

Anisotropic hydrodynamics

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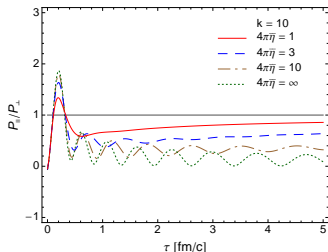
Critical Point and Onset of Deconfinement 2016
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1. Introduction

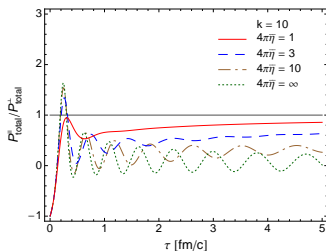
The results of microscopic models suggest that the initial state of matter produced in heavy-ion collisions is highly anisotropic in momentum space (AdS/CFT, CGC, string models,...).

Color-flux-tube model results for different viscosity to entropy density ratios:
Ryblewski+WF, PRD 88, 034028 (2013)

ratio of pressures without the color field



ratio of pressures with the color field



THERMALIZATION \rightarrow HYDRODYNAMIZATION

The standard viscous hydrodynamics is based on the expansion of the underlying microscopic theory in the **Knudsen** and **inverse Reynolds** numbers around the local equilibrium state, this type of expansion may be questioned in the situation where space-time gradients and/or deviations from the local equilibrium are large.

The goal of the anisotropic hydrodynamics program is to create a dissipative hydrodynamics framework that more accurately describes several features such as:

- i) the early time dynamics of the QGP created in heavy-ion collisions
- ii) dynamics near the transverse edges of the nuclear overlap region
- iii) temperature-dependent (and potentially large) shear viscosity to entropy density ratio: $\bar{\eta} = \eta/s$

2. Anisotropic hydrodynamics – an attempt of classification

2.1 Thermodynamic vs. kinetic-theory formulations

Thermodynamic formulation

R. Ryblewski, WF

PRC 83, 034907 (2011), JPG 38 (2011) 015104

1. energy-momentum conservation
 $\partial_\mu T^{\mu\nu} = 0$
2. ansatz for the entropy source, e.g.,
 $\partial(\sigma U^\mu) \propto (\lambda_\perp - \lambda_\parallel)^2 / (\lambda_\perp \lambda_\parallel)$

3. Generalized form of the equation of state based on the **Romatschke-Strickland (RS) form**

generalization of equilibrium/isotropic distributions, frequently used in the studies of anisotropic quark-gluon plasma (here as a modified Boltzmann distribution in the local rest frame)

$$f_{RS} = \exp\left(-\sqrt{\frac{p_\perp^2}{\lambda_\perp^2} + \frac{p_\parallel^2}{\lambda_\parallel^2}}\right) = \exp\left(-\frac{1}{\lambda_\perp} \sqrt{p_\perp^2 + x p_\parallel^2}\right) = \exp\left(-\frac{1}{\Lambda} \sqrt{p_\perp^2 + (1 + \xi) p_\parallel^2}\right)$$

anisotropy parameter $x = 1 + \xi = \left(\frac{\lambda_\perp}{\lambda_\parallel}\right)^2$ and transverse-momentum scale $\lambda_\perp = \Lambda$

2.1 Thermodynamic vs. kinetic-theory formulations

4. Energy-momentum tensor (with single anisotropy parameter)

$$T^{\mu\nu} = (\varepsilon + P_{\perp}) U^{\mu} U^{\nu} - P_{\perp} g^{\mu\nu} - (P_{\perp} - P_{\parallel}) Z^{\mu} Z^{\nu}$$

$\varepsilon(\sigma, x)$ — energy density, $P_{\perp}(\sigma, x)$ — transv. pressure, $P_{\parallel}(\sigma, x)$ — long. pressure
alternatively one may use: $\varepsilon(\Lambda, \xi)$, $P_{\perp}(\Lambda, \xi)$, $P_{\parallel}(\Lambda, \xi)$

U — flow four-vector, Z — beam four-vector, $U^2 = 1$, $Z^2 = -1$, $U \cdot Z = 0$

this form of $T^{\mu\nu}$ follows from the covariant version of RS

$$f_{RS} = \exp\left(-\frac{1}{\Lambda} \sqrt{(p \cdot U)^2 + \xi (p \cdot Z)^2}\right), \quad U = (t/\tau, 0, 0, z/\tau), \quad Z = (z/\tau, 0, 0, t/\tau)$$

5. Several applications have been made to describe the heavy-ion data within this framework

2.2 Two expansion methods

Kinetic-theory formulation

Perturbative approach

Bazov, Heinz, Strickland
PRC 90, 044908 (2014)

$$f = f_{RS} + \delta f$$

- the leading order is still described by the Romatschke-Strickland form (accounting for the difference between the longitudinal and transverse pressures)
- advanced methods of traditional viscous hydrodynamics are used to restrict the form of the correction δf and to derive aHydro equations — non-trivial dynamics included in the transverse plane and, more generally, in (3+1)D case

Molnar, Niemi, Rischke
arXiv:1602.00573

Non-perturbative approach

Nopoush, Ryblewski, Strickland, Tinti, WF

$$f = f_{\text{aniso}} + \dots$$

- all effects due to anisotropy included in the leading order, in the generalised RS form
- (1+1)D conformal case, two anisotropy parameters
 - (1+1)D non-conformal case, two anisotropy parameters + one bulk parameter
 - full (3+1)D case, five anisotropy parameters + one bulk parameter (shear tensor and bulk pressure)

2.3 Non-perturbative approach

Boost-invariant and cylindrically symmetric expansion, (1+1)D non-perturbative approach
as much as possible, the momentum anisotropy is included in the leading order

$$f(x, p) = f_{\text{aniso}} = f_{\text{iso}} \left(\frac{\sqrt{p^\mu \Xi_{\mu\nu} p^\nu}}{\lambda} \right)$$

1. Conformal case, two anisotropy parameters

L. Tinti, WF, Phys.Rev. C89 (2014) 034907

$$\begin{aligned} \Xi^{\mu\nu} &= U^\mu U^\nu + \xi^{\mu\nu} \\ u_\mu \xi^{\mu\nu} &= 0 \quad \xi_\mu^\mu = 0 \\ \xi^{\mu\nu} &= \text{diag}(0, \xi) \quad \xi \equiv (\xi_x, \xi_y, \xi_z) \quad (\text{in the local rest frame}) \end{aligned}$$

2. Non-conformal case, two anisotropy parameters + one bulk parameter

M. Nopoush, R. Ryblewski, M. Strickland, Phys. Rev. C 90 (2014) 014908

$$\begin{aligned} \Xi^{\mu\nu} &= U^\mu U^\nu + \xi^{\mu\nu} - \Delta^{\mu\nu} \Phi \\ u_\mu \xi^{\mu\nu} &= 0 \quad u_\mu \Delta^{\mu\nu} = 0 \quad \xi_\mu^\mu = 0 \quad \Delta_\mu^\mu = 3 \\ \xi^{\mu\nu} &= \text{diag}(0, \xi) \quad \xi \equiv (\xi_x, \xi_y, \xi_z) \quad (\text{in the local rest frame}) \end{aligned}$$

2.3 Non-perturbative approach

equations of motion for $\xi_x, \xi_y, \Phi, \lambda, T, u^i$ for (1+1)d case are obtained by taking moments of the Boltzmann equation in the relaxation time approximation

$$p^\mu \partial_\mu f = p^\mu \frac{u_\mu}{\tau_{\text{eq}}} (f^{\text{eq}} - f) \quad \rightarrow \quad \partial_{\mu_1} \int dP p^{\mu_1} \dots p^{\mu_n+1} f = u_{\mu_1} \int dP p^{\mu_1} \dots p^{\mu_n} \frac{1}{\tau_{\text{eq}}} (f^{\text{eq}} - f)$$

0th moment (1 eq.)

$$\partial_\mu N^\mu = \frac{u_\mu}{\tau_{\text{eq}}} (N_{\text{eq}}^\mu - N^\mu)$$

1st moment (2 eq.)

$$u_\nu \partial_\mu T^{\mu\nu} = u_\nu \frac{u_\mu}{\tau_{\text{eq}}} (T_{\text{eq}}^{\mu\nu} - T^{\mu\nu})$$

Landau matching condition for the energy
(1 eq.)

$$u_\mu T^{\mu\nu} = u_\mu T^{\mu\nu}_{\text{eq}}$$

2nd moment (2 eq.)

$$X_\mu^i X_\nu^j \partial_\lambda \Theta^{\lambda\mu\nu} = X_\mu^i X_\nu^j \frac{u_\lambda}{\tau_{\text{eq}}} (\Theta_{\text{eq}}^{\lambda\mu\nu} - \Theta^{\lambda\mu\nu})$$

two linear combinations of these equations with

$i = 0, 1, 2, 3$

X, Y defined in addition to U and Z

...

2.3 Non-perturbative approach

3A. (3+1) dimensional framework for leading order anisotropic hydrodynamics

L. Tinti, Phys. Rev. C92 (2015) 014908

Testing different formulations of leading order anisotropic hydrodynamics

L. Tinti, R. Ryblewski, W. Florkowski, M. Strickland, Nucl. Phys. A946 (2016) 29

$$\begin{aligned}\Xi^{\mu\nu} &= u^\mu u^\nu + \xi^{\mu\nu} - \Delta^{\mu\nu} \Phi \\ u_\mu \xi^{\mu\nu} &= 0 \quad \xi_\mu^\mu = 0 \quad (5 \text{ parameters in } \xi^{\mu\nu})\end{aligned}$$

3B. Anisotropic matching principle for the hydrodynamic expansion, L. Tinti, arXiv:1506.07164

$$T^{\mu\nu} = \int dP p^\mu p^\nu f_{\text{aniso}}(x, p) = \int dP p^\mu p^\nu f_{\text{iso}} \left(\frac{\sqrt{p^\mu \Xi_{\mu\nu} p^\nu}}{\lambda} \right)$$

Instead of looking at the moments we can derive first the equations for the pressure corrections, following DNMR (Denicol, Niemi, Molnar, Rischke) strategy used for viscous hydrodynamics

This is the latest development for the leading order, that may be supplemented by NLO terms following the approach by Heinz et al.

3 Exact solutions of the Boltzmann equation

new technique: aHydro and viscous hydro predictions are checked against exact solutions of the Boltzmann kinetic equation in the relaxation time approximation, important constraints on the structure of the hydro equations and the form of the kinetic coefficients

G. Baym, Phys. Lett. B138 (1984)

One dimensional expansion

Denicol, Maksymiuk, Ryblewski, Strickland, WF

1. conformal case
2. non-conformal case
3. non-conformal case with quantum statistics
4. mixtures

(1+1)D flow with Gubser symmetry

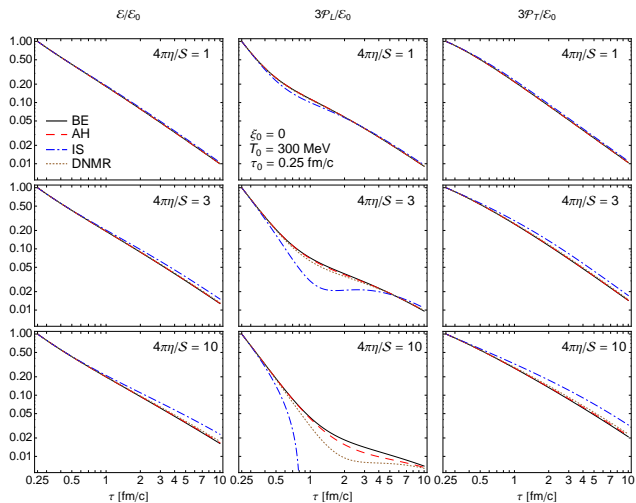
Denicol, Heinz, Martinez, Noronha, Strickland

3.1 Conformal case

Comparisons

WF, R. Ryblewski, M. Strickland, Phys.Rev. C88 (2013) 024903

$m = 0$, boost-invariant, transversally homogeneous system, (0+1) case

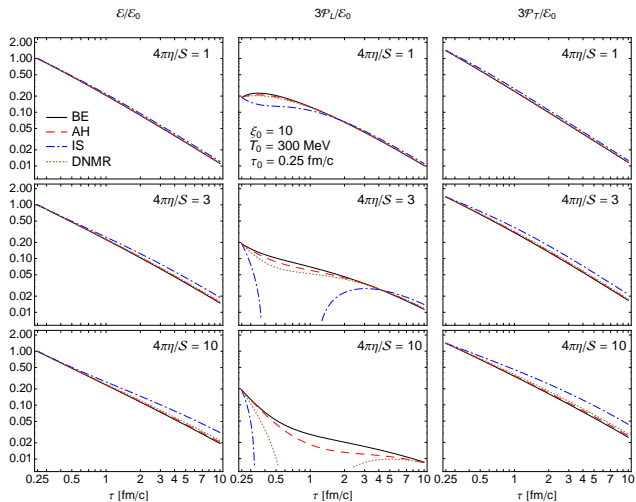


3.1 Conformal case

Comparisons

WF, R. Ryblewski, M. Strickland, Phys.Rev. C88 (2013) 024903

$m = 0$, boost-invariant, transversally homogeneous system, (0+1) case



3.2 Non-conformal case

Various, second order hydro equations for the bulk pressure

$$\tau_{\Pi} \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} - \frac{1}{2} \tau_{\Pi} \Pi \left[\frac{1}{\tau} - \left(\frac{\dot{\zeta}}{\zeta} + \frac{\dot{T}}{T} \right) \right] \quad (A)$$

Muronga, Phys. Rev. C69 (2004) 034903

Heinz, Song, Chaudhuri, Phys. Rev. C73 (2006) 034904

$$\tau_{\Pi} \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} - \frac{4}{3} \tau_{\Pi} \Pi \frac{1}{\tau} \quad (B)$$

Jaiswal, Bhalerao, Pal, Phys. Rev. C87 (2013) 021901

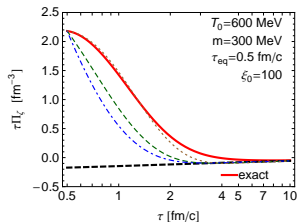
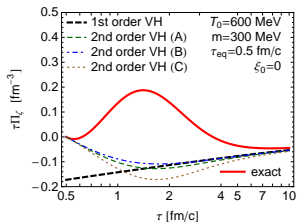
$$\tau_{\Pi} \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} \quad (C)$$

Heinz, Song, Chaudhuri, Phys. Rev. C73 (2006) 034904

3.2 Non-conformal case

Kinetic-theory vs. hydro

WF, E. Maksymiuk, R. Ryblewski, M. Strickland, Phys.Rev. C89 (2014) 054908



exact solution and all 2nd order viscous hydrodynamics variations tend toward the 1st order solution at late times

none of the 2nd order viscous hydrodynamics variations seems to qualitatively describe the early-time evolution of the bulk viscous pressure in all cases

there is something incomplete in the manner in which 2nd order viscous hydrodynamics treats the bulk pressure (neglected shear–bulk coupling)

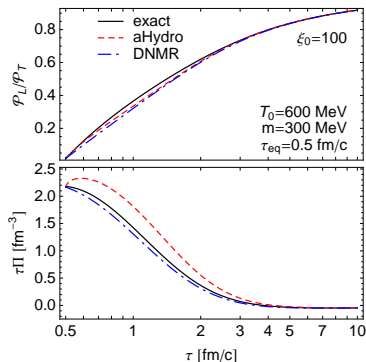
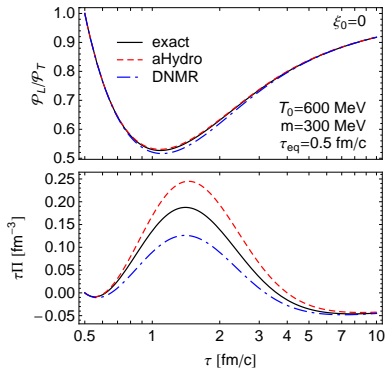
3.2 Non-conformal case

Viscous hydro predictions with shear-bulk coupling

Bulk viscous pressure evolution within full viscous anisotropic hydrodynamics with **SHEAR-BULK COUPLING**

G. Denicol, S. Jeon, C. Gale, Phys.Rev. C90 (2014) 024912

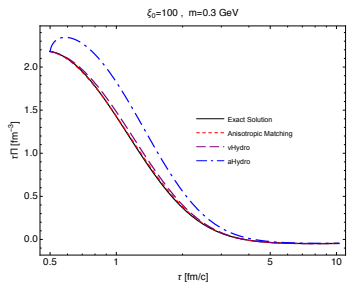
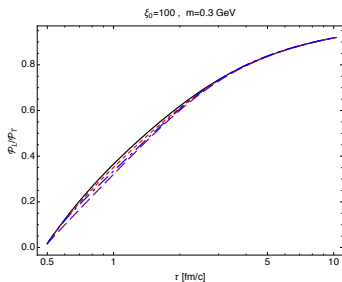
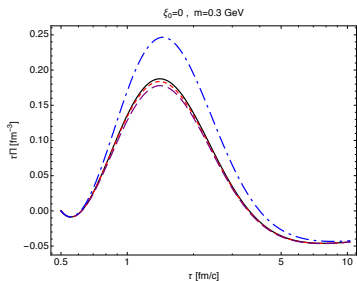
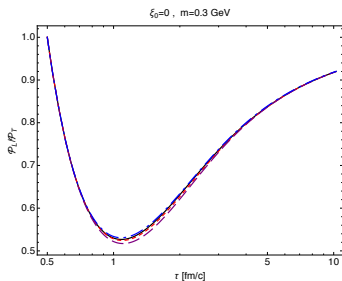
G. Denicol, R. Ryblewski, WF, M.Strickland, Phys.Rev. C90 (2014) 044905



the shear-bulk couplings are extremely important for correct description of the bulk viscous correction

3.2 Non-conformal case

Anisotropic matching principle, Leonardo Tinti, arXiv:1506.07164, PRC in print



4. Summary and conclusions

4. Summary and conclusions

- A new framework of ANISOTROPIC HYDRODYNAMICS (aHydro) has been introduced. At the moment there exist two complementary formulations of aHydro based on the kinetic theory, and a phenomenological approach based on the entropy production form.
- Comparisons of aHydro and viscous hydrodynamics with the exact solutions of the kinetic theory confirmed usefulness of aHydro and gave information about the structure of hydrodynamic equations and the form of the kinetic coefficients (e.g., SHEAR-BULK COUPLING)
- aHydro may be matched with the microscopic models of early stages (as soon as the two pressures become positive), removes negative pressures, reproduces right limits for the entropy production, ...

4. Summary and conclusions

more work has been done so far:

- mixtures of anisotropic fluids, interacting quark and gluon systems with different anisotropies
- comparisons with the Gubser flow, exact solutions of the Boltzmann equation with the transverse flow included
- NLO corrections
- inclusion of the medium dependent masses to reproduce QCD EOS
- photon, dilepton production from early anisotropic phase
- bottomonium suppression
- ...

5. Back-up slides

Dissipative hydrodynamics

the system is never in local equilibrium (diffusion, friction, heat conduction,...)

- energy-momentum conservation

$$\partial_\mu T_{vis}^{\mu\nu} = 0 \quad T_{vis}^{\mu\nu} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu} (\mathcal{P} + \Pi) + \pi^{\mu\nu}$$

- number of equations: 5 + 6 (\mathcal{E} , \mathcal{P} , u^μ (3), Π , $\pi^{\mu\nu}$ (5))
- number of equations: 4 + 1 (equation of state $\mathcal{E}(\mathcal{P})$)
- we need 6 extra equations - different methods possible

$$\begin{aligned} \dot{\Pi} + \frac{\Pi}{\tau_\Pi} &= -\beta_\Pi \theta - \delta_{\Pi\Pi} \Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} \\ \dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu} \omega^{\nu\rangle\gamma} - \delta_{\pi\pi} \pi^{\mu\nu} \theta - \tau_{\pi\pi} \pi_\gamma^{\langle\mu} \sigma^{\nu\rangle\gamma} + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu} \end{aligned}$$

Israel-Stewart equations

$\tau_\Pi \beta_\Pi = \zeta \rightarrow$ bulk viscosity, $\tau_\pi \beta_\pi = \eta \rightarrow$ shear viscosity

Dissipative hydrodynamics

the system is never in local equilibrium (diffusion, friction, heat conduction,...)

- energy-momentum conservation

$$\partial_\mu T_{vis}^{\mu\nu} = 0 \quad T_{vis}^{\mu\nu} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu} (\mathcal{P} + \Pi) + \pi^{\mu\nu}$$

- number of equations: 5 + 6 (\mathcal{E} , \mathcal{P} , $u^\mu(3)$, Π , $\pi^{\mu\nu}(5)$)
- number of equations: 4 + 1 (equations of state $\mathcal{E}(\mathcal{P})$)
- we need 6 extra equations - different methods possible

$$\begin{aligned} \dot{\Pi} + \frac{\Pi}{\tau_\Pi} &= -\beta_\Pi \theta - \delta_{\Pi\Pi} \Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} \\ \dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu} \omega^{\nu\rangle\gamma} - \delta_{\pi\pi} \pi^{\mu\nu} \theta - \tau_{\pi\pi} \pi_\gamma^{\langle\mu} \sigma^{\nu\rangle\gamma} + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu} \end{aligned}$$

New approaches (shear-bulk coupling $\eta - \zeta$)

$\tau_\Pi \beta_\Pi = \zeta \rightarrow$ bulk viscosity, $\tau_\pi \beta_\pi = \eta \rightarrow$ shear viscosity