

Modeling the hadronization processes in HIC (based on the Nambu Jona-Lasinio Lagrangian)

in collaboration with
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CPOD 2016 , Wrocław, Poland May 30th - June 4th, 2016

Motivation:

How to study phase transitions at finite chemical potential (NICA, FAIR)

Lattice results only reliable for $\mu/T \ll 1$:

- ❑ assumptions about continuation to finite μ
- ❑ effective theories which allow for such an extension intrinsically

The **Nambu Jona Lasinio Lagrangian** is such an effective field theory

- ❑ allows for **predictions for finite T** and μ
- ❑ needs as **input only vacuum values** + YM Polyakov loop
- ❑ **shares the symmetries** with the QCD Lagrangian
- ❑ can be « **derived** » from **QCD** Lagrangian



Nambu



Jona-Lasinio

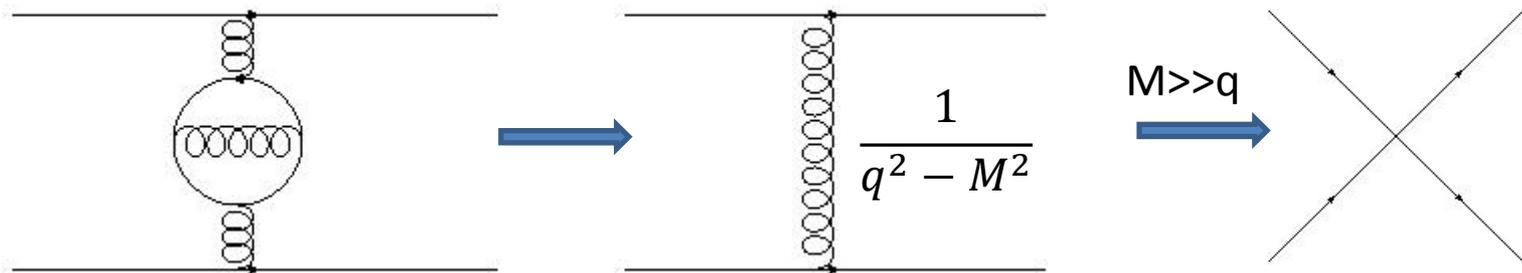
- How one does obtain the NJL Lagrangian?
- How to construct mesons Mesons and Baryons?
- Cross section for elastic scattering and hadronisation
- Expanding plasma: How quarks hadronize?
- Realistic simulations

Conserving the QCD symmetries in an effective Lagrangian

- 1) local $SU_c(3)$ color gauge transformation (by construction)
- 2) global $SU_f(3)$ flavor symmetry
- 3) for massless quarks ONLY:
chiral invariance of QCD Lagrangian: $SU_f(3)_V \times SU_f(3)_A$

However, chiral symmetry is spontaneously broken since quarks have non-zero masses.

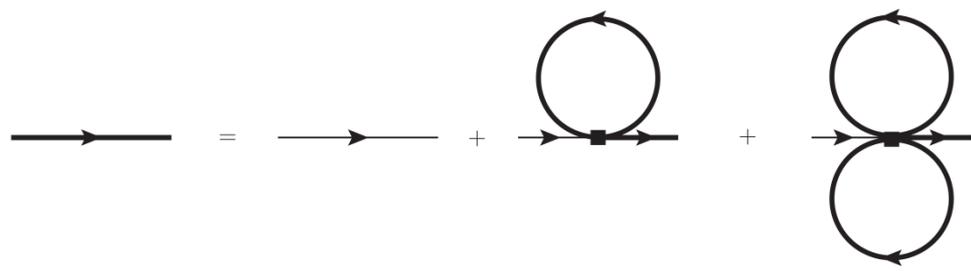
⇒ An *effective Lagrangian* with the **same symmetries** for the quark degrees of freedom can be obtained by discarding the gluon dynamics completely.



NJL Lagrangian

$$\mathcal{L}_{\text{NJL}} = \bar{\Psi}_i (i\gamma_\mu \partial^\mu - \hat{M}_0) \Psi_i - G_c^2 [\bar{\Psi}_i \gamma^\mu \mathbf{T}^a \delta_{ij} \Psi_j] [\bar{\Psi}_k \gamma_\mu \mathbf{T}^a \delta_{kl} \Psi_l] \\ + \mathbf{H} \det_{ij} [\bar{\Psi}_i (1 - \gamma_5) \Psi_j] - \mathbf{H} \det_{ij} [\bar{\psi}_i (1 + \gamma_5) \psi_j]$$

\mathcal{L}_{NJL} : Shares the symmetries with the QCD Lagrangian (color we discuss later)
 Allows for calculating **effective quark masses**:



$$\mathbf{M} = \hat{M}_0 - 4G \langle \bar{\psi} \psi \rangle + 2\mathbf{H} \langle \bar{\psi}' \psi' \rangle \langle \bar{\psi}'' \psi'' \rangle$$

But contains only quarks

no gluons and

no hadrons

So not very obvious how of use for hadronisation.

Polyakov NJL: gluons on a static level

Eur.Phys.J. C49 (2007) 213-217

It is not possible to introduce gluons as dynamical degrees of freedom without spoiling the simplicity of the NJL Lagrangian which allows for real calculations

but

one can introduce gluons through an effective potential for the Polyakov loop

$$\frac{U(\mathbf{T}, \Phi, \bar{\Phi})}{\mathbf{T}^4} = -\frac{b_2(\mathbf{T})}{2} \bar{\Phi} \Phi - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} (\bar{\Phi} \Phi)^3$$

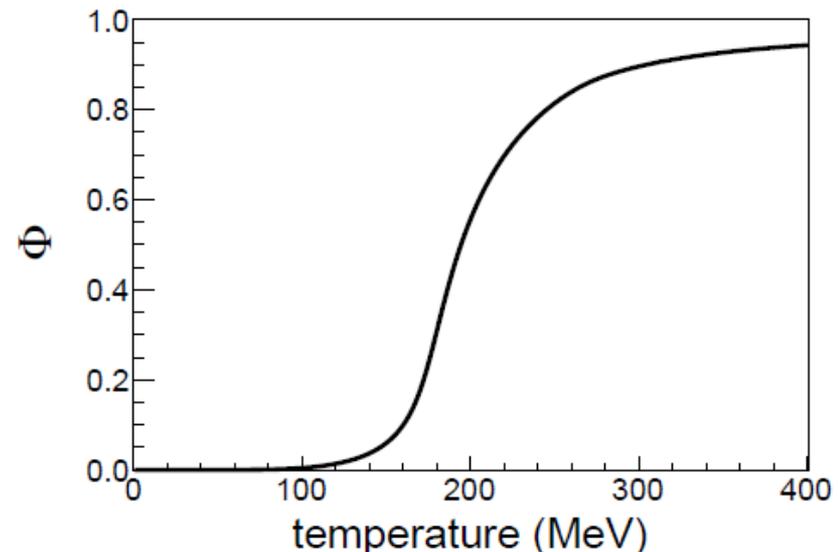
$$b_2(\mathbf{T}) = a_0 + a_1 \frac{\mathbf{T}_0}{\mathbf{T}} + a_2 \left(\frac{\mathbf{T}_0}{\mathbf{T}} \right)^2 + a_3 \left(\frac{\mathbf{T}_0}{\mathbf{T}} \right)^3$$

$$a_0 = 6.75, a_1 = -1.95, a_2 = 2.625, a_3 = -7.44, b_3 = 0.75, b_4 = 7.5$$

Parameters-> right pressure in the SB limit

Φ is the order parameter of the deconfinement transition

$$\Phi = \frac{1}{N_c} \text{Tr}_c \left\langle \mathbf{P} \exp \left(- \int_0^\beta d\tau \mathbf{A}_0(\mathbf{x}, \tau) \right) \right\rangle$$

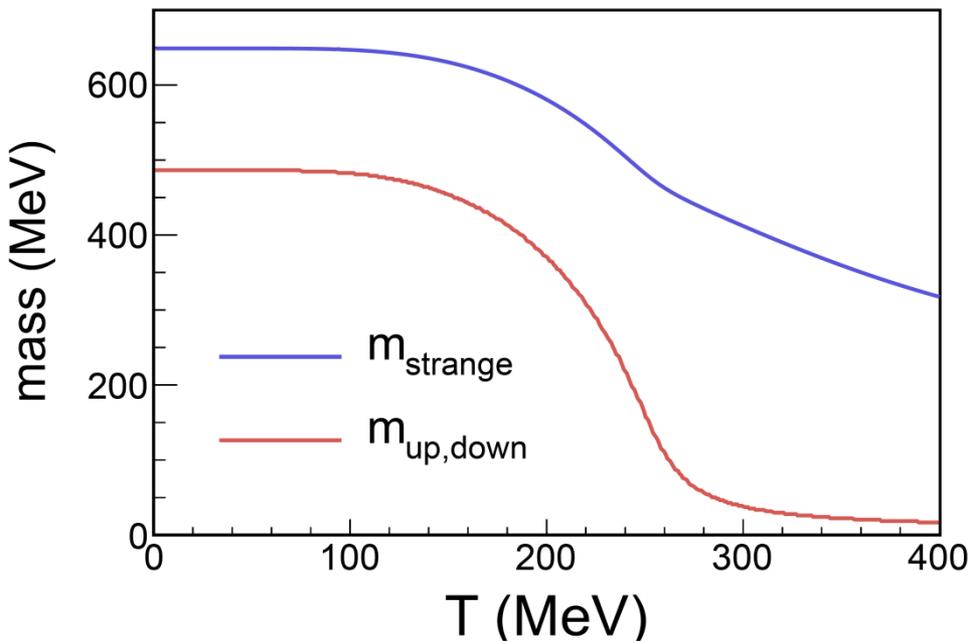


Quark Masses in NJL and PNJL

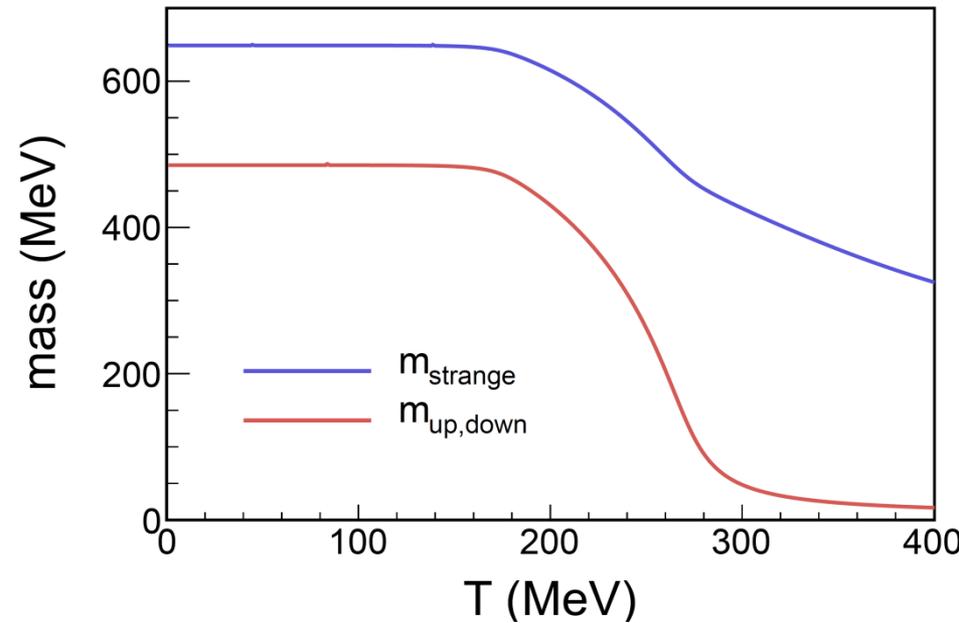
Quark masses are obtained by minimizing the grand canonical potential

$$M = \hat{M}_0 - 4G \langle \bar{\psi}\psi \rangle + 2H \langle \bar{\psi}'\psi' \rangle \langle \bar{\psi}''\psi'' \rangle$$

NJL



PNJL



In PNJL the transition is steeper than in NJL

How can we get mesons?

Quarks are the degrees of freedom of the Lagrangian

To study the phase transition we need mesons

Use a Trick : Fierz transformation of the original Lagrangian

Fierz Transformation allows for a reordering of the field operators in 4 point contact interactions. It is simultaneously applied in Dirac, colour and flavour space

Example in Dirac space:

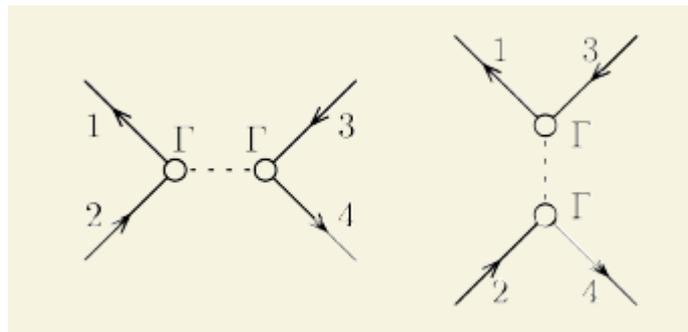
$$(\bar{\chi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\chi) = (\bar{\chi}\chi)(\bar{\psi}\psi) - \frac{1}{2}(\bar{\chi}\gamma^\mu\chi)(\bar{\psi}\gamma_\mu\psi) - \frac{1}{2}(\bar{\chi}\gamma^\mu\gamma_5\chi)(\bar{\psi}\gamma_\mu\gamma_5\psi) - (\bar{\chi}\gamma_5\chi)(\bar{\psi}\gamma_5\psi)$$

Scalar

vector

pseudovector

pseudoscalar



How can we get mesons? II

$$\mathcal{L}_{int} = -G_c^2 [\bar{\Psi}_i \gamma^\mu T^a \delta_{ij} \Psi_j] [\bar{\Psi}_k \gamma_\mu T^a \delta_{kl} \Psi_l]$$

Fierz transformation transforms original Lagrangian to one for mesons

$$\mathcal{L}_{\text{Pseudo scalar}} = G (\bar{\Psi}_i \tau_{il}^a \mathbb{1}_c i\gamma_5 \Psi_l) (\bar{\Psi}_k \tau_{kj}^a \mathbb{1}_c i\gamma_5 \Psi_j) ; \quad G = \frac{N_c^2 - 1}{N_c^2} G_c$$



Singulet in color mixing of flavour

Similar terms can be obtained for

Vector mesons γ_μ

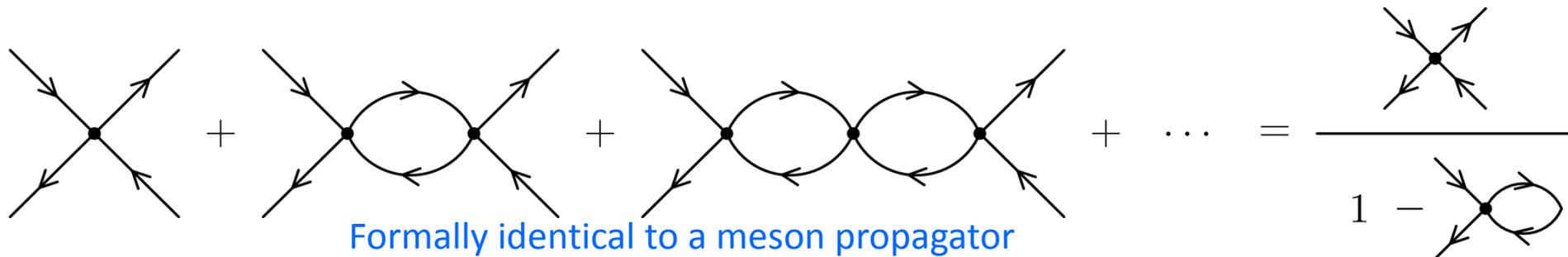
Scalar Mesons 1

Pseudovector mesons $\gamma_\mu \gamma_5$

How can we get mesons? III

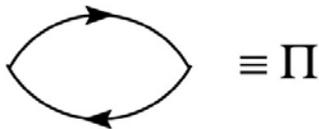
We use \mathcal{K} as a kernel for a Bethe-Salpeter equation (relativistic Lippmann-Schwinger eq.)

$$\mathbf{T}(\mathbf{p}) = \mathcal{K} + \mathbf{i} \int \frac{d^4\mathbf{k}}{(2\pi)^4} \mathcal{K} \mathbf{S}\left(\mathbf{k} + \frac{\mathbf{p}}{2}\right) \mathbf{S}\left(\mathbf{k} - \frac{\mathbf{p}}{2}\right) \mathbf{T}(\mathbf{p})$$



In (P)NJL one can sum up this series analytically:

$$\mathbf{T}(\mathbf{p}) = \frac{2\mathbf{G}_{\text{eff}}}{\mathbf{1} - 2\mathbf{G}_{\text{eff}}\mathbf{\Pi}(\mathbf{p})}, \quad \mathbf{\Pi}(\mathbf{p}_0, \mathbf{p}) = -\frac{1}{\beta} \sum_{\mathbf{n}} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \mathbf{\Omega} \mathbf{S}\left(\mathbf{k} + \frac{\mathbf{p}}{2}\right) \mathbf{\Omega} \mathbf{S}\left(\mathbf{k} - \frac{\mathbf{p}}{2}\right)$$

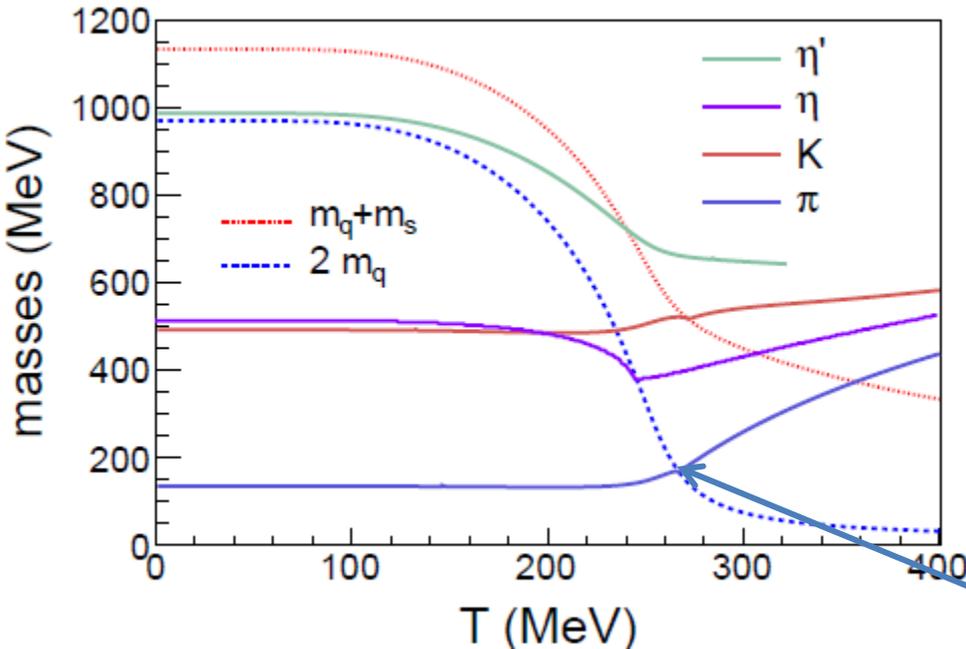


$$\mathcal{K} = \mathbf{\Omega} 2\mathbf{G}_{\text{eff}} \bar{\mathbf{\Omega}} \quad \mathbf{\Omega} = \mathbf{1}_c \otimes \tau^a \otimes \{\mathbf{1}, \mathbf{i}\gamma_5, \gamma_\mu, \gamma_5\gamma_\mu\}$$

How to get mesons? IV

The **meson pole mass** and the **width** one obtains by solving:

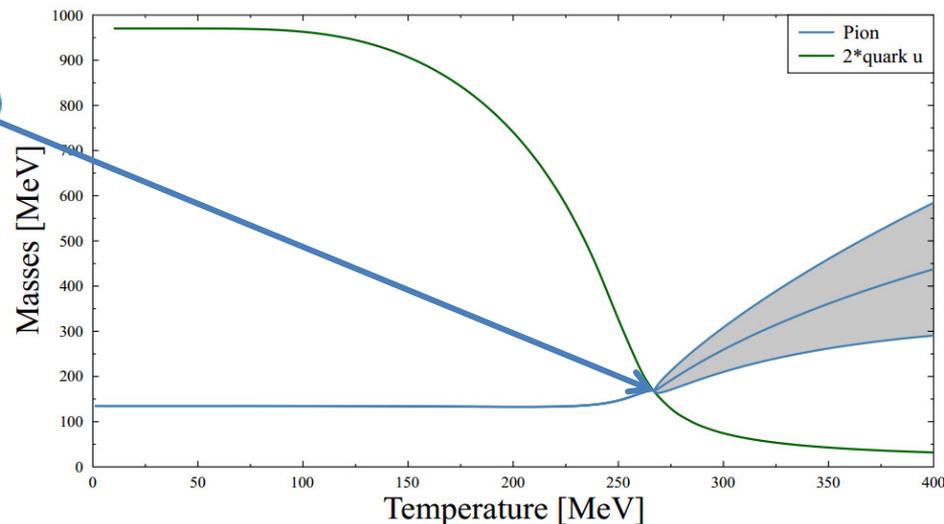
$$1 - 2G_{\text{eff}} \Pi(p_0 = M_{\text{meson}} - i\Gamma_{\text{meson}}/2, \mathbf{p} = \mathbf{0}) = 0$$



masses of pseudoscalar mesons
and of quarks at $\mu = 0$

At $T=0$ physical and calculated mass
agree quite well

When mesons become unstable they
develop a width



Looking back

We have seen that the NJL model describes quite well meson properties
For this one has to fix the 5 parameters of the model

Λ = upper cut off of the internal momentum loops

G_c = coupling constant

M_0 = bare mass of u,d and s quarks

H = coupling constant 't Hooft term

These parameters have been adjusted to reproduce

Masses of π and K in the vacuum, as well as the η - η' mass splitting
 π decay constant, $q\bar{q}$ condensate (-241 MeV)³

Therefore:

All properties

Of masses, cross sections etc. at finite μ and T

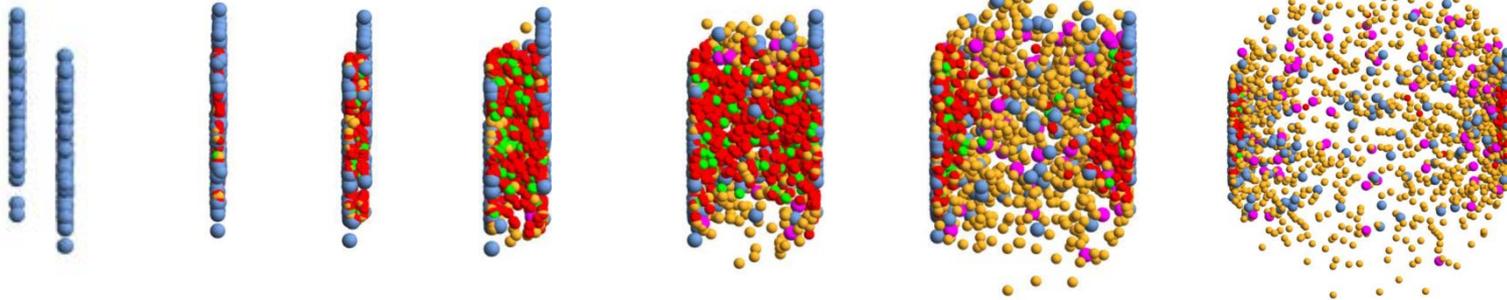
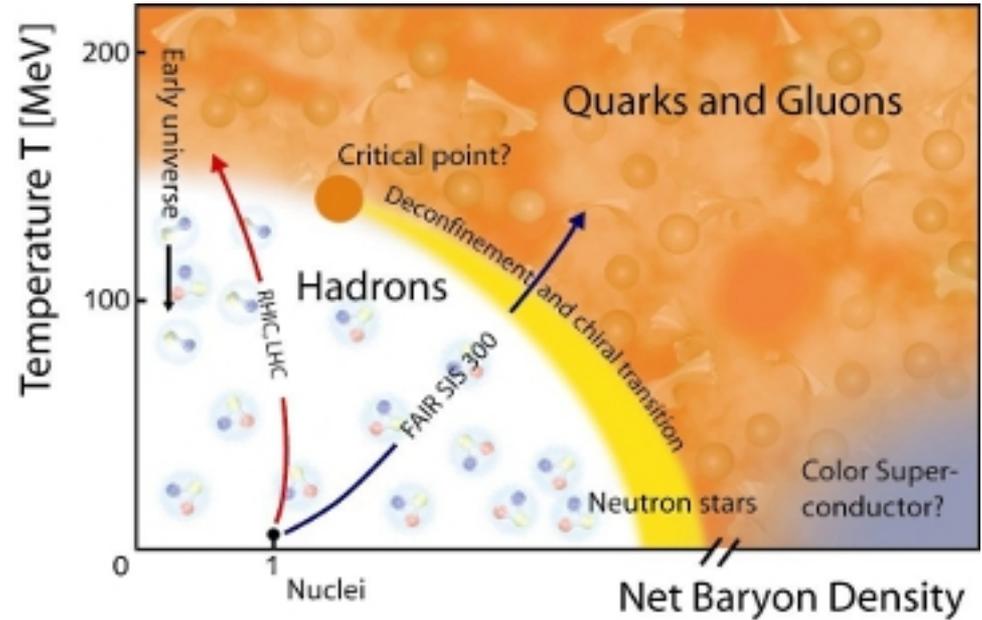
follow without any new parameters from ground state observables.

Baryons can be calculated in an analogous way.

circumstantial evidence:

For beam energies $> \approx 100$ AGeV
a plasma of quark and gluons (QGP)
is formed

The challenge:
How to come from quarks to
hadrons



- Antibaryons (229)
- Mesons (3661)
- Quarks (1499)
- Gluons (175)

As PHSD calculations see a heavy ion reaction
is there local equilibrium?

Courtesy:
P. Moreau 2015

Masses close to the tricritical point

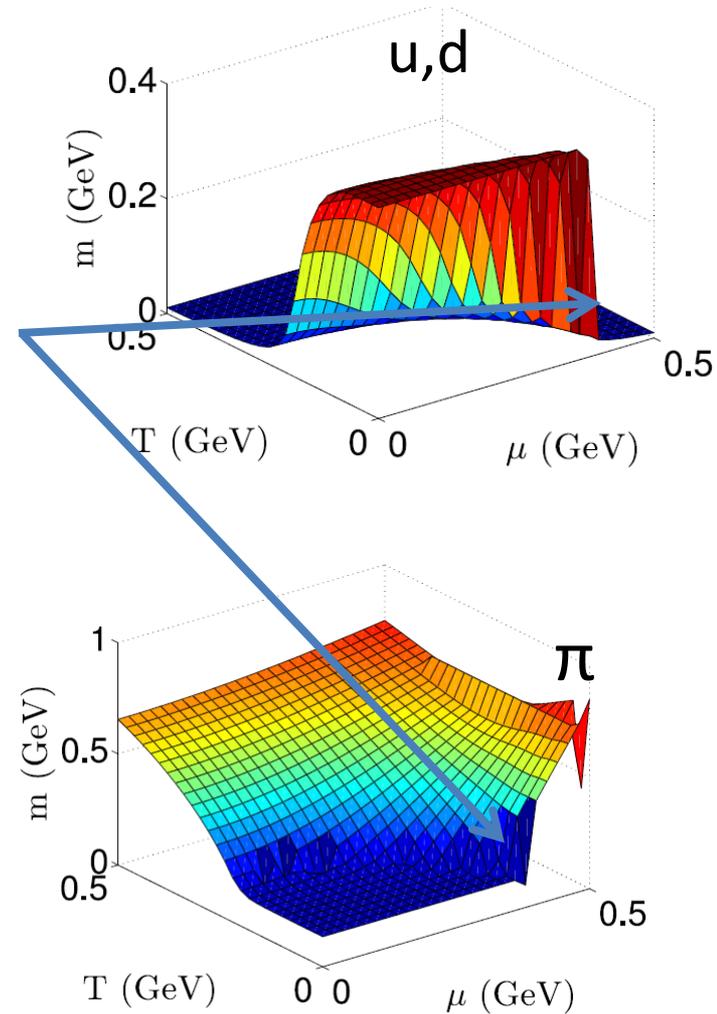
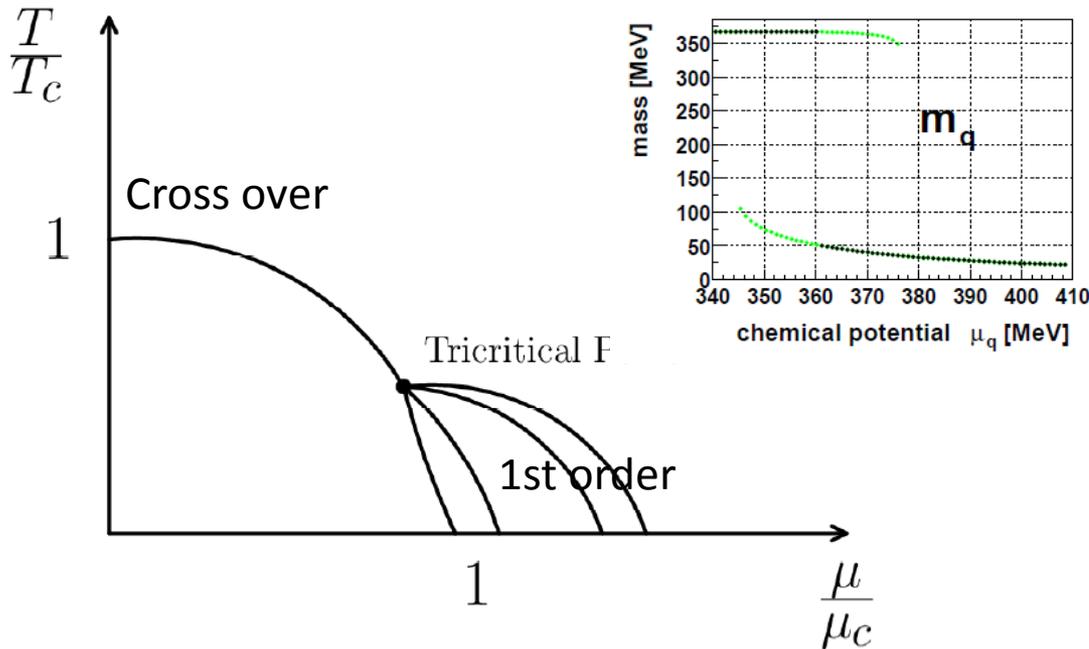
NJL Lagrangian:

transition between quarks and hadrons

Cross over at $\mu = 0$

1st order transition $\mu \gg 0$

sudden change of q and meson mass

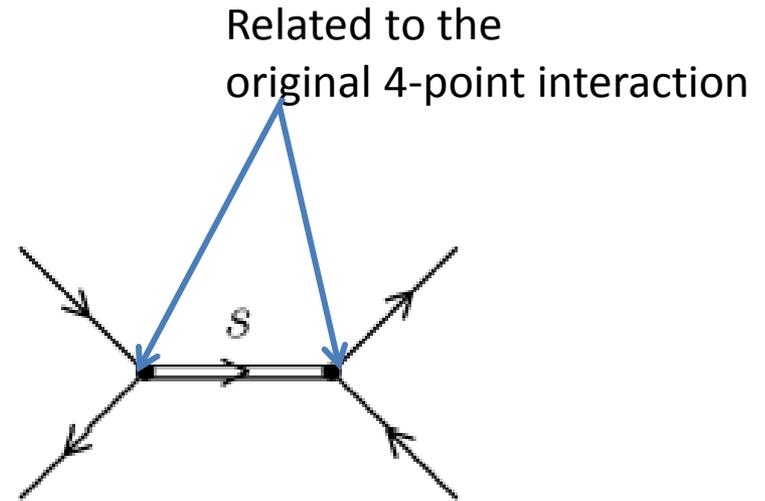
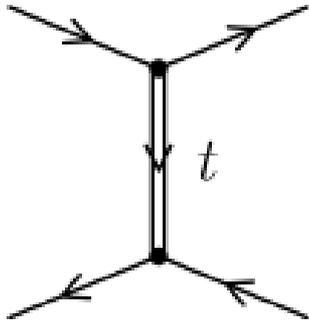


Details have not been explored yet

Cross sections

Having the Lagrangian we can derive in the usual way the Feynman rules and can calculate cross sections

Example: $u\bar{u} \rightarrow u\bar{u}$ matrix elements



But also

elastic cross sections like $uu \rightarrow uu$

hadronization cross sections $q\bar{q} \rightarrow MM$ $M=\pi, K, \eta, \eta', \rho \dots$

hadronization cross sections $Diq Diq \rightarrow$ baryons + q etc

$u\bar{u} \rightarrow u\bar{u}$

Cross sections

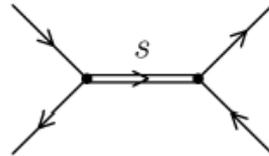
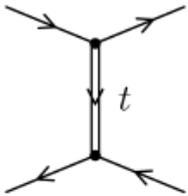
Phys.Rev. C53 (1996) 410-429

$$\begin{aligned}
 -i\mathcal{M}_s &= \delta_{c_1,c_2}\delta_{c_3,c_4}\bar{v}(p_2)Tu(p_1) \left[i\mathcal{D}_s^S(p_1+p_2) \right] \bar{u}(p_3)Tv(p_4) \\
 &+ \delta_{c_1,c_2}\delta_{c_3,c_4}\bar{v}(p_2)(i\gamma_5 T)u(p_1) \left[i\mathcal{D}_s^P(p_1+p_2) \right] \bar{u}(p_3)(i\gamma_5 T)v(p_4)
 \end{aligned}$$

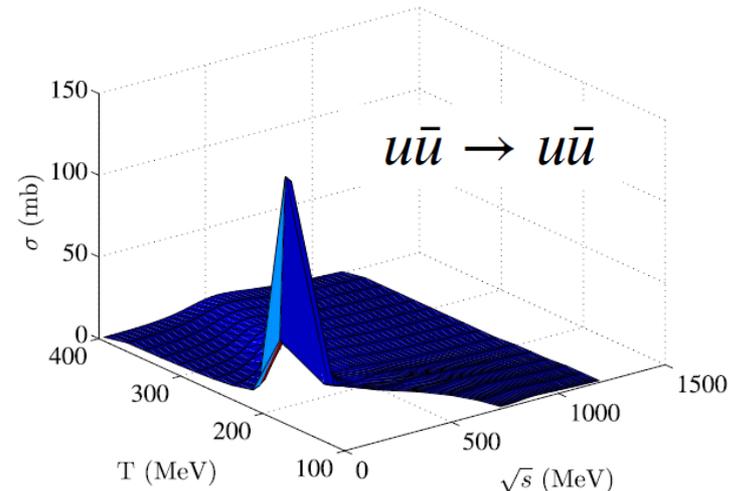
D= meson propagator

$$D(p_0, \vec{p}) \propto \frac{2G}{1 - 2G\Pi(p_0, \vec{p})}$$

$$\begin{aligned}
 -i\mathcal{M}_t &= \delta_{c_1,c_3}\delta_{c_2,c_4}\bar{u}(p_3)Tu(p_1) \left[i\mathcal{D}_t^S(p_1-p_3) \right] \bar{v}(p_2)Tv(p_4) \\
 &+ \delta_{c_1,c_3}\delta_{c_2,c_4}\bar{u}(p_3)(i\gamma_5 T)u(p_1) \left[i\mathcal{D}_t^P(p_1-p_3) \right] \bar{v}(p_2)(i\gamma_5 T)v(p_4) \quad .
 \end{aligned}$$

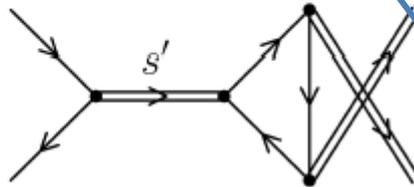
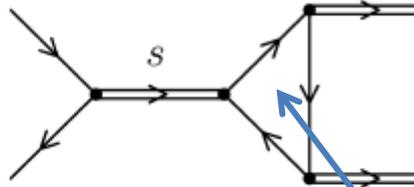
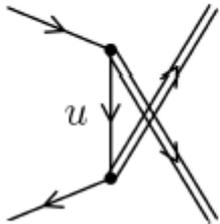
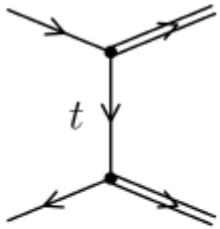


Cross section up to 100 mb
close to cross over
due to resonant s-channel
otherwise small (5-10 mb)



Hadronization cross sections

$$q\bar{q} \rightarrow MM$$



$$-iM_s = g_{Mqq'}^2 f_s \bar{v}_2 u_1 \Gamma_\nu (i\mathcal{D}^s_M) \Gamma_{q_1 q_2 q_3}^\nu + \dots$$

$$-iM_t = g_{Mqq'}^2 f_t \bar{v}_2 \Gamma_\nu \frac{i(\not{p}_1 - \not{p}_3 + m_t)}{(p_1 - p_3)^2 - m_t^2} \Gamma^\nu u_1$$

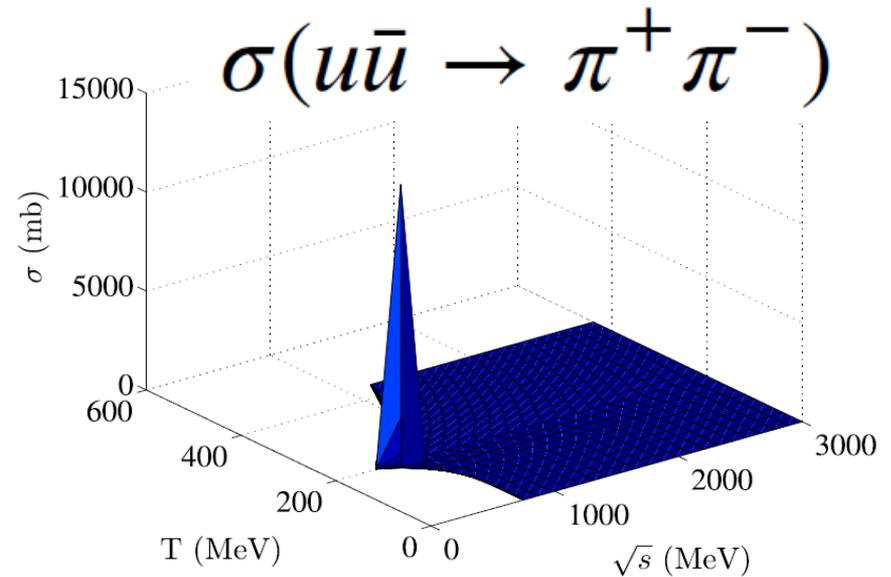
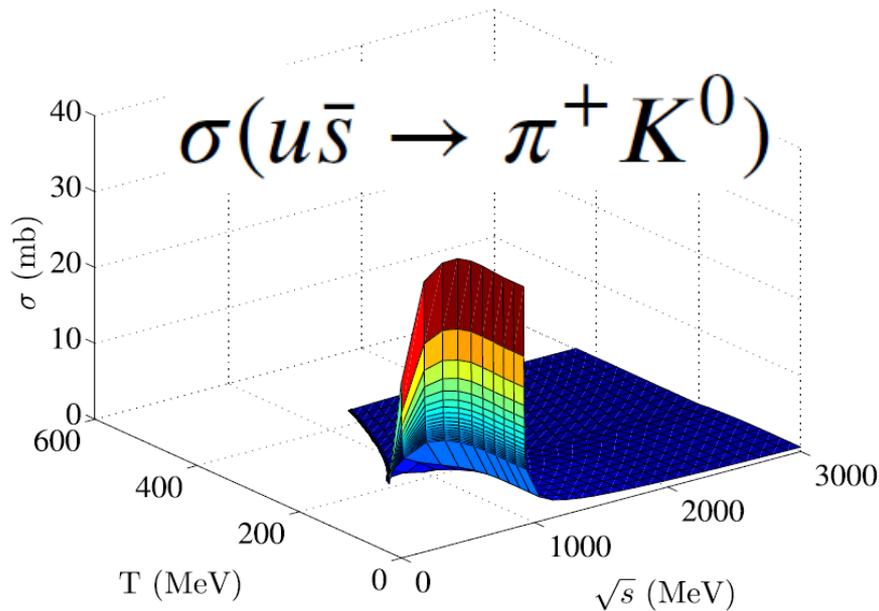
$$-iM_u = g_{Mqq'}^2 f_u \bar{v}_2 \Gamma_\nu \frac{i(\not{p}_1 - \not{p}_4 + m_t)}{(p_1 - p_4)^2 - m_t^2} \Gamma^\nu u_1$$

$\Gamma_{q_1 q_2 q_3}$ triangle vertex

Γ_ν appropriate γ matrix

Hadronization cross sections

These s-channel resonances create as well very large hadronization cross section close to T_c



Consequence:

If an expanding plasma comes to T_c **quarks are converted into hadrons**

despite of the NJL Lagrangian does not contain confinement

How to make a transport theory out of NJL

Using 7 parameters fitted to ground state properties of mesons and baryons
the NJL model allows for calculating

Quark masses (T, μ)

Hadron masses (T, μ)

Elastic cross sections (T, μ)

Hadronization cross sections (T, μ)

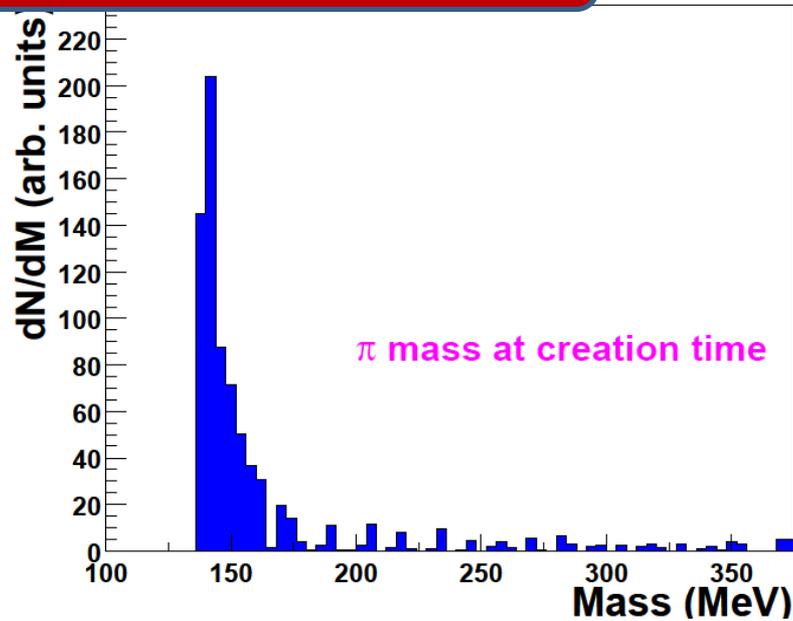
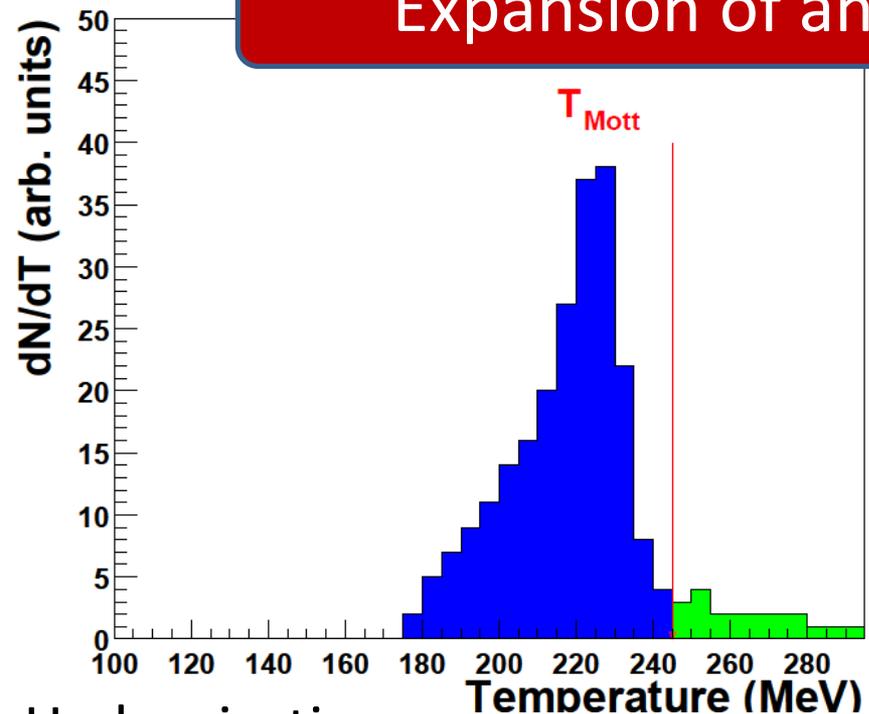
So we have all ingredients for a transport theory

Problem:

With a mass of 2 MeV and temperatures > 200 MeV the quarks
move practically with the speed of light.

So we have to construct a fully relativistic transport theory (all details in
Phys.Rev. C87 (2013) , 034912)

Expansion of an equilibrated plasma

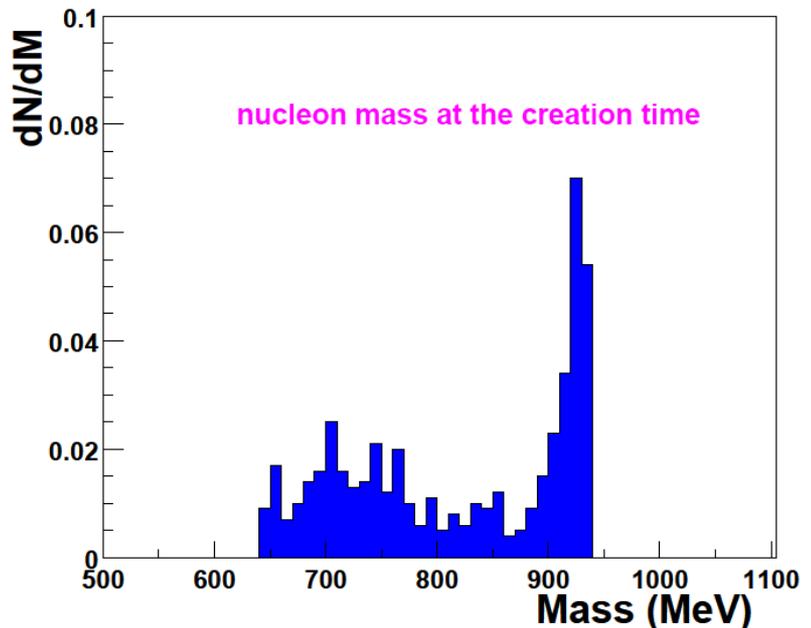


Hadronization:

Not at a fixed T but broad T distribution

Particles are produced over a wide mass range

Come to vacuum mass during expansion



Expansion of a plasma

For realistic calculations we use the **initial configuration of the PHSD approach** and compare NJL with PHSD calculations

NJL

$$m_q \approx 5 \text{ MeV}$$

no gluons

g fix

Hadronization by cross section

$$q\bar{q} \rightarrow m_1 + m_2$$

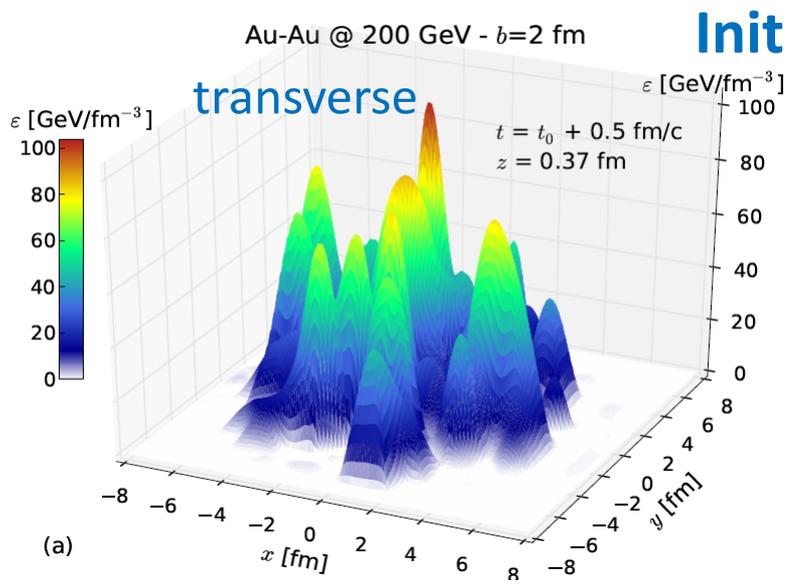
PHSD

$$400 \text{ MeV} \leq m_q \leq 800 \text{ MeV}$$

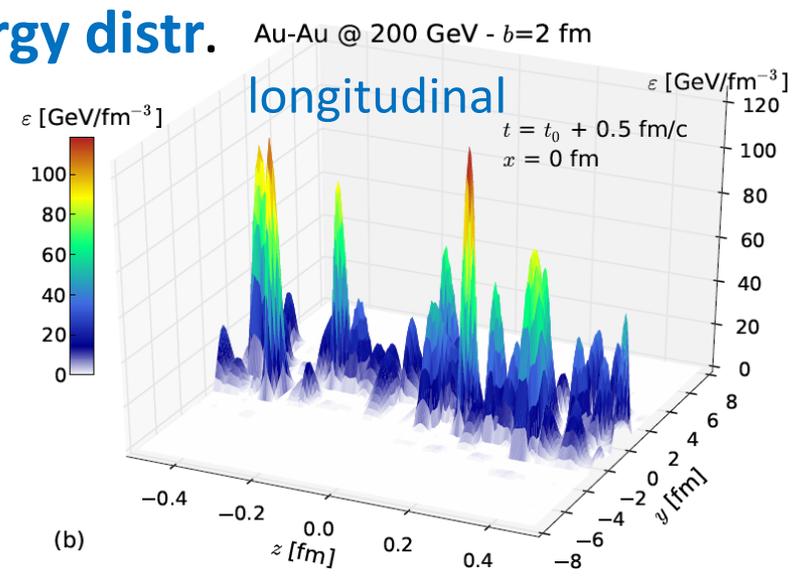
gluons

g running (T/T_c)

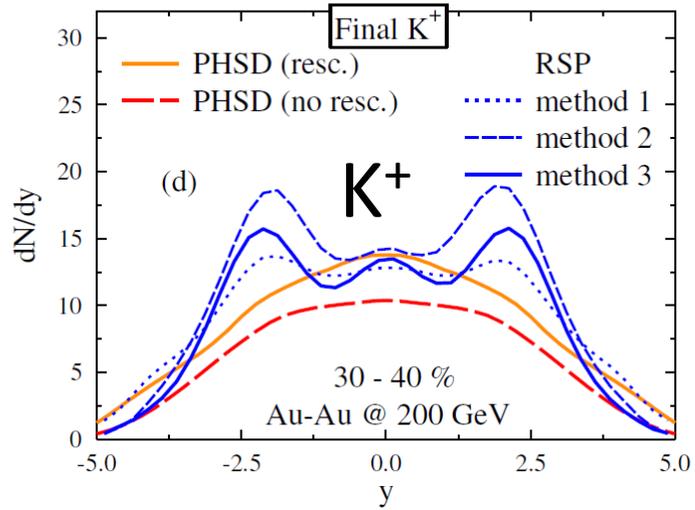
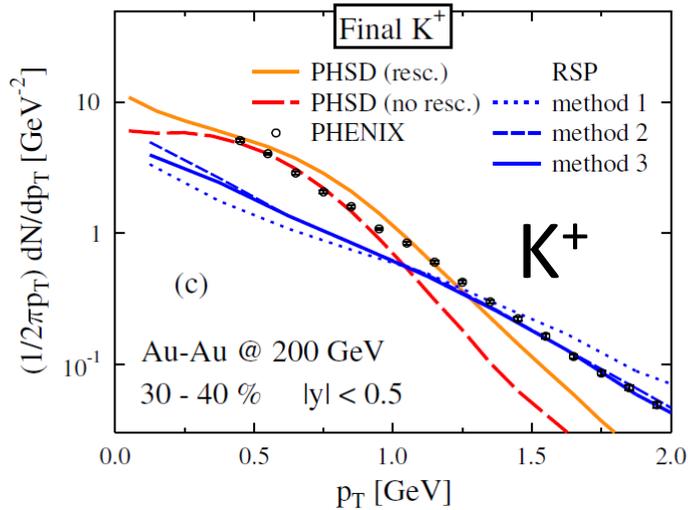
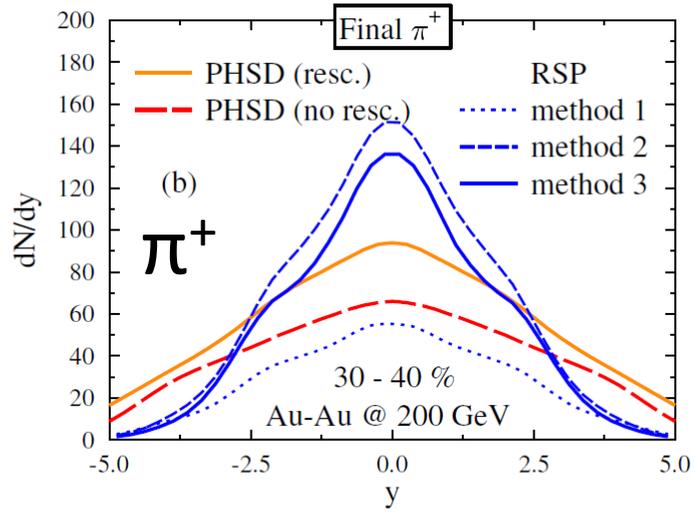
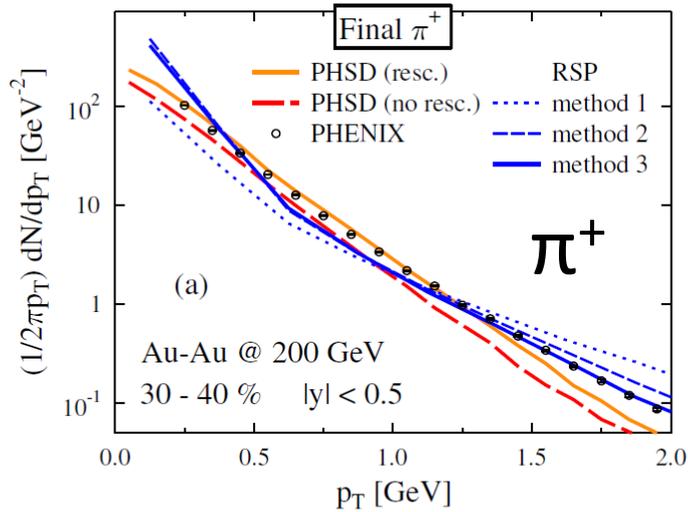
$$q\bar{q} \rightarrow m \text{ (or "string")}; qqq \rightarrow b \text{ (or "string")}$$



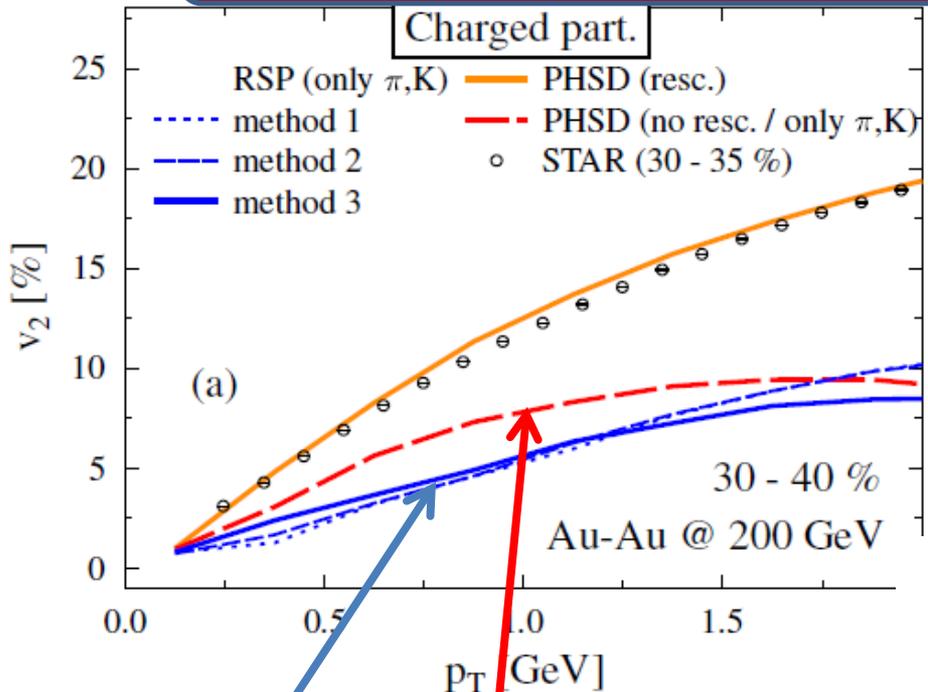
Initial energy distr.



Expansion of a plasma with PHSD initial cond. I



Expansion of a plasma with PHSD initial cond. II



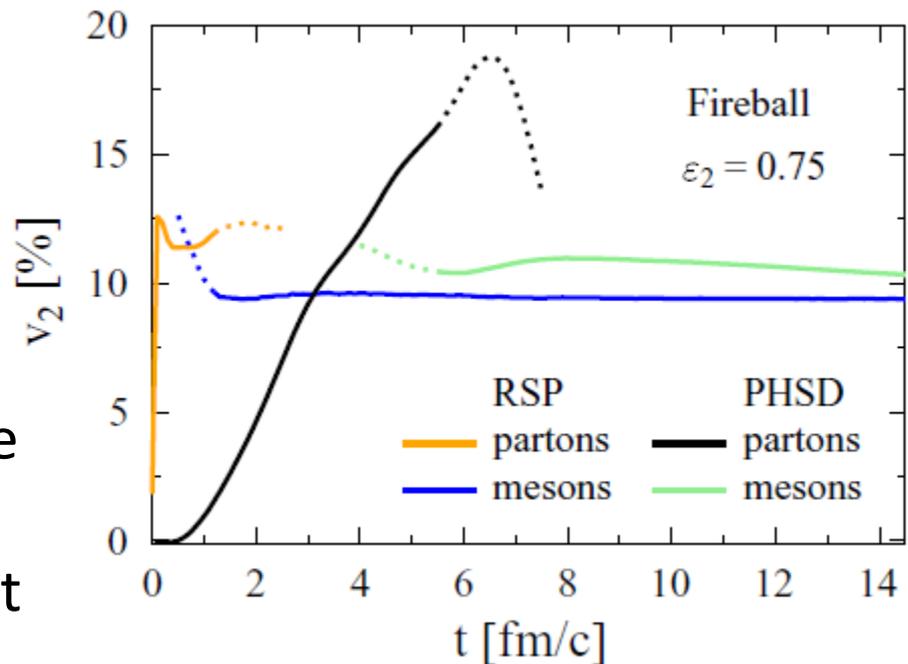
Expanding almond shaped fireball as initial condition

NJL (RSP) has no hadronic rescattering

without rescattering

NJL (RSP) and PHSD have about the same v_2

Time evolution completely different



Summary of our long way

Starting point: NJL Lagrangian which shares the symmetries with QCD

Fierz transformation \rightarrow color less meson channel and qq channels

Bethe Salpeter equation in $q\bar{q} \rightarrow$ mesons as pole masses

Bethe Salpeter equation in qq \rightarrow diquarks as pole masses

(diquark-quark Bethe Salpeter equation \rightarrow baryons as pole masses)

All masses described (10% precision) by 7 parameters fitted to ground state properties
(PNJL needs additional parameters to fix the Polyakov loop)

Extension of all masses to finite T and μ **without any new parameter**
cross section (elastic and hadronisation) **without any new parameter**

Relativistic molecular dynamics approach based on constraints gives
time evolution equations of particles in a 6+1 dim. phase space

Studies of hadronization in realistic plasmas:

No sudden transition between quarks and hadrons

experimental results reasonably well reproduced (quite astonishing)

Almost all ready to study first order phase transition