Probing the nature of phases across the phase transition at finite isospin chemical potential

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Introduction : Why μ_I ?

- Protons convert to neutrons and neutrinos via electron capture in the core of neutron stars, leading to high baryon density with considerable I_3, i.e., finite isospin.

- Using μ_u and μ_d as light quark chemical potentials, one has $\mu_B = 3(\mu_u + \mu_d)/2$ & $\mu_I = (\mu_u - \mu_d)/2$ or alternatively, $\mu_u = \mu_B/3 + \mu_I$ & $\mu_d = \mu_B/3 - \mu_I$.

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- For QCD with two flavours, quarks are two component spinors, leading to a quark matrix :

$$M = \begin{pmatrix} \not{D}(\mu_I) + m & \lambda\gamma_5 \\ -\lambda\gamma_5 & \not{D}(-\mu_I) + m \end{pmatrix}$$

where λ is introduced as an isospin-breaking term to study SSB in $\lambda \to 0$ limit.

- The fermion determinant can be shown to be real, and for staggered fermions the usual techniques work to simulate the theory, as shown by Kogut-Sinclair (PRD '02), who also obtained first numerical results for the phase transition.

- Employing staggered fermions on 8^4 lattices with $a = 0.299(2)$ fm & lattice quark mass $ma = 0.025$, corresponding to $m_\pi \simeq$ 260 MeV, Endrődi (PRD '15) investigated the phase structure.

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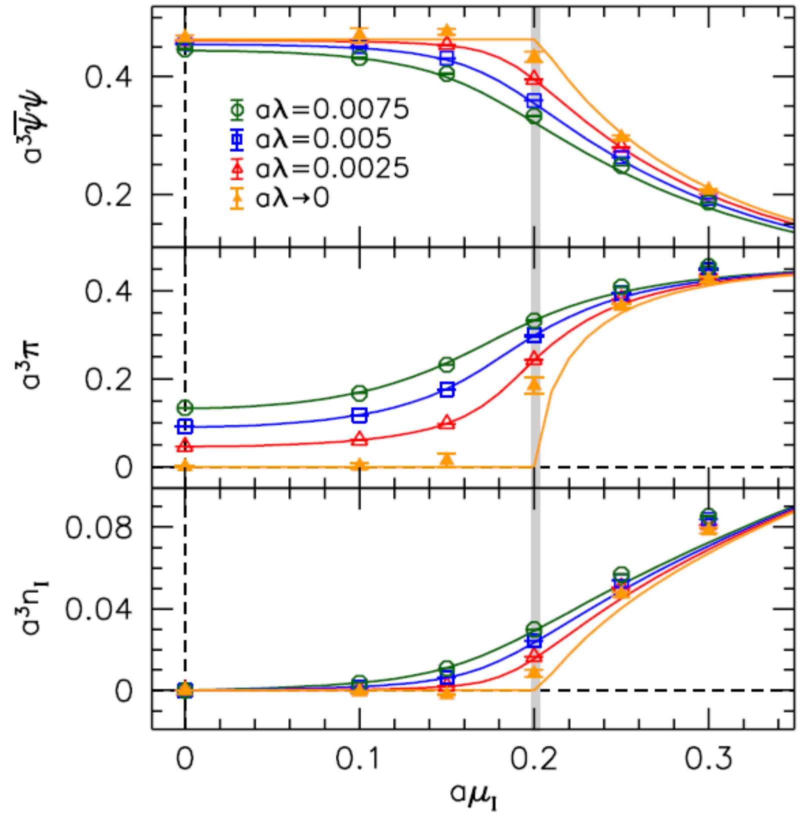
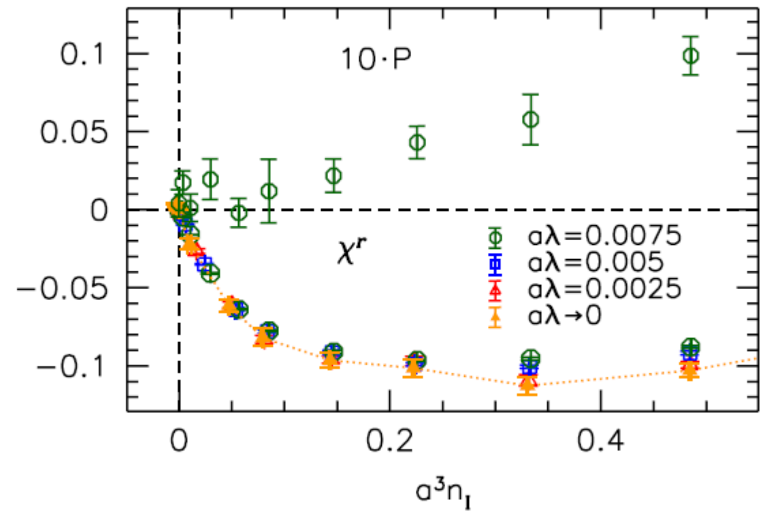
- He computed the chiral condensate, the pion condensate, and the isospin density by using,

$$\langle\bar{\psi}\psi\rangle = \frac{T}{V}\frac{\partial\ logZ}{\partial m}, \langle\pi\rangle = \langle\bar{\psi}_u\gamma_5\psi_d - \bar{\psi}_d\gamma_5\psi_u\rangle = \frac{T}{V}\frac{\partial\ logZ}{\partial\lambda}, \langle n_I\rangle = \frac{T}{V}\frac{\partial\ logZ}{\partial\mu_I}$$

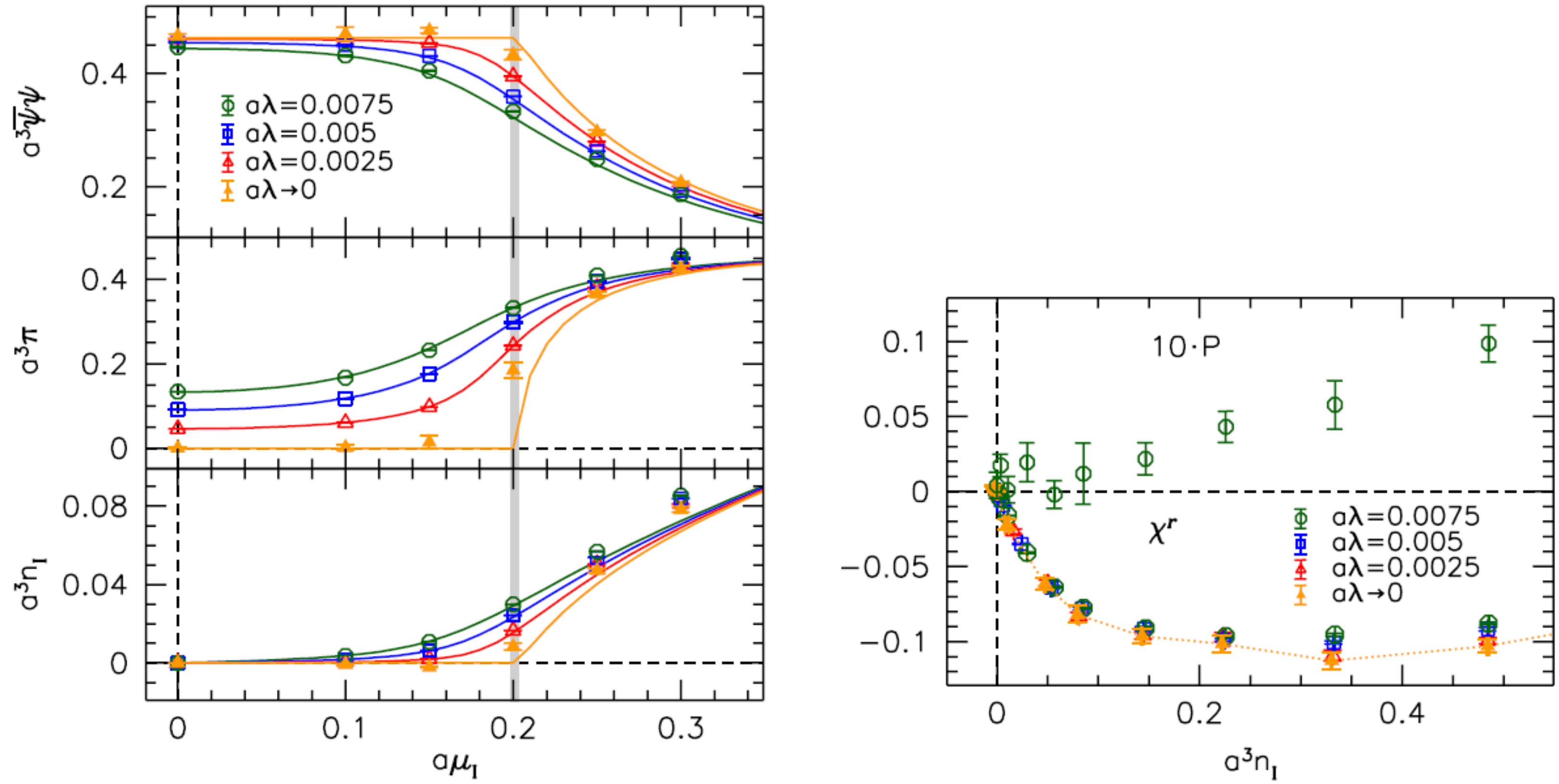
and obtained $a\mu_I^c \simeq 0.2$.



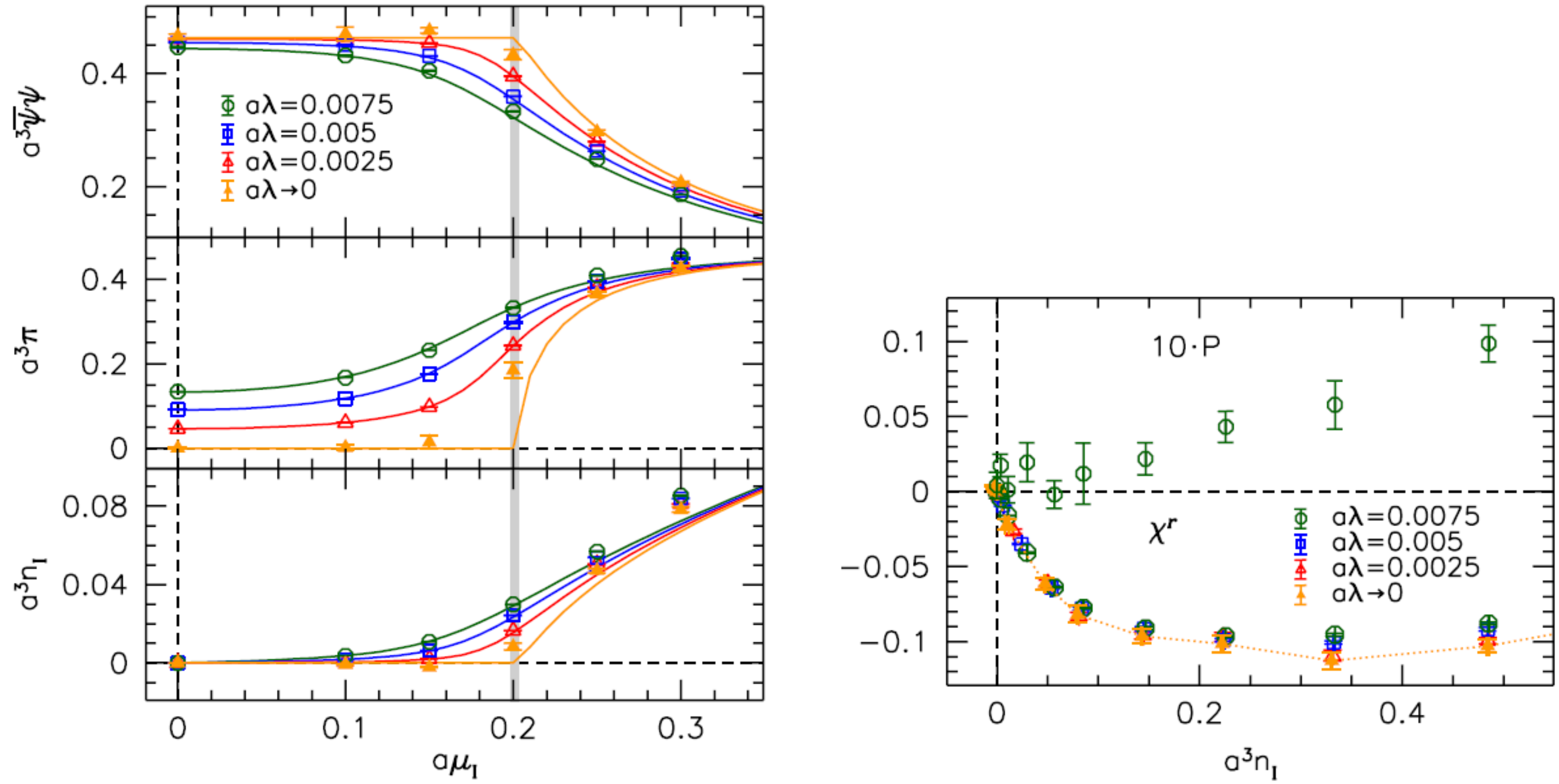
$a^3\bar{\psi}\psi$
0.4
0.2
aλ=0.0075
aλ=0.005
aλ=0.0025
aλ→0
$a^3\pi$
0.4
0.2
0
a^3n_I
0.08
0.04
0
0
0.1
0.2
0.3
$a\mu_I$



0.1
0.05
0
−0.05
−0.1
10·P
χ^r
aλ=0.0075
aλ=0.005
aλ=0.0025
aλ→0
0
0.2
0.4
a^3n_I



- $\lambda \to 0$ extrapolation in yellow, points (linear), line (chiral theory fit).
- Grey vertical band denotes $m_\pi/2$.
- Pion condensate & Isospin density become nonzero around $\mu_I^c \simeq m_\pi/2$, where chiral condensate drops rapidly as well.



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- Pion condensate & Isospin density become nonzero around $\mu_I^c \simeq m_\pi/2$, where chiral condensate drops rapidly as well. • Polyakov loop displayed in the upper half of the left panel shows deconfinement to occur there as well.

Introduction II : Nature of Probe

- High μ_I phase appears to have restored chiral symmetry and deconfinement. Leading candidate for χSB – topological excitations.

- Successful phenomenology built on Instanton-fermion couplings. (Schafer-Shuryak RMP '98, Diakanov hep-ph/9602375)

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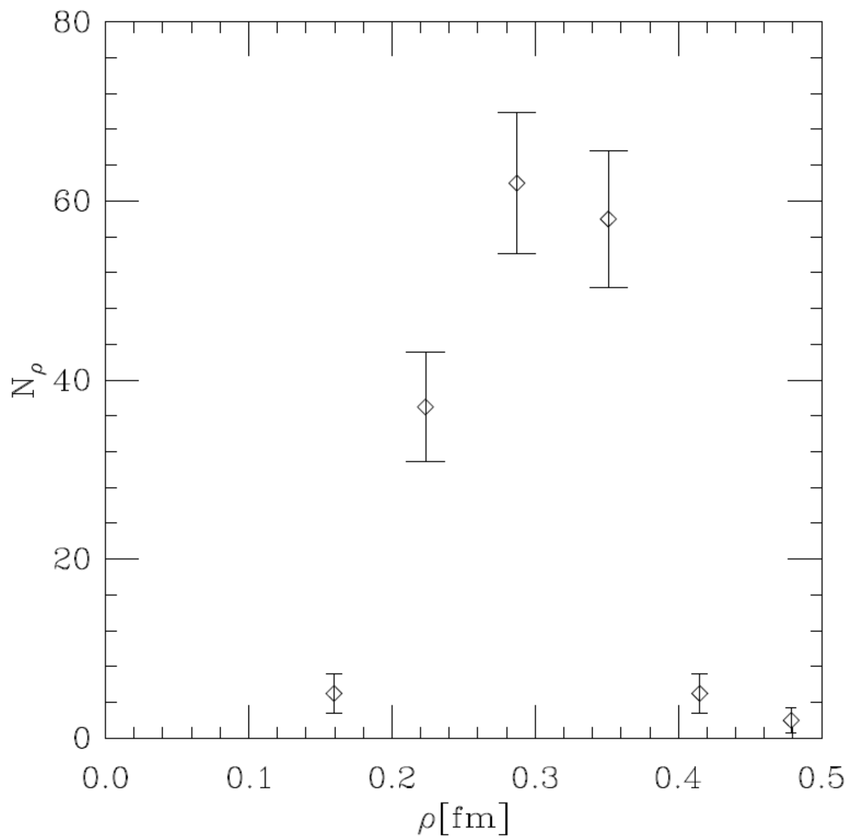
- LQCD simulations also support it : Instanton-distribution peaks at a radius $\rho = 0.3$ fm (DeGrand-Hasenfratz PRD '01).



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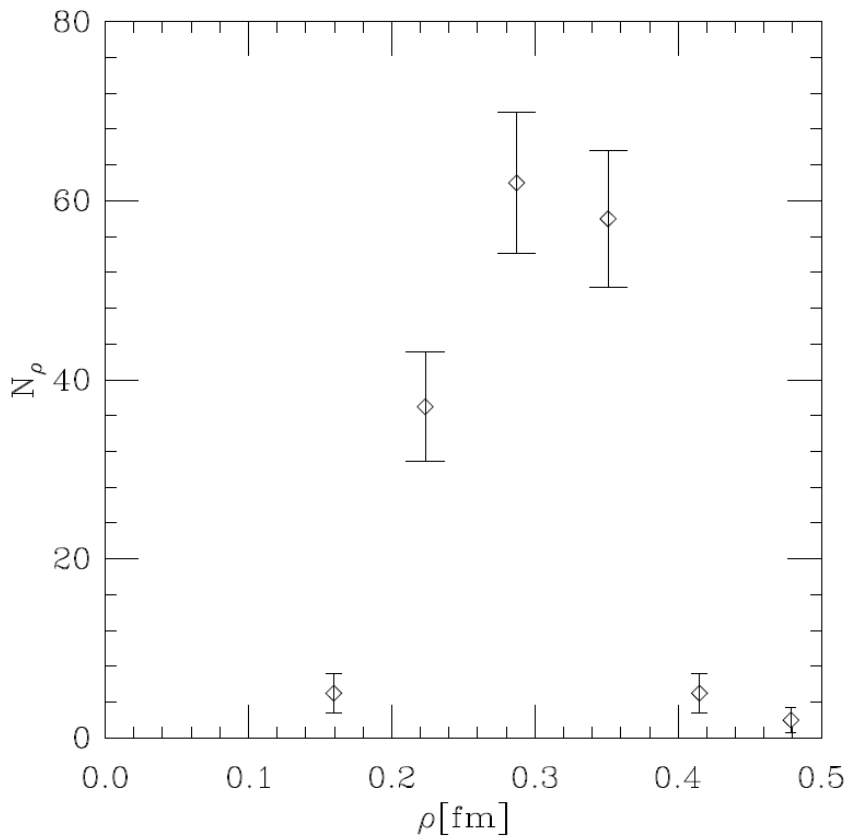

♠ Note that Overlap Dirac operator, which has *exact* chiral symmetry on the lattice as well as an index theorem, was used for the analysis above.

♣ The Overlap Dirac operator spectra has also been used to understand the nature of the high temperature phase.

♢ Number of low eigen modes do get depleted as $T \uparrow$. (Edwards-Heller-Kiskis-Narayanan, PRL '99, NPB (PS) '00, PRD '01; Gavai-Gupta-Lacaze, PRD '02)

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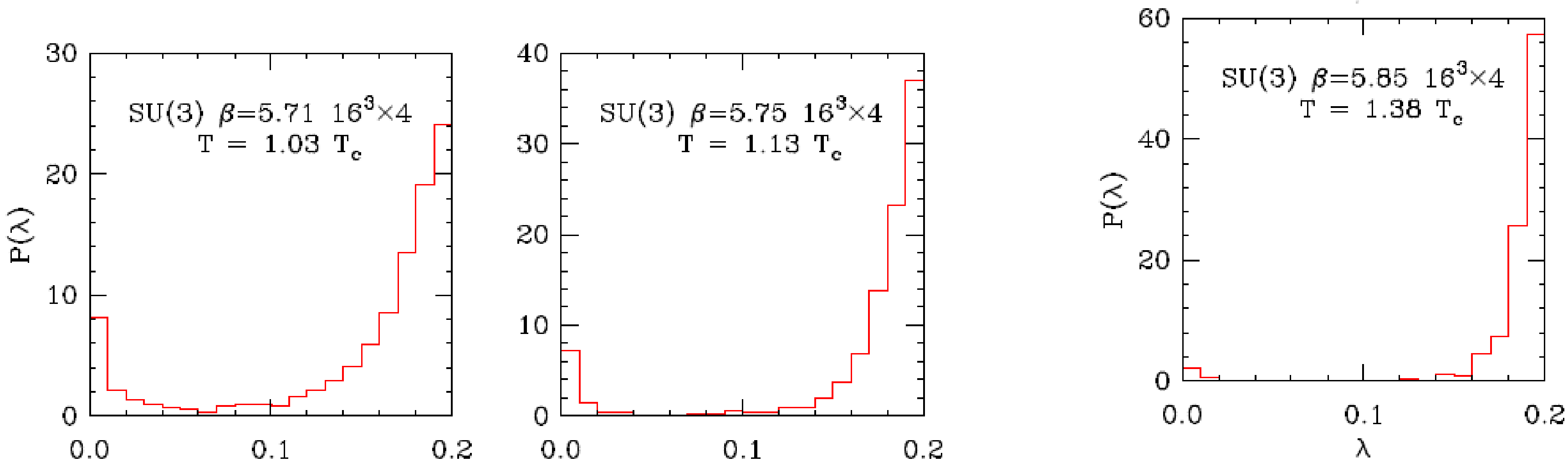

♢ Furthermore, a gap appears to separate the low modes from others.

Employing Exact Chiral Lattice Fermions

Overlap Dirac-Neuberger fermions possess satisfy the following correlator equalities in the chirally symmetric phase : $C_S(z) = -C_{PS}(z) \quad \text{and} \quad C_V(z) = C_{AV}(z)$.

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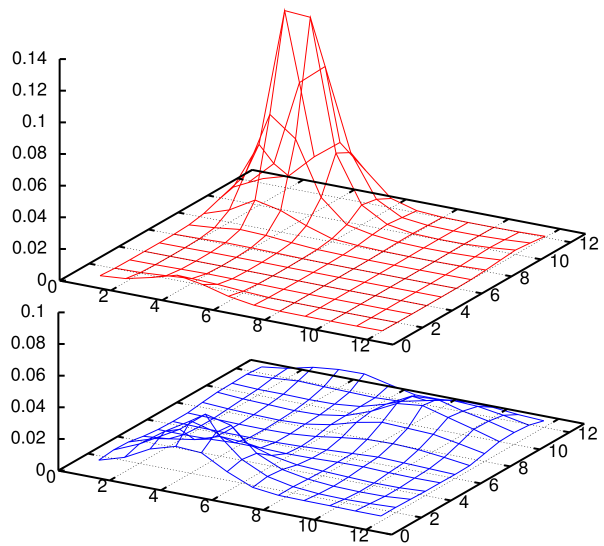

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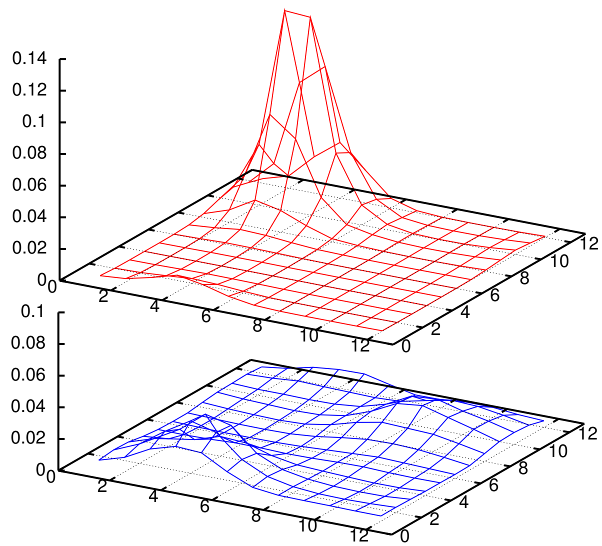

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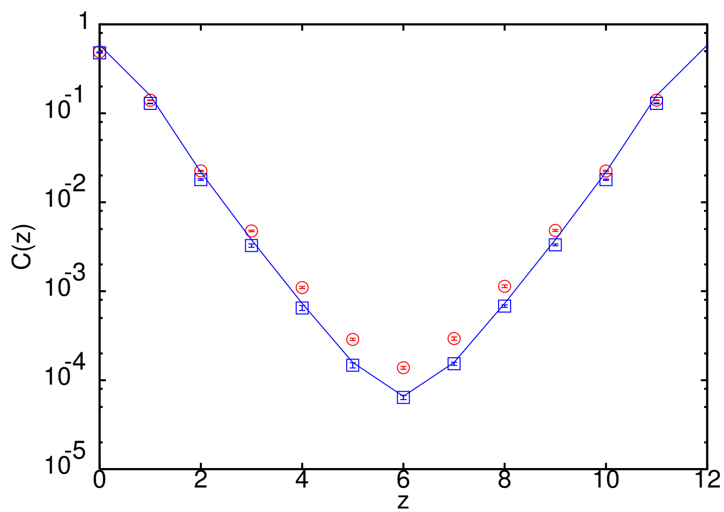

♡ Vector and Axial vector correlators equal above T_c but Pseudoscalar and Scalar equal *only* without zero modes ($T = 1.5T_c$ above).

Our Results

- We employed the Arnoldi method to extract the eigenvalues of Overlap Dirac operator (defined on dynamical configurations with nonzero μ_I but without an explicit μ_I in the operator itself), demanding a residue $r = ||DX - \eta|| \leq 10^{-10}$.
- Extracted ∼500 eigenvalues from each configuration.

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- Extracted $\sim$500 eigenvalues from each configuration.

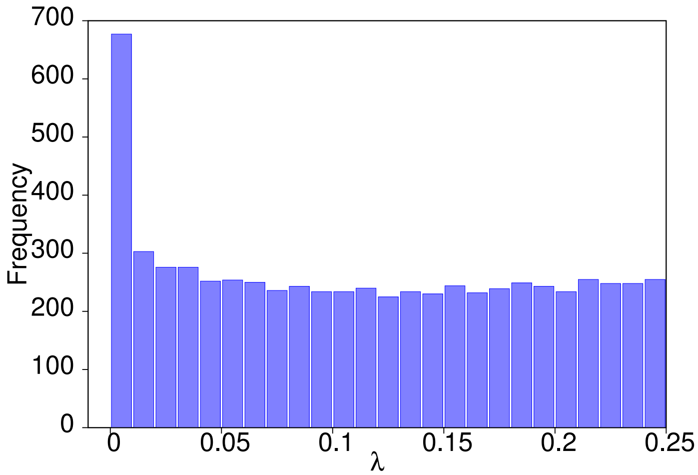
- Used a larger $24^3 \times 6$ lattice and a Symanzik improved action with 2 stout steps. Quark mass was tuned to have the physical pion mass.

- $a\mu_I^c = 0.1$ there, which again corresponds to μ_I^c being $m_\pi/2$.

- Computations made at two μ_I values, below and above the transition and two different λ — the isospin breaking parameter in the quark matrix.

$\lambda = 0.0006$

♡ We examined Overlap Dirac eigenmodes[†] for $\mu_I/\mu_I^c \simeq 0.5$ and 1.5, corresponding to $a\mu_I = 0.05$ and 0.15 respectively.



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[†]Eigenvalue λ is complex for D_{OV}. We display $|\lambda|$ distribution.

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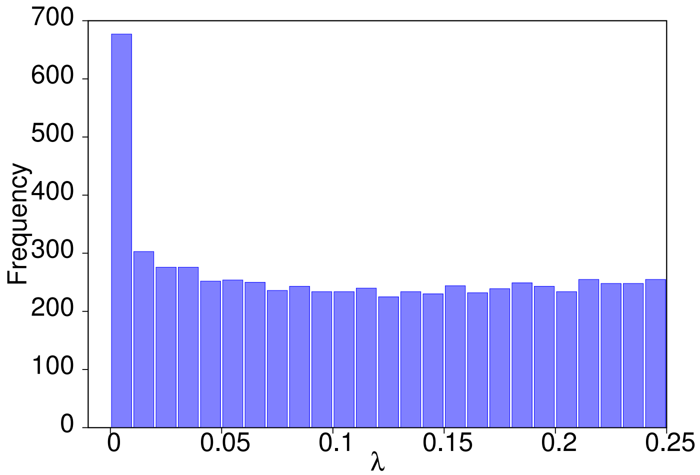

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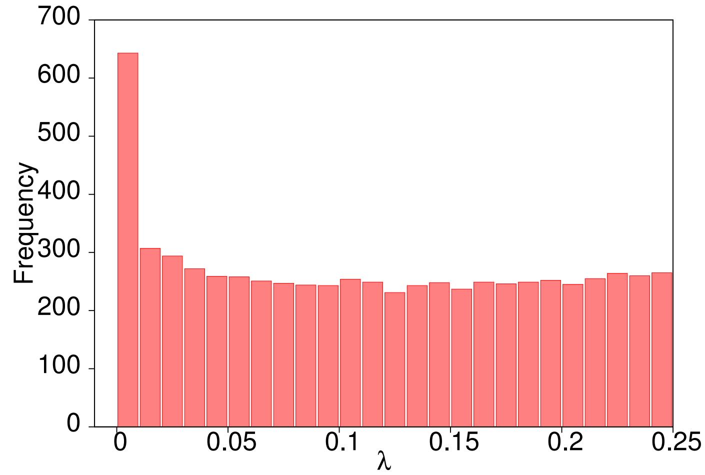
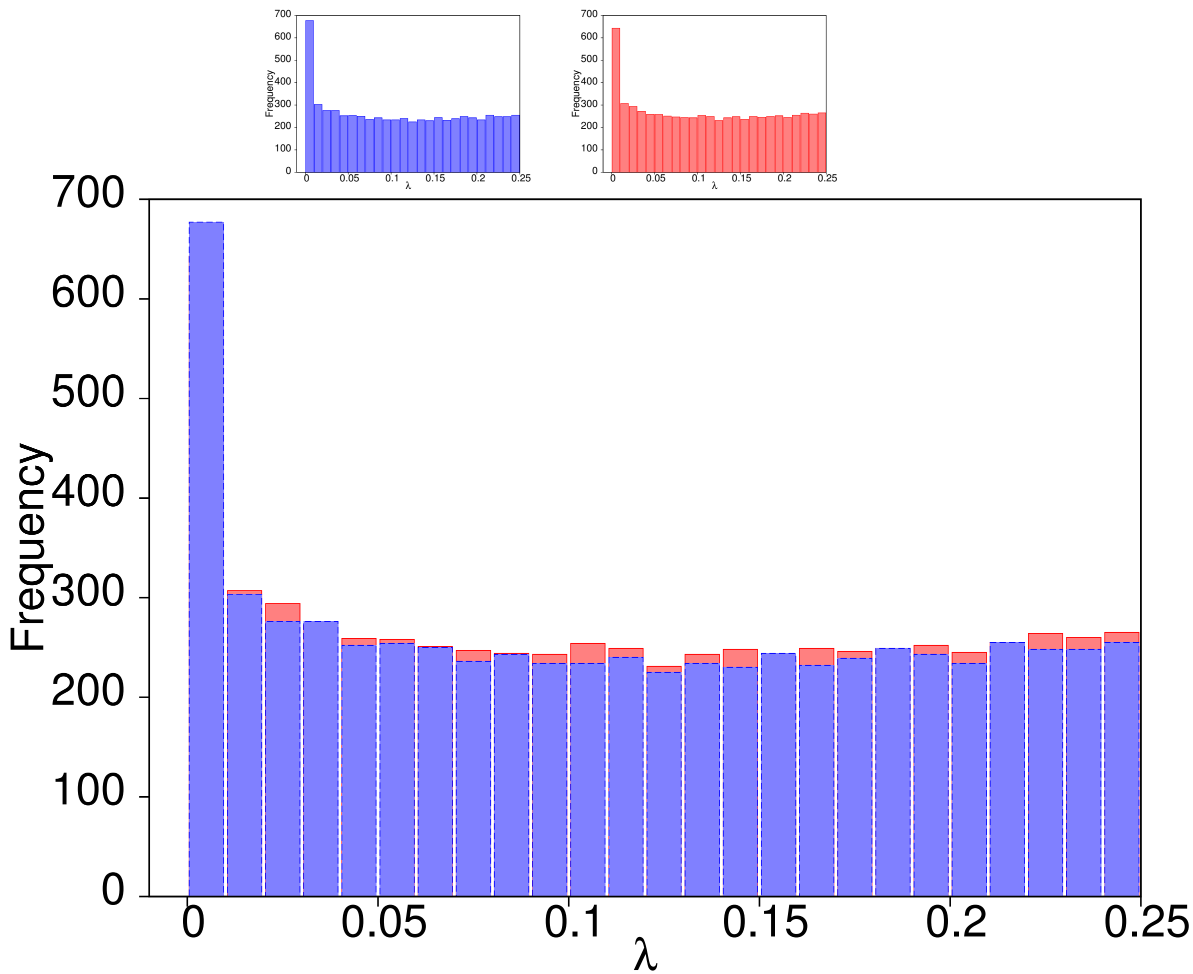


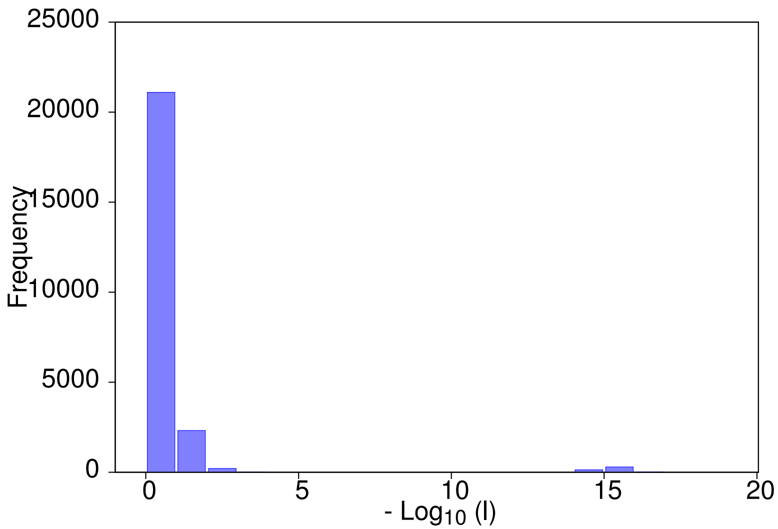
♡ $\mu_I/\mu_I^c = 1.5$: Surprisingly similar distribution as in the lower phase.

† Eigenvalue λ is complex for D_{ov}. We display $|\lambda|$ distribution.



♡ As expected, the overlap is indeed significant. Alternatively, the surprise is confirmed to be not an illusion.

♡ Looking at the eigenvalue distribution on a log scale, one can easily identify the zero modes from the gap in the spectrum. Explicit chirality checks needed to confirm their nature.



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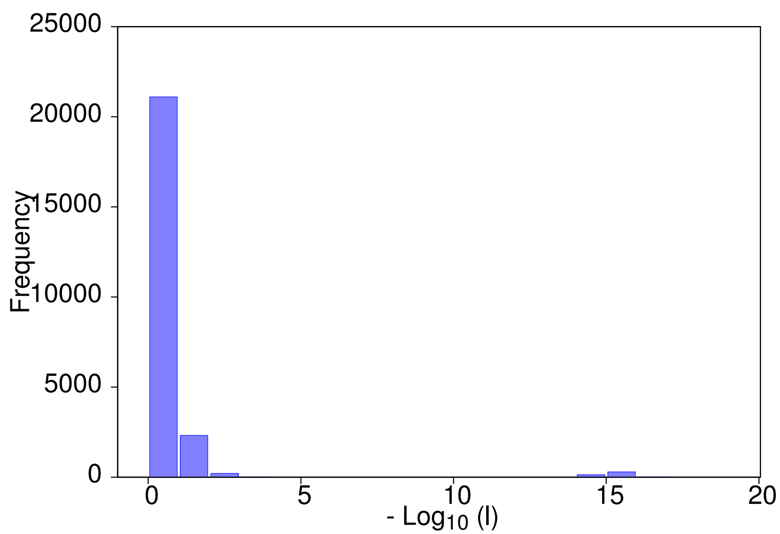

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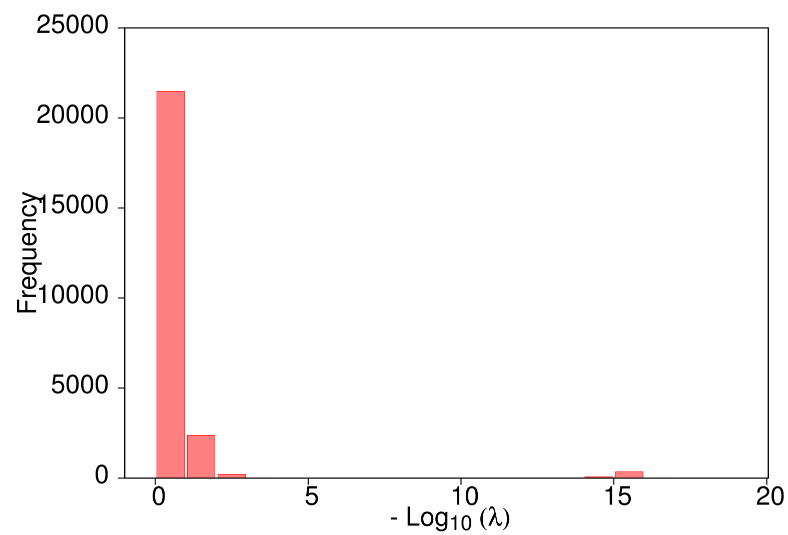
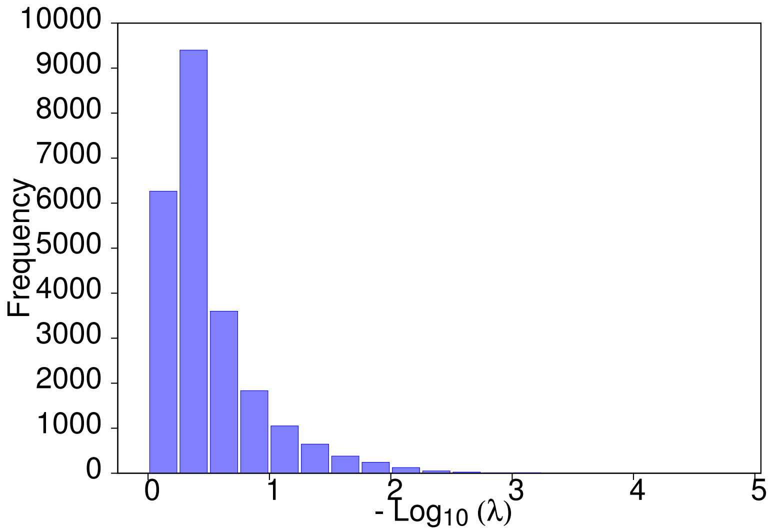


♡ $\mu_I/\mu_I^c = 1.5$: The number of zero modes still appear to be roughly the same.

♡ Zooming in on the eigenvalue distribution on the log scale to see if the near-zero modes have any difference which was missed.



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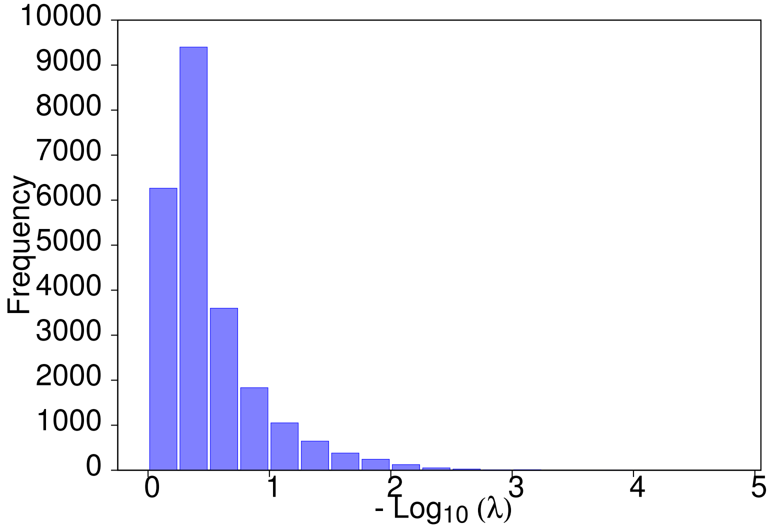
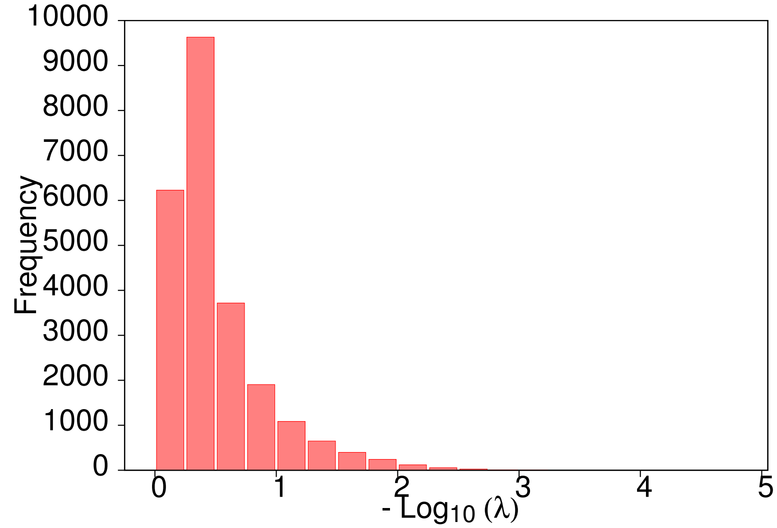
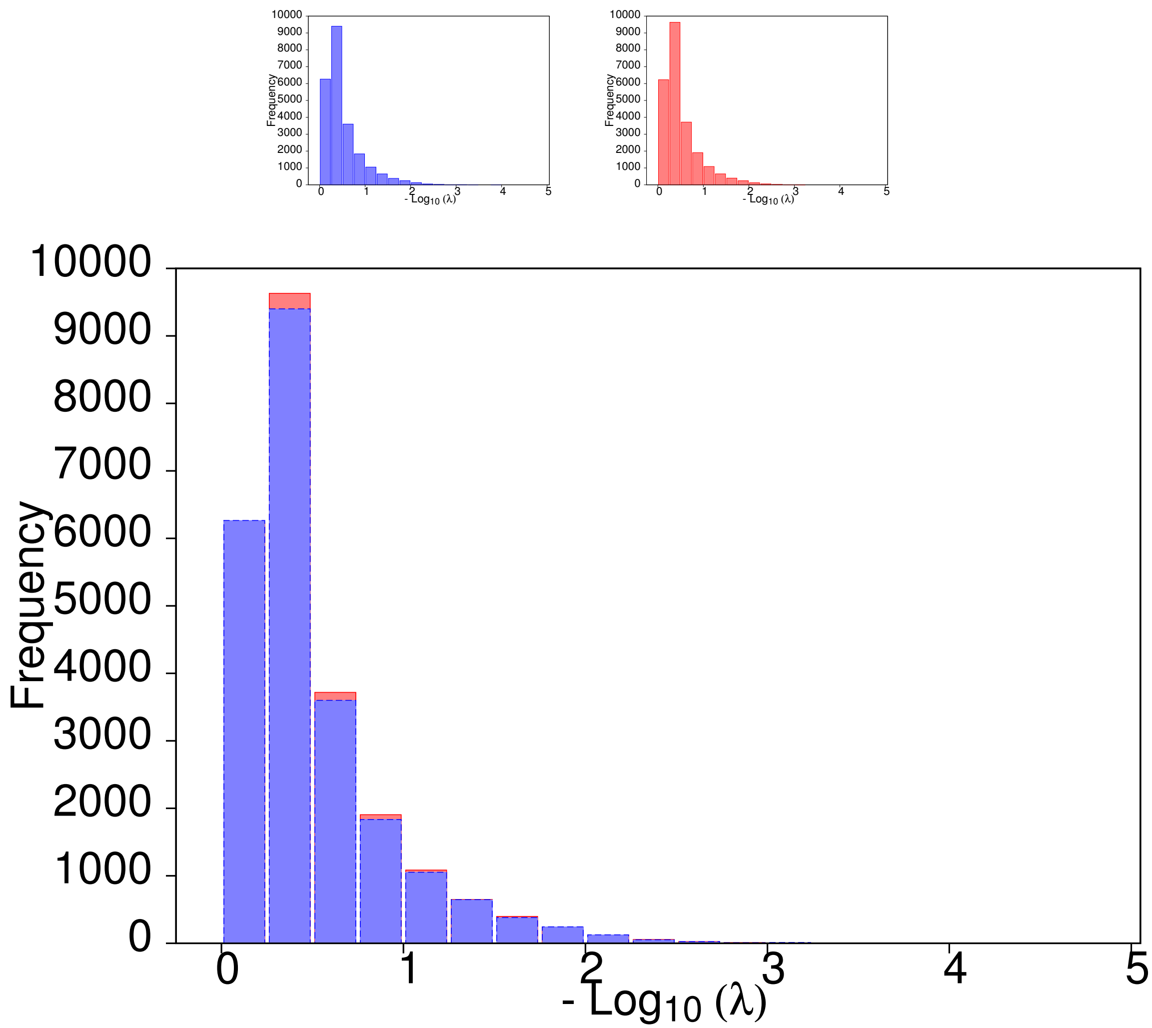




♠ $\mu_I/\mu_I^c = 0.5$: Nice smooth fall-off is seen.

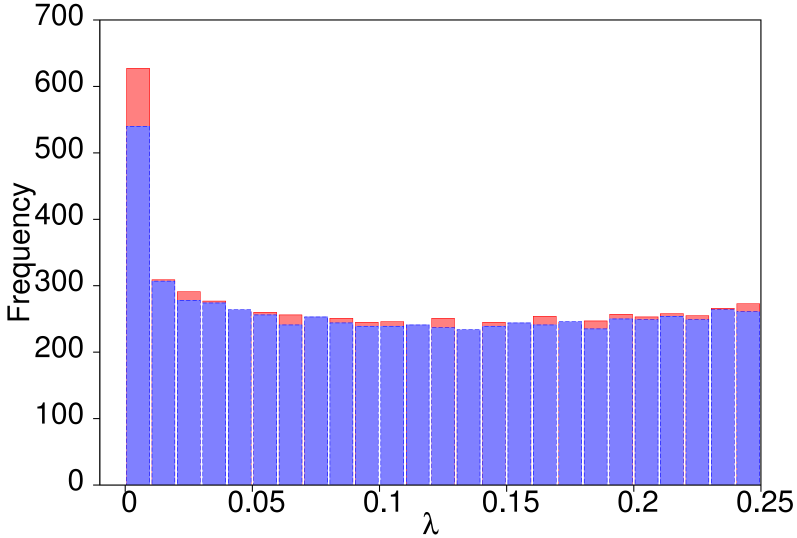
♡ $\mu_I/\mu_I^c = 1.5$: Similarity in the distribution as in the lower phase still indicated.



♡ No visible difference in the near-zero mode distributions.,

$\lambda = 0.0025$

♡ Again we examined Overlap Dirac eigenmodes for $\mu_I/\mu_I^c \simeq 0.5$ and 1.5, corresponding to $a\mu_I = 0.05$ and 0.15 respectively.



♠ Similar overlap as before.

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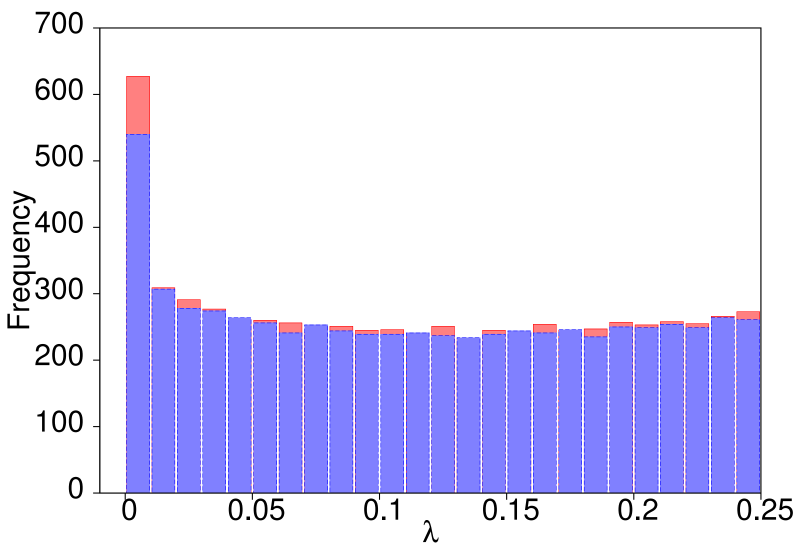

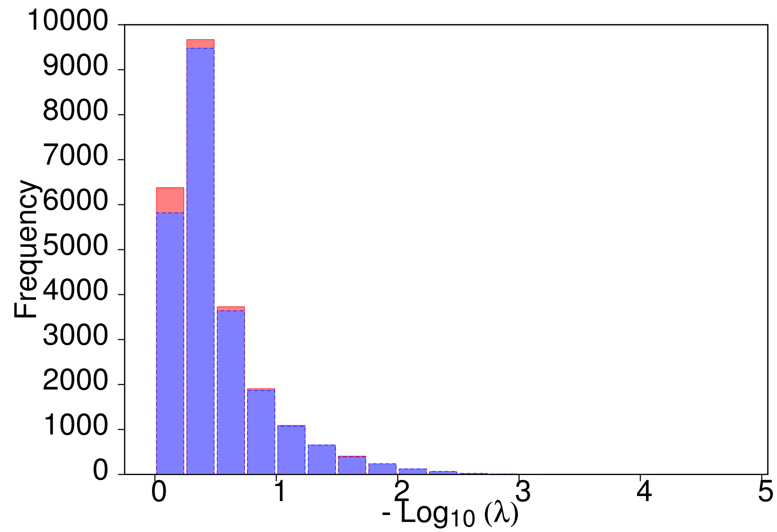

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♡ The near-zero distribution

What about Zero Modes?

♣ Nonzero modes are doubly degenerate for Overlap fermions as a result of the chiral symmetry.

♢ Zero modes are *not* degenerate & come with specific chirality, +ve or -ve.

♣ Act as a direct measure of topology.

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• For $T \neq 0$, Gavai-Gupta-Lacaze (PRD '02) found

T/T_c	N_{zero}
1.25	18
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• For $\mu_I \neq 0$, we find for *same* number of configs (50) :

μ_I/μ_I^c	$N_{zero}^{0.0006}$	$N_{zero}^{0.0025}$
0.5	426	416
1.5	451	310

• No variation across μ_I for $\lambda = 0.0006$ & a mild one for $\lambda = 0.0025$ (25% reduction)

Summary

- We investigated the eigenvalue distribution for chirally exact Overlap Dirac operator for $\mu_I/\mu_I^c = 0.5$ & 1.5, *i. e.*, below and above the isospin phase transition.

- The distribution of zero and near-zero modes is nearly the same for both at $\lambda = 0.0006$, with a 25 % reduction in former at $\lambda = 0.0025$.

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- The distribution of zero and near-zero modes is nearly the same for both at $\lambda = 0.0006$, with a 25 % reduction in former at $\lambda = 0.0025$.

- This should be contrasted with the earlier $T \neq 0$ results, where too these modes were present above the transition but decreased sharply as one moved away from the transition.

- Further investigations are going on to pin down the changes in the near-zero modes more quantitatively in an effort to understand the difference in T and μ_I directions.