Correlated Fluctuations Near the QCD critical Point

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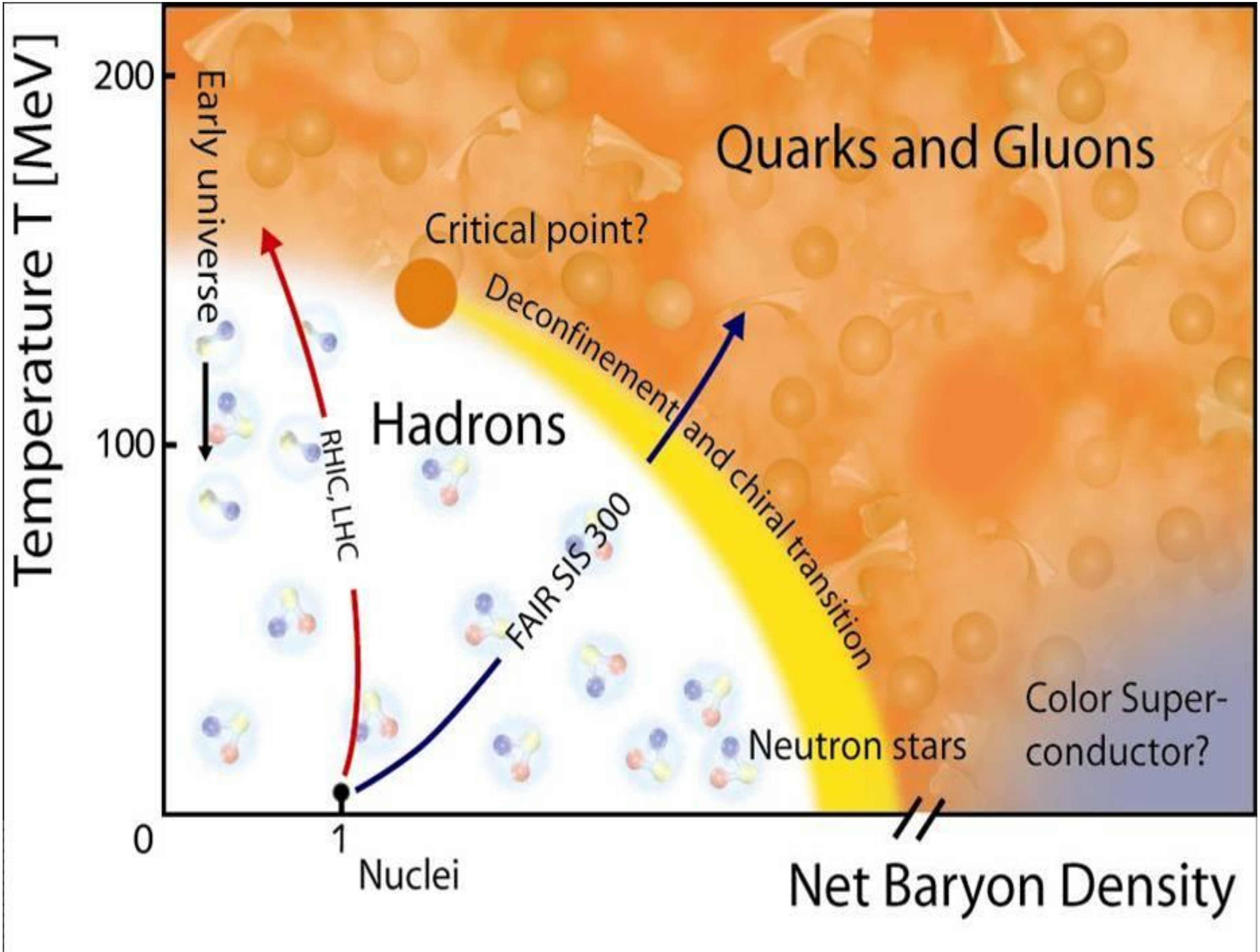
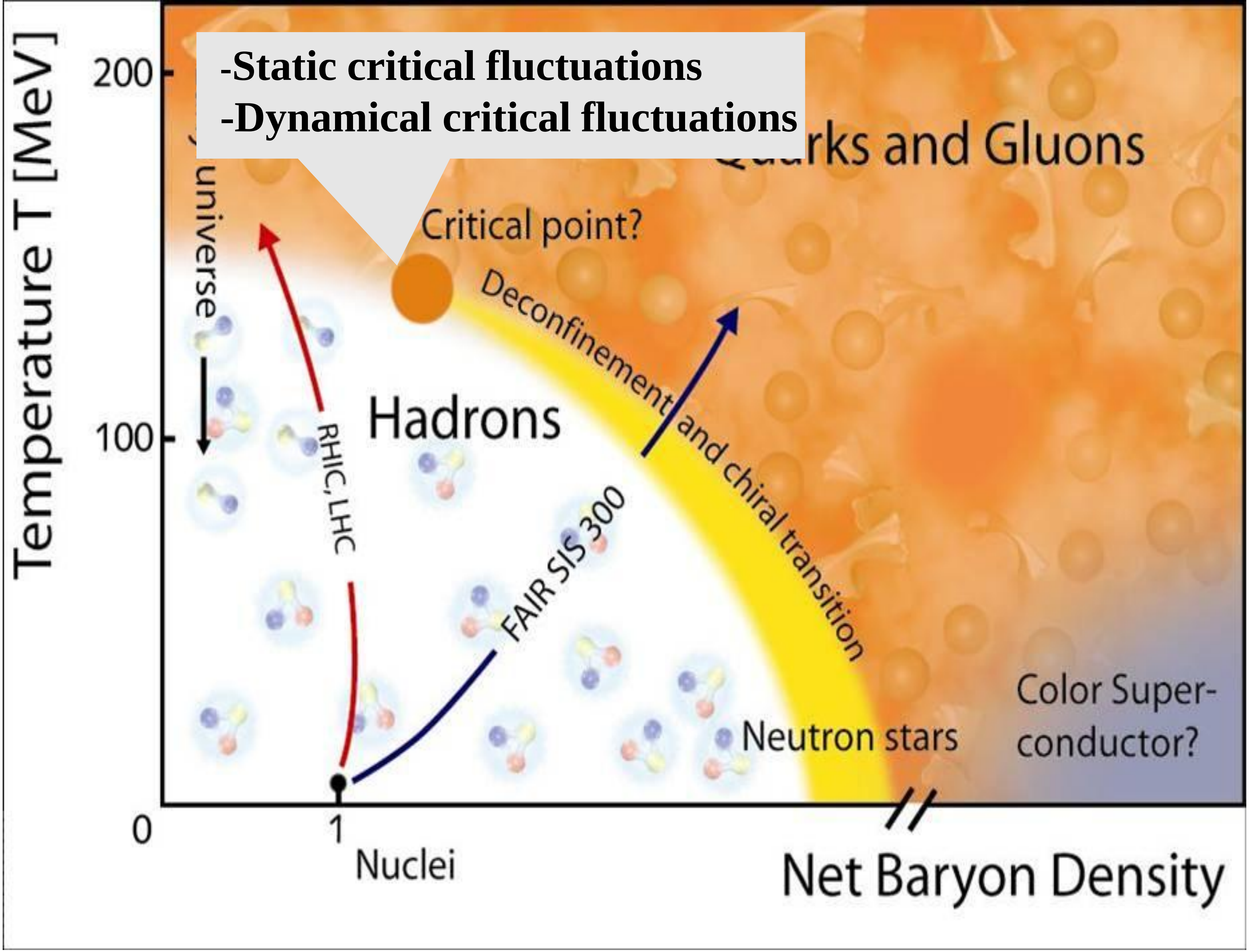
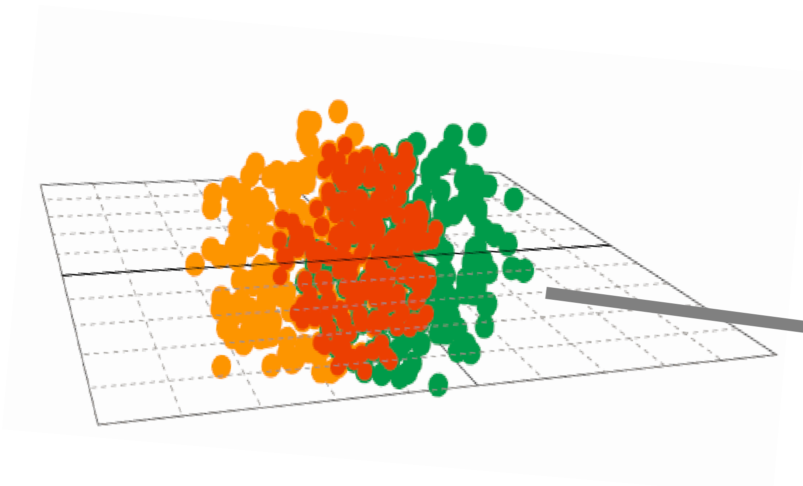
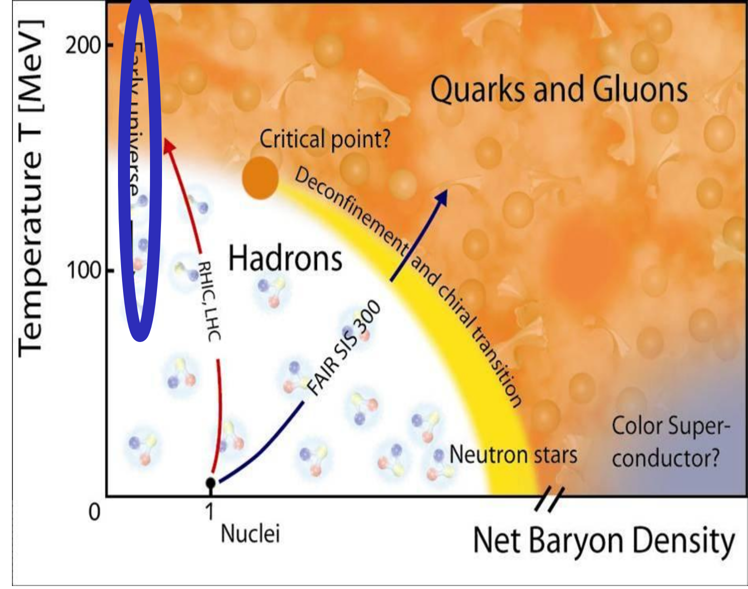

Temperature T [MeV]
200
100
0
1
Nuclei
Net Baryon Density
Early universe
RHIC, LHC
Quarks and Gluons
Critical point?
Hadrons
Deconfinement and chiral transition
FAIR SIS 300
Neutron stars
Color Super-
conductor?

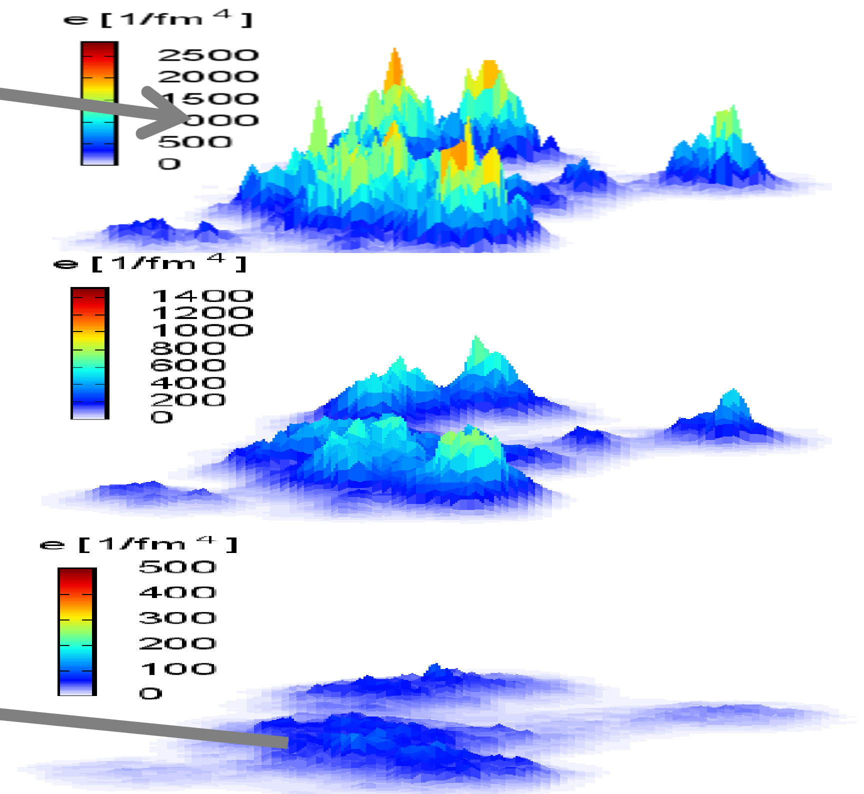


-Static critical fluctuations
-Dynamical critical fluctuations
Quarks and Gluons
Critical point?
Deconfinement and chiral transition
universe
Hadrons
RHIC, LHC
FAIR SIS 300
Neutron stars
Color Super-
conductor?
Temperature T [MeV]
200
100
0
1
Nuclei
Net Baryon Density

Initial state fluctuations & final state Correlations





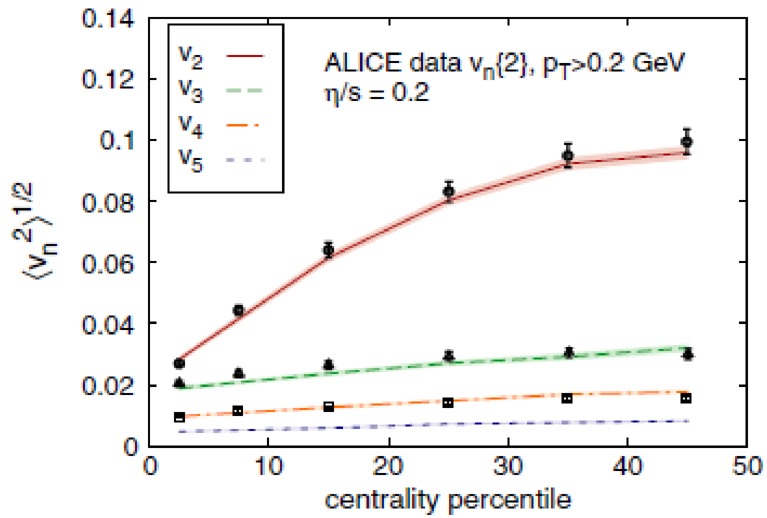


$$E\frac{dN}{d^3p} = \frac{dN}{dy p_T dp_T d\varphi}$$

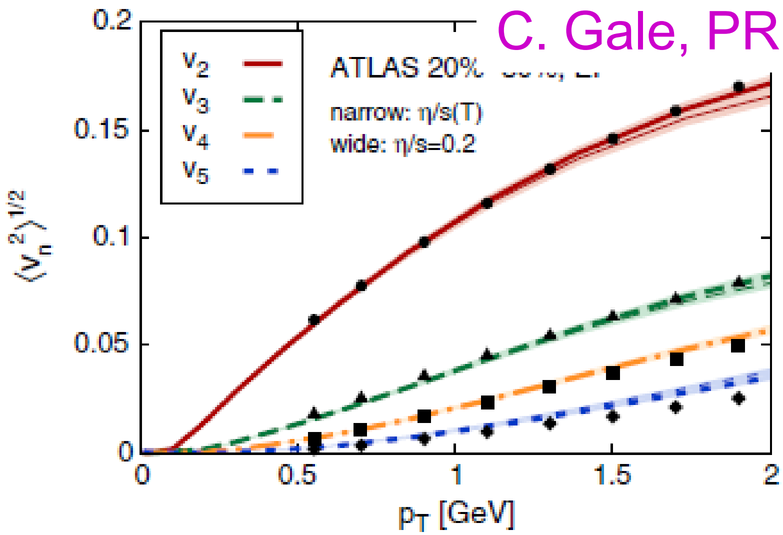
$$= \frac{1}{2\pi}\frac{dN}{dy p_T dp_T}[1+2v_1(p_T,b)\cos(\varphi)+2v_2(p_T,b)\cos(2\varphi)$$

$$+2v_3(p_T,b)\cos(3\varphi)......]$$

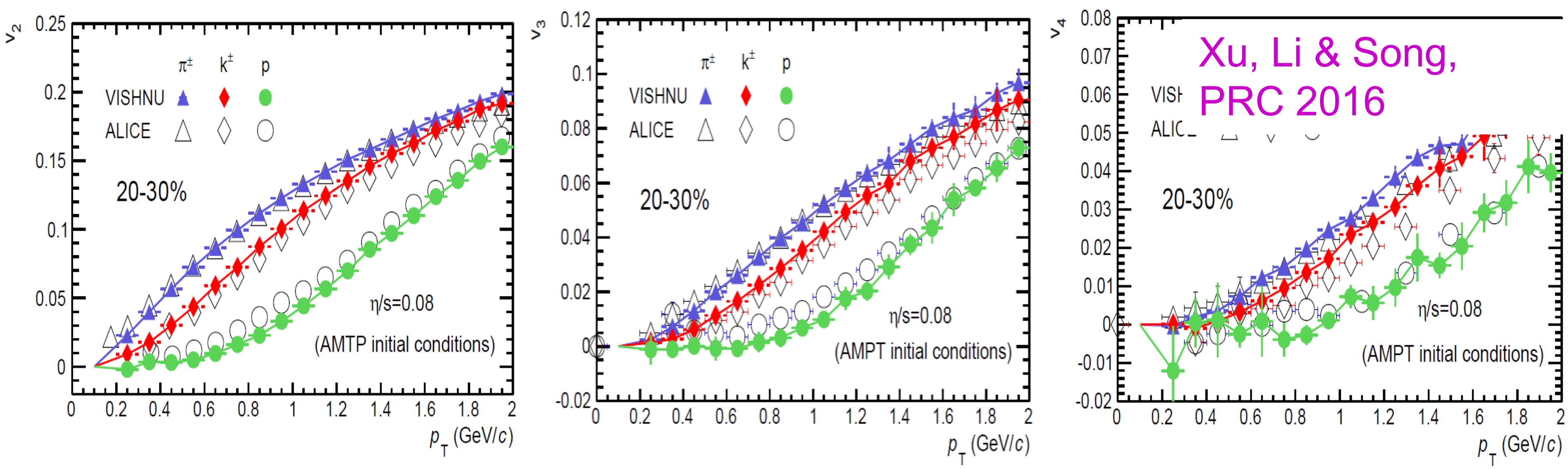
Observables for initial state fluctuations (I)



C. Gale, PRL2013

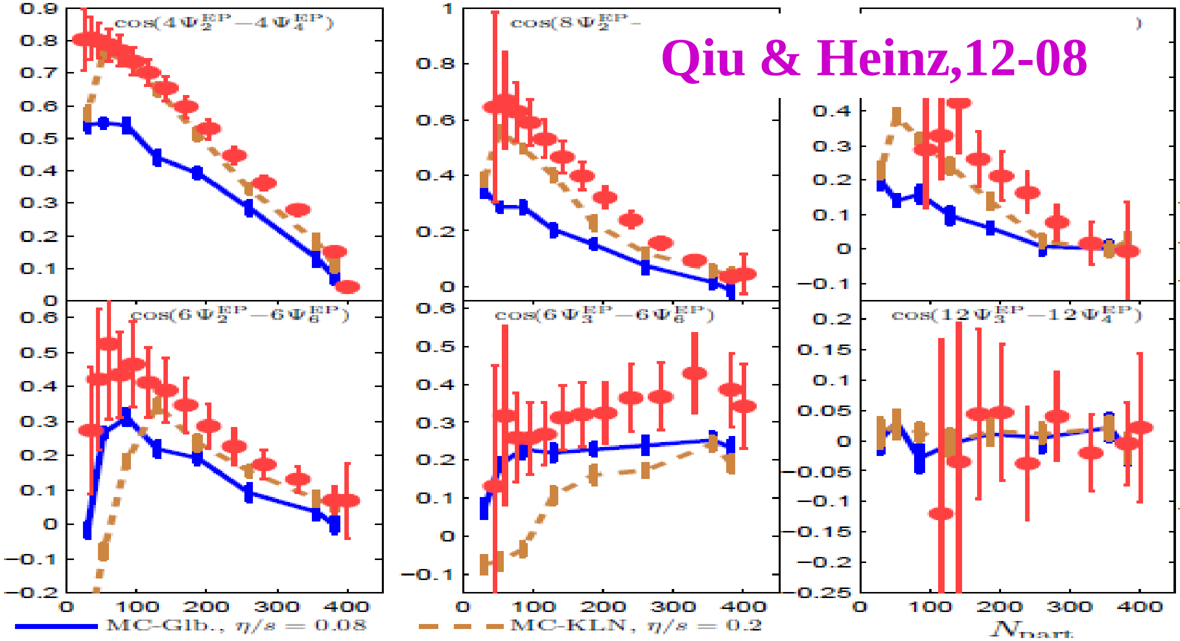


Xu, Li & Song, PRC 2016

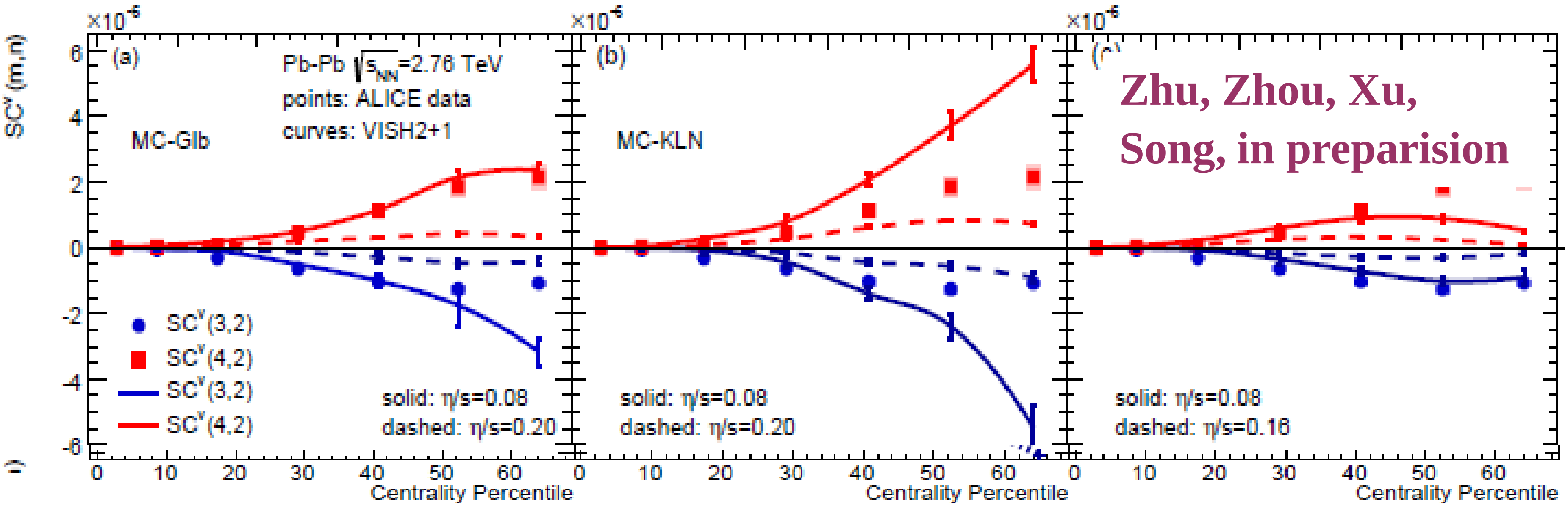


-Vn (integrated, differential, PID) are nice described by e-b-e hydrodynamics and hybrid model

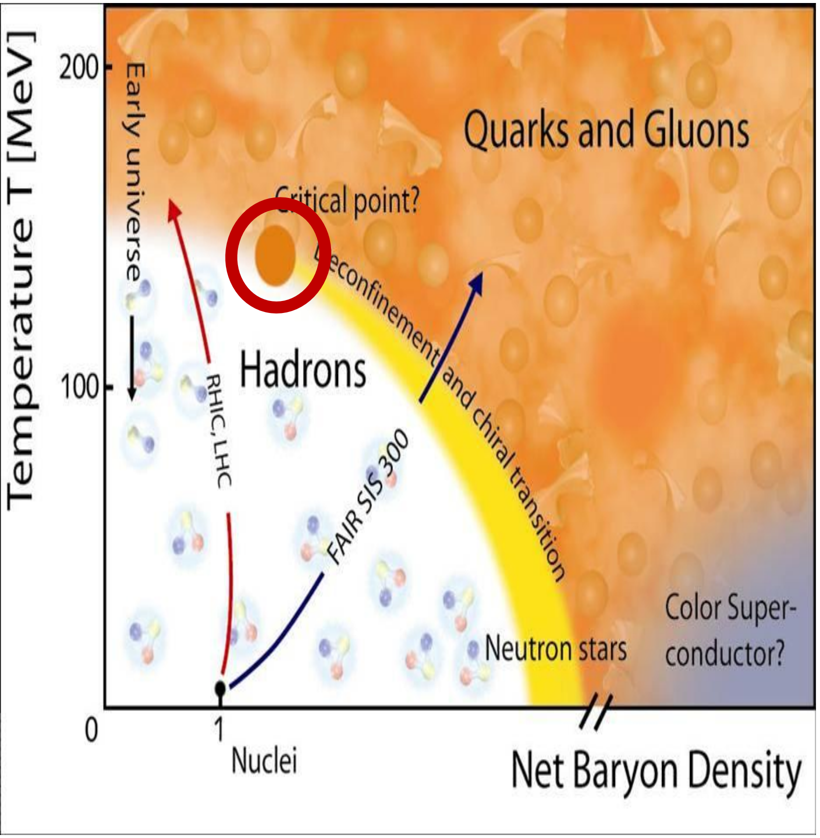
Observables for initial state fluctuations (II)



-the correlations between flow angles & correlations between flow harmonics are qualitatively described by e-b-e hydrodynamics with different initial conditions



Correlated fluctuations near the QCD critical point



Initial State Fluctuations

-QGP fireball evolutions smear-out the initial fluctuations

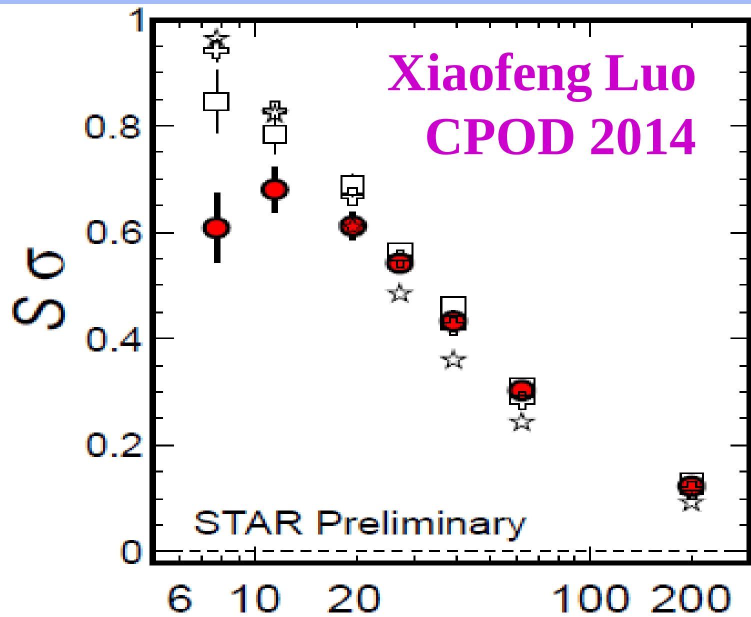
-uncorrelated (in general)

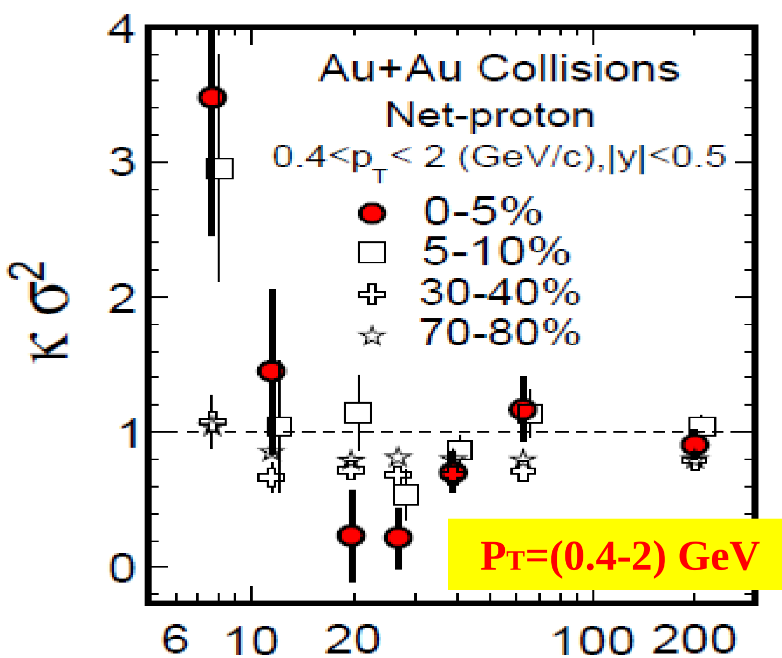
Fluctuations near the critical point

-dramatically increase near Tc

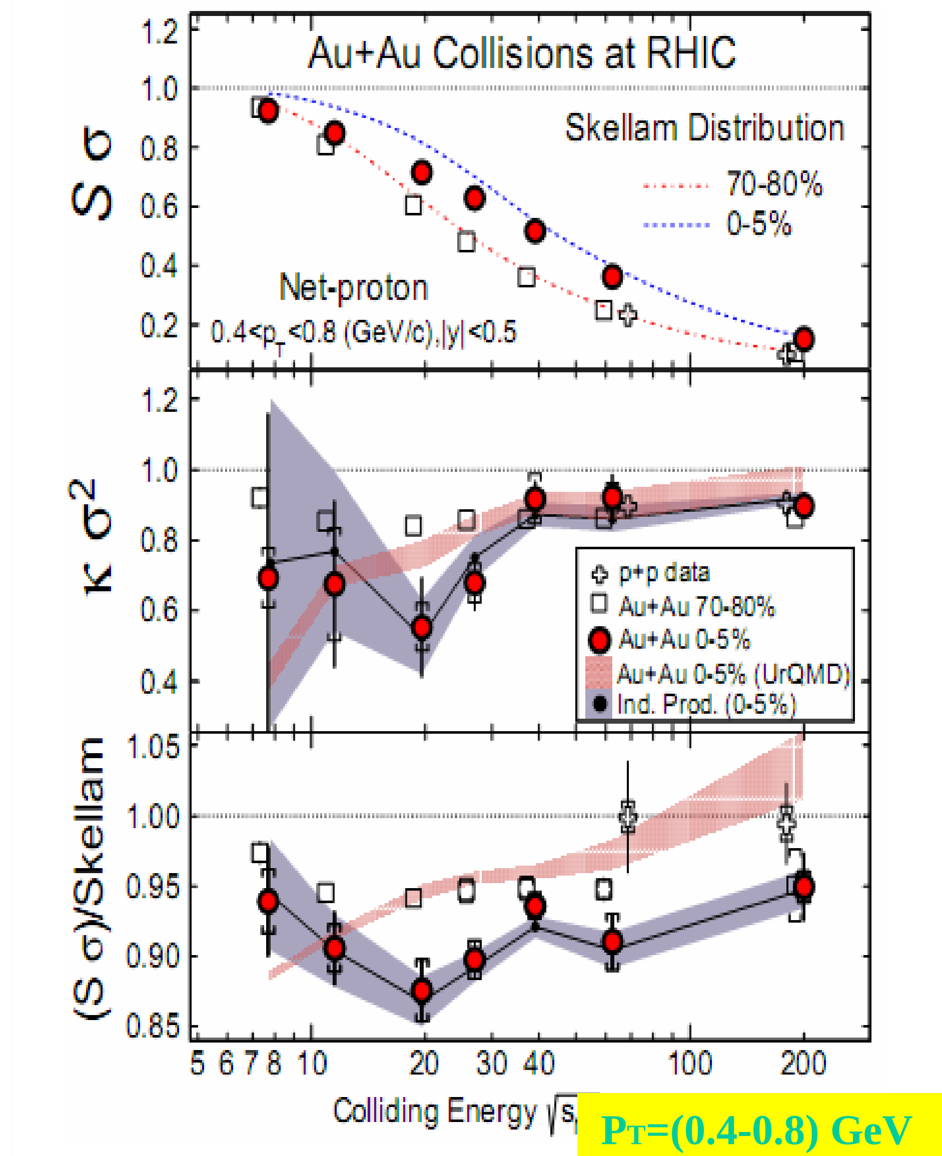
-Strongly correlated

STAR BES: Cumulant ratios





STAR PRL 2014



Theoretical predictions on critical fluctuations

Stephanov PRL 2009

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \cdots\right]$$

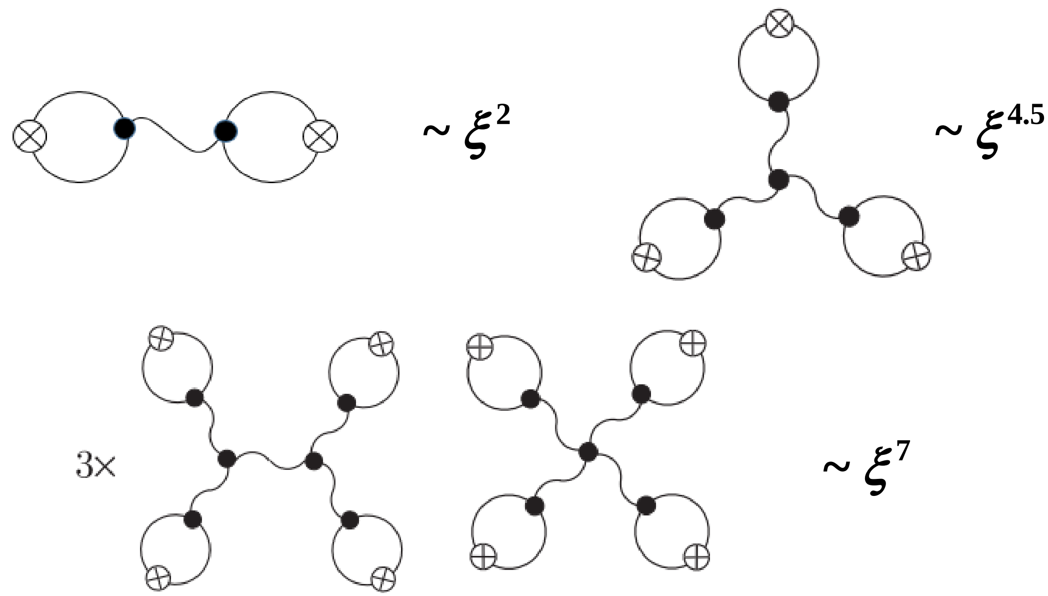
$$\langle\sigma_0^2\rangle = \frac{T}{V}\xi^2 \qquad \langle\sigma_0^3\rangle = \frac{2\lambda_3 T}{V}\xi^6; \qquad \langle\sigma_0^4\rangle_c = \frac{6T}{V}[2(\lambda_3\xi)^2 - \lambda_4]\xi^8.$$

Critical Fluctuations of particles :

$$\langle(\delta N)^2\rangle \sim \xi^2$$

$$\langle(\delta N)^3\rangle \sim \xi^{4.5}$$

$$\langle(\delta N)^4\rangle \sim \xi^7$$



At critical point : $\xi \sim \infty$ **(infinite medium)**

Finite size & finite evolution time: $\xi < O(2-3\text{fm})$

It is important to address the effects from dynamical evolutions

Dynamical Modeling near the QCD critical point

Chiral Hydrodynamics (I)

K. Paech, H. Stocker and A. Dumitru, PRC2003

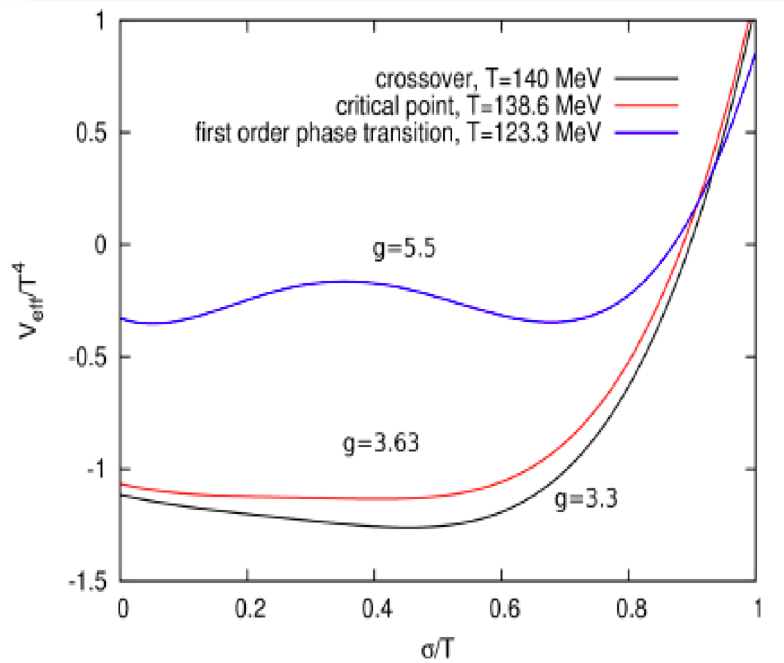
$$L = \bar{q}[i\gamma - g(\sigma + i\gamma_5 \tau\pi)]q + \frac{1}{2}[\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi] - U(\sigma, \pi)$$

$$\begin{cases} \partial_\mu \partial^\mu \sigma + \frac{\partial U_{eff}}{\partial \sigma} + g < \bar{q}q >= 0 \\ \partial_\mu T^{\mu\nu}_{fluid} = S^\nu \qquad S^\nu = -(\partial^2 u + \frac{\partial U_{eff}}{\partial u})\partial^\nu \sigma \end{cases}$$

the order of the phase transition is in charged by the coupling g.

σ order parameter

quark & anti-quark is treated as the heat bath (fluid), which interact with the chiral field via effective mass $g\sigma$



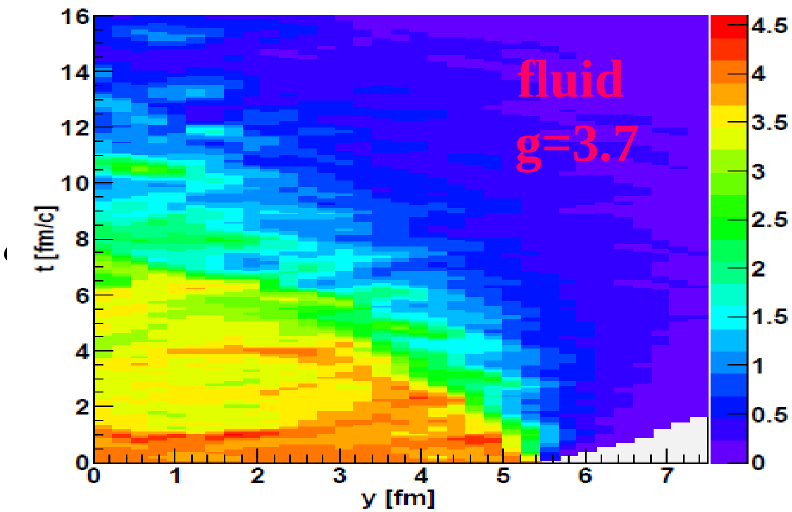
Chiral Hydrodynamics (II)

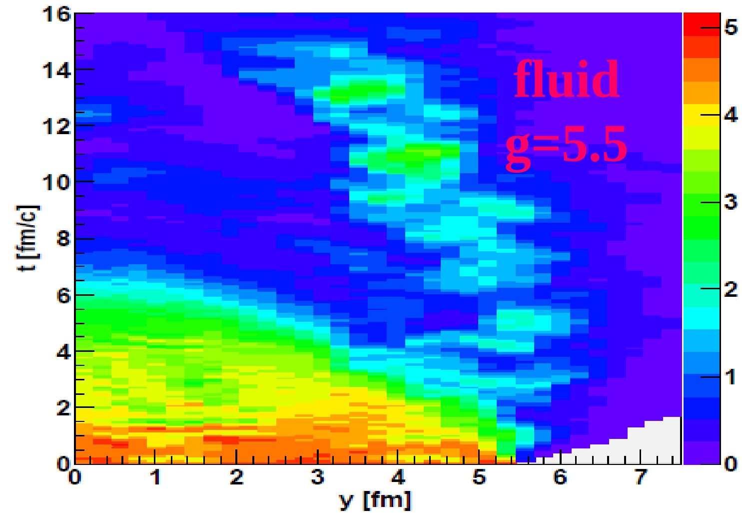
K. Paech, H. Stocker and A. Dumitru, PRC2003

$$L = \bar{q}[i\gamma - g(\sigma + i\gamma_5 \tau\pi)]q + \frac{1}{2}[\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi] - U(\sigma, \pi)$$

$$\begin{cases} \partial_\mu \partial^\mu \sigma + \frac{\partial U_{eff}}{\partial \sigma} + g < \bar{q}q >= 0 \\ \partial_\mu T^{\mu\nu}_{fluid} = S^\nu \end{cases} \qquad S^\nu = -(\partial^2 u + \frac{\partial U_{eff}}{\partial u})\partial^\nu \sigma$$

(fluctuation is introduced by initial state)

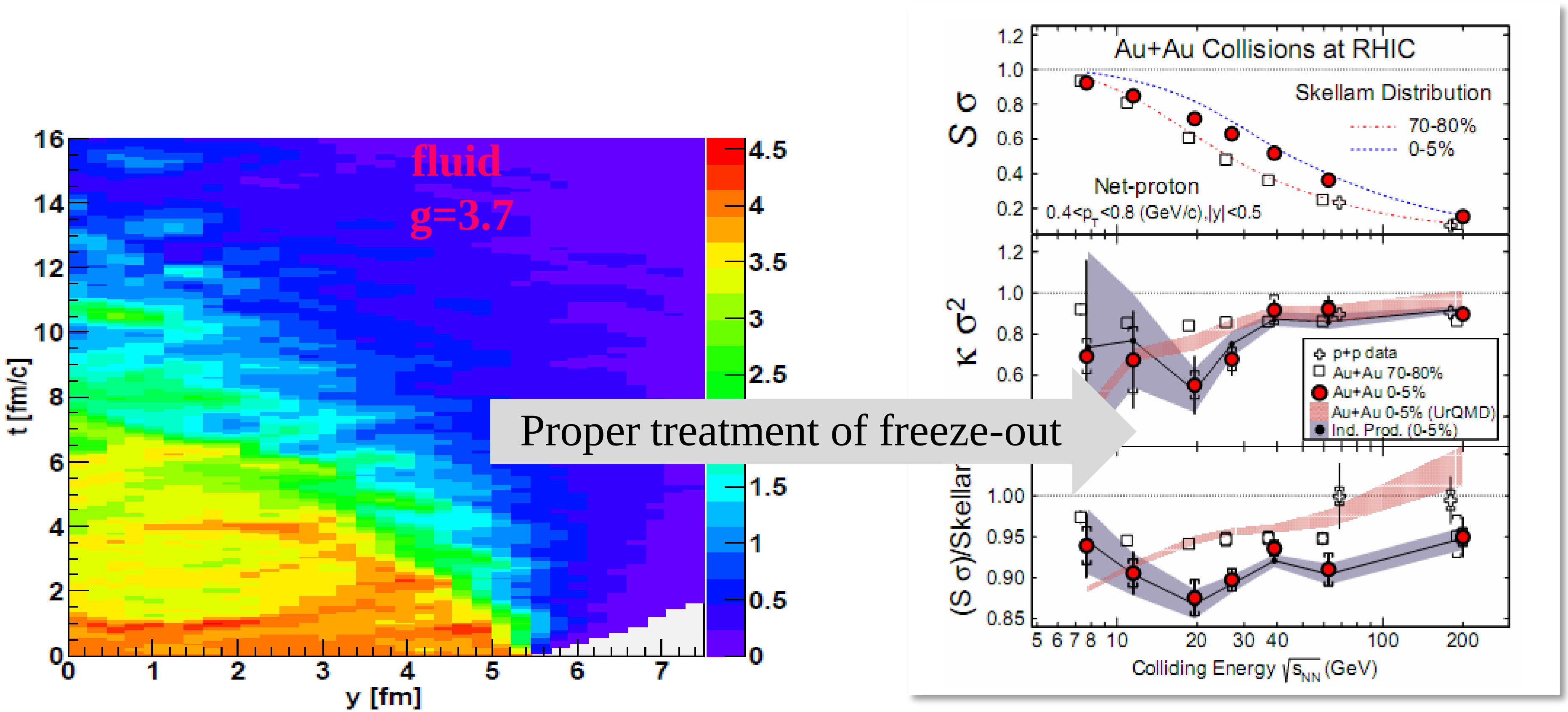




-Chiral fluid dynamics with dissipation & noise Nahrgang,et al., PRC 2011

-Chiral fluid dynamics with a Polyakov loop (PNJL)

Herold, et al., PRC 2013



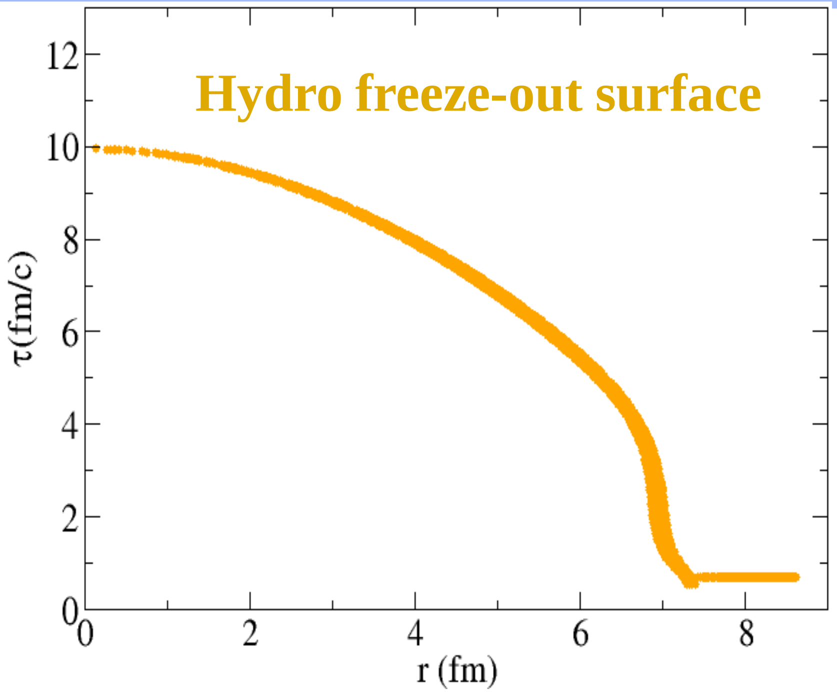
From dynamical evolution to experimental observables, it is important to properly treat the freeze-out procedure with an external field

Freeze-out scheme near T_{cr}

& static critical fluctuations

Jiang, Li & Song, arXiv:1512.06164[nucl-th]

Particle emissions near T_{cr} with external field



Jiang, Li & Song, arXiv: 1512.06164[nucl-th]

Particle emissions in traditional hydro

$$E\frac{dN}{d^3p}=\int_\Sigma \frac{p_\mu d\sigma^\mu}{2\pi^3} f(x,p)$$

Particle emissions near T_{cr}

$$M \longrightarrow g\sigma(x)$$

$$f(x,p)=f_0(x,p)[1-g\sigma(x)/(\gamma T)]$$
$$= f_0+\delta f$$

$$\langle \delta f_1 \delta f_2 \rangle_\sigma = f_{01} f_{02} f_{03} \left(\frac{g^2}{\gamma_1\gamma_2} \frac{1}{T^3} \right) \langle \sigma_1 \sigma_2 \rangle_c ,$$

$$\langle \delta f_1 \delta f_2 \delta f_3 \rangle_\sigma = f_{01} f_{02} f_{03} \left(-\frac{g^3}{\gamma_1\gamma_2\gamma_3} \frac{1}{T^3} \right) \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c ,$$

$$\langle \delta f_1 \delta f_2 \delta f_3 \delta f_4 \rangle_\sigma = f_{01} f_{02} f_{03} f_{04} \left(\frac{g^4}{\gamma_1\gamma_2\gamma_3\gamma_4} \frac{1}{T^4} \right) \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c .$$

For stationary & infinite medium:

$$\left\langle (\delta N)^2 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^2 \int d^3p_1 d^3x_1 \int d^3p_2 d^3x_2 \frac{f_{01} f_{02}}{\gamma_1 \gamma_2} \frac{g^2}{T^2} \langle \sigma_1 \sigma_2 \rangle_c \,,$$

$$\left\langle (\delta N)^3 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^3 \int d^3p_1 d^3x_1 \int d^3p_2 d^3x_2 \int d^3p_3 d^3x_3 \frac{f_{01} f_{02} f_{03}}{\gamma_1 \gamma_2 \gamma_3} \left(-\frac{g^3}{T^3} \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c\right),$$

$$\left\langle (\delta N)^4 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^4 \int d^3p_1 d^3x_1 \int d^3p_2 d^3x_2 \int d^3p_3 d^3x_3 \int d^3p_4 d^3x_4 \frac{f_{01} f_{02} f_{03} f_{04}}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \frac{g^4}{T^4} \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c \,.$$

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega[\sigma] = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4\right],$$

$$\langle \sigma_1 \sigma_2 \rangle_c = T D(x_1 - x_2),$$

$$\langle \sigma_1 \sigma_2 \sigma_3 \rangle_c = -2T^2 \lambda_3 \int d^3z D(x_1 - z) D(x_2 - z) D(x_3 - z),$$

$$\langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c = -6T^3 \lambda_4 \int d^3z D(x_1 - z) D(x_2 - z) D(x_3 - z) D(x_4 - z) + 12 T^3 \lambda_3^2 \int d^3u \int d^3v D(x_1 - u) D(x_2 - u) D(x_3 - v) D(x_4 - v) D(u - v).$$

$$\left\langle \delta n_{p_1} \delta n_{p_2} \right\rangle_c = \frac{f_{01} f_{02}}{\omega_{p_1} \omega_{p_2}} \frac{G^2}{T} \frac{V}{m_\sigma^2}. \qquad \left\langle \delta n_{p_1} \delta n_{p_2} \delta n_{p_3} \right\rangle_c = \frac{2\lambda_3}{V^2 T} \frac{f_{01} f_{02} f_{03}}{\omega_{p_1} \omega_{p_2} \omega_{p_3}} \left(\frac{G}{m_\sigma^2}\right)^3.$$

$$\left\langle \delta n_{p_1} \delta n_{p_2} \delta n_{p_3} \delta n_{p_4} \right\rangle_c = \frac{6}{V^3 T} \frac{f_{01} f_{02} f_{03} f_{04}}{\omega_{p_1} \omega_{p_2} \omega_{p_3} \omega_{p_4}} \left(\frac{G}{m_\sigma^2}\right)^4 \left[2\left(\frac{\lambda_3}{m_\sigma}\right)^2 - \lambda_4\right].$$

--the results in Stephanov PRL09 are reproduced

CORRELATED particle emissions along the freeze-out surface

$$\left\langle (\delta N)^2 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^2 \left(\prod_{i=1,2} \left(\frac{1}{E_i} \int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i \right) \right) \frac{f_{01} f_{02}}{\gamma_1 \gamma_2} \frac{g^2}{T^2} \langle \sigma_1 \sigma_2 \rangle_c \,,$$

$$\left\langle (\delta N)^3 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^3 \left(\prod_{i=1,2,3} \left(\frac{1}{E_i} \int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i \right) \right) \frac{f_{01} f_{02} f_{03}}{\gamma_1 \gamma_2 \gamma_3} \left(-\frac{g^3}{T^3} \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c \right) ,$$

$$\left\langle (\delta N)^4 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^4 \left(\prod_{i=1,2,3,4} \left(\frac{1}{E_i} \int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i \right) \right) \frac{f_{01} f_{02} f_{03} f_{04}}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \frac{g^4}{T^4} \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c$$

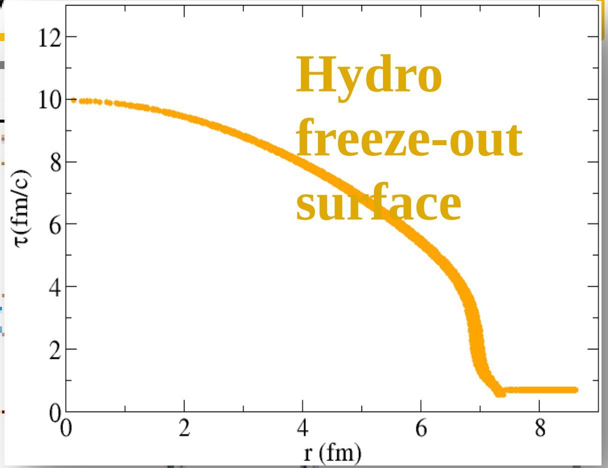
$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega[\sigma] = \int d^3x \left[\frac{1}{2} (\nabla\sigma)^2 + \ldots \right.$$

$$\langle \sigma_1 \sigma_2 \rangle_c = T D(x_1 - x_2),$$

$$\langle \sigma_1 \sigma_2 \sigma_3 \rangle_c = -2T^2 \lambda_3 \int d^3z D(x_1 - z) D(x_2 - z) D(x_3 - z \ldots$$

$$\langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c = -6T^3 \lambda_4 \int d^3z D(x_1 - z) D(x_2 - z) D(x_3 \ldots$$

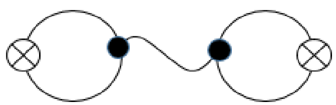
$$+12T^3 \lambda_3^2 \int d^3u \int d^3v D(x_1 - u) D(x_2 - u) D(x_3 - v) D(x_4 - v) D(u - v).$$



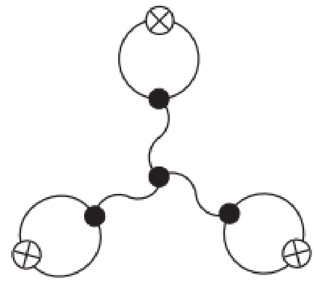
For simplicity: We assume that the correlated sigma field only influence the particle emissions near Tc, which does not influence the evolution of the bulk matter

-- Static critical fluctuations along the freeze-out surface

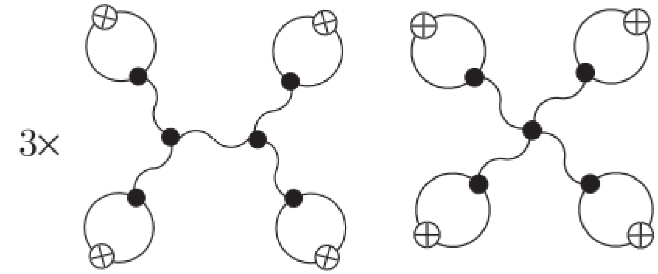
The choice of input parameters



$g_{\sigma pp}$ ξ



$g_{\sigma pp}$ ξ λ_3



$g_{\sigma pp}$ ξ λ_3 λ_4

- $g_{\sigma pp} \sim (0, 10)$

  phenomenological model

- $\xi \sim 3\text{fm}$ (max value)

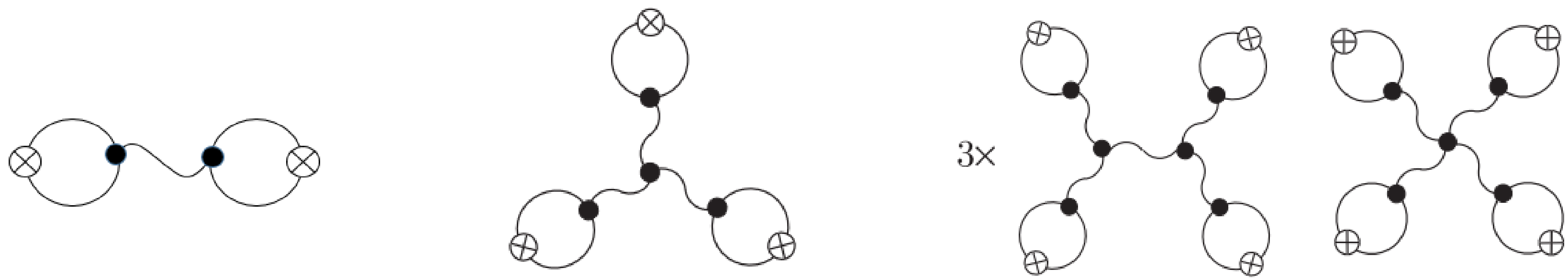
  near the critical point, critical slowing down

- $\lambda_3 \sim (0, 8)$, $\lambda_4 \sim (4, 20)$

  lattice simulation of the effective potential around critical point.

A. Andronic, et al. NPA (2006); M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009); S. P. Klevansky, Rev. Mod. Phys, Vol, 64, No.3 (1992); W. Fu, Y-x, Liu, Phys. Rev. D 79, 074011(2009); M. M. Tsypin, Phys. Rev. Lett. 73, 2015 (1994); M. M. Tsypin, Phys. Rev. B 55, 8911 (1997).; B. Berdnikov and K. Rajagopal, Phys. Rev. D 61, 105017 (2000).

The choice of input parameters



$g_{\sigma pp}$ ξ | $g_{\sigma pp}$ ξ λ_3 | $g_{\sigma pp}$ ξ λ_3 λ_4

$\sqrt{s_{NN}}[GeV]$		7.7	11.5	19.6	27	39	62.4	200
para-I	g	3.2	2.5	2.3	2.2	2	1.8	1
	λ_3	6	4	3	2	0	0	0
	λ_4	14	13	12	11	10	9	8
	ξ	1	2	3	3	2	1	0.5
para-II	g	3.2	2.5	2.3	2.2	2	1.8	1
	λ_3	6	4	3	2	2	1.5	1
	λ_4	14	13	12	11	10	9	8
	ξ	1	2.5	4	4	3	2	1
para-III	g	2.8	1.8	1.7	1.6	1	0.5	0.1
	λ_3	6	4	3	2	2	1.5	1
	λ_4	14	13	12	11	10	9	8
	ξ	1	2	3	3	2	1	0.5

- $g_{\sigma pp} \sim (0, 10)$

  phenomenological model

- $\xi \sim 3\text{fm}$ (max value)

  near the critical point, critical s

- $\lambda_3 \sim (0,8),\ \lambda_4 \sim (4,20)$

  lattice simulation of the effective point.

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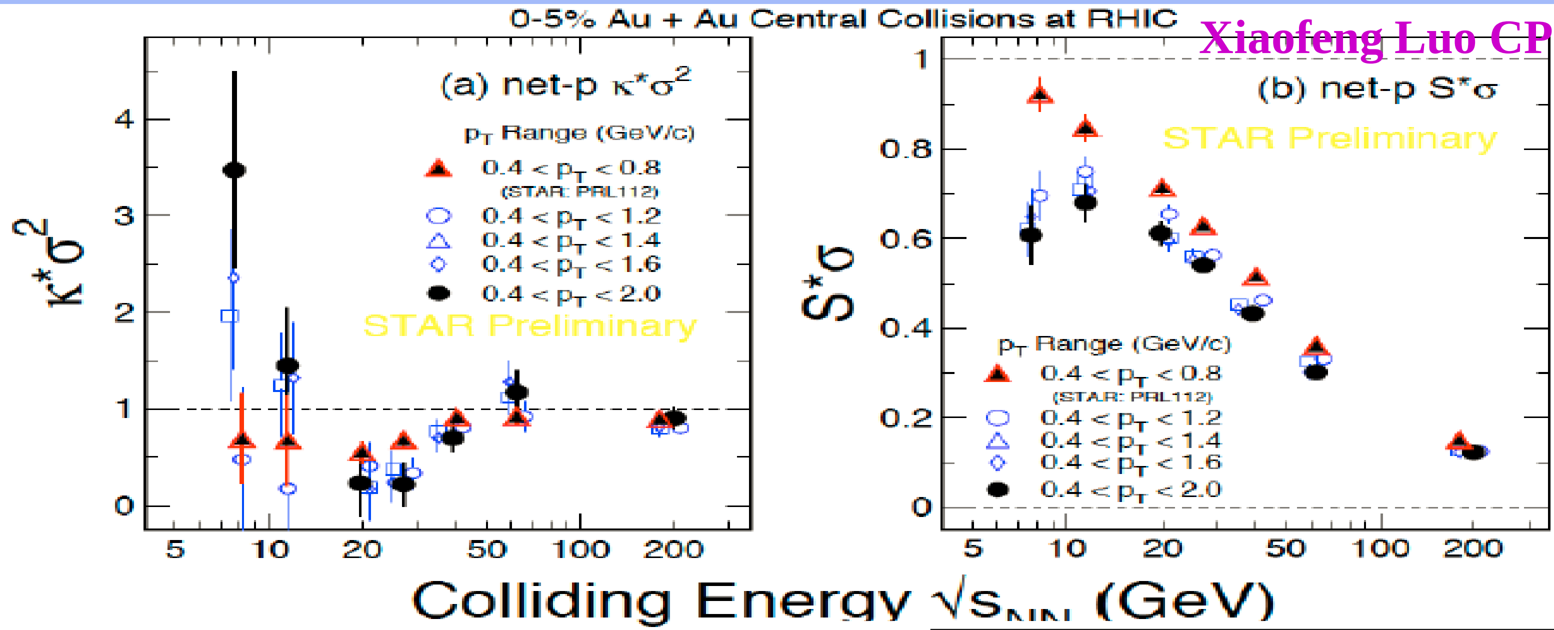
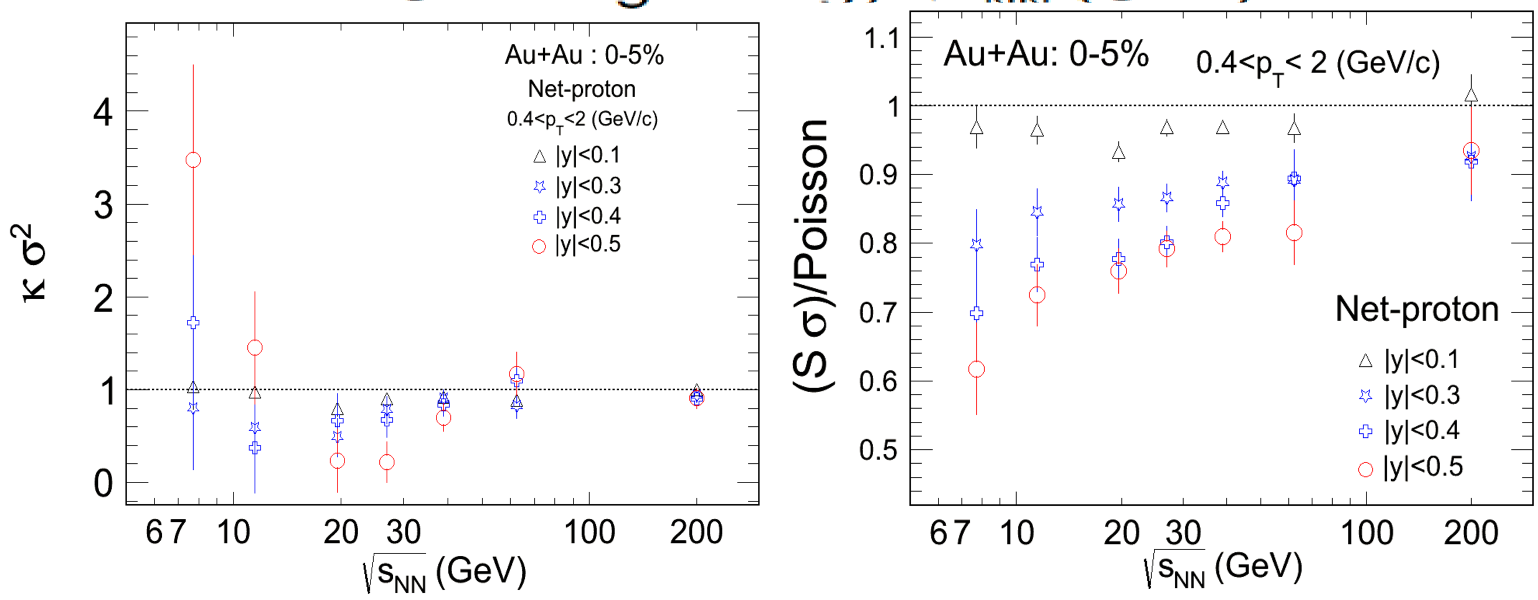
Jiang, Li & Song, arXiv: 1512.06164

Comparison with the experimental data

STAR data (acceptance dependence)

Xiaofeng Luo CPOD 2014

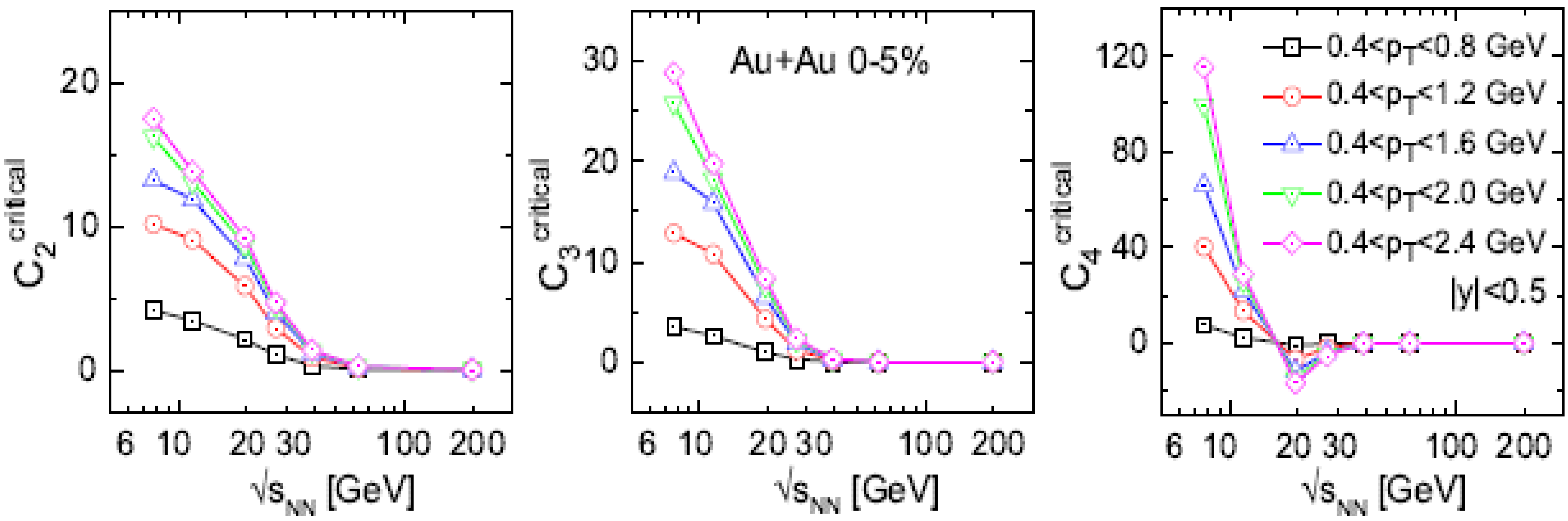




-Wider p_T or y acceptance lead to more pronounce fluctuation signals

Transverse momentum acceptance dependence

Jiang, Li & Song, arXiv: 1512.06164



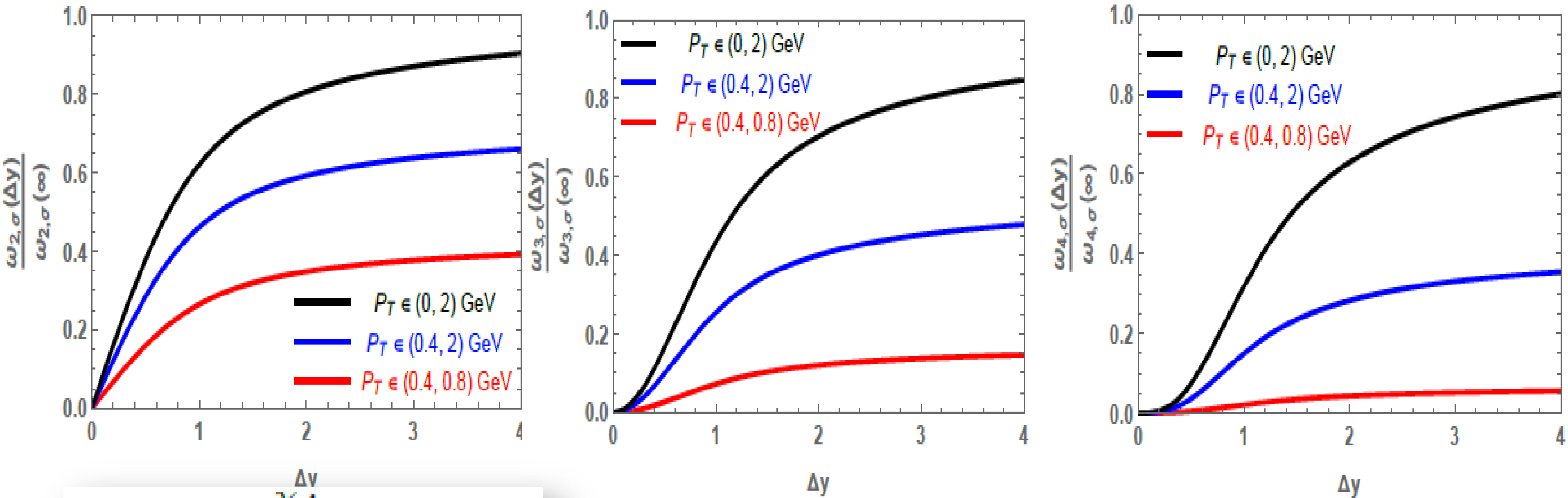
-The critical fluctuations are significantly enhanced with the p_T ranges increased to 0.4-2.0 GeV

- At lower collision energies, the dramatically increased mean value of net protons also leads to dramatically enhanced critical fluctuations

-Critical fluctuations are influenced by both the mean value (average number) of net protons within specific acceptance window and the correlation length.

Rapidity acceptance dependence

Ling & Stephanov PRC2016



$$(\delta f_A)_\sigma = -\frac{\chi_A}{\gamma_A} g\sigma(x_A),$$

$$\langle\sigma(x)\sigma(y)\rangle \to T\xi^2\delta^3(x-y)$$

$$\langle\sigma(x)\sigma(y)\sigma(z)\rangle \to -2\tilde{\lambda}_3 T^{3/2}\xi^{9/2}\delta^6(x,y,z)$$

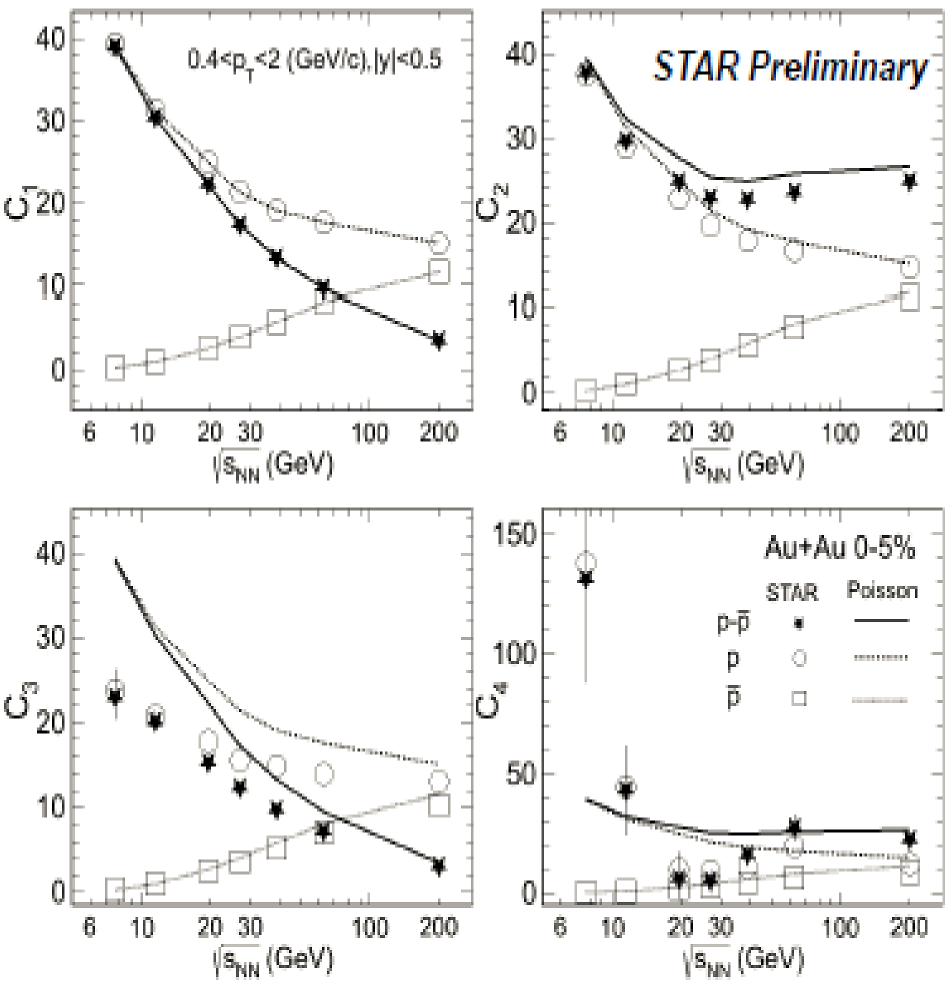
$$\langle\sigma(x)\sigma(y)\sigma(z)\sigma(w)\rangle_c \to 6(2\tilde{\lambda}_3^2 - \tilde{\lambda}_4)T^2\xi^7\delta^9(x,y,z,w)$$

- freeze-out surface: Blast Wave model:

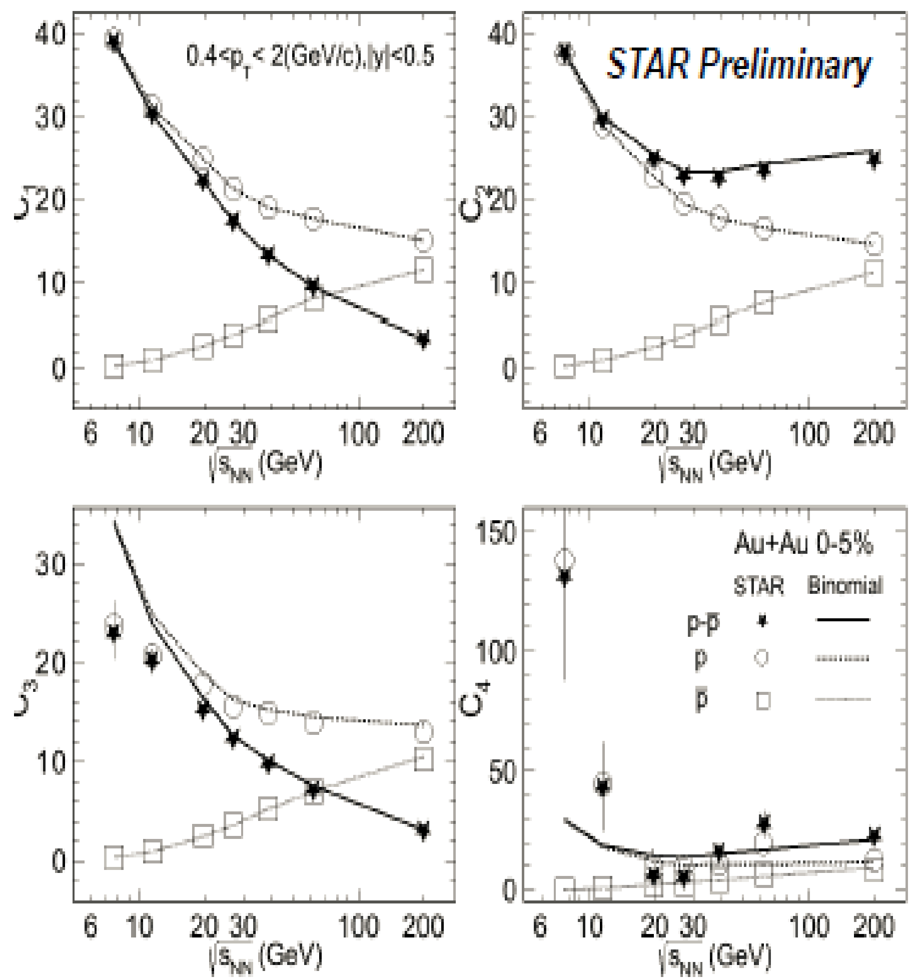
-The dependence on transverse momentum acceptance is very significant

- extension the rapidity coverage will significantly increase the magnitude of critical fluctuations

Cumulants vs. Poisson



Cumulants vs. Binomial



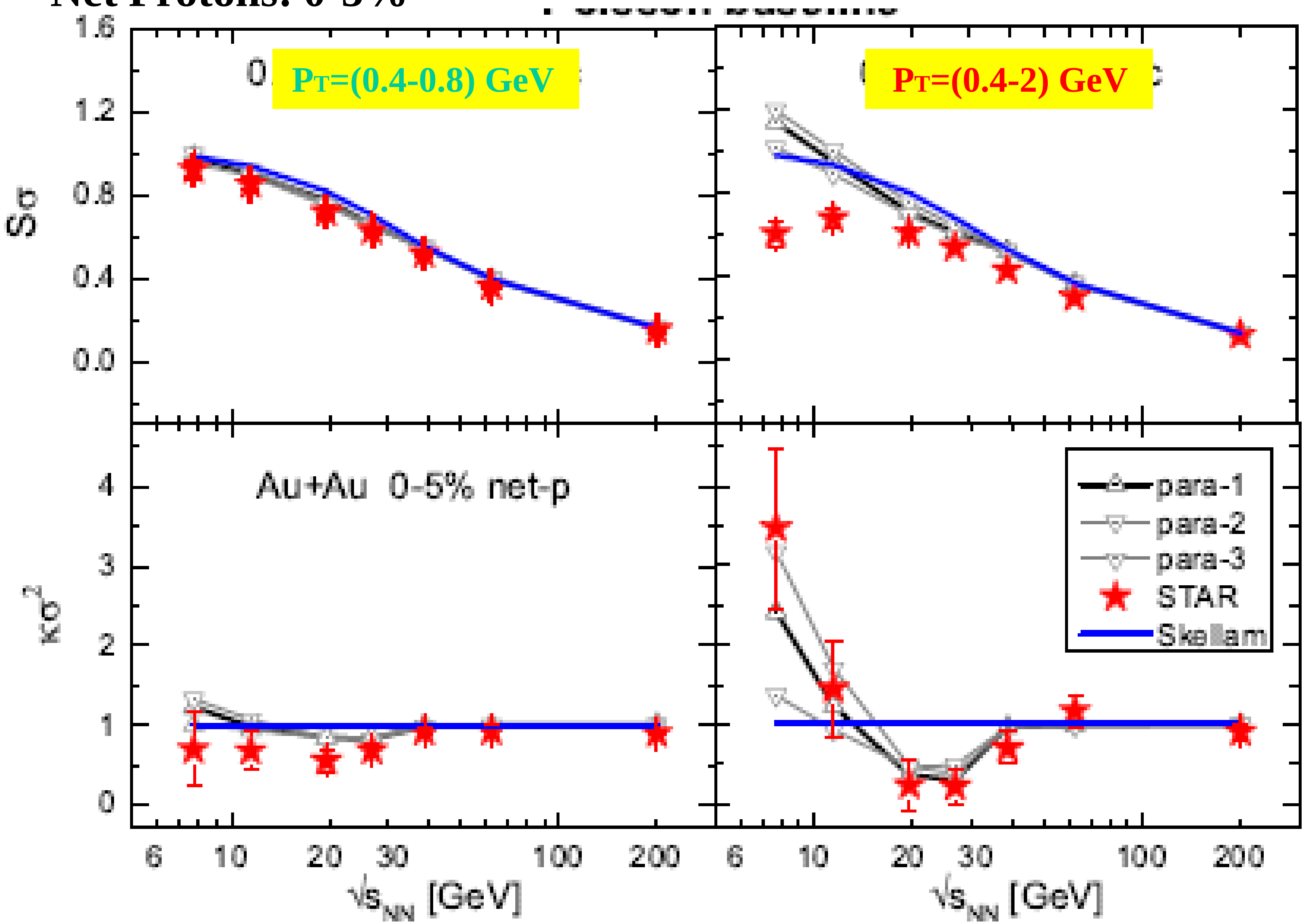
Fluctuations measured in experiment:

critical fluct. + non-critical (thermal) fluct. + …

$\kappa\sigma^2, S\sigma$: (Model + Poisson baselines)

Jiang, Li & Song, arXiv: 1512.06164

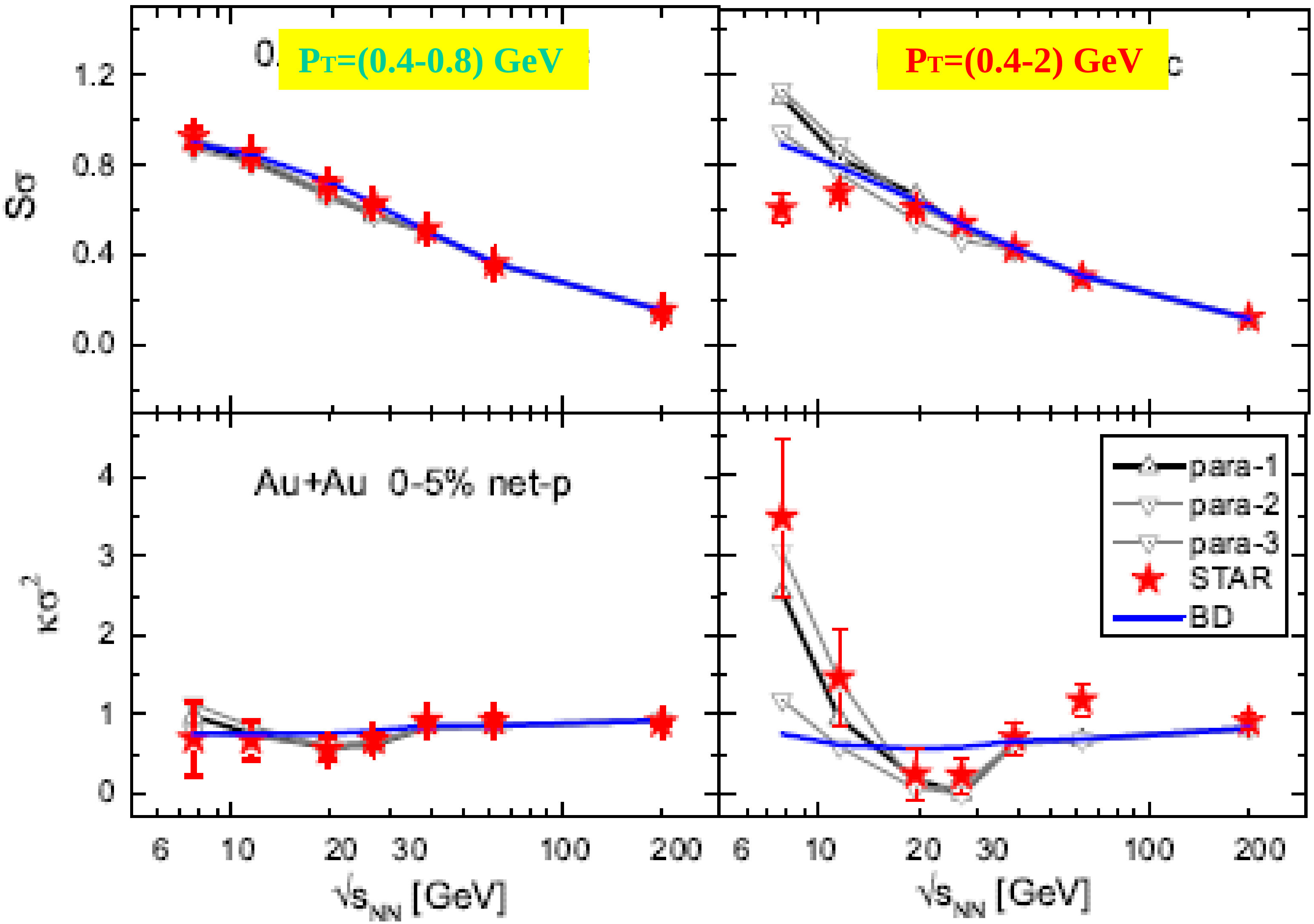
Net Protons: 0-5%



$\kappa\sigma^2, S\sigma$ (Model + Binomial baselines)

Jiang, Li & Song, arXiv: 1512.06164

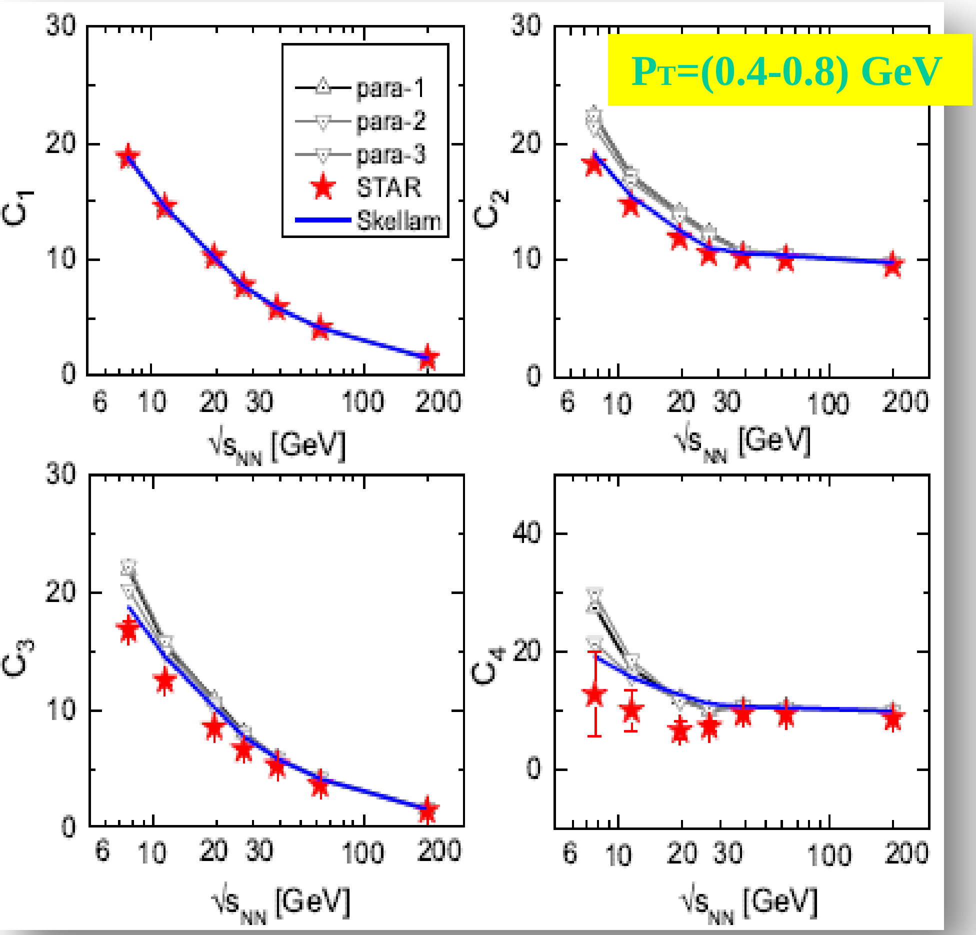
Net Protons: 0-5%

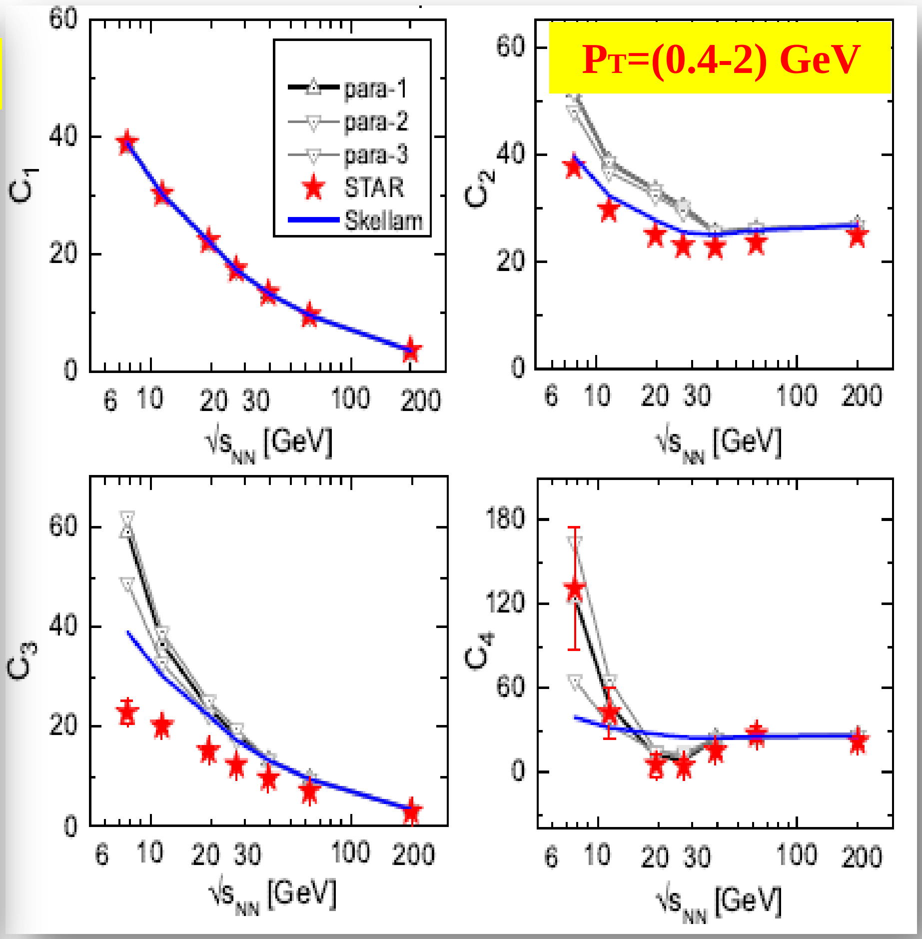


C_1 C_2 C_3 C_4: (Model + Poisson baselines)

Net Protons 0-5%

Jiang, Li & Song, arXiv: 1512.06164



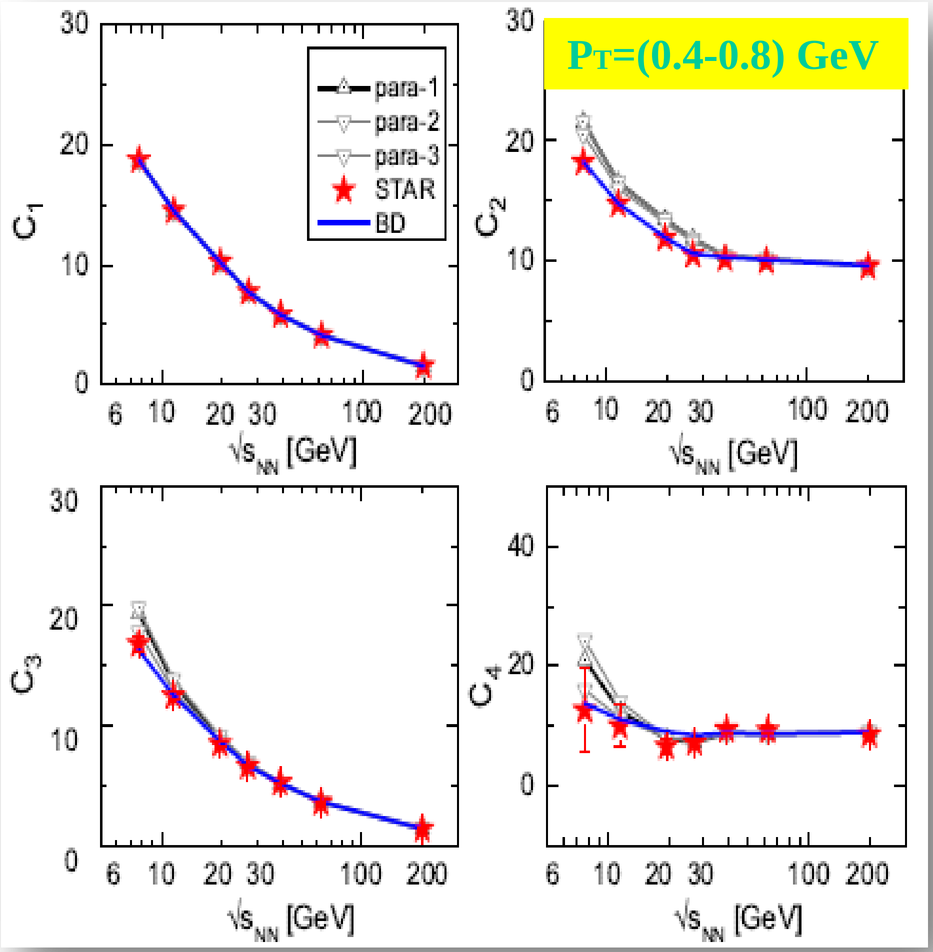


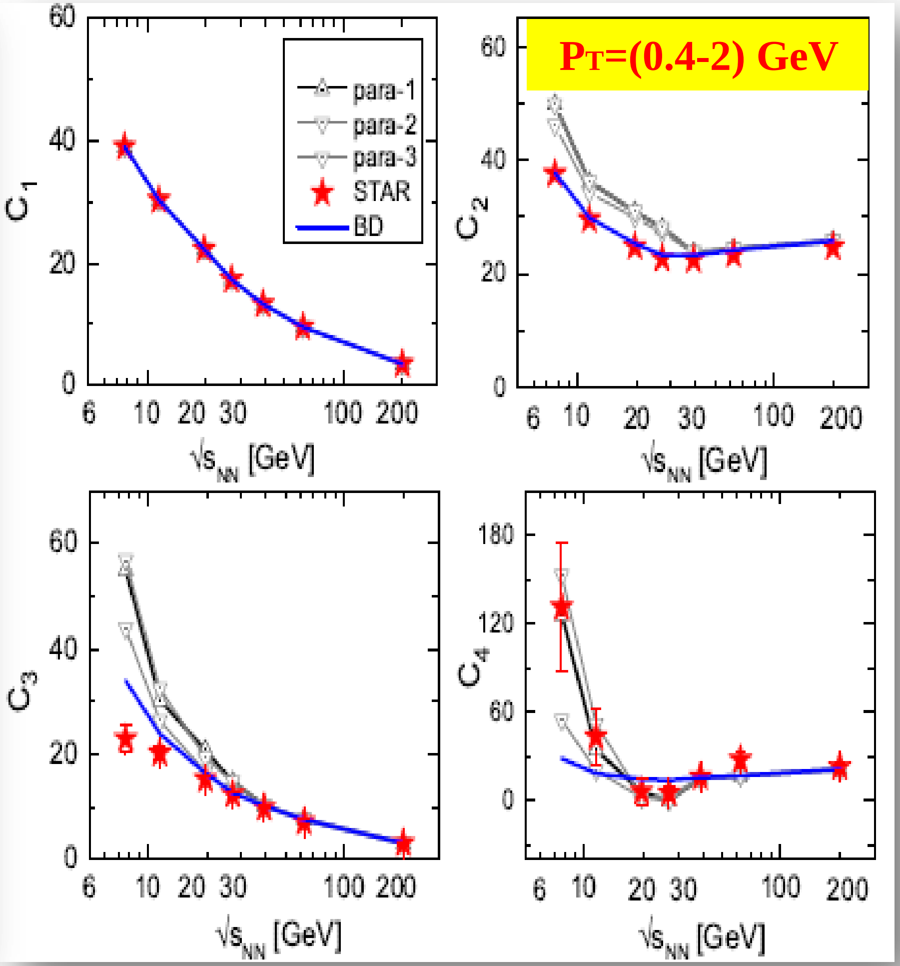
Critical fluctuations give positive contribution to C_2 , C_3; well above the poisson baselines, can NOT explain/describe the C_2 , C_3 data

C_1 C_2 C_3 C_4: (Model + Binomial baselines)

Net Protons 0-5%

Jiang, Li & Song, arXiv: 1512.06164





Critical fluctuations give positive contribution to C_2 , C_3; well above the binomial baselines, can NOT explain/describe the C_2 , C_3 data

C_1 C_2 C_3 C_4: Pt-(0.4-2) GeV (Model + Poisson baselines)

P_T=(0.4-2) GeV

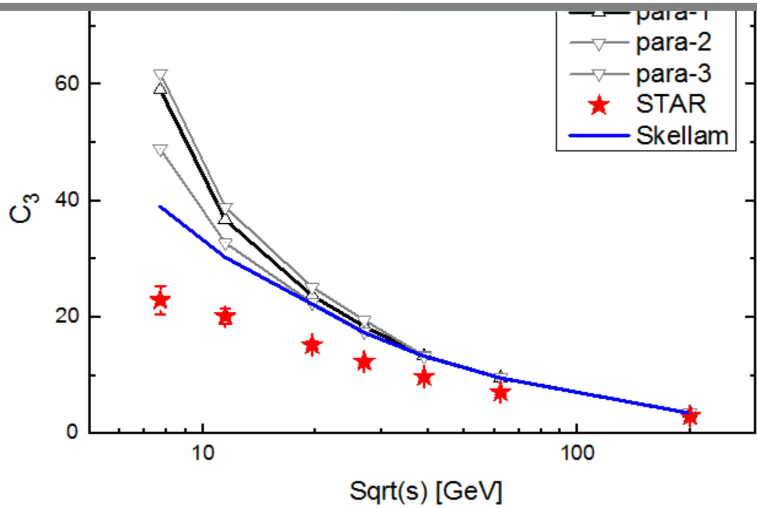
$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega[\sigma] = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4\right],$$

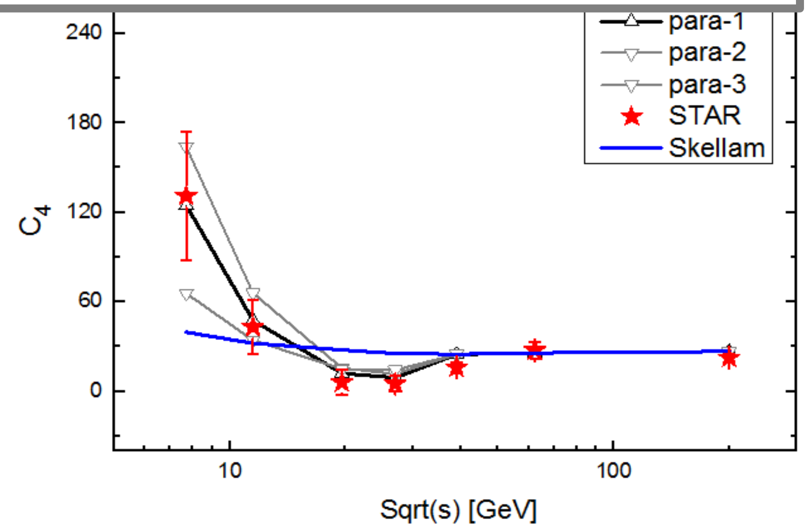
$$\langle\sigma_1\sigma_2\rangle_c = TD(x_1 - x_2),$$

$$\langle\sigma_1\sigma_2\sigma_3\rangle_c = -2T^2\lambda_3 \int d^3z D(x_1 - z)\, D(x_2 - z) D(x_3 - z),$$

$$\langle\sigma_1\langle\ P[\sigma] \sim \exp\{-\Omega[\sigma]/T\},\ \int d^3z D(x_1 - z)\, D(x_2 - z) D(x_3 - z)\, D(x_4 - z)$$

$$+12T^3\lambda_3^2 \int d^3u \int d^3v D(x_1 - u)\, D(x_2 - u) D(x_3 - v)\, D(x_4 - v)\, D(u - v).$$





The contributions from STATIC critical fluctuations to C_2 , C_3 are always positive (Both this model & early Stephanov PRL09 framework)

Dynamical Critical Fluctuations

Real time evolution of non-Gaussian cumulants

Mukherjee, Venugopalan & Yin PRC 2015

Zero mode of the sigma field:

$$\sigma \equiv \frac{1}{V}\int d^3x\, \sigma(\boldsymbol{x}),$$

Fokker_Planck equations:

$$\partial_\tau P(\sigma;\tau) = \frac{1}{(m_\sigma^2 \tau_{\rm eff})}\left\{\partial_\sigma \left[\partial_\sigma \Omega_0(\sigma) + V_4^{-1}\partial_\sigma\right] P(\sigma;\tau)\right\}$$

Coupled equations for higher order cumulants:

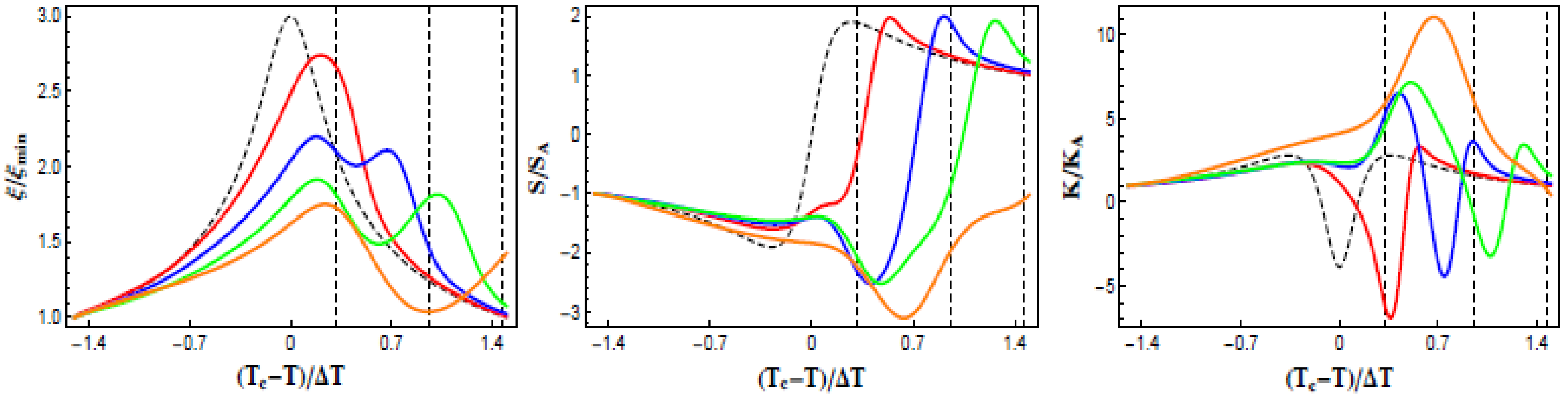
$$\partial_\tau \kappa_2(\tau) = -2\,\tau_{\rm eff}^{-1}\,(b^2)\left[\left(\frac{\kappa_2}{b^2}\right)F_2(M) - 1\right]\left[1+\mathcal{O}(\epsilon^2)\right],$$

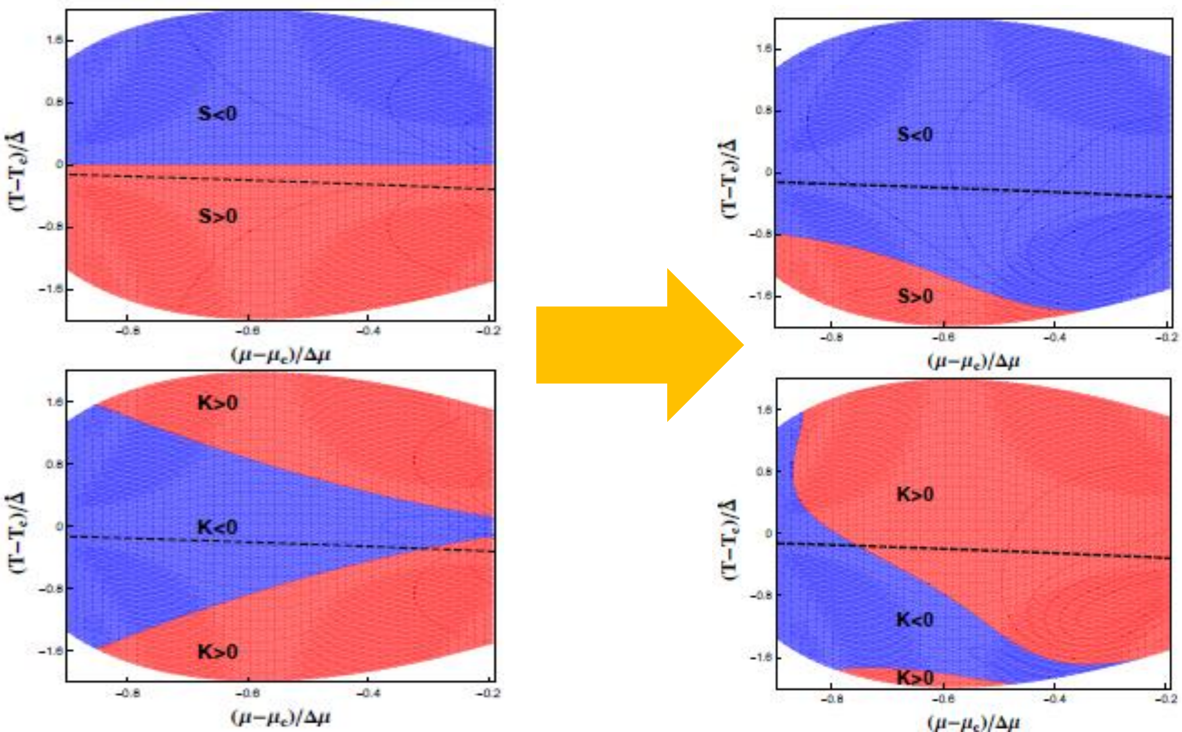
$$\partial_\tau \kappa_3(\tau) = -3\,\tau_{\rm eff}^{-1}\,(\epsilon\, b^3)\left[\left(\frac{\kappa_3}{\epsilon\, b^3}\right)F_2(M) + \left(\frac{\kappa_2}{b^2}\right)^2 F_3(M)\right]\left[1+\mathcal{O}(\epsilon^2)\right]$$

$$\partial_\tau \kappa_4(\tau) = -4\,\tau_{\rm eff}^{-1}\,(\epsilon^2\, b^4)\left\{\left(\frac{\kappa_4}{\epsilon^2\, b^4}\right)F_2(M) + 3\left(\frac{\kappa_2}{b^2}\right)\left(\frac{\kappa_3}{\epsilon\, b^3}\right)F_3(M) + \left(\frac{\kappa_2}{b^2}\right)^3 F_4\right\} \times \left[1+\mathcal{O}(\epsilon^2)\right]$$

Real time evolution of non-Gaussian cumulants

Mukherjee, Venugopalan & Yin PRC 2015





-Critical slowing down limits growth of correlation length

-Non-Gaussian cumulants do not follow growth of the correlation length

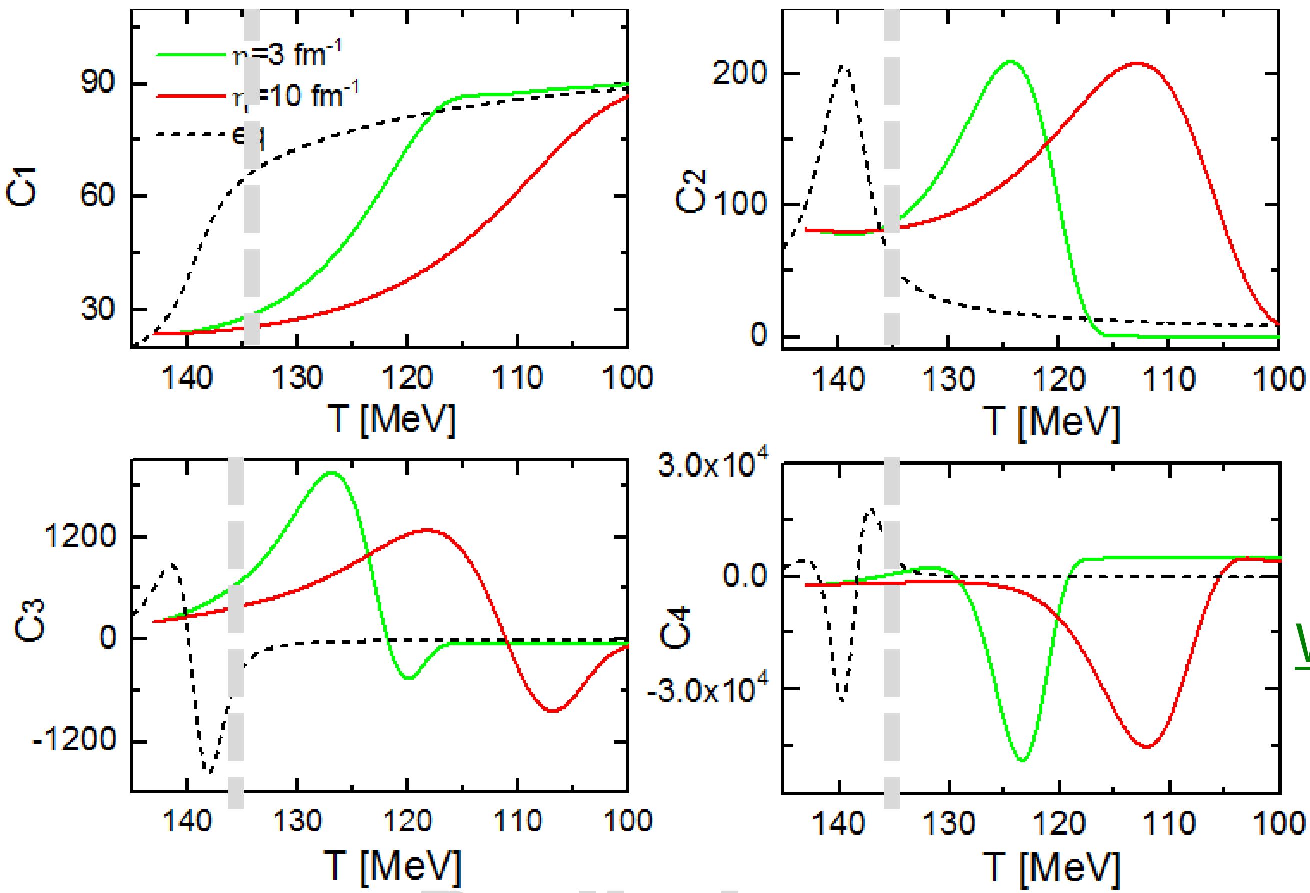
-Sign of the non -Gaussian cumulants can be different from the equilibrium one

Dynamical critical fluctuations of the sigma field

Langevin dynamics: $\partial^\mu \partial_\mu \sigma(t,x) + \eta \partial_t \sigma(t,x) + V'_{eff}(\sigma) = \xi(t,x)$

with effective potential from linear sigma model with constituent quarks

$$V_{eff}(\sigma) = U(\sigma) + \Omega_{\bar{q}q}(T,\sigma) = \frac{\lambda^2}{4}\left(\sigma^2 - \nu^2\right)^2 - h_q \sigma - U_0 - 2d_q T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + \exp\left(-\frac{E}{T}\right)\right)$$



Preliminary

$$\frac{T(t)}{T_0} = \left(\frac{t}{t_0}\right)^{-0.45}$$

-The sign of C_3 is different from the equilibrium one due to the memory effects

Work in the near future

Coupling sigma field with particles; Study the dynamical critical fluctuations of net protons

Summary and outlook

RHIC BES Experiment:

STAR BES give exiting results on the net proton cumulants with p_T=(0.4-2) GeV, showing its potential of discovery the QCD critical point

Static critical fluctuations:

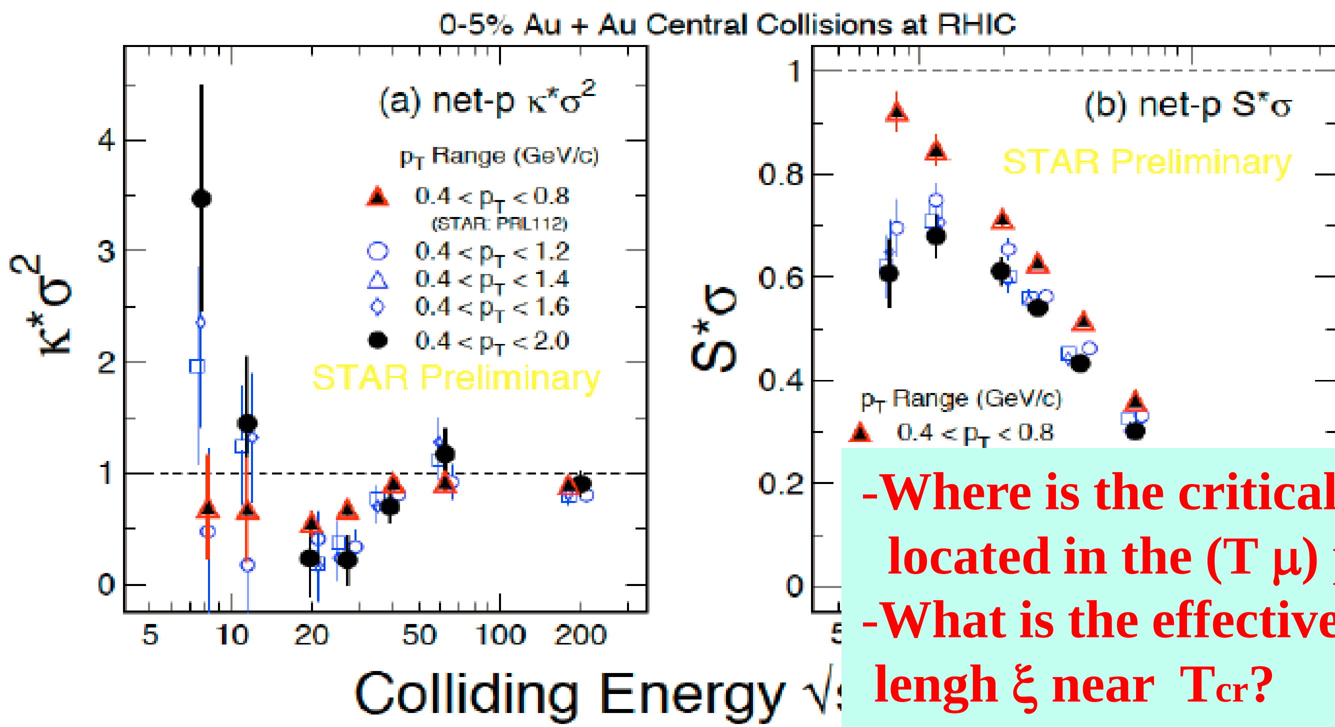
-qualitatively explain the acceptance dependence of critical fluctuations

-C_4 and $\kappa\sigma^2$ can be reproduced through tuning the parameters of the model

-However C_2 , C_3 are well above the poisson/BN baselines, which can NOT explain/describe the data

Dynamical critical fluctuations:

-Sign of the C_3, C_4 cumulants can be different from the equilibrium one due to the memory effects



-Where is the critical points located in the (T μ) plane ?

-What is the effective correlation lengh ξ near T_{cr}?

… … … …

-Full development of the dynamical model near the critical point is needed

-microscopic/ macroscopic evolution of the bulk of matter, together with the evolution of the order parameter field

-proper treatment of freeze-out with the order parameter field

-interactions between thermal & critical fluctuations

… ……

-Thermal (non-critical) fluctuation baselines

Thank You

Boltzmann approach with external field

Stephanov PRD 2010

$$\mathcal{S} = \int d^3x \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - U(\sigma)) - \int ds M(\sigma),$$

$$\begin{cases} \partial^2 \sigma + dU/d\sigma + (dM/d\sigma) \int_p f/\gamma = 0. \\ \frac{p^\mu}{M} \frac{\partial f}{\partial x^\mu} + \partial^\mu M \frac{\partial f}{\partial p^\mu} + \mathcal{C}[f] = 0, \end{cases}$$

-analytical solution with perturbative expansion, please refer to Stephanov PRD 2010

Stationary solution for the Boltamann equation with external field

$$f_\sigma(\boldsymbol{p}) = e^{\mu/T} e^{-\gamma(\boldsymbol{p})M/T}.$$

Effective particle mass: $M = M(\sigma) = g\sigma$