Critical point and Onset of Deconfinement 2016, 30 May, University of Wrocław

Novel picture of the soft modes at the QCD critical point based on the FRG method

Takeru Yokota (Dept. of Phys., Kyoto Univ., Japan)

Collaborators

Teiji Kunihiro (Dept. of Phys., Kyoto Univ.)

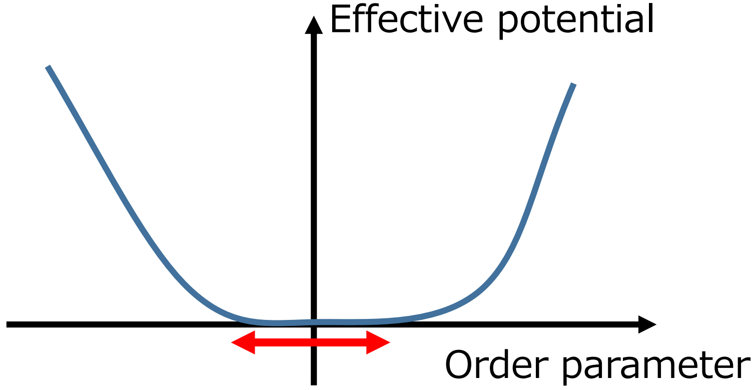
Kenji Morita (YITP, Kyoto Univ.)

Reference: (To be published in PTEP) arXiv:1603.02147

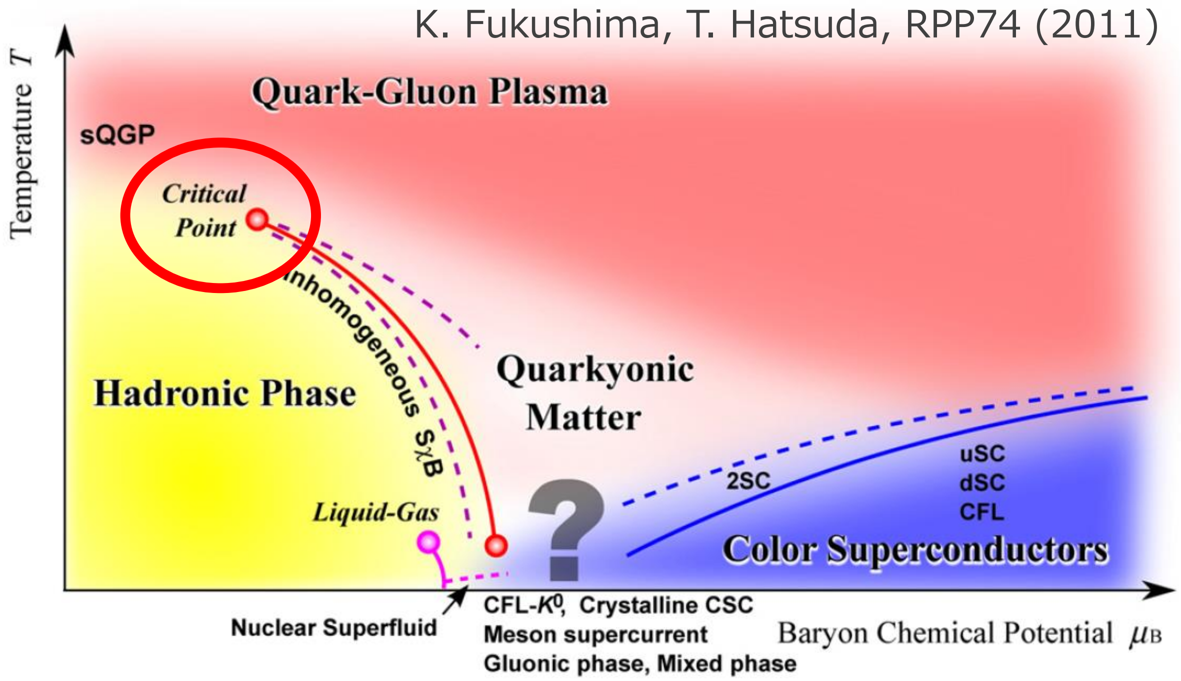
The QCD critical point and the soft mode

- The phase transition is of second order at the QCD critical point (CP) and the effective potential becomes flat.

At a CP



Flat potential



- At a CP, the mode corresponding to the fluctuation of the order parameter becomes gapless and long lived (soft mode).

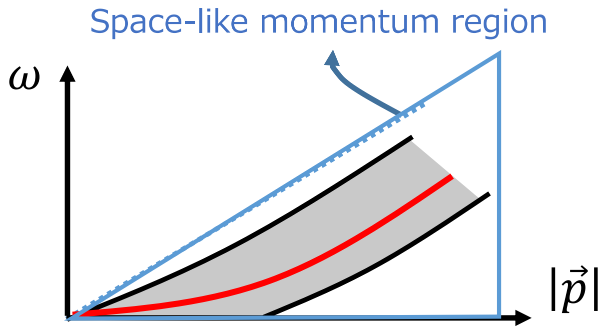
- What is the soft mode at the QCD CP?

What is the soft mode at the QCD CP?

Prediction in previous researches: Hydrodynamical modes

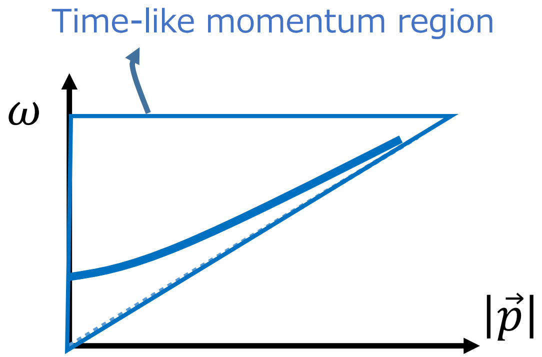
H. Fujii, M. Ohtani, PRD70 (2004) — TDGL and RPA for NJL model
D. T. Son, M. A. Stephanov, PRD70 (2004) — Langevin equation
Y. Hatta, T. Ikeda, PRD67 (2003) — QCD effective potential

- In the case of nonzero current quark mass
- These modes are interpreted as particle-hole excitation mode and their dispersion relations have supports in the space-like momentum region.



Dispersion relation of hydrodynamical modes

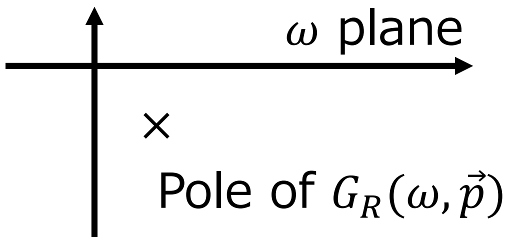
The sigma mesonic mode does not become soft in the case of off chiral limit, in contrast to the case of chiral limit.



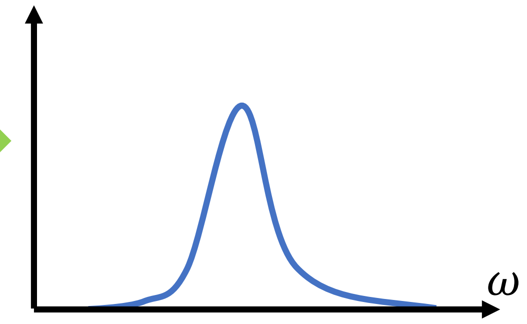
Dispersion relation of sigma mesonic modes

How to investigate the soft mode

- Spectral functions are useful to get information about the excited modes.



$\rho(\omega,\vec{p}) = 2\mathrm{Im}G_R(\omega,\vec{p})$



$$G_R(\omega,\vec{p}) = \int d^4x e^{i\omega t - i\vec{p}\cdot\vec{x}} \left[i\theta(t) \left\langle \left[\hat{O}(t,\vec{x}), \hat{O}(0,\vec{0}) \right] \right\rangle_{T,\mu} \right]$$

  - The investigation of soft mode using the sigma channel spectral function ρ_σ has been done using RPA analysis for NJL model. $\hat{O} = \bar{\psi}\psi$

    H. Fujii, M. Ohtani, PRD70 (2004)

- Recently a method to calculate meson spectral functions using functional renormalization group (FRG) has been developed.

  R. Tripolt, N. Strodthoff, L. Smekal, J. Wambach, PRD89 (2014); R. Tripolt, L. Smekal, J. Wambach, PRD90 (2014)

  K. Kamikado, N. Strodthoff, L. Smekal, J. Wambach, EPJ C74 (2014)

In this research, we have made an intensive investigation of **meson spectral functions** around the QCD CP using **the FRG method**, developed recently, for exploring the nature of the **soft mode**.

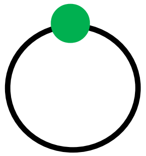
Functional renormalization group

- Functional renormalization group (FRG) is a nonperturbative method to calculate the effective action.

C. Wetterich, PLB 301 (1993)

Effective average action (EAA)

Regulator (an appropriate function)

Wetterich equation: $$\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{STr}\left[\frac{\partial_k R_k}{\Gamma_k^{(2)}[\phi] + R_k}\right] = \frac{1}{2}$$ 

$$\Gamma_{k=\Lambda}[\phi] = S_\Lambda[\phi]$$

UV scale

Coarse graining (**scale parameter** $k \to 0$)

$$\Gamma_{k=0}[\phi] = \Gamma[\phi]$$

Effective action

Quark-meson model and Local potential approximation

- We employ the 2-flavor quark-meson model and our calculation is based on the local potential approximation (LPA).

  - 2-flavor quark-meson model (in imaginary-time formalism)

D. U. Jungnickel, C. Wetterich, PRD53 (1996)

$$S_\Lambda\left[\overline{\psi},\psi,\phi=(\sigma,\vec{\pi})\right]=\int_0^{\frac{1}{T}}d\tau\int d^3\vec{x}\left\{\overline{\psi}\left(\not{\partial}+g_s(\sigma+i\vec{\tau}\cdot\vec{\pi}\gamma_5)-\mu\gamma_0\right)\psi+\frac{1}{2}(\partial_\mu\phi)^2+V_\Lambda(\phi^2)-\underline{c\sigma}\right\}$$

Effect of the current quark mass

  - Ansatz for the form of EAA in LPA

$$\Gamma_k\left[\overline{\psi},\psi,\phi=(\sigma,\vec{\pi})\right]=\int_0^{\frac{1}{T}}d\tau\int d^3\vec{x}\left\{\overline{\psi}\left(\not{\partial}+g_s(\sigma+i\vec{\tau}\cdot\vec{\pi}\gamma_5)-\mu\gamma_0\right)\psi+\frac{1}{2}(\partial_\mu\sigma)^2+\frac{1}{2}(\partial_\mu\pi^a)^2\underline{+U_k(\phi^2)}-c\sigma\right\}$$

k dependence

U_k is the lowest order of the derivative expansion of the flow of meson terms and this approximation is expected to be a good starting point for describing the behavior of low-momentum modes.

Two point Green's function and the flow equations

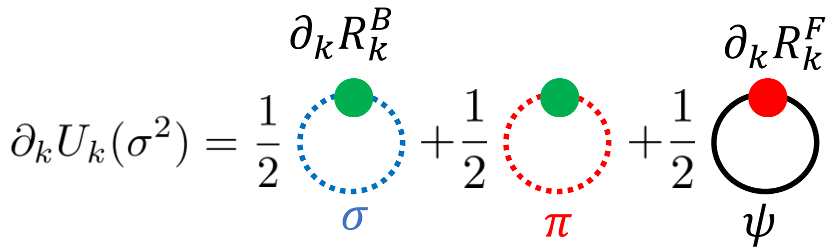
- Definition of $G_{\sigma,k}$ and $G_{\pi,k}$:

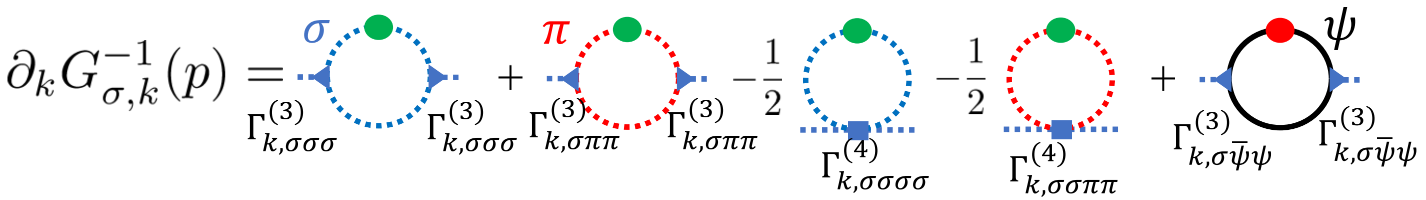
$$\left.\frac{\delta^2\Gamma_k}{\delta\sigma(p)\delta\sigma(q)}\right|_{\sigma=\sigma_0} = (2\pi)^4\delta^{(4)}(p+q)G_{\sigma,k}^{-1}(p) \qquad \left.\frac{\delta^2\Gamma_k}{\delta\pi^a(p)\delta\pi^a(q)}\right|_{\sigma=\sigma_0} = (2\pi)^4\delta^{(4)}(p+q)G_{\pi,k}^{-1}(p)$$

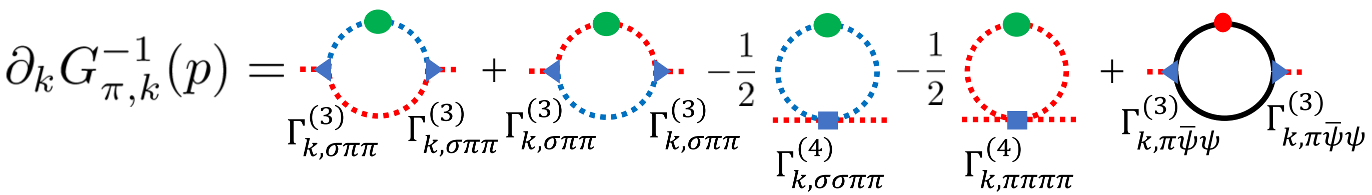
σ_0 → Average of σ (chiral condensate)

  - $G_{\sigma,k}$ and $G_{\pi,k}$ become meson two-point Green's functions at $k \to 0$.

- Using Wetterich eq. and the ansatz for EAA in LPA, the flow equations for U_k , $G_{\sigma,k}$ and $G_{\pi,k}$ are obtained.







Quark propagator $= \left[\gamma_\mu p^\mu - \mu\gamma_0 + m_q(\sigma) + R_k^F(p)\right]^{-1}$ $\qquad m_q(\sigma) = g_s\sigma_0$

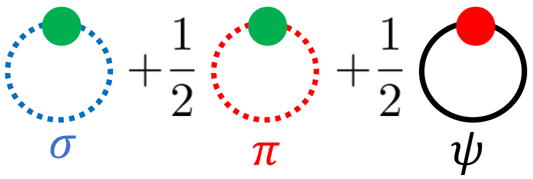
Sigma propagator $= \left[p^2 + m_{\sigma,k}^2(\sigma) + R_k^B(p)\right]^{-1}$ $\qquad m_{\sigma,k}^2(\sigma) = \frac{\partial^2 U_k}{\partial\sigma^2}(\sigma)$

Pion propagator $= \left[p^2 + m_{\pi,k}^2(\sigma) + R_k^B(p)\right]^{-1}$ $\qquad m_{\pi,k}^2(\sigma) = \frac{1}{\sigma}\frac{\partial U_k}{\partial\sigma}(\sigma)$

Calculation of the mesonic spectral functions

① Solve the flow equation for U_k.

$$\partial_k U_k(\sigma^2) = \frac{1}{2}\,[\sigma] + \frac{1}{2}\,[\pi] + \frac{1}{2}\,[\psi]$$

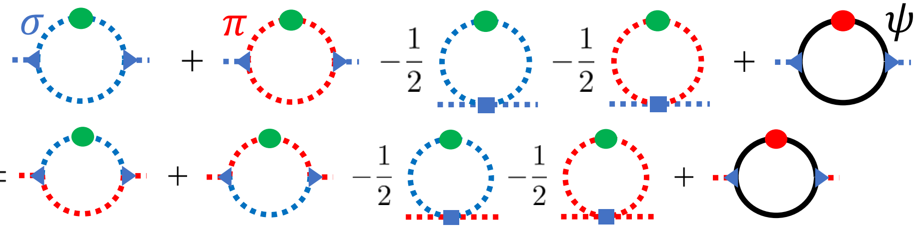


- Effective potential $F[\sigma]/V = U_0(\sigma^2) - c\sigma$ gives the chiral condensate σ_0 and thermal quantities.

② Evaluate the RHSs of the flow equations for $G_{\sigma,k}$ and $G_{\pi,k}$ using U_k, and integrate them with respect to k.
The retarded Green's functions G_σ^R and G_π^R are obtained by analytic continuation for Matsubara frequencies.

$$\partial_k G^{-1}_{\sigma,k}(p) = \ldots + \ldots - \frac{1}{2}\ldots - \frac{1}{2}\ldots + \ldots$$

$$\partial_k G^{-1}_{\pi,k}(p) = \ldots + \ldots - \frac{1}{2}\ldots - \frac{1}{2}\ldots + \ldots$$



- Since we employ the imaginary-time formalism, analytic continuation from Matsubara frequencies to real frequencies is needed to calculate the retarded Green's functions.

→ In our case , it is known that the analytic continuation can be done easily in the flow equations.

R. Tripolt, N. Strodthoff, L. Smekal, J. Wambach, PRD89 (2014); R. Tripolt, L. Smekal, J. Wambach, PRD90 (2014)

③ Obtain the spectral functions from the imaginary parts of G_σ^R and G_π^R.

$$\rho_\sigma(\omega,\vec{p}) = 2\mathrm{Im}G_\sigma^R(\omega,\vec{p})$$
$$\rho_\pi(\omega,\vec{p}) = 2\mathrm{Im}G_\pi^R(\omega,\vec{p})$$

Initial condition and parameters

$$S_\Lambda\left[\overline{\psi},\psi,\phi=(\sigma,\vec{\pi})\right]=\int_0^{\frac{1}{T}}d\tau\int d^3\vec{x}\left\{\overline{\psi}\left(\not{\partial}+g_s(\sigma+i\vec{\tau}\cdot\vec{\pi}\gamma_5)-\mu\gamma_0\right)\psi+\frac{1}{2}(\partial_\mu\phi)^2+V_\Lambda(\phi^2)-c\sigma\right\}$$

$$V_\Lambda(\phi^2)=\frac{1}{2}m_\Lambda^2\phi^2+\frac{1}{4!}\lambda(\phi^2)^2$$

Λ	m_Λ/Λ	λ_Λ	c/Λ^3	g_s
1000MeV	0.794	2.00	0.00175	3.2

R. Tripolt, L. Smekal, J. Wambach, PRD90 (2014)

Vacuum value

M_q	M_π	M_σ	σ_0
286MeV	137MeV	496MeV	93MeV

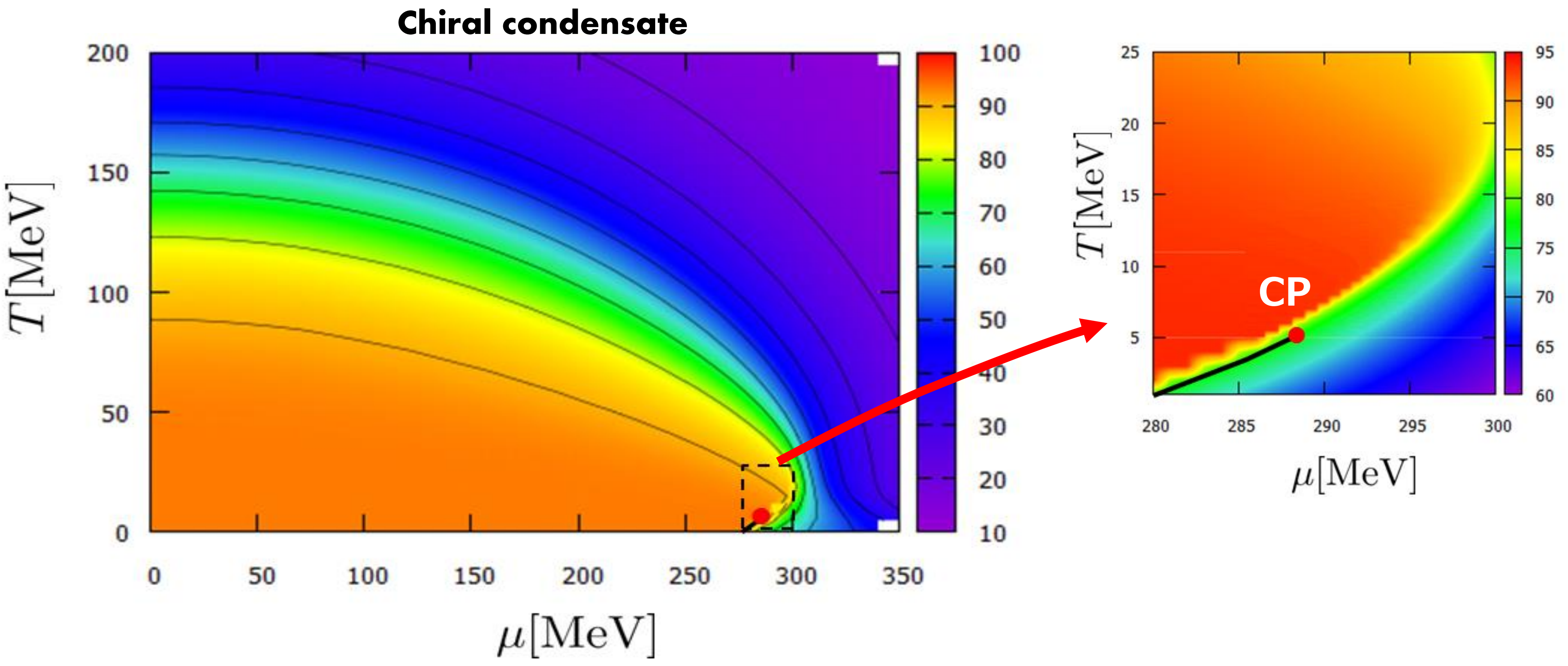
M_q is the constituent quark mass.

$$M_q=g_s\sigma_0$$

M_π and M_σ are the screening masses.

$$M_\pi=\frac{1}{\sigma_0}\frac{\partial U_0}{\partial\sigma}(\sigma_0)\qquad M_\sigma=\frac{\partial^2 U_0}{\partial\sigma^2}(\sigma_0)$$

The location of the QCD CP



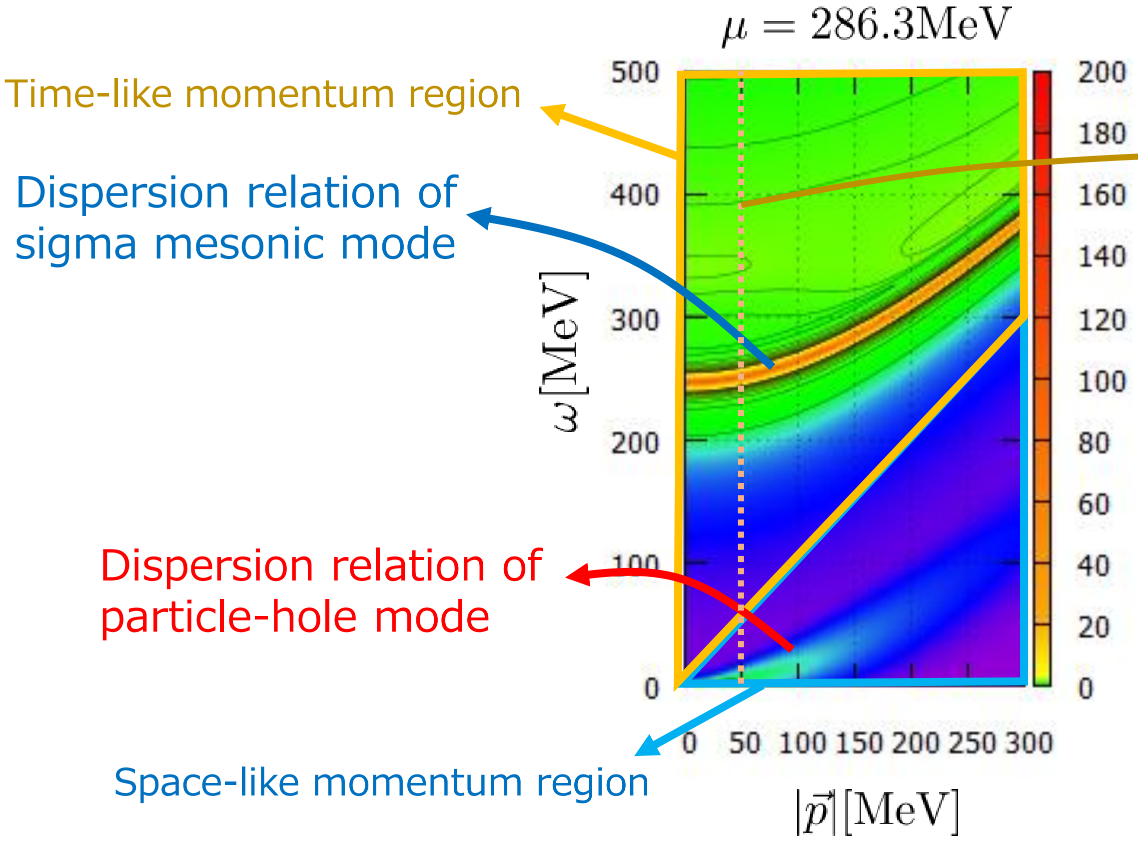
- The location of the CP is determined by the min. point of the sigma screening mass M_σ.

$\Rightarrow$ $(T_c, \mu_c) = (5.1 \pm 0.1\text{MeV}, 286.7 \pm 0.2\text{MeV})$

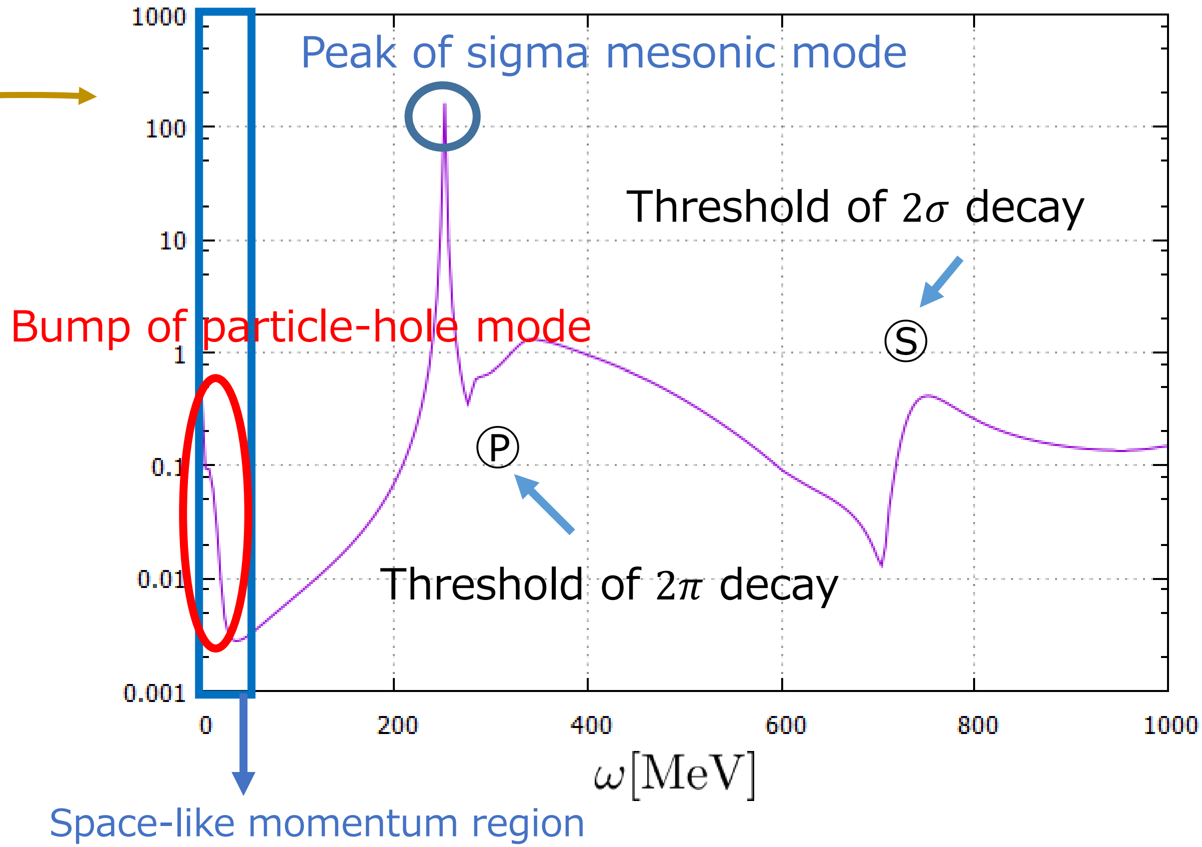
An example of the sigma channel spectral function ρ_σ near the QCD CP

- We fix $\boldsymbol{T = 5.1\ \mathrm{MeV}}$ below. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t = 286.69\ \mathrm{MeV}}$)

Contour map of ρ_σ

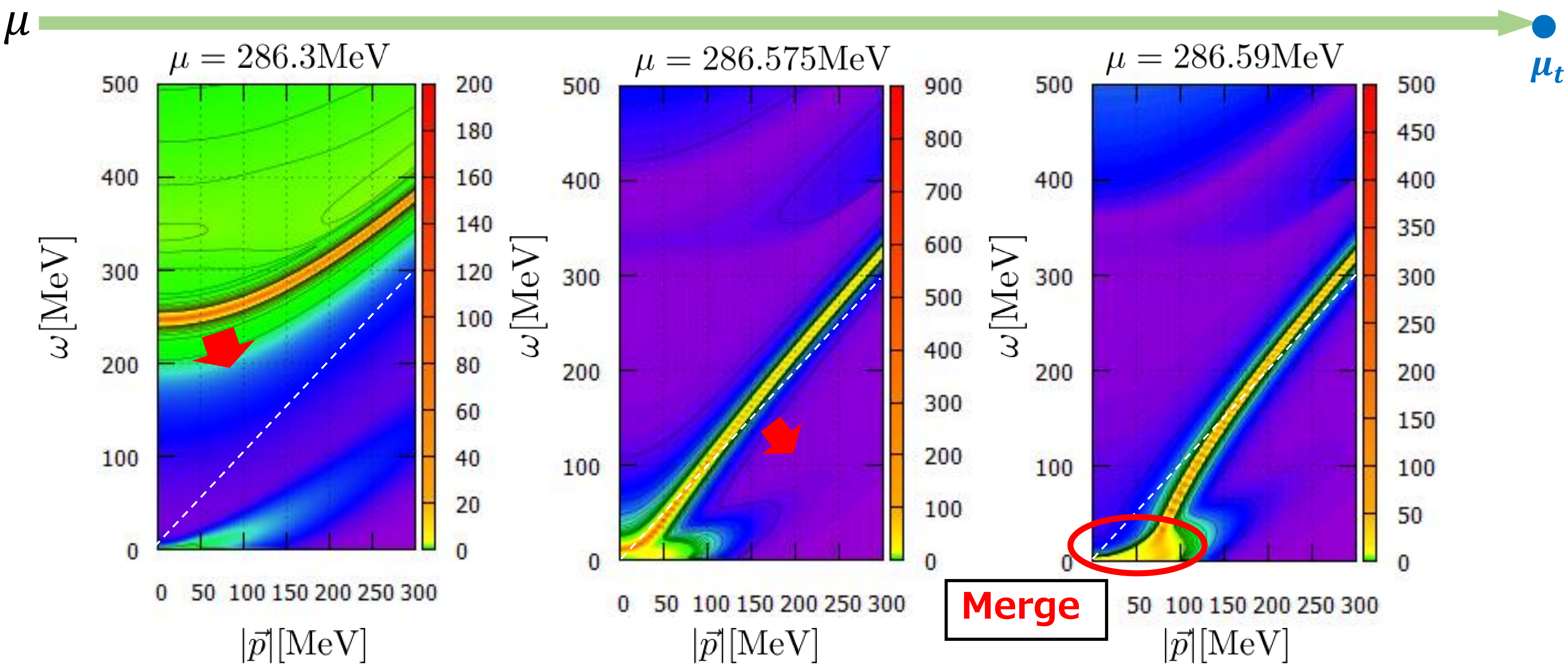


ρ_σ when $|\vec{p}| = 50\ \mathrm{MeV}$



Contour maps of the sigma channel spectral function ρ_σ near the QCD CP

- We fix $\boldsymbol{T} = \mathbf{5.1}$ **MeV**. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t} = \mathbf{286.69}$ **MeV**)



- As the system approaches the QCD CP, the dispersion relation of sigma mesonic mode shifts to low-energy region and merges with the bump of the particle-hole modes in the space-like momentum region.

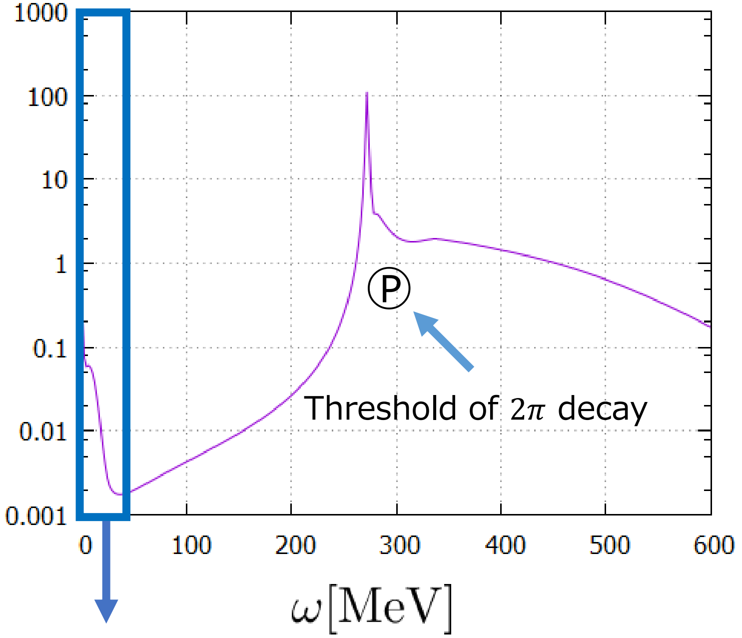
Position and strength of peaks and bumps of ρ_σ near the QCD CP

- We fix $\boldsymbol{T} = \mathbf{5.1\ MeV}$ and $|\vec{\boldsymbol{p}}| = \mathbf{50\ MeV}$. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t} = \mathbf{286.69\ MeV}$)

μ

$\boldsymbol{\mu_t}$

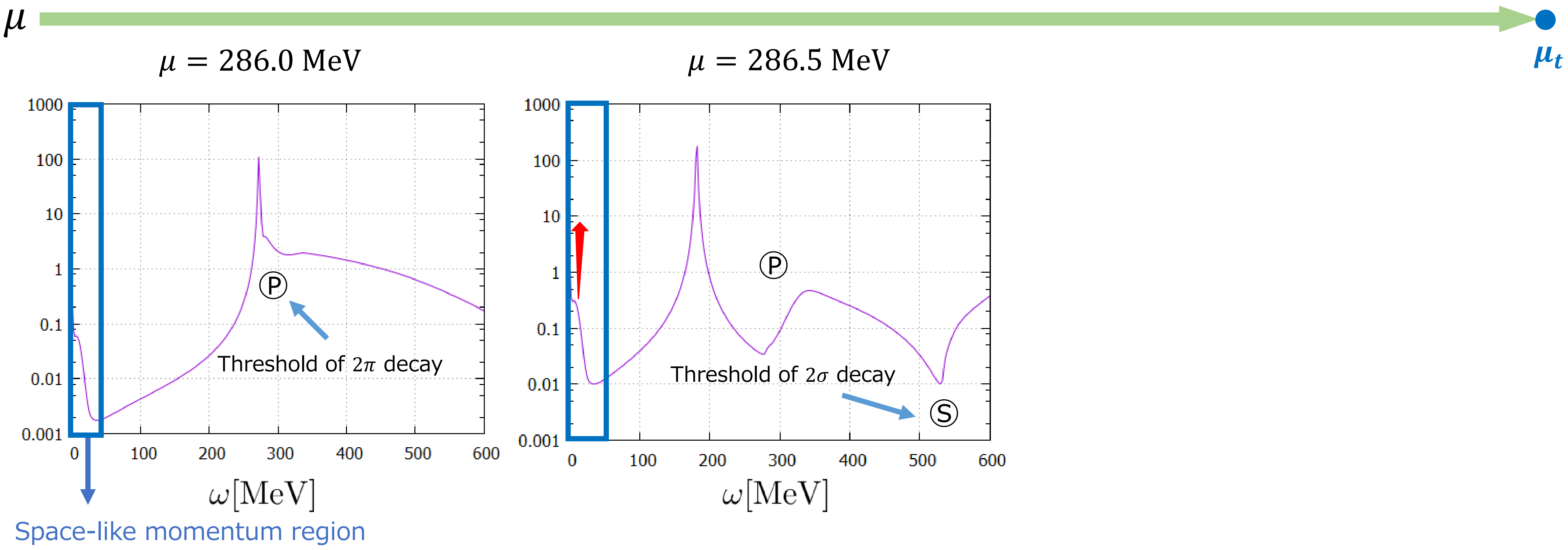
$\mu = 286.0\ \mathrm{MeV}$



Space-like momentum region

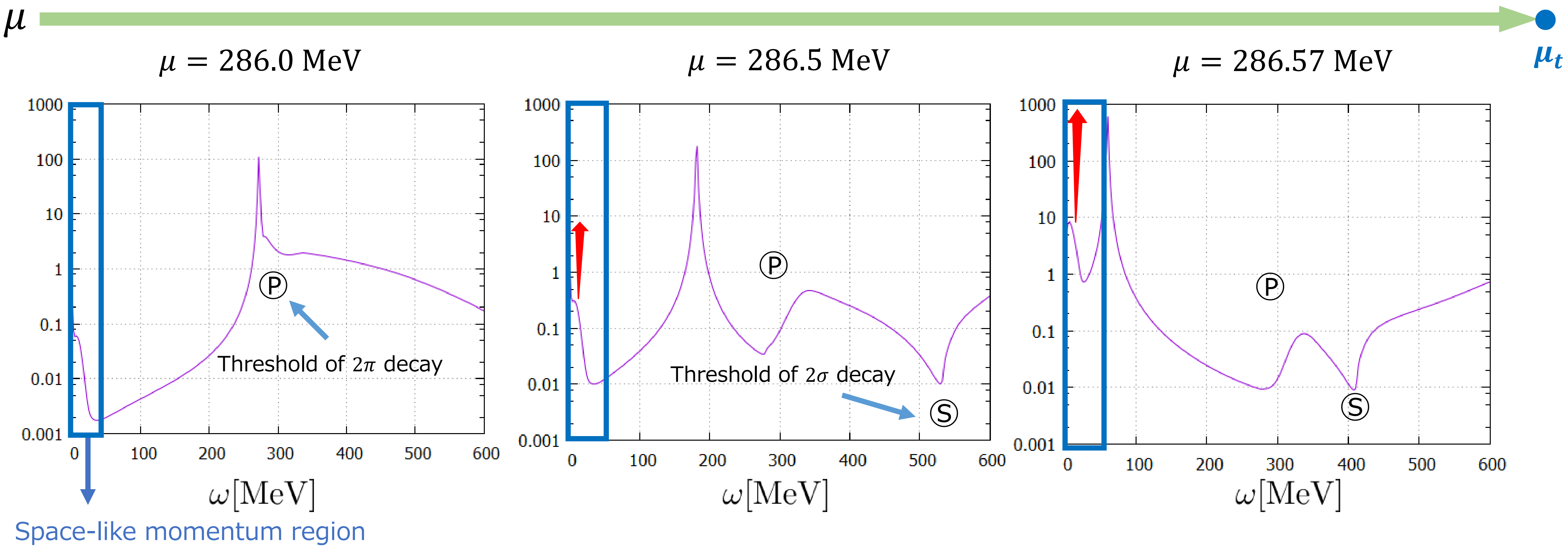
Position and strength of peaks and bumps of ρ_σ near the QCD CP

- We fix $\boldsymbol{T} = \mathbf{5.1\ MeV}$ and $|\vec{\boldsymbol{p}}| = \mathbf{50\ MeV}$. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t} = \mathbf{286.69\ MeV}$)



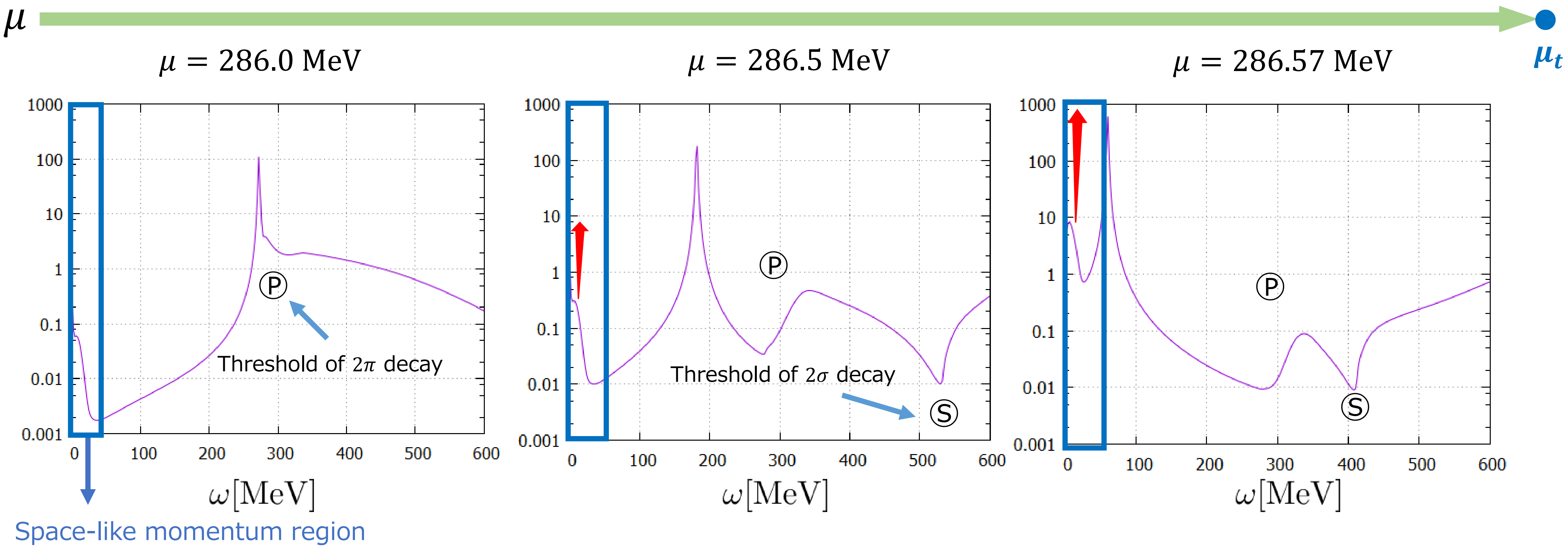
Position and strength of peaks and bumps of ρ_σ near the QCD CP

- We fix $\boldsymbol{T} = \mathbf{5.1\ MeV}$ and $|\vec{\boldsymbol{p}}| = \mathbf{50\ MeV}$. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t} = \mathbf{286.69\ MeV}$)



Position and strength of peaks and bumps of ρ_σ near the QCD CP

- We fix $\boldsymbol{T} = \mathbf{5.1\ MeV}$ and $|\vec{\boldsymbol{p}}| = \mathbf{50\ MeV}$. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t} = \mathbf{286.69\ MeV}$)



- The particle-hole mode becomes soft.

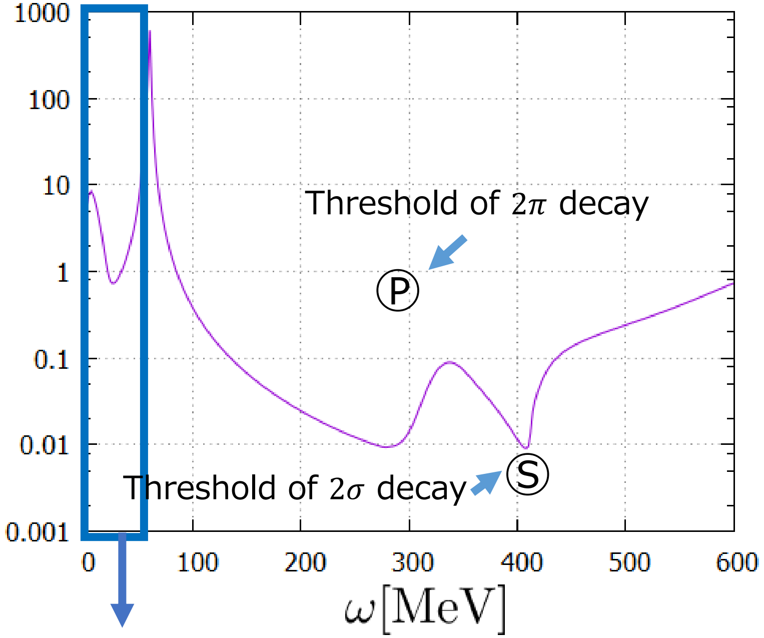
Position and strength of peaks and bumps of ρ_σ near the QCD CP
-In much closer to CP-

- We fix $\boldsymbol{T} = \mathbf{5.1\ MeV}$ and $|\vec{\boldsymbol{p}}| = \mathbf{50\ MeV}$. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t} = \mathbf{286.69\ MeV}$)

μ

$\boldsymbol{\mu_t}$

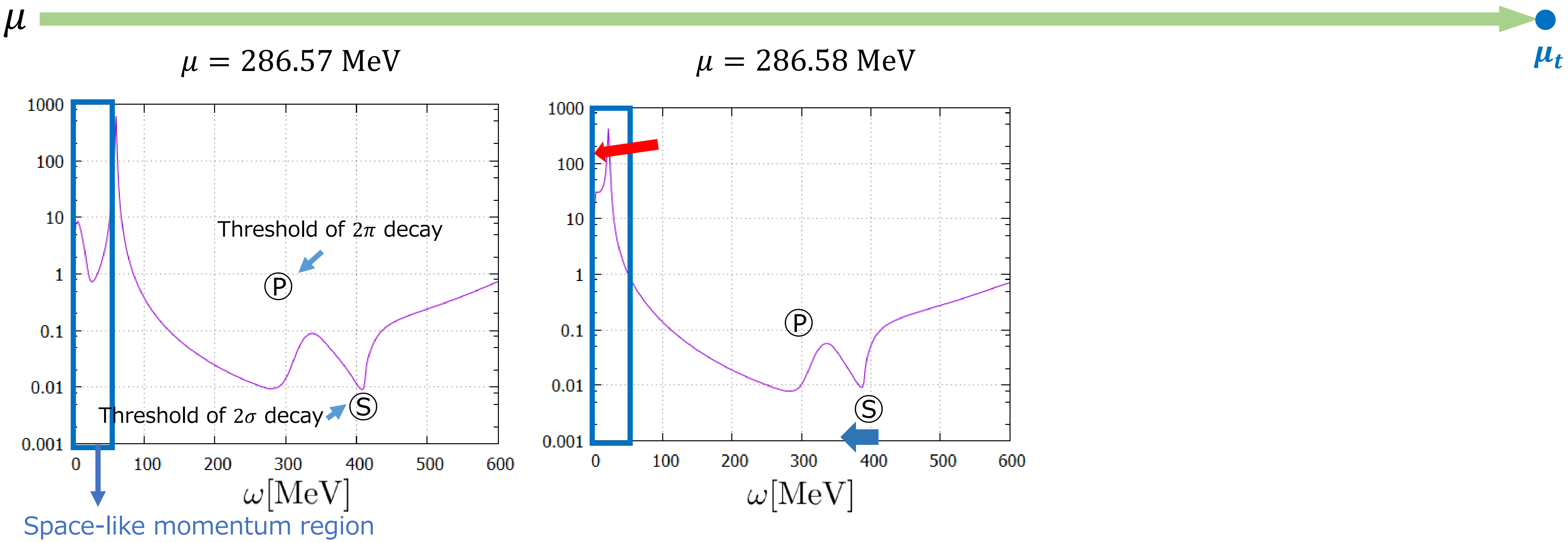
$\mu = 286.57\ \mathrm{MeV}$



Space-like momentum region

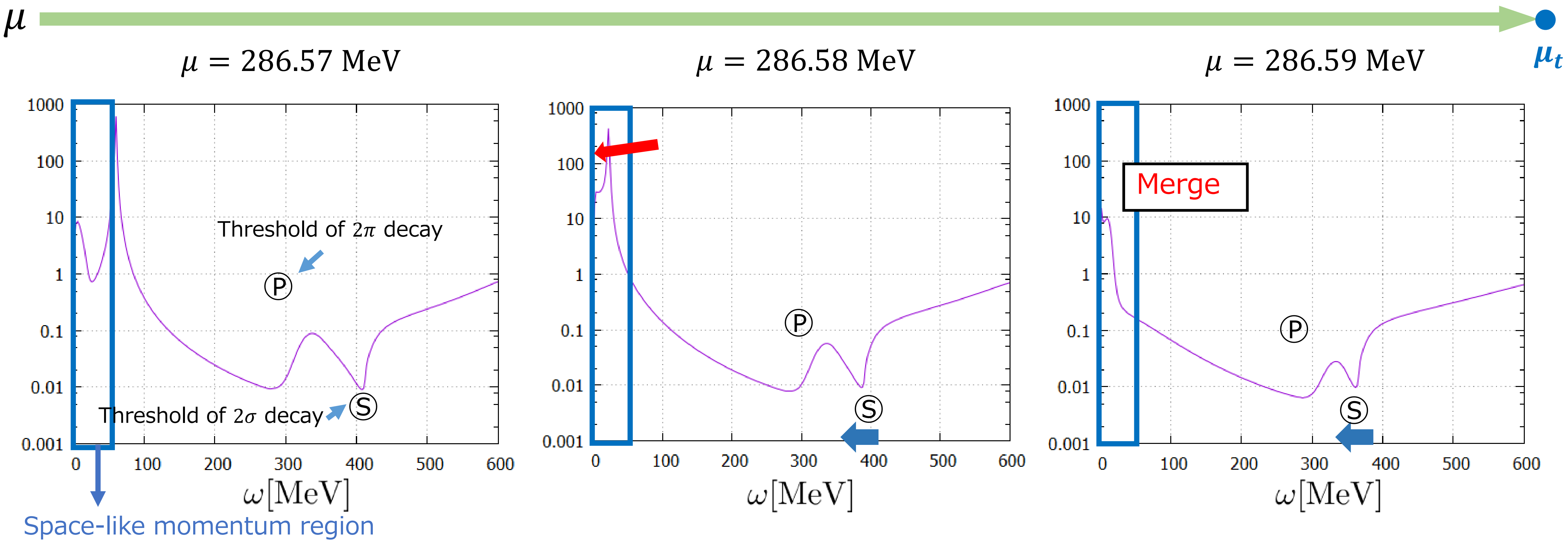
Position and strength of peaks and bumps of ρ_σ near the QCD CP
-In much closer to CP-

- We fix $\boldsymbol{T} = \mathbf{5.1\ MeV}$ and $|\vec{\boldsymbol{p}}| = \mathbf{50\ MeV}$. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t} = \mathbf{286.69\ MeV}$)



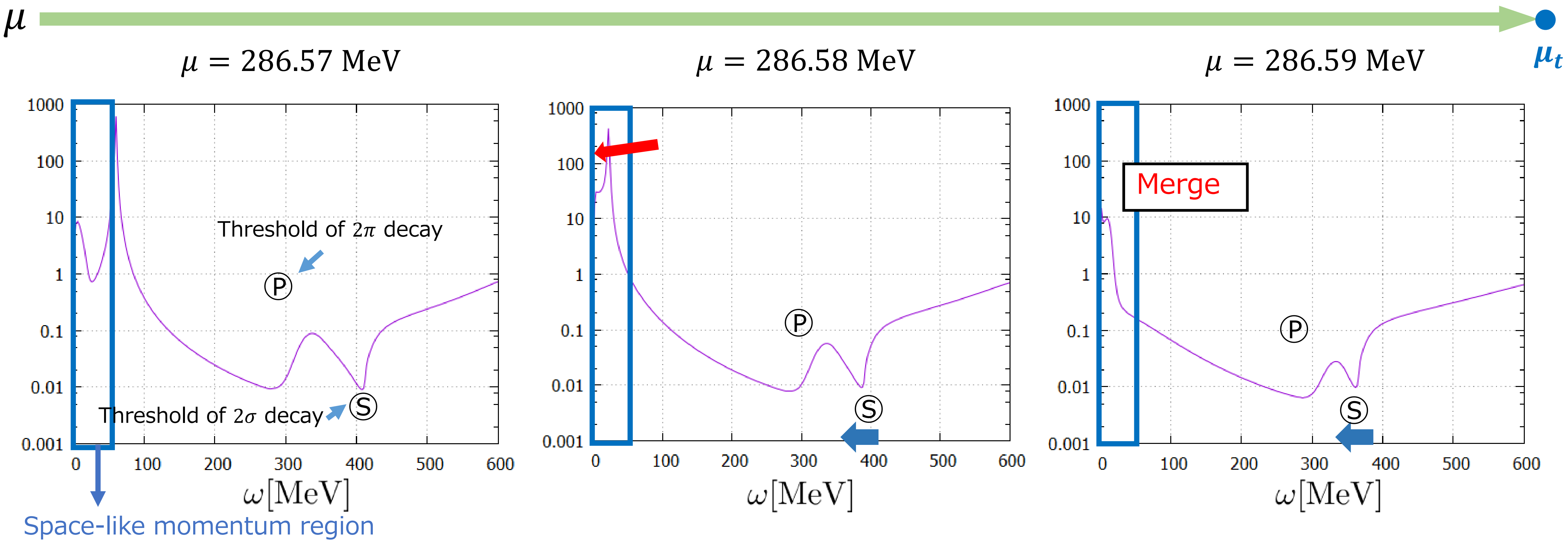
Position and strength of peaks and bumps of ρ_σ near the QCD CP
-In much closer to CP-

- We fix $\boldsymbol{T} = \mathbf{5.1\ MeV}$ and $|\vec{\boldsymbol{p}}| = \mathbf{50\ MeV}$. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t} = \mathbf{286.69\ MeV}$)



Position and strength of peaks and bumps of ρ_σ near the QCD CP
-In much closer to CP-

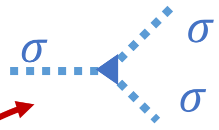
- We fix $\boldsymbol{T} = \mathbf{5.1\ MeV}$ and $|\vec{\boldsymbol{p}}| = \mathbf{50\ MeV}$. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t} = \mathbf{286.69\ MeV}$)

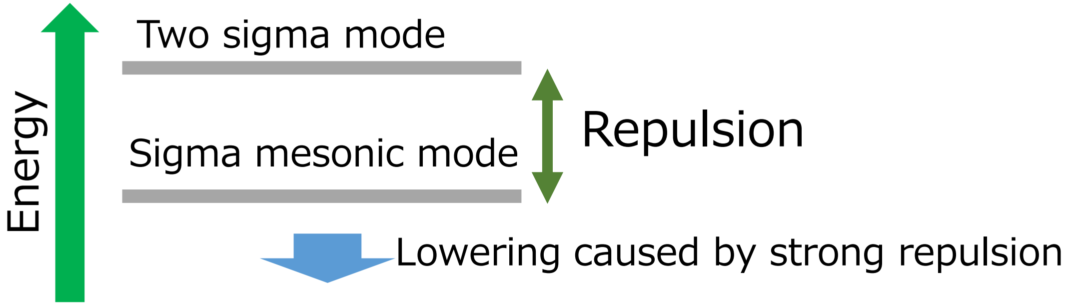


- The sigma mesonic mode penetrates into the space-like region and merges with the bump of the particle–hole mode. → The softening of the sigma mesonic mode
- The threshold of two sigma mode also shifts to lower energy.

Interpretation with level repulsion

- Does the level repulsion between sigma mode and two sigma mode leads to the softening of the sigma mode?

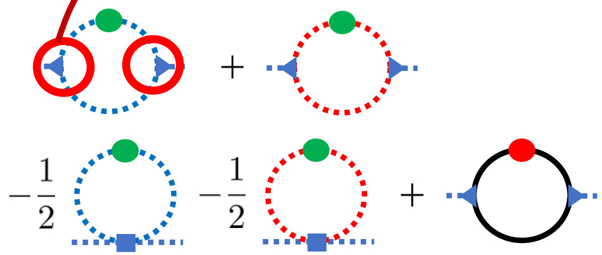


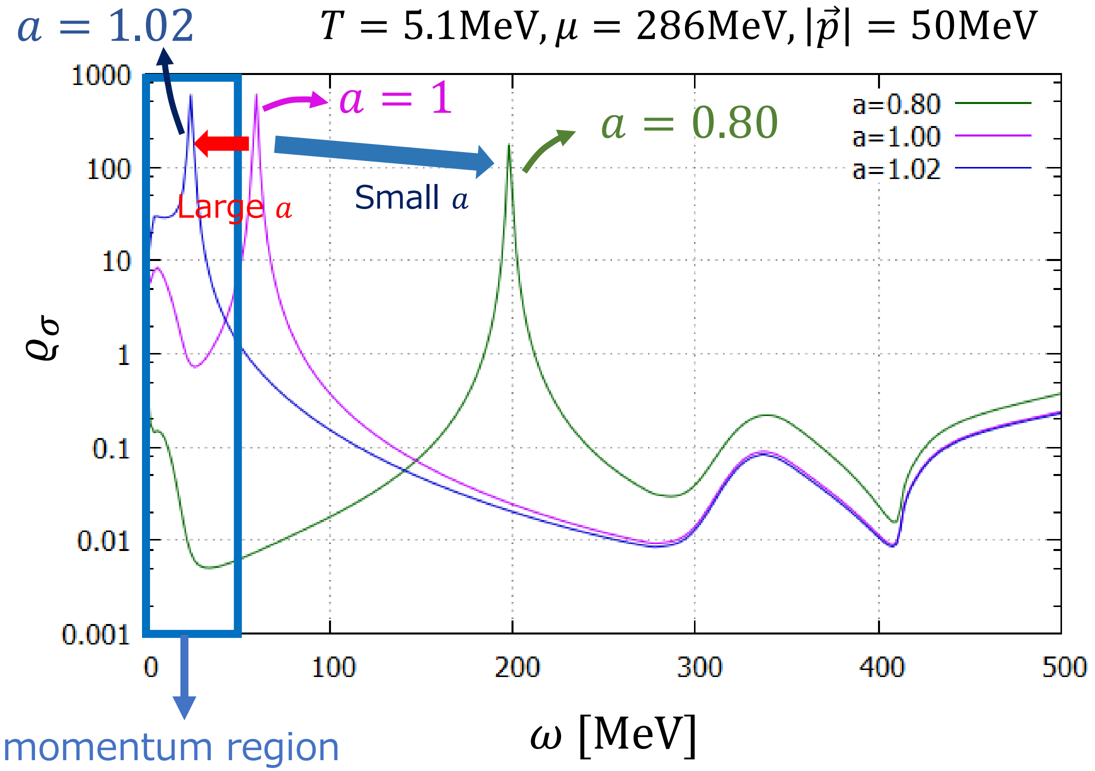


- The three point vertex $\Gamma^{(3)}_{\sigma\sigma\sigma}$ will determine the strength of repulsion between the sigma mesonic and two sigma modes.
  - We have investigated the behavior of ρ_σ when the strength of $\Gamma^{(3)}_{\sigma\sigma\sigma}$ is changed to $a\Gamma^{(3)}_{\sigma\sigma\sigma}$ by hand in the flow equations.

$$\Gamma^{(3)}_{\sigma\sigma\sigma} \to a\Gamma^{(3)}_{\sigma\sigma\sigma}$$

$\partial_k G^{-1}_{\sigma,k}(\omega,\vec{p}) =$





- Actually the strength of $\Gamma^{(3)}_{\sigma\sigma\sigma}$ strongly affects the level of the sigma mesonic mode.

Problem about the group velocity of the sigma mesonic mode

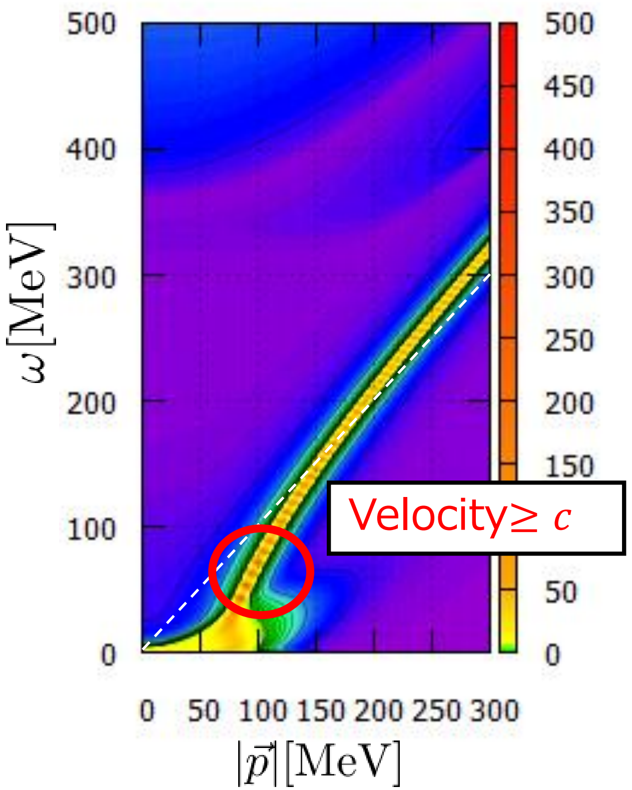
- There is a region where the group velocity of the sigma mesonic mode exceeds the speed of light.

  - One of the possibility of the causes is the employment of LPA.

  → Improvement of the approximation, such as the inclusion of wave-function renormalizations might cure the problem.

  → One of the future works

$T = 5.1$ MeV $\quad \mu = 286.59$MeV

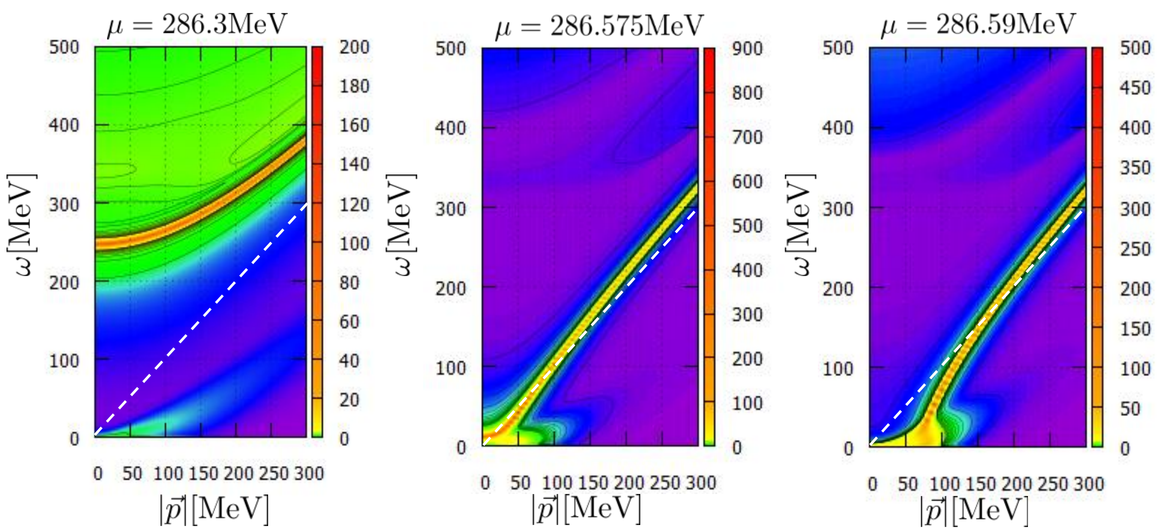


Summary

- We have made an intensive investigation of **sigma channel spectral function** around the QCD CP using **the FRG method** for exploring the nature of the **soft mode**.

- As the system approaches the QCD CP, ρ_σ in the space-like momentum region enhances.

  → The softening of the particle-hole mode



- In addition, the sigma mesonic mode penetrates into the space-like momentum region and merges with the bump of the particle–hole mode.

  → This suggests that **the sigma mesonic mode also becomes soft** at the QCD CP.

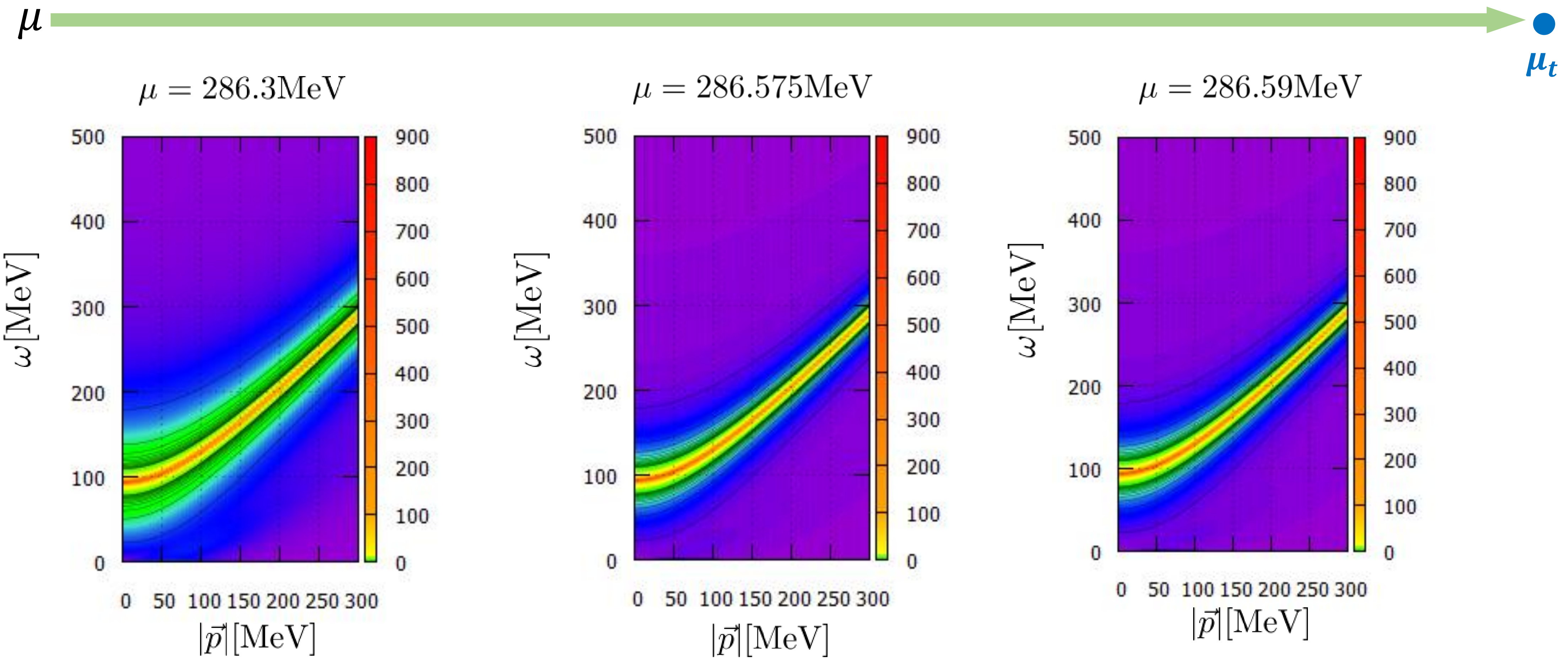
- Future work

Calculating the spectral functions using more improved method than LPA
(e.g. the inclusion of wave-function renormalization)
to confirm the pictures and investigate the cause of the problem of the group velocity.

Back up

Contour maps of the pion channel spectral function ρ_π near the QCD CP

- We fix $\boldsymbol{T} = \mathbf{5.1}$ **MeV** below. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t} = \mathbf{286.69}$ **MeV**)

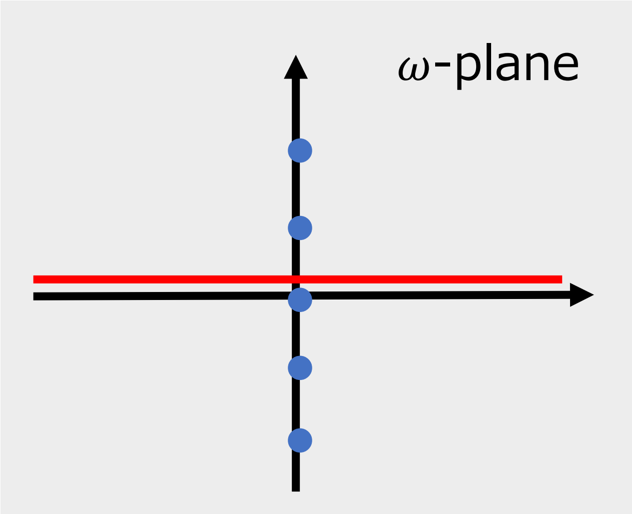


Analytical continuation

R. Tripolt, N. Strodthoff, L. Smekal, J. Wambach, PRD89 (2014); R. Tripolt, L. Smekal, J. Wambach, PRD90 (2014)
K. Kamikado, N. Strodthoff, L. Smekal, J. Wambach, EPJ C74 (2014)

- In our case, analytic continuation can be realized easily in the flow eq. after Matsubara sum.

  - This is possible when frequency independent regulator are used.
  - The replacement $n_{B,F}(E+i\omega_n) \to n_{B,F}(E)$ $(\omega_n = 2\pi n T)$ before $i\omega_n \to \omega + i\varepsilon$ is needed to realize the analyticity in the upper-half plane.



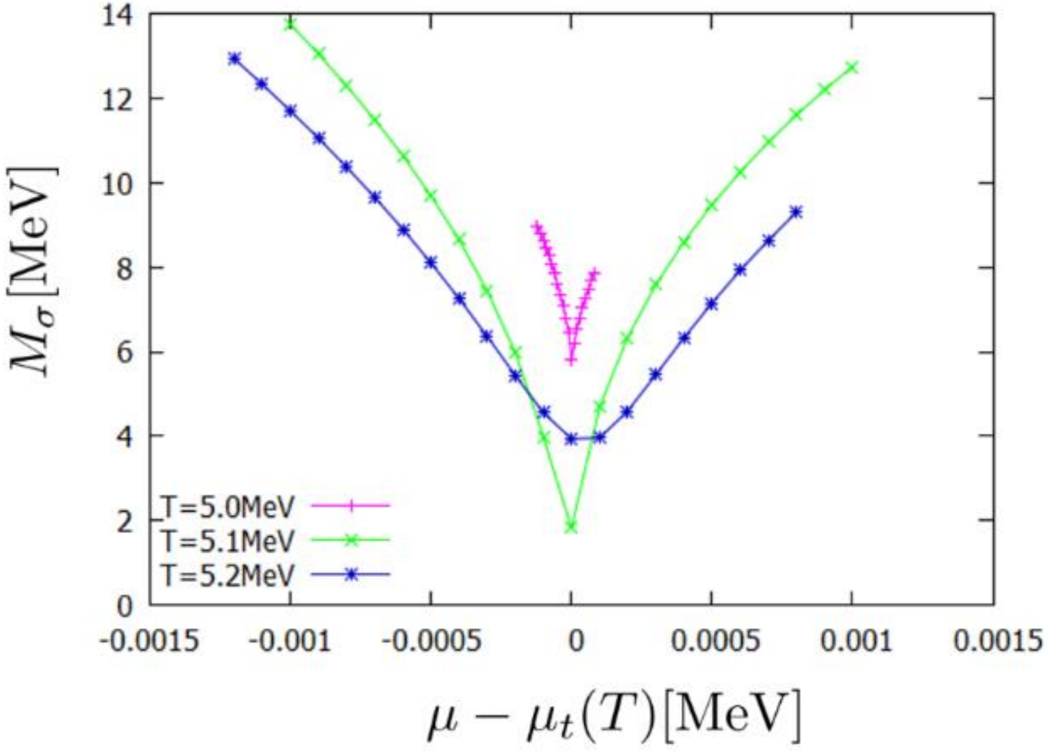
- Spectral function

$$\rho_{ab}(\omega,\vec{p}) = 2\mathrm{Im}G_R^{ab}(\omega,\vec{p}) = 2\mathrm{Im}\frac{1}{\Gamma^{(2)}_{ab}(\omega,\vec{p})}$$

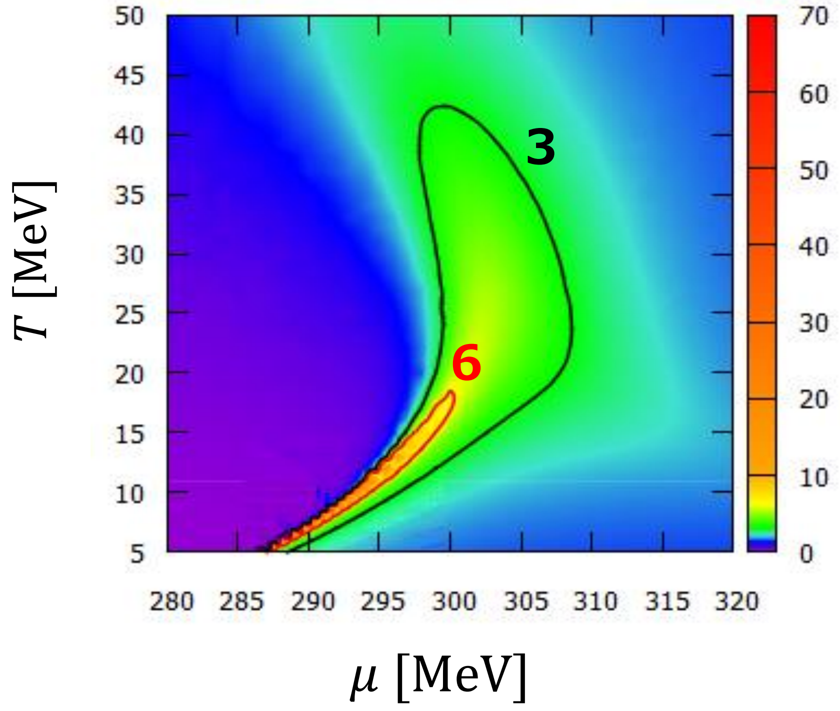
$$\frac{\delta^2\Gamma[\phi]}{\delta\phi_a(p)\delta\phi_b(p')} = (2\pi)^4\delta^{(4)}(p+p')\Gamma^{(2)}_{ab}(p)$$

The location of the QCD CP

M_σ



Quark number susceptibility
(normalized by those of a free quark gas)



- Quark number susceptibility enhances near the QCD CP as well as the chiral susceptibility M_σ^{-2}.