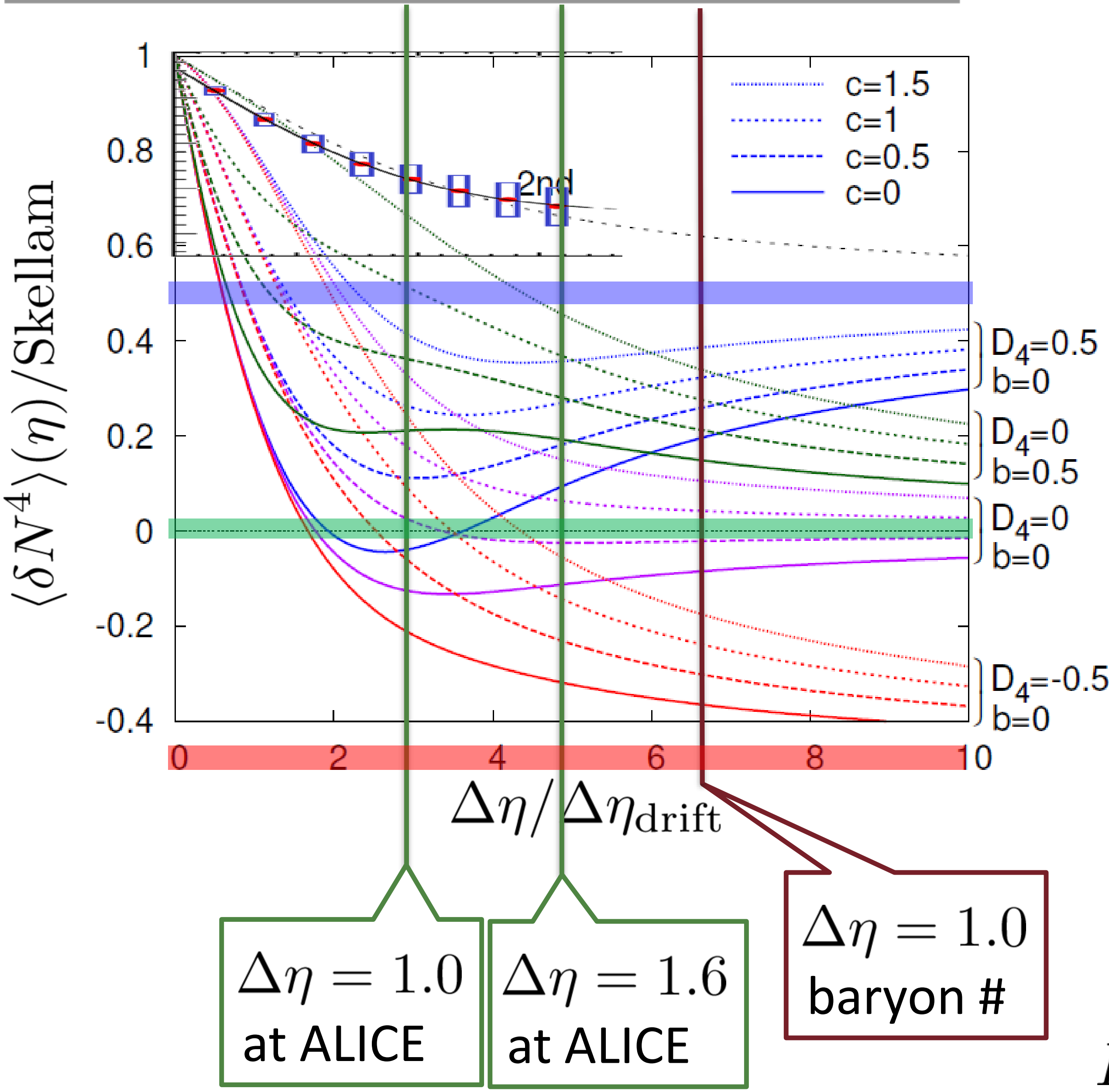


Initial Condition

$$D_4 = \frac{\langle Q^4_{(\rm net)} \rangle_c}{\langle Q_{(\rm tot)} \rangle}$$

$$b = \frac{\langle Q^2_{(\rm net)} Q_{(\rm tot)} \rangle_c}{\langle Q_{(\rm net)} \rangle}$$

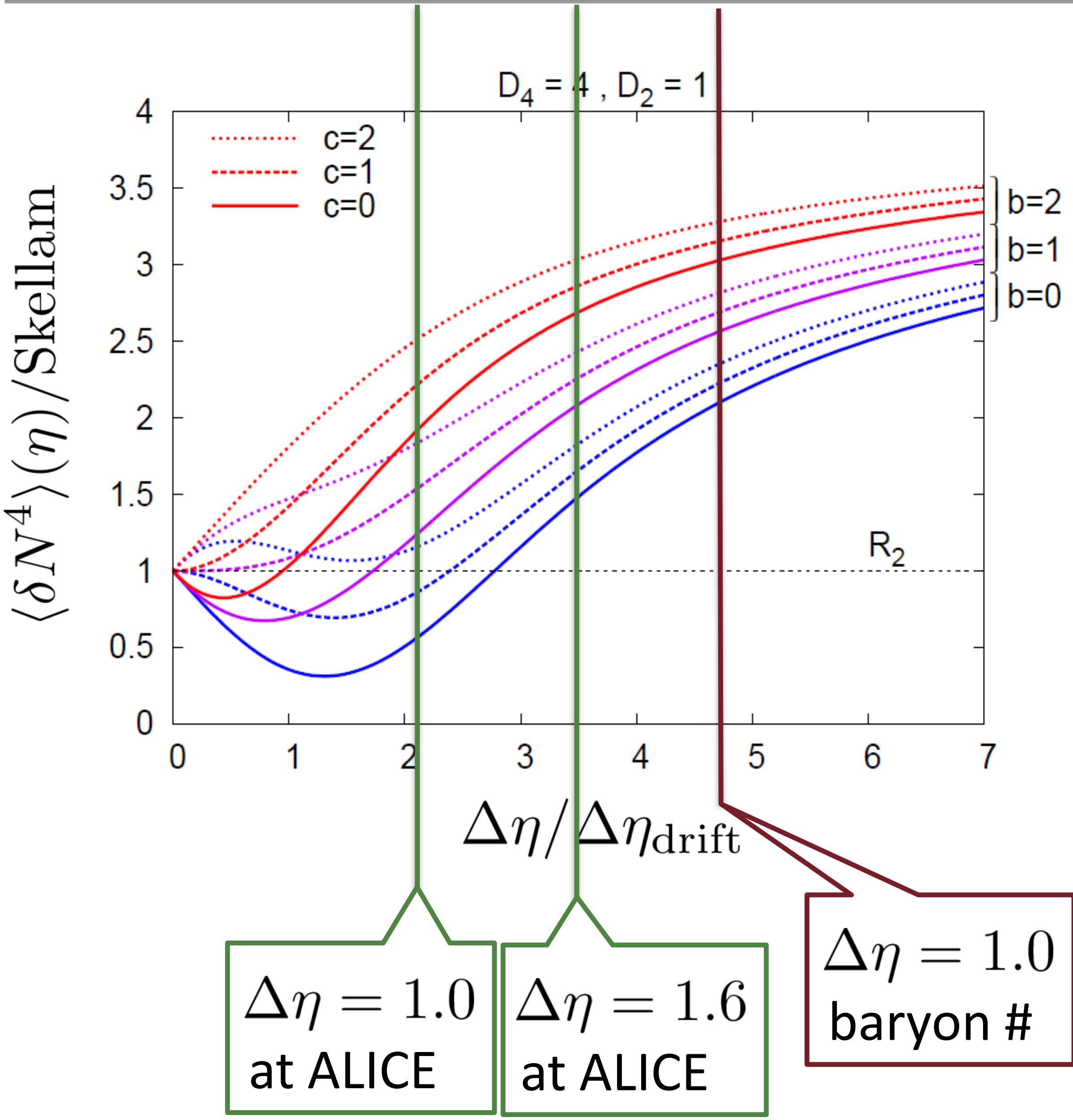
$$c = \frac{\langle Q^2_{(\rm tot)} \rangle_c}{\langle Q_{(\rm tot)} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\rm net)} \rangle_c}{\langle Q_{(\rm tot)} \rangle} = 0.5$$

$$D \sim M^{-1}$$

4^{th} order : w/ Critical Fluctuation

MK, Asakawa, PLB(2014)
MK, NPA(2015)



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

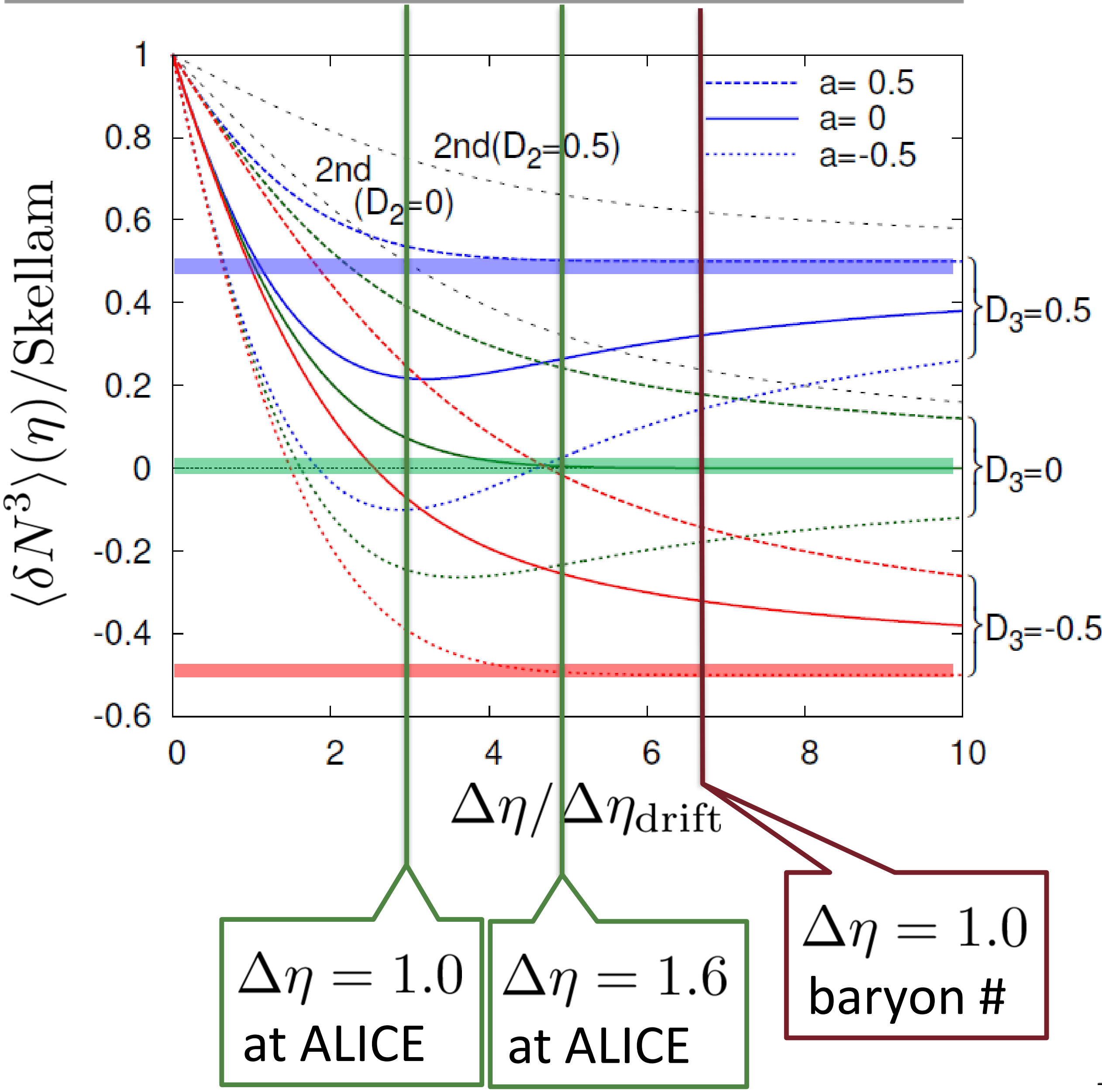
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

$$D \sim M^{-1}$$

Δη Dependence: 3rd order

MK, Asakawa, PLB(2014)
MK, NPA(2015)



Initial Condition

$$D_3 = \frac{\langle Q^3_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{net})}\rangle}$$

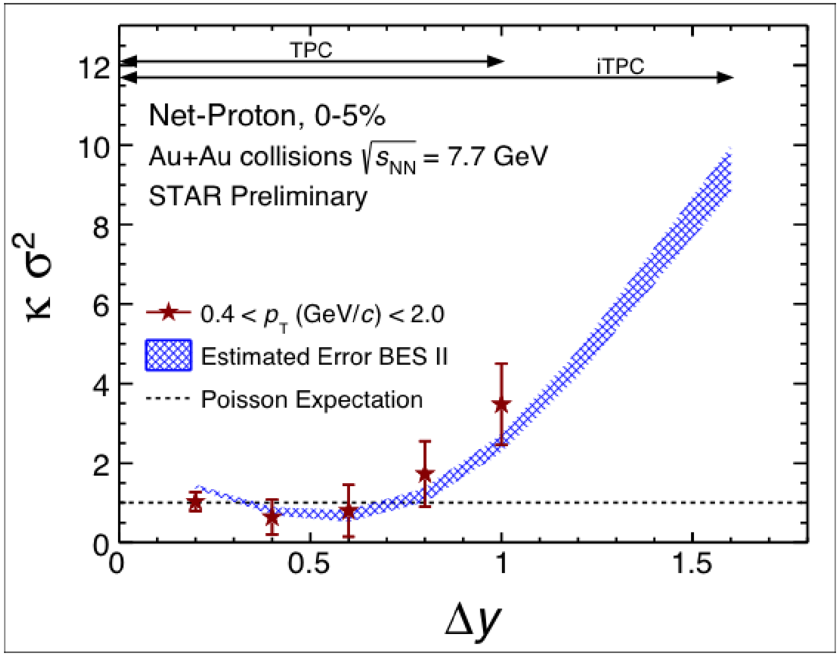
$$a = \frac{\langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{net})}\rangle}$$

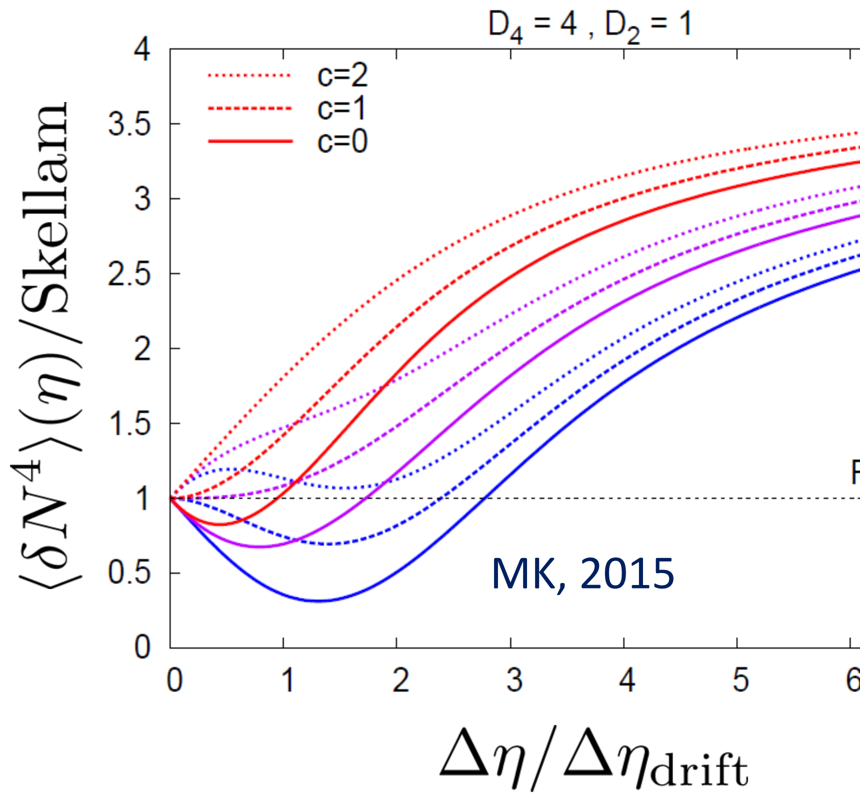
$$D_2 = \frac{\langle Q^2_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} = 0.5$$

$$D \sim M^{-1}$$

Δη Dependence @ STAR

X. Luo, CPOD2014

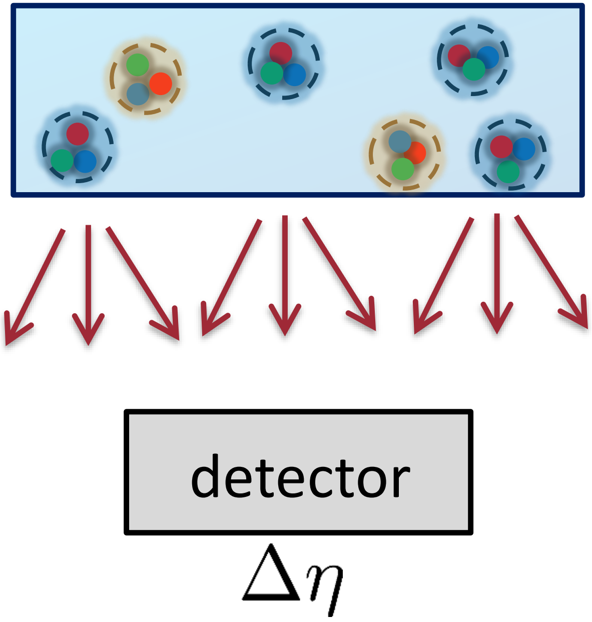




Non-monotonic dependence on Δy ?

Very Low Energy Collisions

- Large contribution of global charge conservation
- Violation of Bjorken scaling



Fluctuations at low √s should be interpreted carefully!

Summary of 1st Part

- Effect of thermal blurring provoked by rapidity conversion is not negligible with $\Delta y=1.0$.

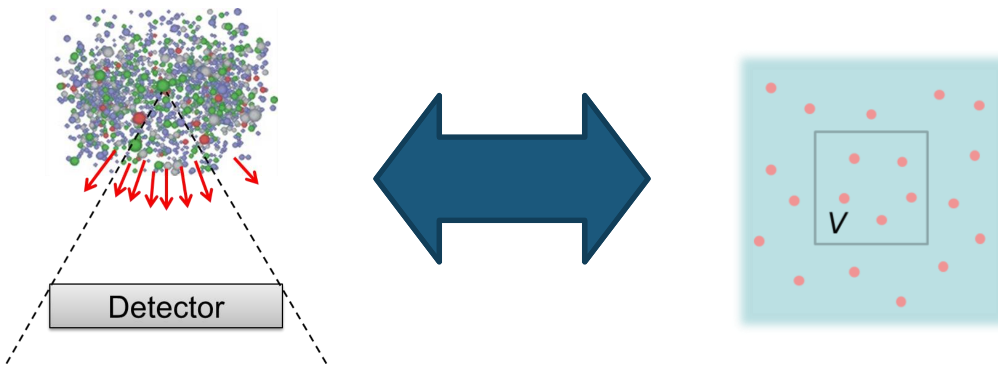
- Higher order cumulants can behave characteristically as functions of Δy.

- This behavior can be used to constrain
  - the magnitude of thermal blurring, and
  - fluctuations in the early stage.

- The study of centrality dependence is also interesting.

Two Problems

in connecting experiments with theories



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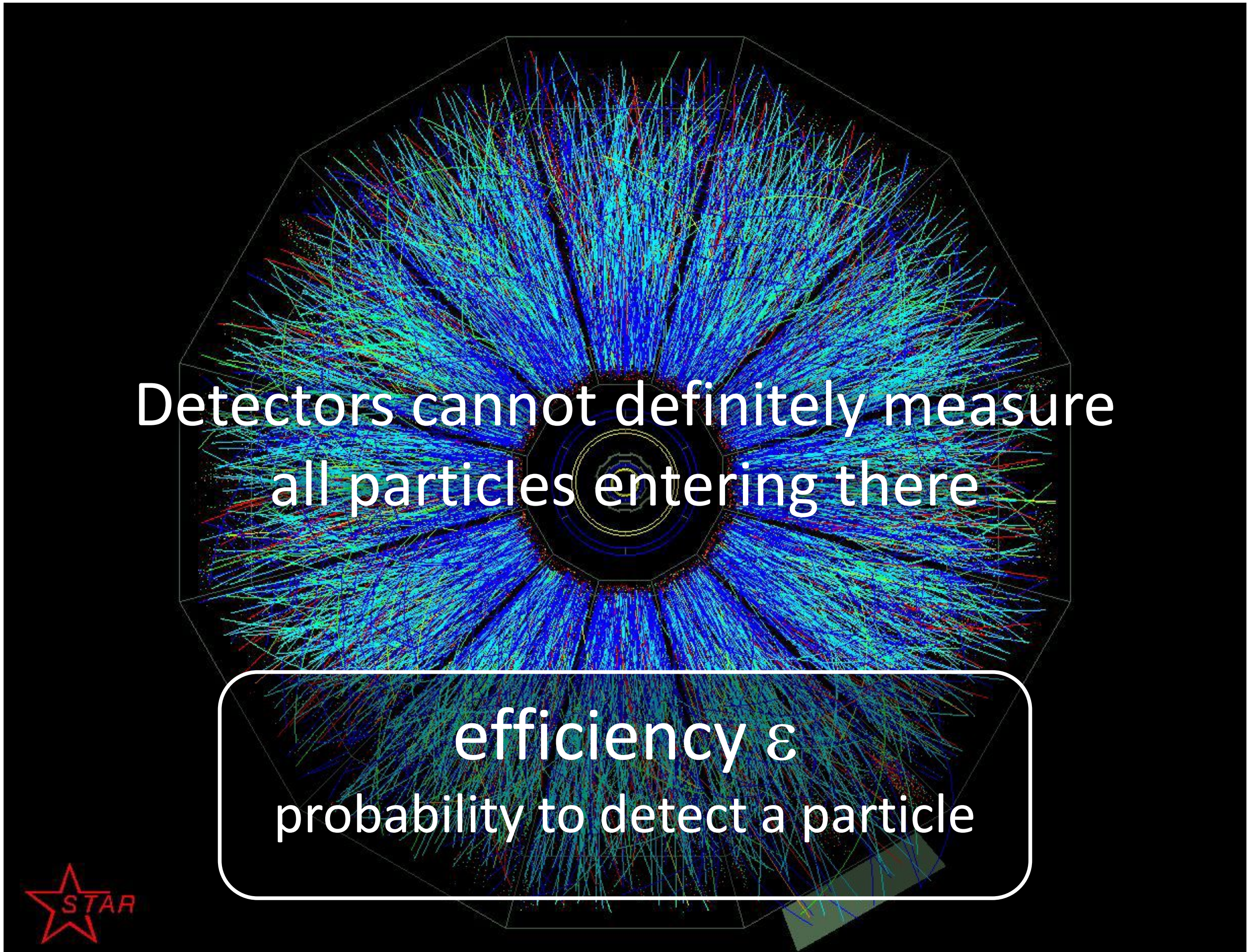
Efficiency



Detectors cannot definitely measure all particles entering there

efficiency ε

probability to detect a particle

Efficiency correction is essential for all observables in experiments.

The Binomial Model

MK, Asakawa, 2012; 2012
Bzdak, Koch, 2012

When efficiency for individual particles are **independent**

$$P_{\rm obs}(n) = \sum_N B_p(n;N) P(N)$$

dist. func. of **observed** particle #

binmial dist. func.

dist. func. of **original** particle #

The cumulants connected with each other

$$\langle n^m \rangle_{\rm c} \Longleftrightarrow \langle N^m \rangle_{\rm c}$$

Caveat: Effects of nonvanishing correlations: Holtzman+ 2016

$$\langle n^m \rangle_c \Longleftrightarrow \langle N^m \rangle_c$$

$$\langle N^m \rangle_c \Rightarrow \langle n^m \rangle_c$$

$$\begin{aligned}
\langle n \rangle &= \xi_1 \langle N \rangle, \\
\langle n^2 \rangle_c &= \xi_1^2 \langle N^2 \rangle_c + \xi_2 \langle N \rangle, \\
\langle n^3 \rangle_c &= \xi_1^3 \langle N^3 \rangle_c + 3\xi_1\xi_2 \langle N^2 \rangle_c + \xi_3 \langle N \rangle, \\
\langle n^4 \rangle_c &= \xi_1^4 \langle N^4 \rangle_c + 6\xi_1^2\xi_2 \langle N^3 \rangle_c + (3\xi_2^2 + 4\xi_1\xi_3) \langle N^2 \rangle_c + \xi_4 \langle N \rangle,
\end{aligned}$$

$$\langle n^m \rangle_c \Rightarrow \langle N^m \rangle_c$$

$$\begin{aligned}
\langle N \rangle &= \xi_1^{-1} \langle n \rangle, \\
\langle N^2 \rangle_c &= \xi_1^{-2} \langle n^2 \rangle_c - \xi_2\xi_1^{-3} \langle n \rangle, \\
\langle N^3 \rangle_c &= \xi_1^{-3} \langle n^3 \rangle_c - 3\xi_2\xi_1^{-4} \langle n^2 \rangle_c + (3\xi_2^2\xi_1^{-5} - \xi_3\xi_1^{-4}) \langle n \rangle, \\
\langle N^4 \rangle_c &= \xi_1^{-4} \langle n^4 \rangle_c - 6\xi_2\xi_1^{-5} \langle n^3 \rangle_c + (15\xi_2^2\xi_1^{-6} - 4\xi_3\xi_1^{-5}) \langle n^2 \rangle_c \\
&\quad - (15\xi_2^3\xi_1^{-7} - 10\xi_2\xi_3\xi_1^{-6} + \xi_4\xi_1^{-5}) \langle n \rangle,
\end{aligned}$$

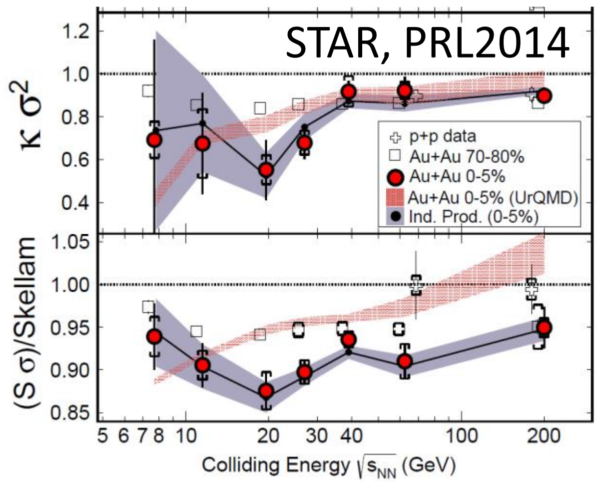
Formulas using factorial moments: Bzdak, Koch, 2012

Summing up Multiple Variables

- **Net-particle number**

$$N_{p,\mathrm{net}} = N_p - N_{\bar{p}}$$

MK, Asakawa, 2012; Bzdak, Koch, 2012

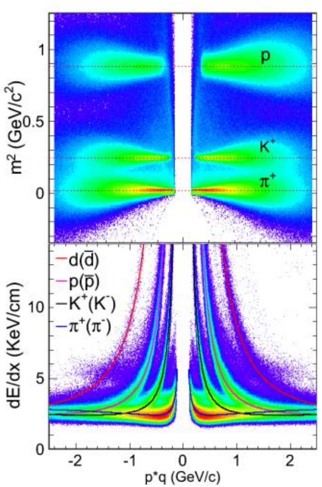


average (common) efficiency for p and $\bar{p}$

cf, Nonaka+, 1604.06212

- **Multi-particle species**
  - net-electric charge
  - p_T dependent efficiency

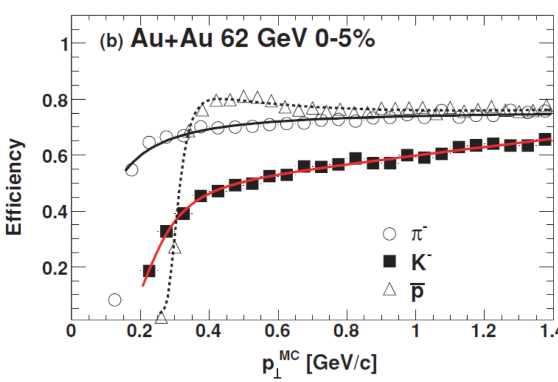
Bzdak, Koch, 2015; Luo, 2014



STAR, net proton

$p_T < 0.8\mathrm{GeV}$ TPC ε~80%

$p_T > 0.8\mathrm{GeV}$ TPC+TOF ε~50%



Efficiency Correction with Factorial Moments

Bzdak, Koch, 2015; Luo, 2014

Simple relations
b/w **factorial** moments

$$\langle n_i^m \rangle_{\mathrm{f}} = \epsilon_i^m \langle N_i^m \rangle_{\mathrm{f}},$$
$$\langle n_i^m n_j^l \rangle_{\mathrm{f}} = \epsilon_i^m \epsilon_j^l \langle N_i^m N_j^l \rangle_{\mathrm{f}}, \cdots$$

n_i: observed particle #
N_i: original particle #
ε_i: efficiency

① Calculate all factorial moments of n_i
② Translate it into original moments N_i
③ Construct the cumulant N_i

Problem

Number of f-moments for nth order with M variables $\binom{M+n}{n}$-1→ M^n ⇒ **Require huge numerical power**

New Formulas for Efficiency Correction

MK, PRC,2016
[1602.01234]

$$\langle Q\rangle_c = \langle\langle q_{(1)}\rangle\rangle_c,$$

$$\langle Q^2\rangle_c = \langle\langle q_{(1)}^2\rangle\rangle_c - \langle\langle q_{(2)}\rangle\rangle_c,$$

$$\langle Q^3\rangle_c = \langle\langle q_{(1)}^3\rangle\rangle_c - 3\langle\langle q_{(2)}q_{(1)}\rangle\rangle_c + \langle\langle 3q_{(2,1|2)} - q_{(3)}\rangle\rangle_c,$$

$$\langle Q^4\rangle_c = \langle\langle q_{(1)}^4\rangle\rangle_c - 6\langle\langle q_{(2)}q_{(1)}^2\rangle\rangle_c + 12\langle\langle q_{(2,1|2)}q_{(1)}\rangle\rangle_c + 6\langle\langle q_{(1,1|2)}q_{(2)}\rangle\rangle_c - 4\langle\langle q_{(3)}q_{(1)}\rangle\rangle_c - 3\langle\langle q_{(2)}^2\rangle\rangle_c + \langle\langle -18q_{(2,1,1|2,2)} + 6q_{(2,1,1|3)} + 4q_{(3,1|2)} + 3q_{(2,2|2)} - q_{(4)}\rangle\rangle_c,$$

$$Q = \sum_{i=1}^{M} a_i N_i$$

linear comb
original par

$$q_{(\cdots)} = \sum_{i=1}^{M} c_{(\cdots)}^{(i)} n$$

linear comb
observed p

Numerical Cost

For nth order and M variables

- F-moment method $\sim \mathcal{O}(M^n)$
- Our method $\sim \mathcal{O}(M)$

for $M \to \infty$

Derivation

(1) Cumulant expansion

$$\ln\langle e^N \rangle = \sum_{m=1}^{\infty} \frac{1}{m!} \langle N^m \rangle_{\rm c}$$

(2) "Linearity" of binomial distribution

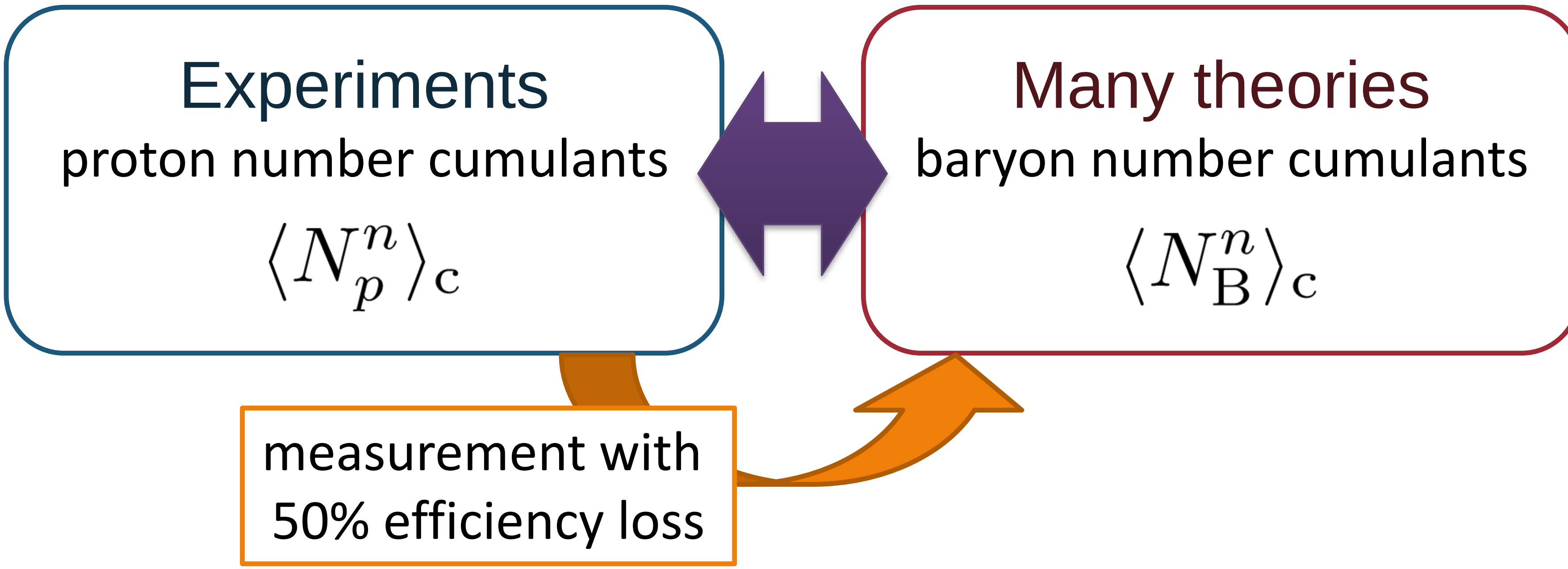
$$\langle n^m \rangle_{\rm c,binomial} = \xi_n(p) N \quad \text{for} \quad B_{p,N}(n)$$

$$\tilde{K}(\theta) = \ln\langle e^{k_{\rm binomial}(\theta)N} \rangle = \sum_m \frac{1}{m!} \langle (k_{\rm binomial}(\theta)N)^m \rangle_{\rm c}$$

(3) Treating multi-variable dist. func.

Proton v.s. Baryon Number Cumulants

MK, Asakawa, 2012; 2012



- The difference would be large.
- Reconstruction of $\langle N_B^n \rangle_c$ is possible using the binomial model.
- The use of binomial model is justified by “isospin randomization.”

Summary of 2nd Part

- Efficiency correction of fluctuations is a nontrivial subject. The binomial model is a solution.

- The new formulas will drastically reduce the numerical cost required for the efficiency corrections.

- Efficiency correction with realistic p_T-dependent efficiency can be carried out with the new formulas.

Summary

Critical Point

Still many many things to do for the search of the QCD CP using fluctuations.

A lot of careful, steady and honest researches are needed.

But, after hard efforts, the gift from God will be delivered!

A Coin Game

① Bet 50 PLN
② You get head coins of

A. 50 x 2 PLN

B. 20 x 5 PLN

Same expectation value.

A Coin Game

① Bet 50 PLN
② You get head coins of

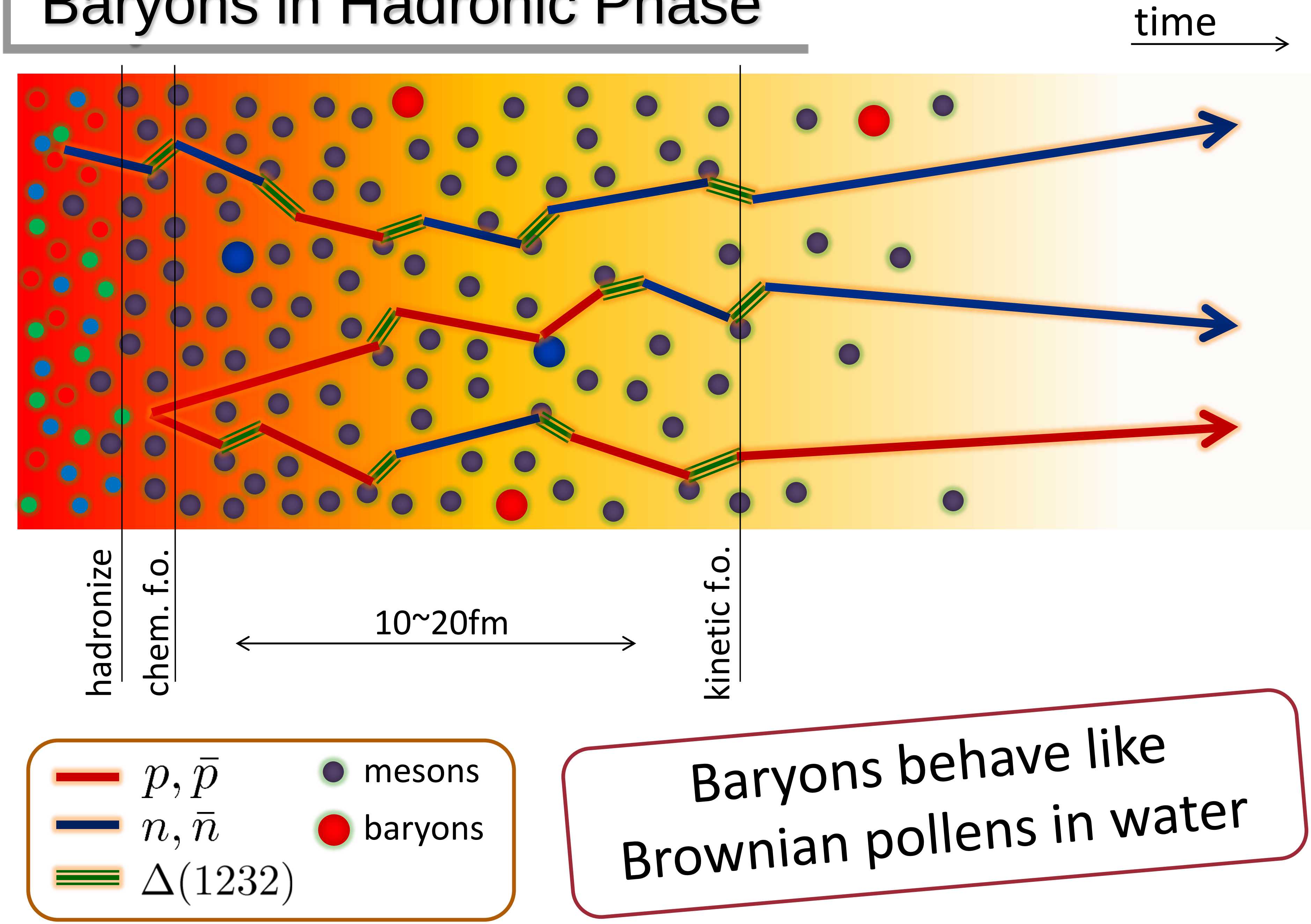
A. 50 x 2 PLN

B. 20 x 5 PLN

C. 1 x 100 PLN

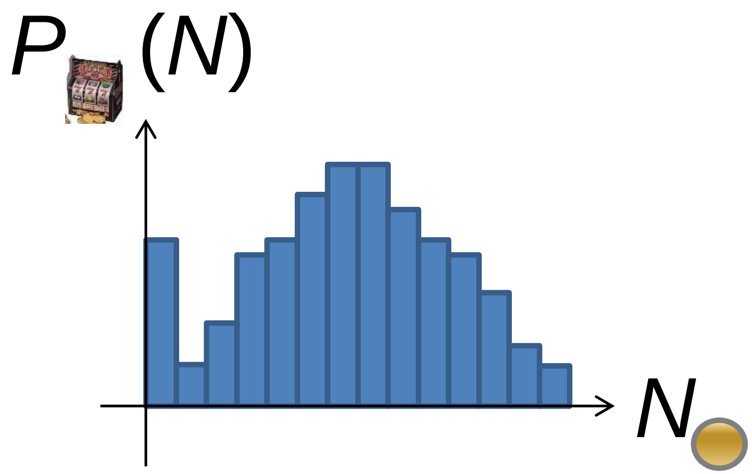
Same expectation value,
but different fluctuation

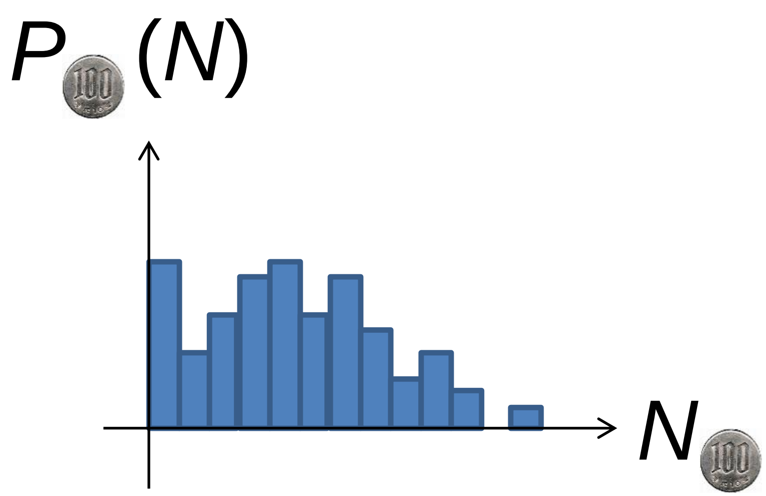
Baryons in Hadronic Phase



Baryons behave like Brownian pollens in water

Slot Machine Analogy

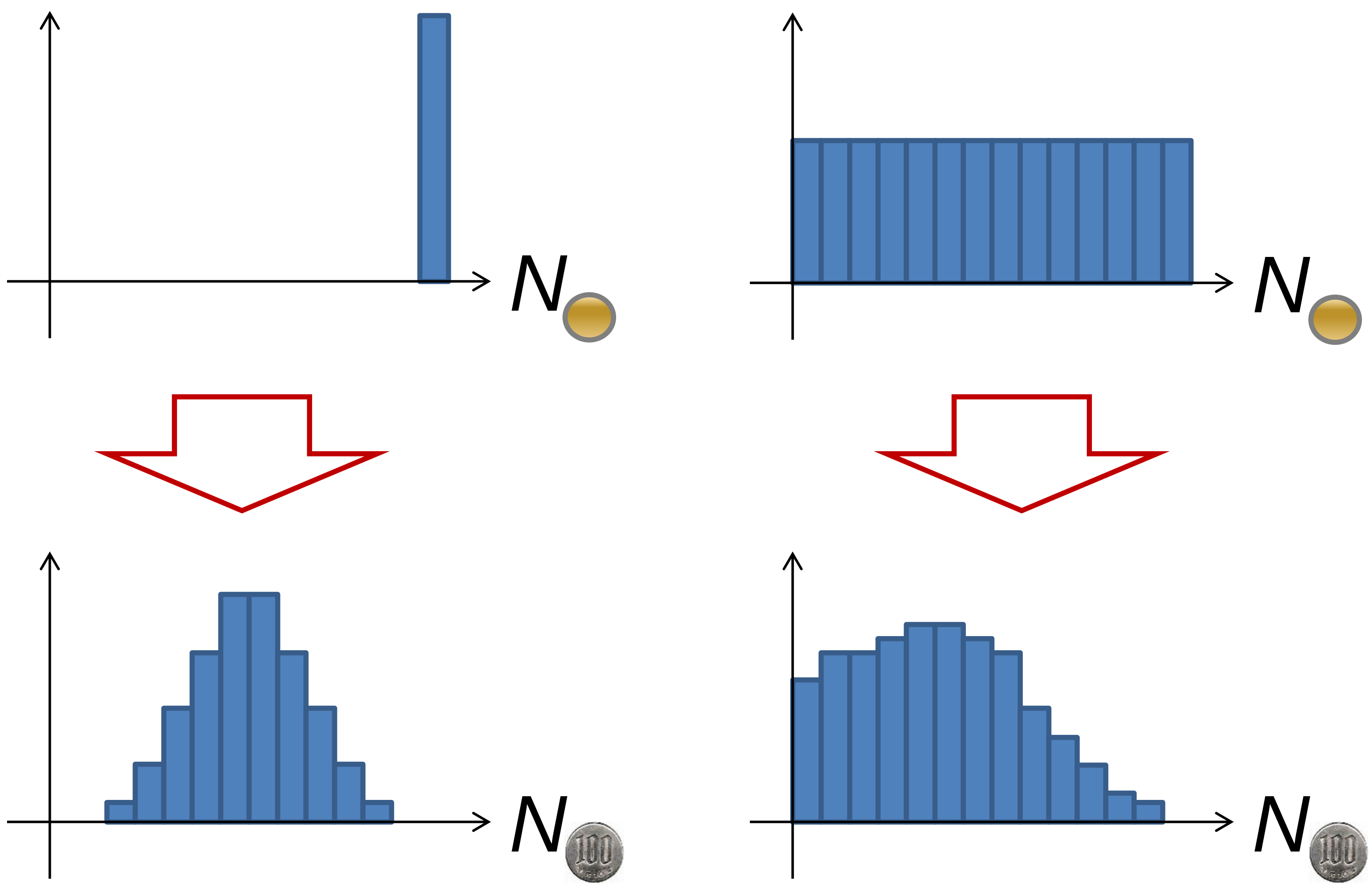




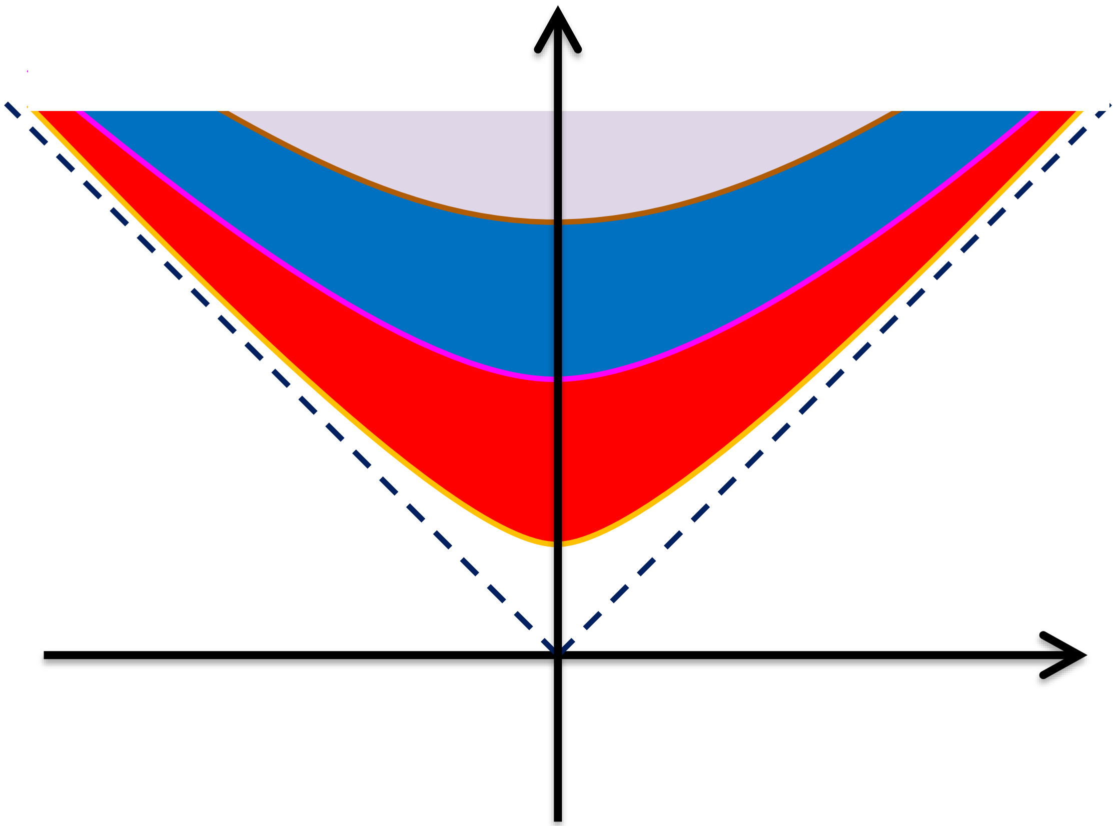
Extreme Examples

Fixed # of coins

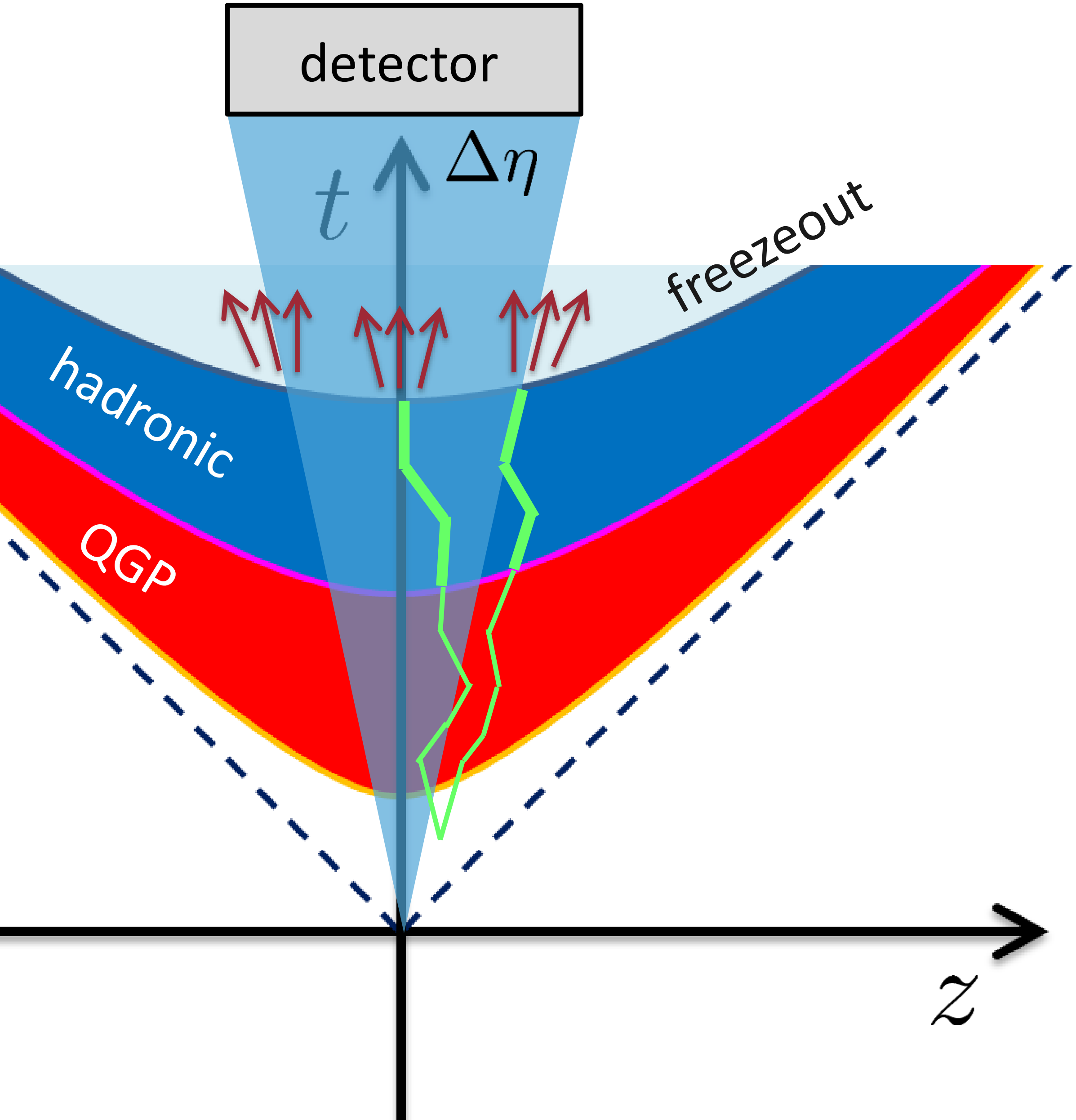
Constant probabilities



Time Evolution of Fluctuations



Time Evolution of Fluctuations



Particle # in Δη

① continues to change until kinetic freezeout due to diffusion.

② changes due to a conversion y → η at kinetic freezeout

"Thermal Blurring"

Future Studies

Experimental side:

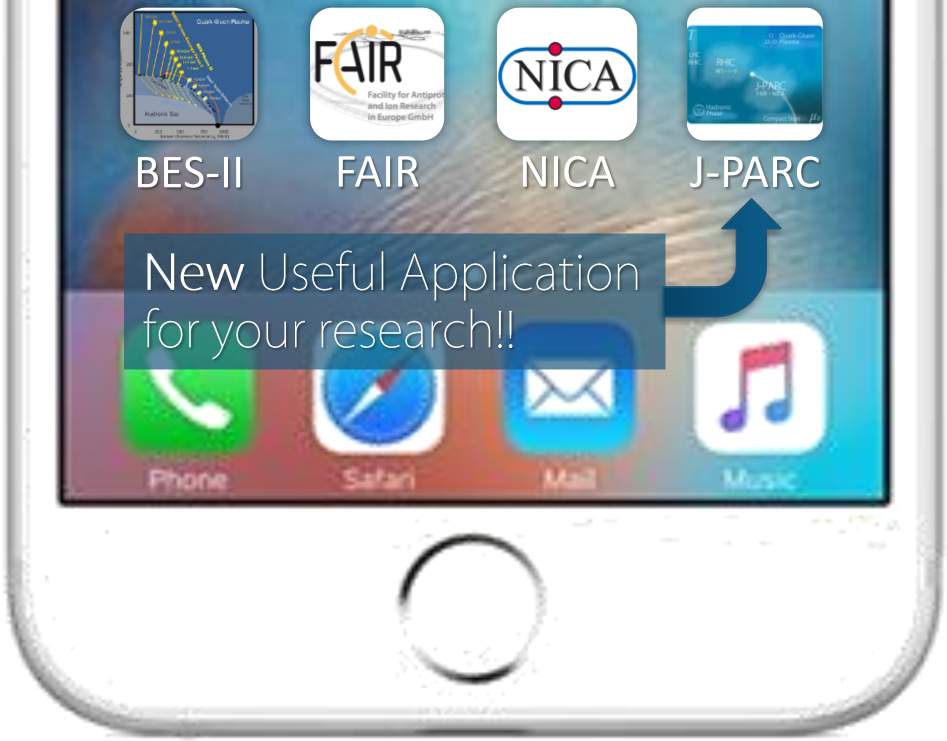
- rapidity window dependences
- baryon number cumulants
- BES for SPS- to LHC-energies

Theoretical side:

- rapidity window dependences in dynamical models
- description of non-equilibrium non-Gaussianity
- accurate measurements on the lattice

Both sides:

- Compare theory and experiment carefully
- Let's accelerate our understanding on fluctuations!


BES-II
FAIR
NICA
J-PARC
New Useful Application
for your research!!
Phone
Safari
Mail
Music