Across the deconfinement

C. Bonati

claudio.bonati@df.unipi.it

Dipartimento di Fisica & INFN, Pisa (Italy)

Work in collaboration with

M. D'Elia, M. Mariti, M. Mesiti,

F. Negro, F. Sanfilippo

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Outline

The deconfinement transition at zero baryonic density

The deconfinement/chiral symmetry restoration transition at vanishing baryon density has been extensively studied using Lattice QCD and its properties are by now well established.

It is not a "phase transition" but just a smooth analytical crossover; as a consequence all "critical properties" are observable dependent.

In the following all Lattice QCD results will be related to the chiral symmetry restoration aspects.

Critical temperature:

$T_c|_{\mathrm{BW}} = 152(5)\,\mathrm{MeV}$ Aoki et al. Phys. Lett. B **643**, 46 (2006)

$T_c|_{\mathrm{hotQCD}} = 154(9)\,\mathrm{MeV}$ Bazavov et al. Phys. Rev. D **85** 054503 (2012)

The sign problem at nonvanishing density

Lattice QCD $\sim$ statistical mechanics with energy $=$ euclidean 4d action

Fermions cannot be directly simulated but they can be integrated out:
$S_{eff}(A) = S_G(A) - \log\det[m + \not{D}(A)]$

We need $\det[m + \not{D}(A)] > 0 \ \forall A$ to use importance sampling in Lattice QCD Monte-Carlo simulations.

In all lattice discretizations a relation of the following form holds true:

$$\gamma_5 \not{D}(\mu)\gamma_5 = \not{D}(-\mu^*)^\dagger$$

If $\Re(\mu) = 0$ this relation implies that if $\lambda \in \sigma[\not{D}(A)]$ then also $\lambda^* \in \sigma[\not{D}(A)]$, so that $\det[m + \not{D}(A)] > 0$.

For real μ values $\det[\not{D}(A)] \notin \mathbb{R}^+$: sign problem.

Possible way out of the sign problem

Most used (partial) solutions:

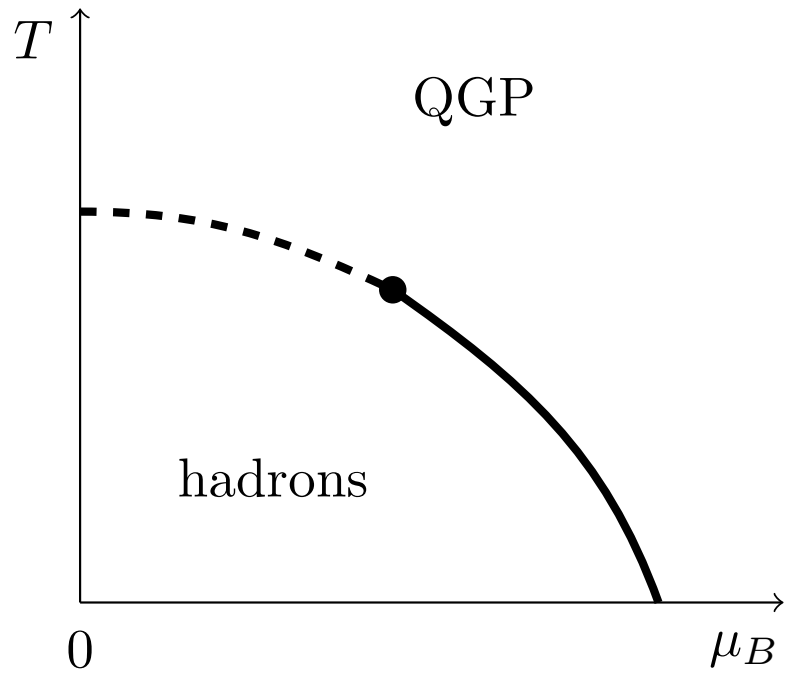
reweighting First sample the distribution at $\mu = 0$ then reweight it to $\mu \neq 0$. Problem: the overlap between the two distributions goes to zero exponentially as $V \to \infty$.

Taylor expansion Expand everything in powers of μ. The coefficients can typically be computed at $\mu = 0$. Problem: the higher the coefficient the more noisy the estimator the worse the scaling with the volume of the signal to noise ratio.

imaginary μ Perform simulation at imaginary chemical potential (where there is no sign problem) then analytically continue to real μ. Problem: systematical errors of the continuation.

Other possibilities: Lefschetz thimble, complex Langevin, canonical simulations, strong coupling methods, dual variables, density of state methods, ...

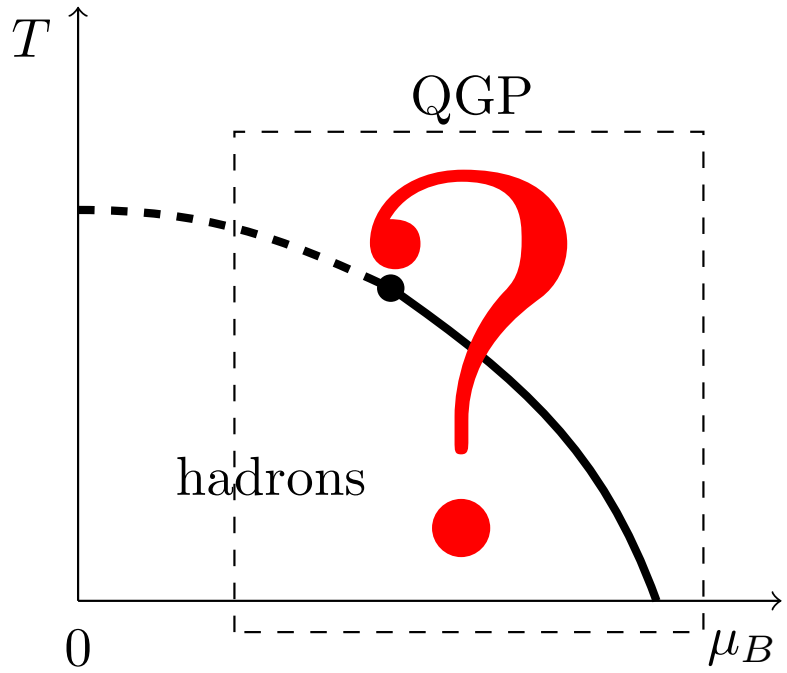
The expected phase diagram in the $T - \mu_B$ plane



Main features:

- analytic crossover for $\mu = 0$ (no known symmetries to break, it would be a real transition for massless quarks)
- first order transition for $T = 0$ (simple argument based on light particles counting)
- a second order transition somewhere in the middle

The known phase diagram in the $T - \mu_B$ plane



The region that can be reliably explored by Lattice QCD is the region of "small" μ_B, where the results obtained by using different methods can be tested again each other.

A well posed problem is the determination of $T_c(\mu_B)$ in this region.

General parametrization of $T_c(\mu_B)$

Since $Z(\mu_B) = Z(-\mu_B)$, in general $T_c(\mu_B)$ is an even function of μ_B. For small μ_B we thus expect

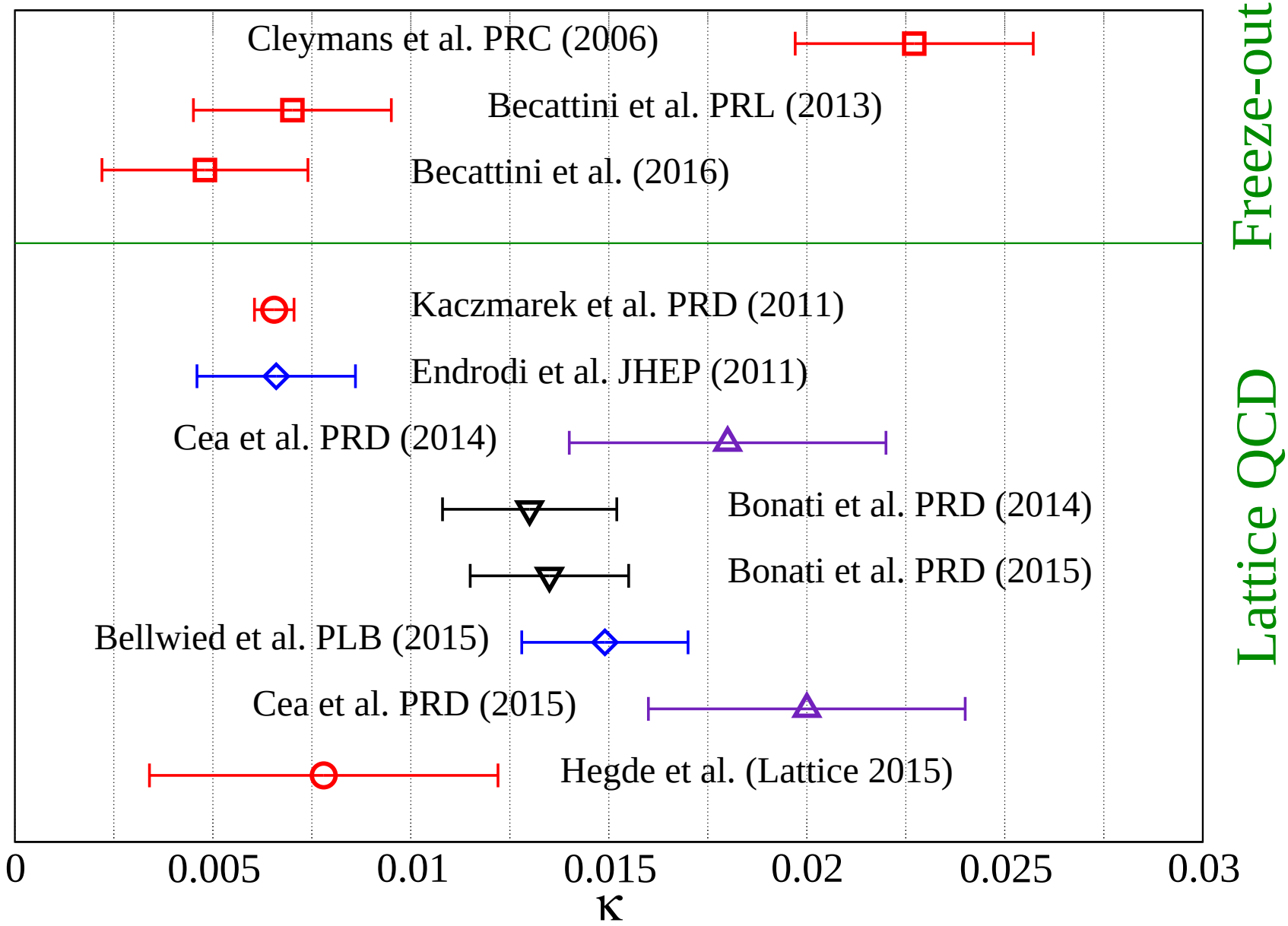
$$T_c(\mu_B) = T_c(0)\left(1 - \kappa\left(\frac{\mu_B}{T_c(0)}\right)^2 + c\left(\frac{\mu_B}{T_c(0)}\right)^4 + \cdots\right) =$$

$$= T_c(0)\left(1 - \kappa\left(\frac{\mu_B}{T_c(\mu_B)}\right)^2 + c'\left(\frac{\mu_B}{T_c(\mu_B)}\right)^4 + \cdots\right)$$

κ is the curvature of the critical line in the $T - \mu_B$ plane at $\mu_B = 0$ and it is an equilibrium property of QCD.

The precise relation between κ and the curvature κ_f of the freeze-out line is nontrivial from the theoretical point of view. On the other hand it seems reasonable that $\kappa \approx \kappa_f$.

Various determinations of κ



A remainder on the quark chemical potentials

The relations between the conserved charges and the number operators are

$$B = (N_u + N_d + N_s)/3$$
$$Q = (2N_u - N_d - N_s)/3$$
$$S = -N_s$$

The quark chemical potentials are defined in such way that

$$B\mu_B + Q\mu_Q + S\mu_S = N_u\mu_u + N_d\mu_d + N_s\mu_s$$

thus

$$\mu_u = \mu_B/3 + 2\mu_Q/3$$
$$\mu_d = \mu_B/3 - \mu_Q/3$$
$$\mu_s = \mu_B/3 - \mu_Q/3 - \mu_S$$

Typical setups and strangeness neutrality

In (almost) all simulations $\mu_Q \equiv 0$ and one of the two following extreme cases is used:

$$1) \quad \mu_u = \mu_d = \mu_B/3; \quad \mu_s = 0$$
$$2) \quad \mu_u = \mu_d = \mu_B/3; \quad \mu_s = \mu_B/3$$

that correspond to

$$1) \quad \mu_S = \mu_B/3$$
$$2) \quad \mu_S = 0$$

If we want to impose strangeness neutrality ($\langle N_s \rangle = 0$) we obtain

$$0 = \frac{\partial \log Z(\mu_B, \mu_S)}{\partial \mu_S} \simeq \left.\frac{\partial \log Z}{\partial \mu_S \partial \mu_S}\right|_{\mu=0} \mu_S + \left.\frac{\partial \log Z}{\partial \mu_S \partial \mu_B}\right|_{\mu=0} \mu_B + \cdots$$

from which we get a relation between μ_B and μ_S.

Strangeness neutrality and Q/B ratio

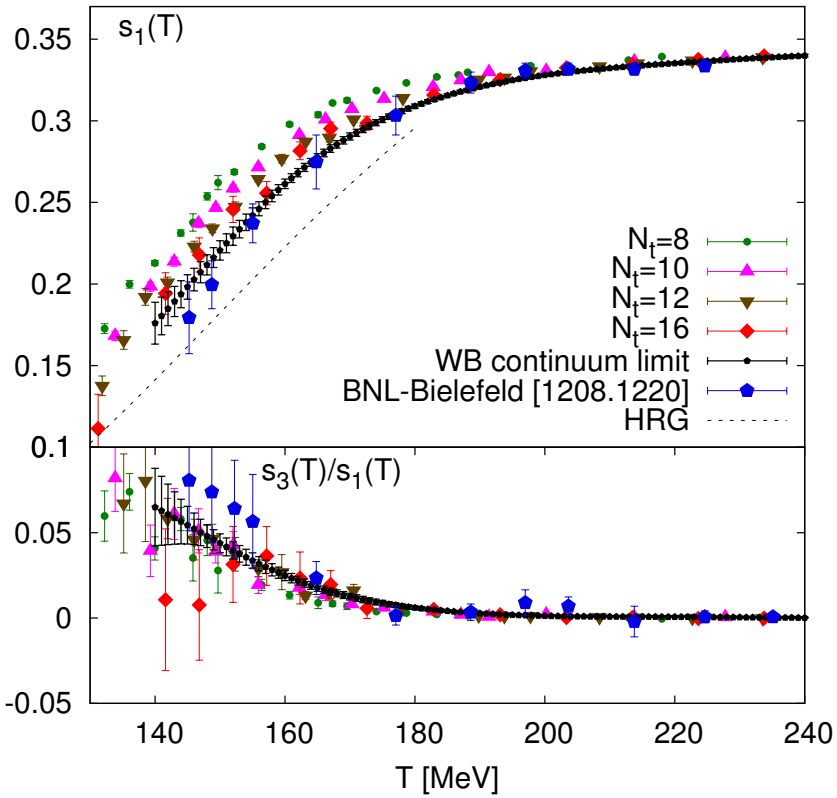
Explicitly one can write

$$\frac{\mu_S}{T} = s_1(T)\frac{\mu_B}{T} + s_3(T)\left(\frac{\mu_B}{T}\right)^3 + \cdots$$

At $T \approx T_c$ we have $\mu_S \simeq \mu_B/4$ and thus $\mu_s \simeq \mu_B/12 = \mu_u/4$.

In a similar way μ_Q can be fixed by imposing $\langle N_Q \rangle = r\langle N_B \rangle$, where $r = Z/A \simeq 0.4$, obtaining:

$$\frac{\mu_Q}{T} = q_1(T)\frac{\mu_B}{T} + q_3(T)\left(\frac{\mu_B}{T}\right)^3 + \cdots$$



Borsányi et al. PRL (2013)

Bazavov et al. PRL (2012)

Kaczmarek et al. PRD (2011)

If the transiton with $m_\ell \equiv m_u = m_d = 0$ is second order, since the baryon number does not break chiral symmetry, for $m_\ell \approx 0$ we can define the scaling variables

$$t \simeq \frac{1}{t_0}\left(\frac{T - T_c(0)}{T_c(0)} + \kappa\left(\frac{\mu_B}{T_c(0)}\right)^2\right) \qquad h \simeq \frac{1}{h_0}\frac{m_\ell}{m_s}$$

and thermodynamical observables have the scaling form $\phi = \phi(t, h)$, thus

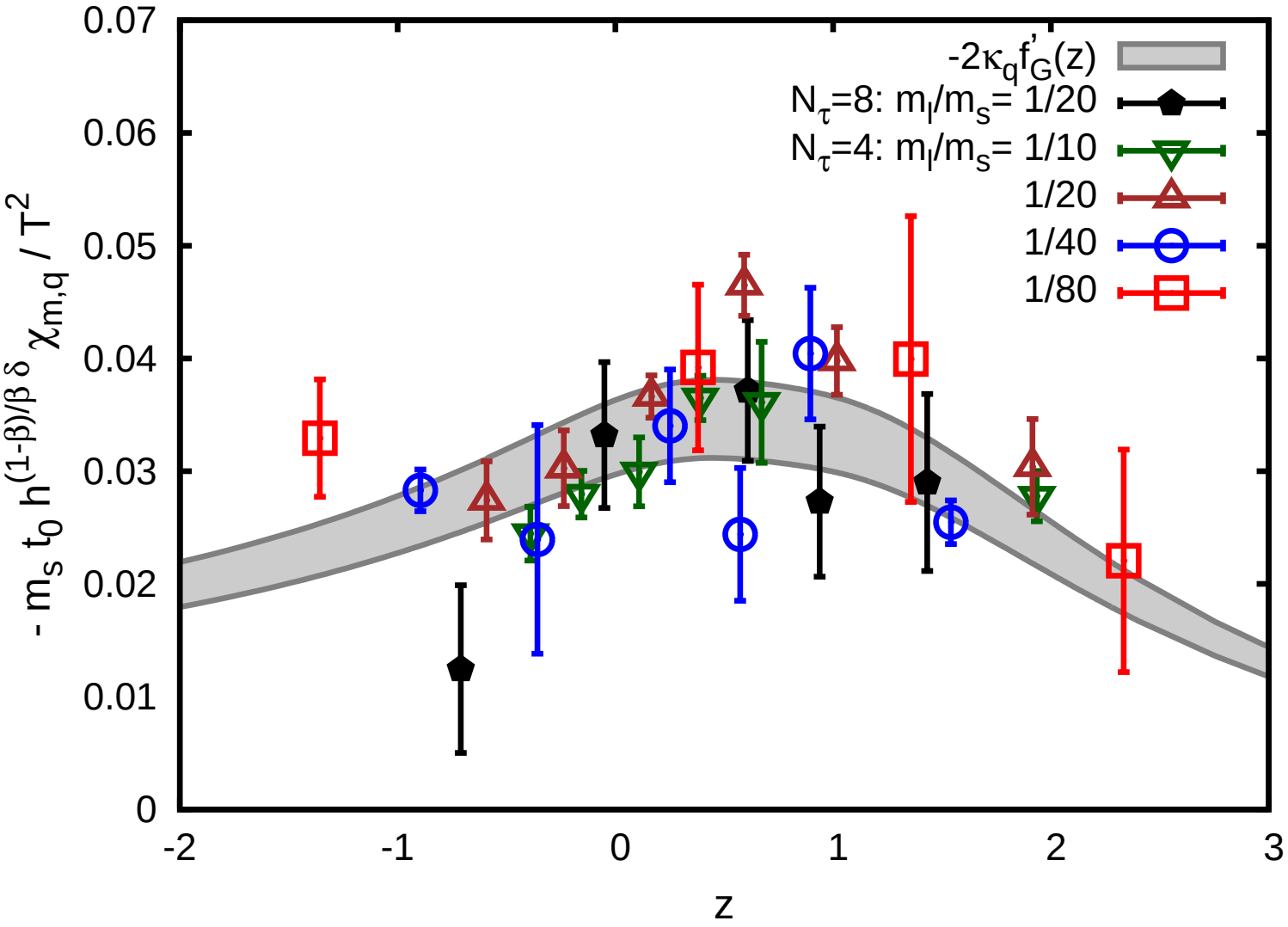
$$\kappa = T_c(0)\left.\frac{\partial\phi/\partial\mu_B^2}{\partial\phi/\partial T}\right|_{\substack{\mu_B=0 \\ T=T_c}} = \frac{t_0}{\partial_t\phi|_{\substack{\mu_B=0 \\ t=0}}}\left.\frac{\partial\phi}{\partial(\mu_B/T)^2}\right|_{\substack{\mu_B=0 \\ t=0}}$$

For $\partial_t\phi$ the scaling function of the $O(4)$ model was used.

κ defined in this way is the curvature in the chiral limit $m_\ell = 0$.

Chemical potentials setup: $\mu_u = \mu_d = \mu_B/3$ and $\mu_s = 0$.
$N_f = 2 + 1$ p4 staggered action, $N_t = 4, 8$.

Kaczmarek et al. PRD (2011)



$$z = t/h^{1/(\beta\delta)}$$

Kaczmarek et al. PRD (2011)

Possible sources of systematic errors:

- the $N_f = 2$ transition is really second order? No consensus on this point. From the theoretical point of view "the phase transition can be second order, with $O(4)$ critical exponents" Pisarski, Wilczek PRL (1984); it can also be first order.
- the values of t_0 and h_0 are not stable between $N_t = 4$ and $N_t = 8$ (they change by around 50%)
- large correction to the chiral scaling behaviour are observed in the magnetic equation of state, that have been taken into account by adding analytical terms.

Hegde et al. Lattice 2015

The basic idea is the same of Kaczmarek et al. PRD (2011) but with some differences

$$t \simeq \frac{1}{t_0}\left(\frac{T - T_c(0)}{T_c(0)} + \vec{\mu}^T \mathbf{K} \vec{\mu}\right) \qquad h \simeq \frac{1}{h_0}\frac{m_\ell}{m_s}$$

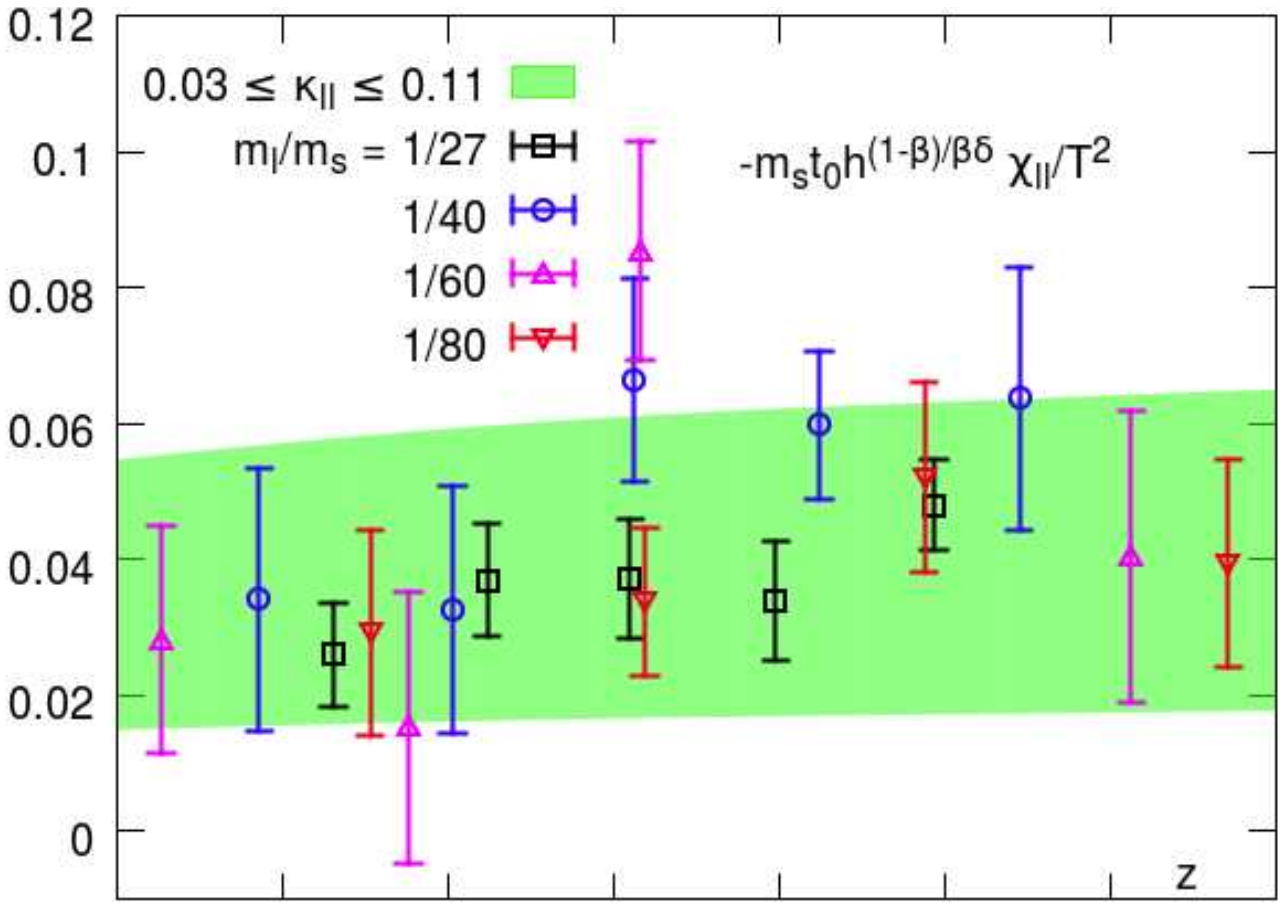
where $\vec{\mu} = (\mu_\ell, \mu_s)$ and $\mathbf{K}$ is a 2×2 curvature matrix.

For $\partial_t \phi$ the scaling function of the $O(4)$ model was used.

Again $\mathbf{K}$ defined in this way is the curvature in the chiral limit $m_\ell = 0$.

Chemical potentials setup: $\mu_\ell \equiv \mu_u = \mu_d$ and μ_s.
$N_f = 2 + 1$ HISQ action, $N_t = 6$ no continuum limit.

Hegde et al. Lattice 2015



$$z = t/h^{1/(\beta\delta)}$$

Endrodi et al. JHEP (2011)

Take your favorite observable $\phi(T, \mu_B)$ that is monotone at $\mu_B = 0$ and let ϕ_0 be its value at the $\mu_B = 0$ transition. Then define $T_c(\mu_B)$ by the equation $\phi(T_c(\mu_B), \mu_B) \equiv \phi_0$. Then

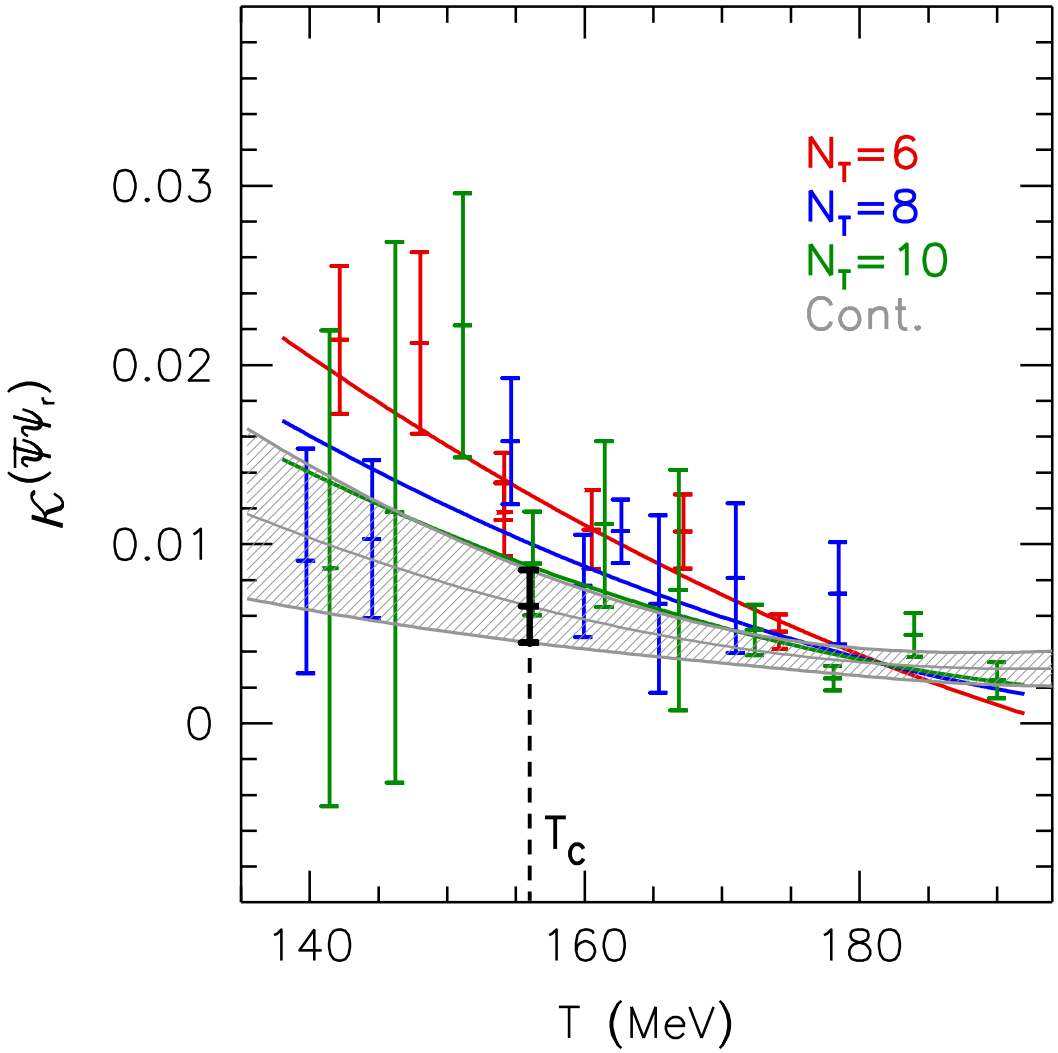
$$\kappa \equiv -T_c(0) \left.\frac{\mathrm{d}T_c(\mu_B)}{\mathrm{d}\mu_B^2}\right|_{\mu_B=0} = T_c(0) \left.\frac{\partial\phi/\partial\mu_B^2}{\partial\phi/\partial T}\right|_{\substack{\mu_B=0 \\ T=T_c}}$$

Problem: why should it work? Partial solution: when the method by Kaczmarek et al. works, the method by Endrodi et. al also works (and it had to give the same answer, compare equations).

Other possible source of systematics: common fit for T an a.

Chemical potentials setup: $\mu_u = \mu_d = \mu_B/3$ and $\mu_s = 0$.
$N_f = 2 + 1$ stout improved fermions with physical masses, $N_t = 6, 8, 10$.

Endrodi et al. JHEP (2011)



Only statistical errors considered!

(see also comments in Bellwied et al. PLB (2015)).

Bellwied et al. PLB (2015)

Simulations with imaginary chemical potential. The procedure is the following

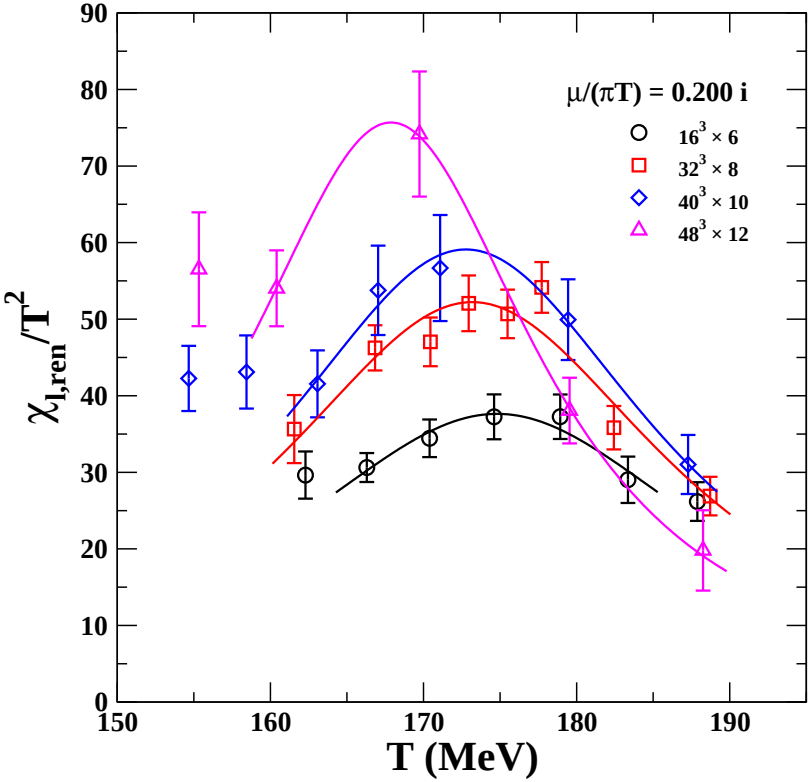
1. fix some value imaginary value for μ (or μ/T)
2. perform simulations at different T values
3. identify $T_c(\mu)$ with standard methods (e.g. peak of susceptibility)
4. fit $T_c(\mu)$ with the expected functional form

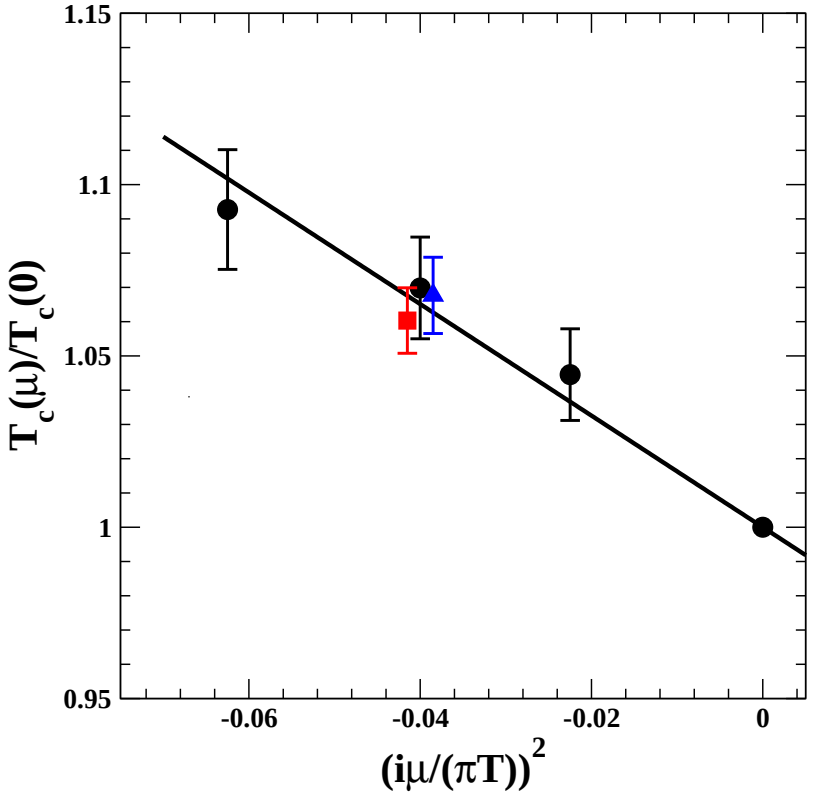
Chemical potential setup: mostly $\mu_Q = 0$ and strangeness neutrality, some tests with $Q/B = 0.4$. $\mu_B^I \lesssim 300\,\mathrm{MeV}$
$N_f = 2+1+1$ stout action with physical masses, $N_t = 10, 12, 16$.

Principal source of systematical errors: extrapolation to vanishing chemical potential.

Cea et al. PRD (2014-2015)

Simulations with imaginary chemical potential.





Chemical potential setup $\mu_u^I = \mu_d^I = \mu_s^I$, $\mu_B^I \lesssim 380\,\mathrm{MeV}$
$N_f = 2+1$ HISQ with $m_s \sim$physical and $m_\ell/m_s = 1/20$, $N_t = 6, 8, 10, 12$

Principal source of systematical errors: extrapolation to vanishing chemical potential.

Bonati et al. PRD (2014-2015)

Two different procedures for the continuum limit

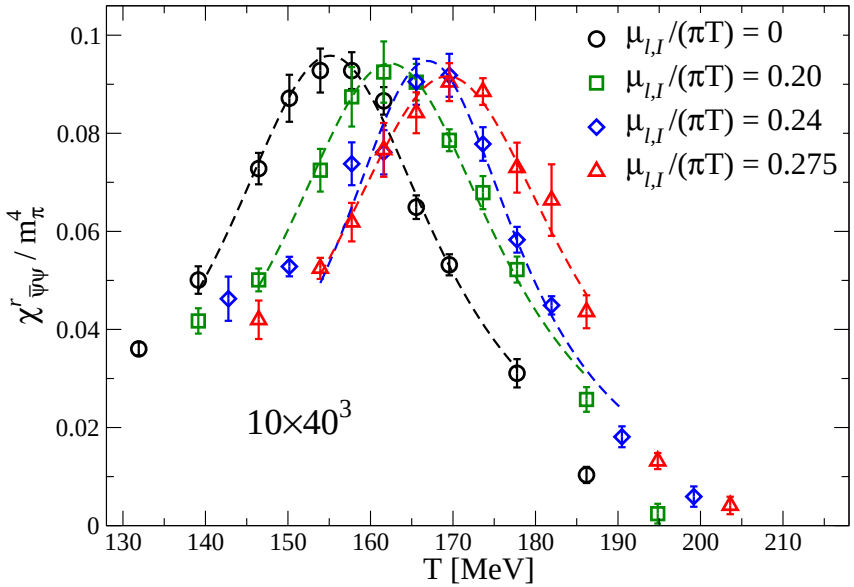
1. fix lattice spacing a
2. for each μ^I compute $T_c(\mu^I, a)$
3. compute the curvature at fixed a: κ_a
4. try several a values and extrapolate κ_a to continuum

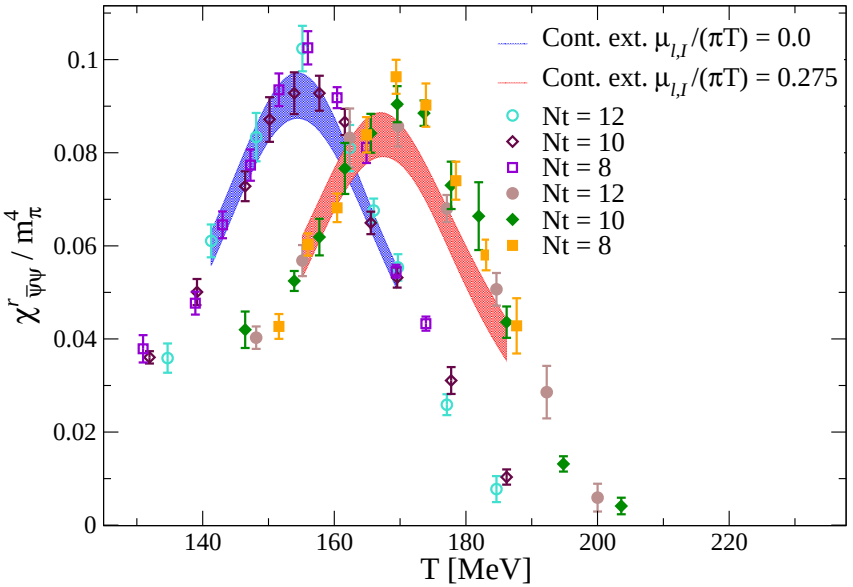
1. fix lattice spacing a
2. for each μ^I compute $\bar{\psi}\psi^r(\mu^I, a)$ and $\chi^r_{\bar{\psi}\psi}(\mu^I, a)$
3. try several a values and extrapolate $\bar{\psi}\psi^r(\mu^I, a)$ and $\chi^r_{\bar{\psi}\psi}(\mu^I, a)$ to continuum
4. find $T_c(\mu^I, a = 0)$ using the results of the previous point
5. compute κ

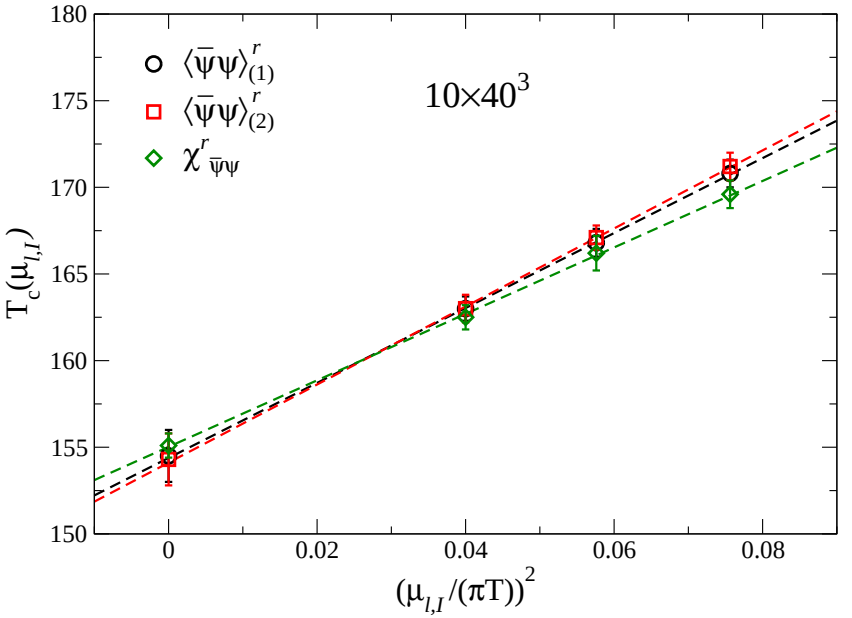
Chemical potentials setup: $\mu^I_u = \mu^I_d = \mu^I_B/3,\ \mu^I_s = 0$ $\quad \mu^I_u = \mu^I_d = \mu^I_s = \mu^I_B/3$ $\qquad \mu^I_B \lesssim 400\,\mathrm{MeV}$

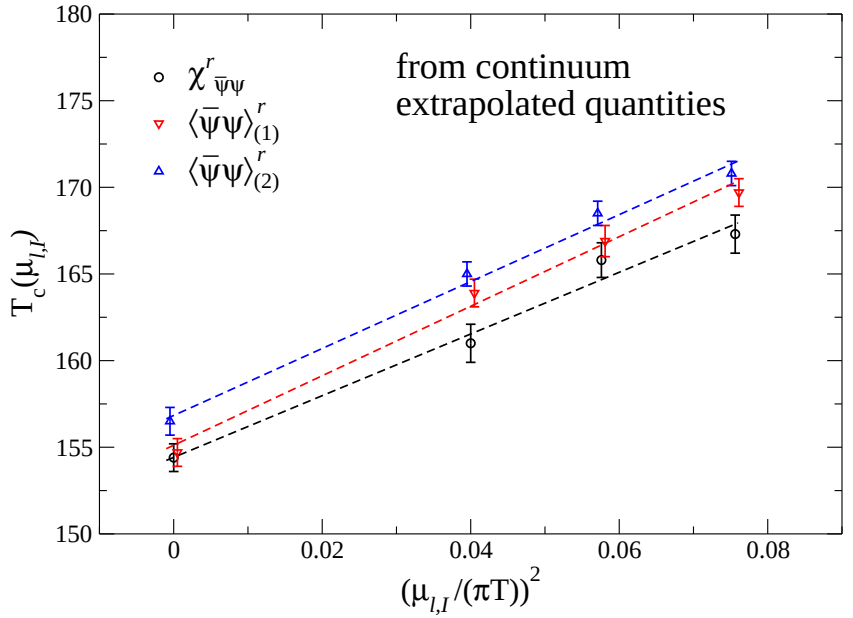
$N_f = 2 + 1$ stout imp. fermions with physical masses, $N_t = 6, 8, 10, 12$.

Bonati et al. PRD (2014-2015)

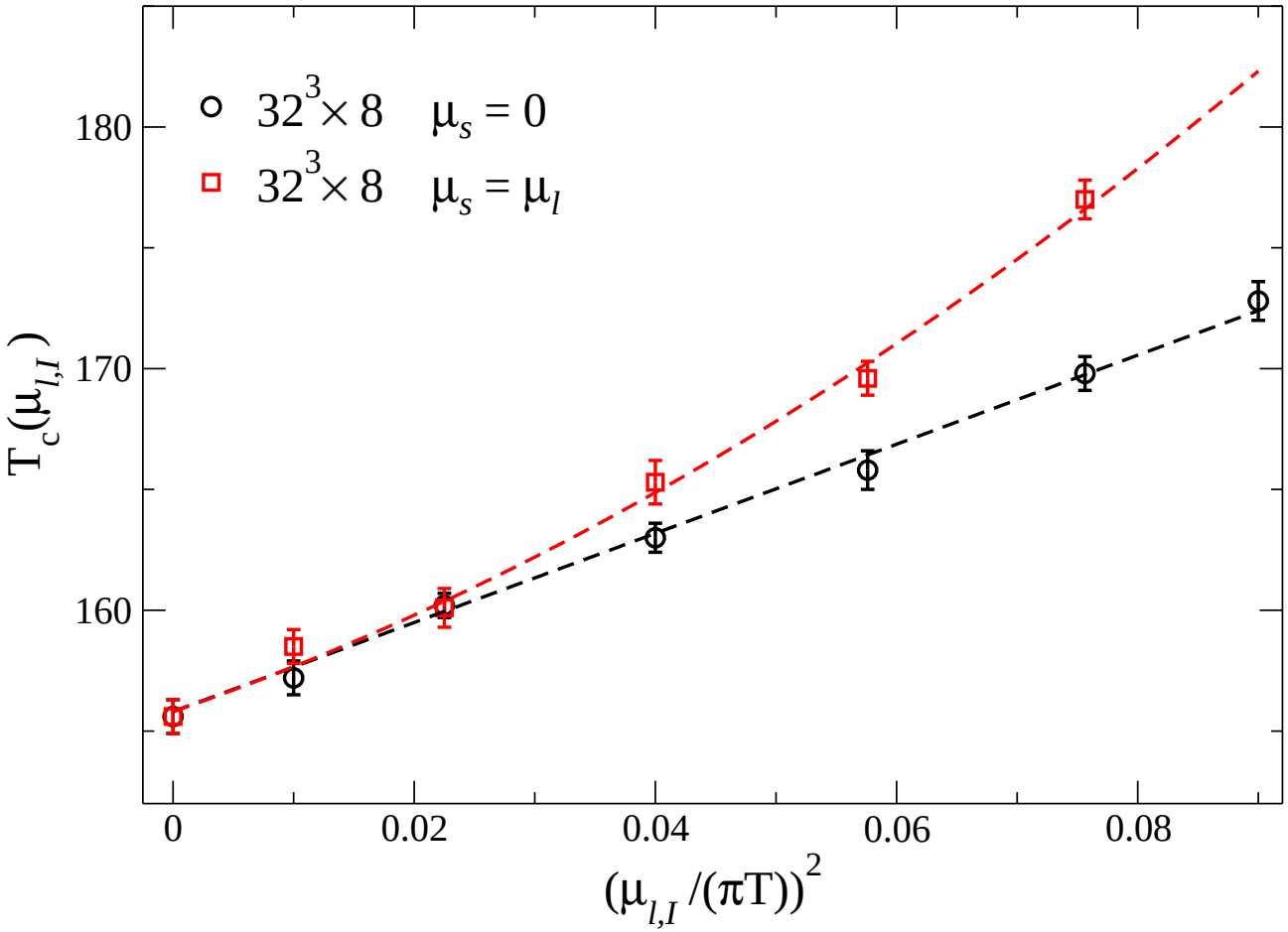






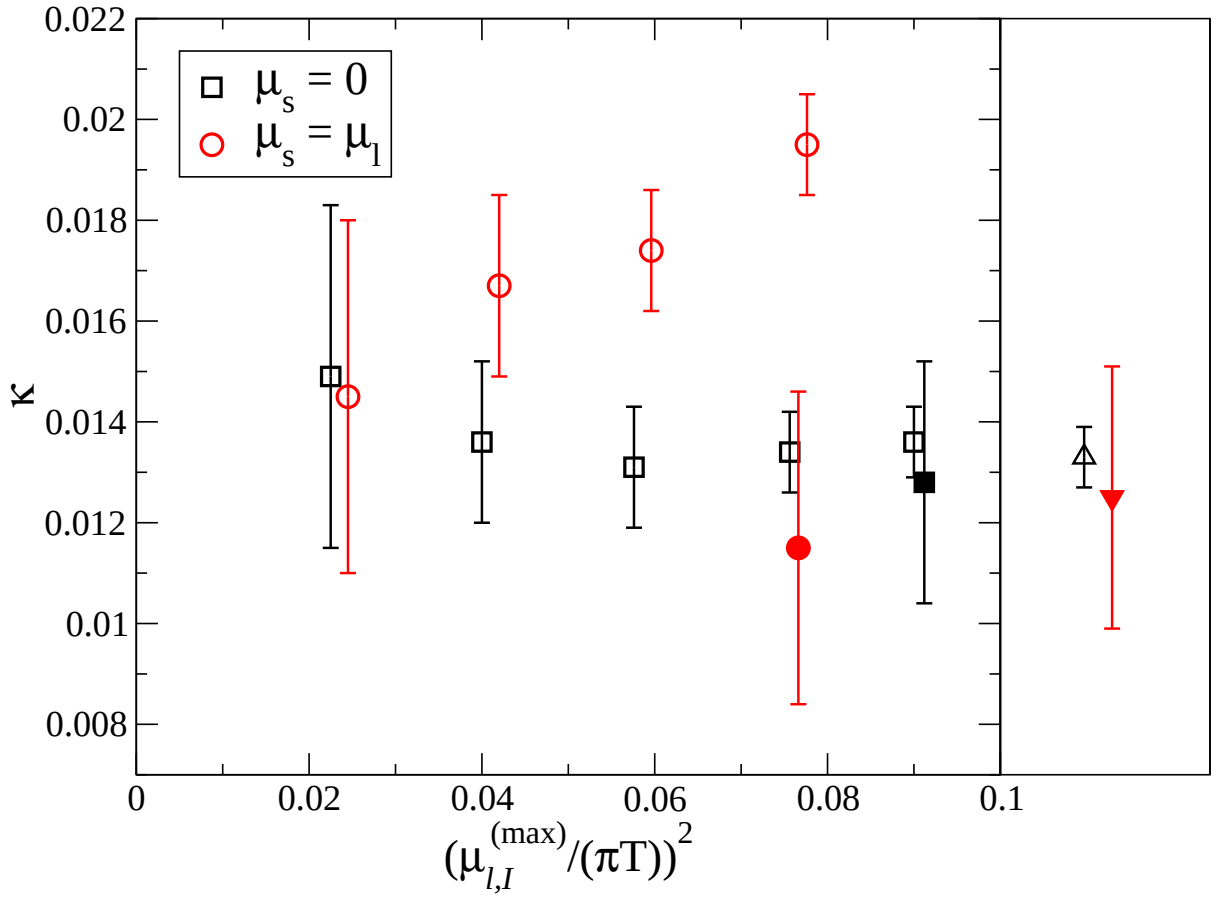


Bonati et al. PRD (2014-2015) μ-systematics



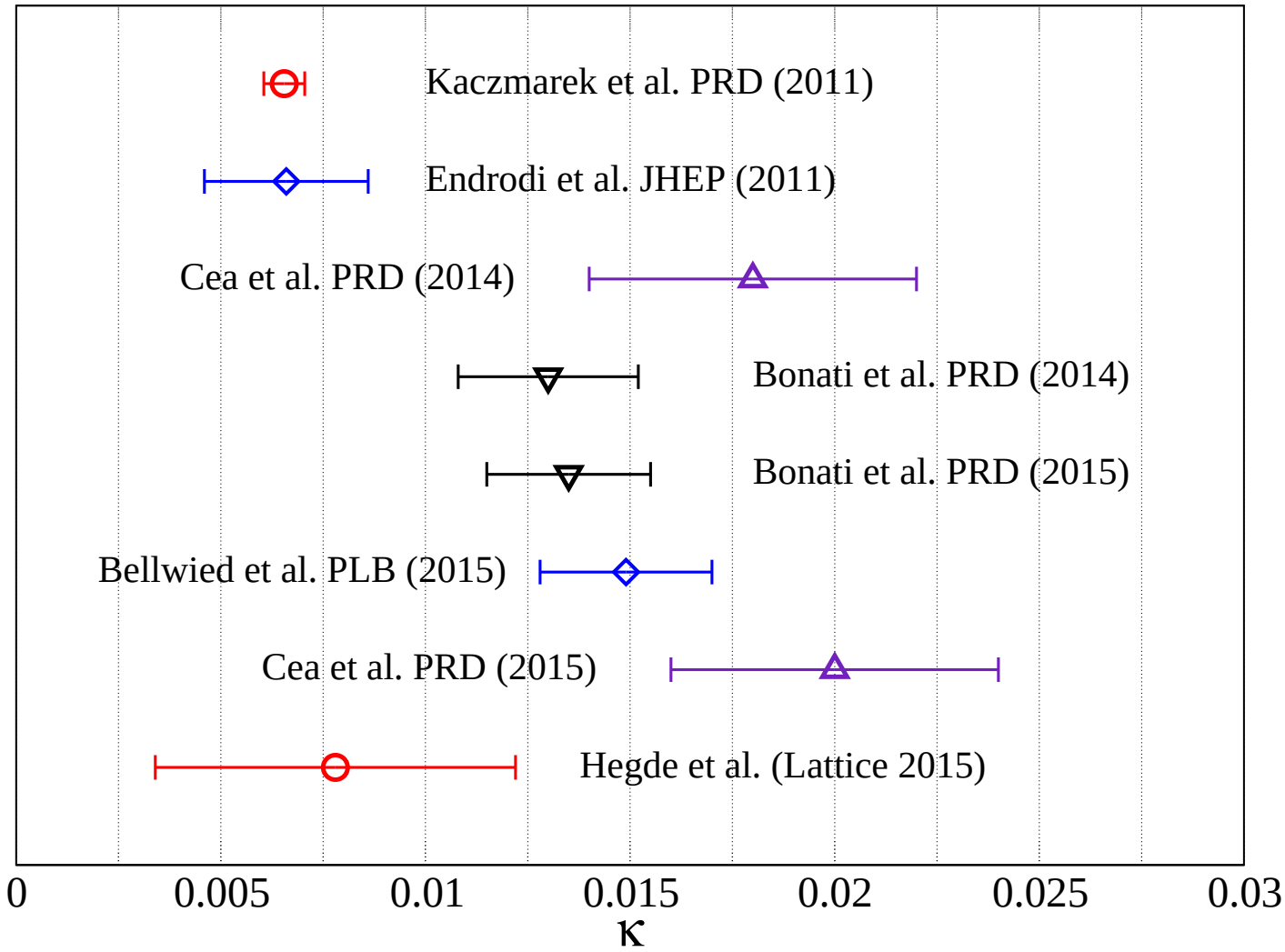
Possible explanation for the different behaviour: the different location of the Roberge Weiss transition.

Bonati et al. PRD (2014-2015) μ-systematics

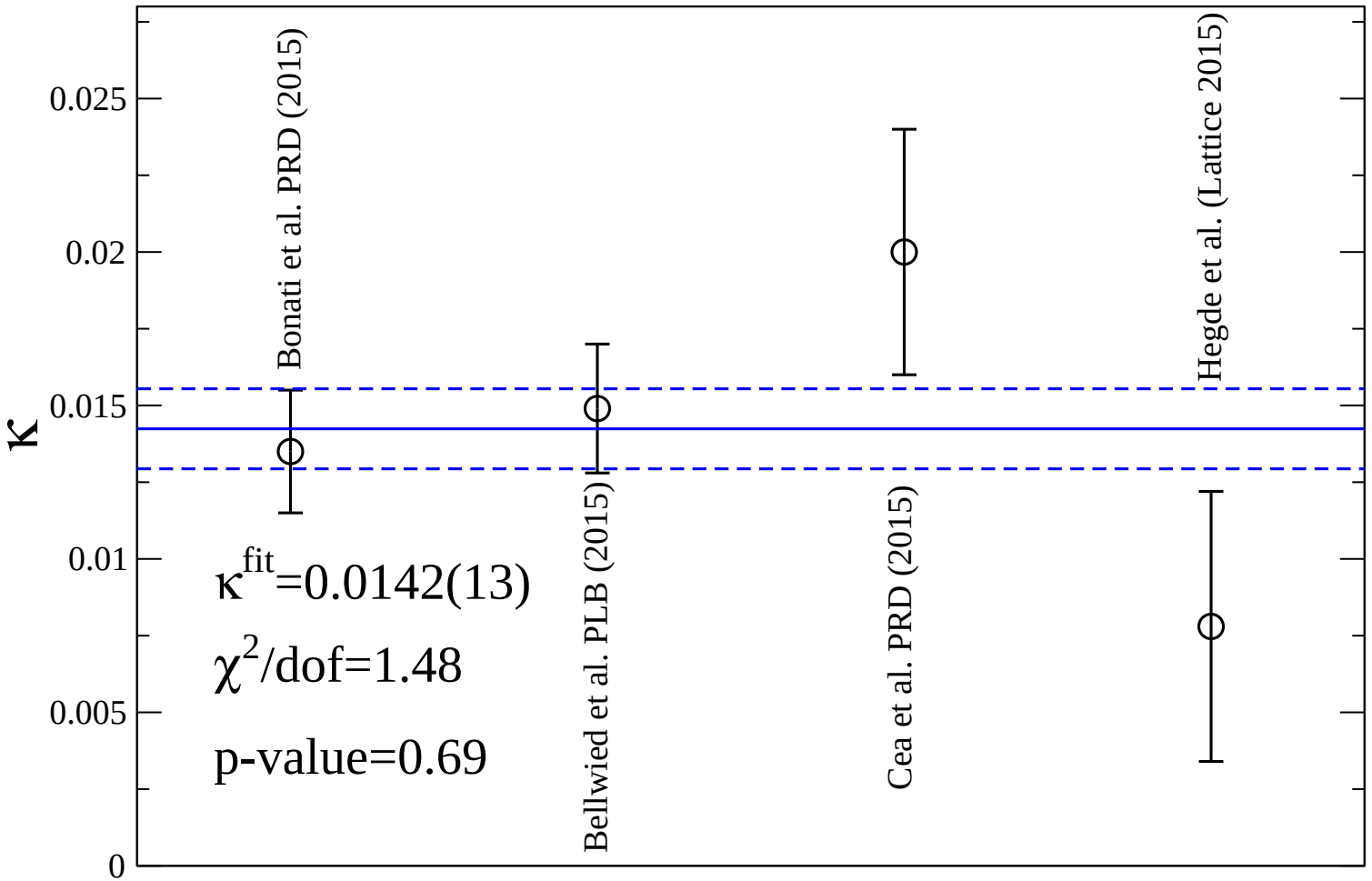


Empty symbols: purely quadratic fit. Filled symbols: also quartic correction. Right panel: combined fit (i.e. fixing a common value for $T_c(0)$) to both data sets when a quartic correction is used for the $\mu_s = \mu_l$ data. The open (filled) triangle corresponds to $\mu_s = 0$ ($\mu_s = \mu_l$).

κ from LQCD summary



Fit fun


0.025
0.02
0.015
0.01
0.005
0
κ
Bonati et al. PRD (2015)
Bellwied et al. PLB (2015)
Cea et al. PRD (2015)
Hegde et al. (Lattice 2015)
κ^fit=0.0142(13)
χ^2/dof=1.48
p-value=0.69

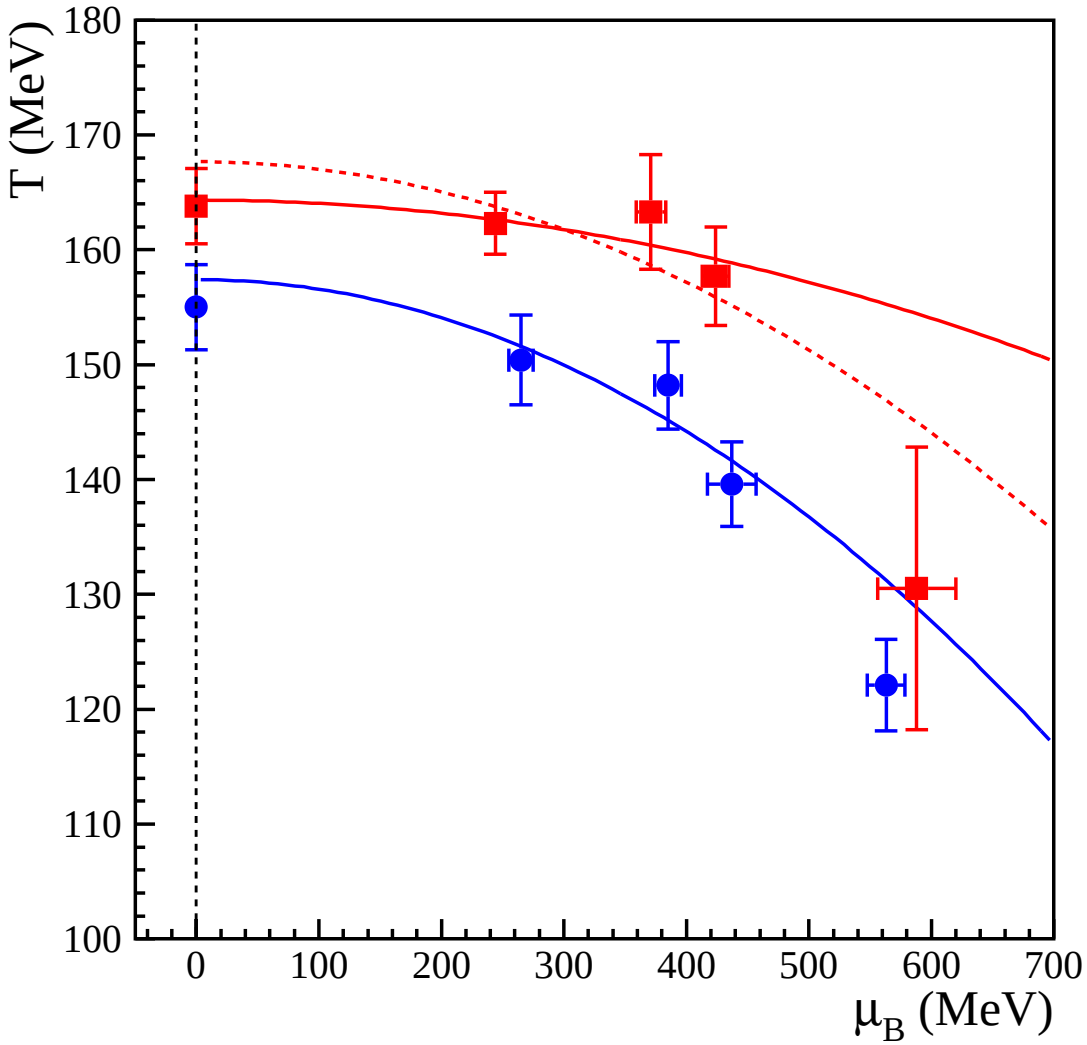
Conclusions

- The study of the critical line of QCD at finite density is an interesting topics, where great advances are in principle possible both on the theoretical (e.g. sign problem solution?) and on the experimental side (e.g. signal of critical point in heavy ions collisions?).
- Recent lattice QCD computations are in reasonable agreement with each other but some fine points have to be further investigated: why does Taylor expansion/analytic continuation underestimate/overestimate κ?
- Relation between κ and the curvature of the freeze-out curve? How far can equilibrium physics go?

Thank you for your attention!

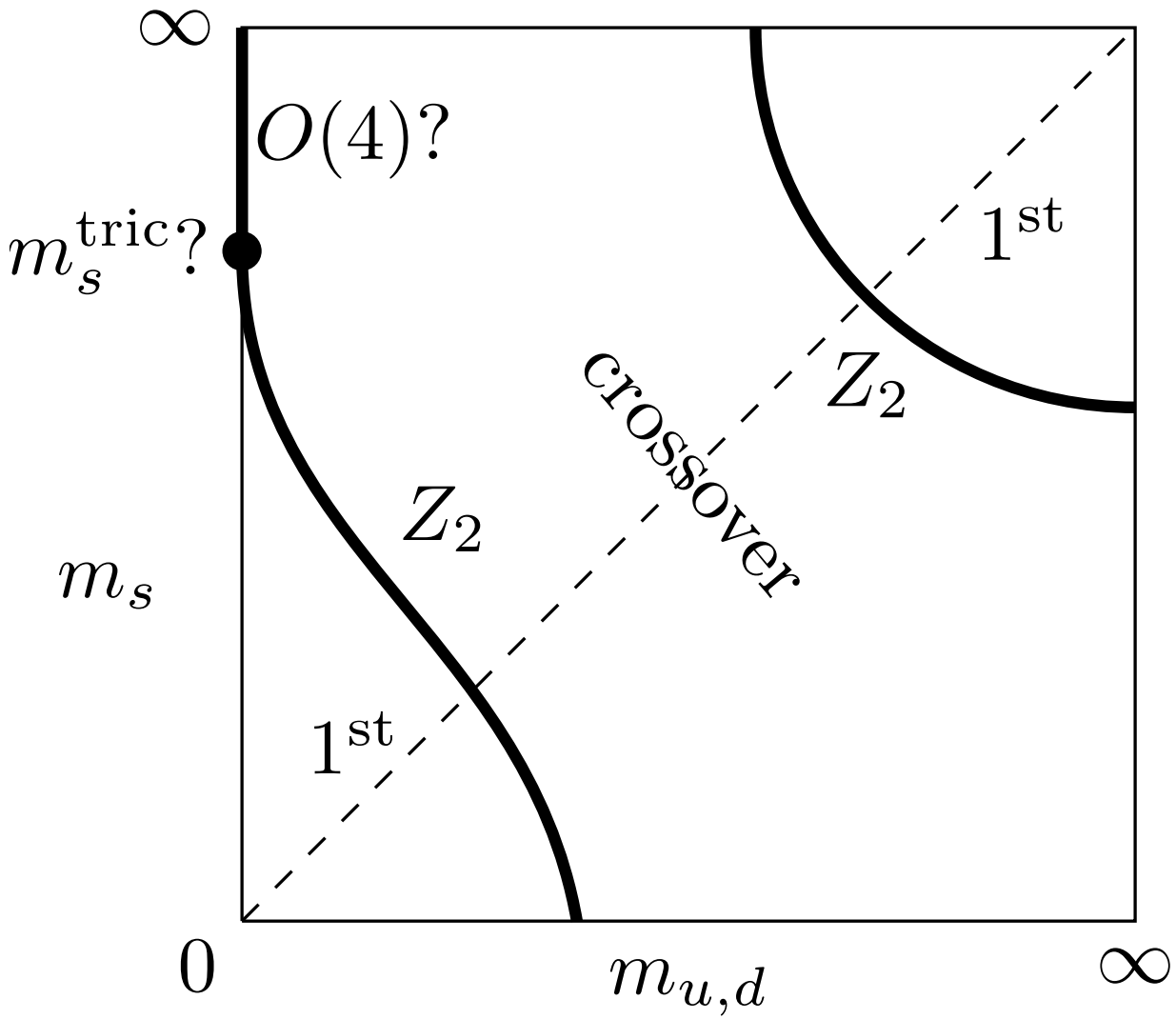
Backup slides with something more

“real” fit range problem

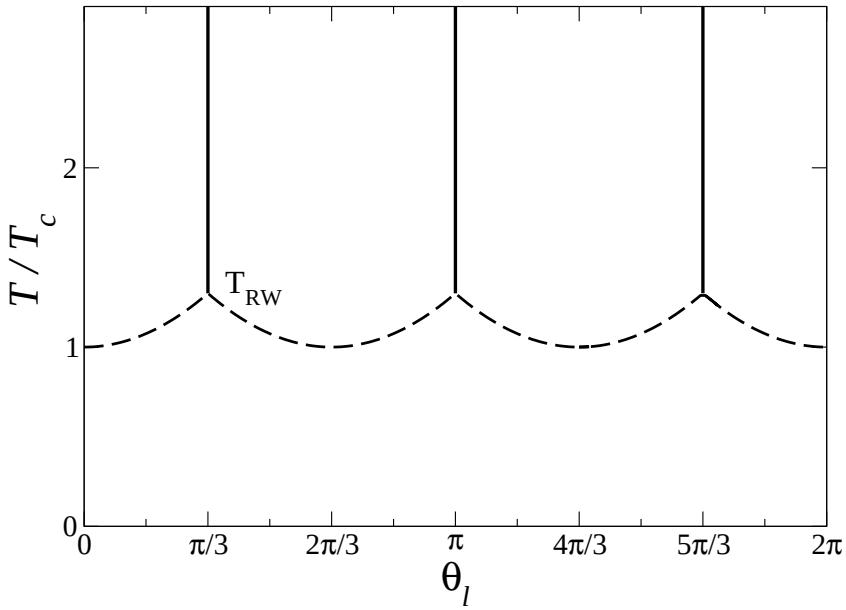


from Becattini et al. (2016)

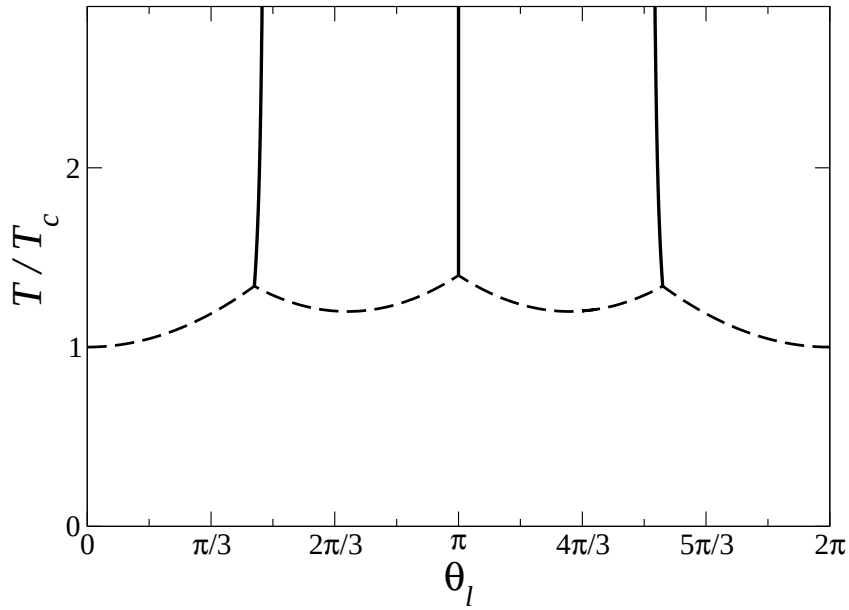
The "Columbia plot"



Phase diagram at imaginary chemical potential

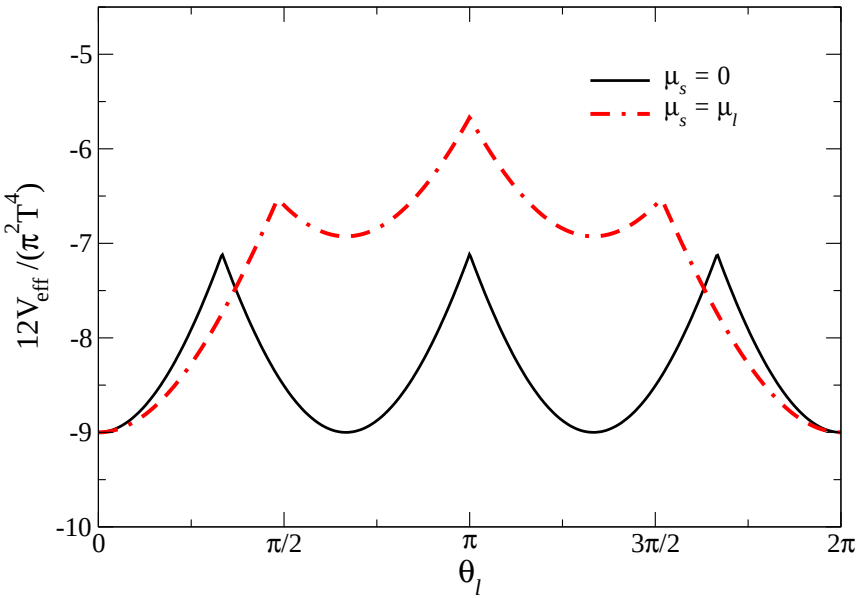


$\mu_s = \mu_\ell,\ \theta_\ell = \mu_\ell^I / T$

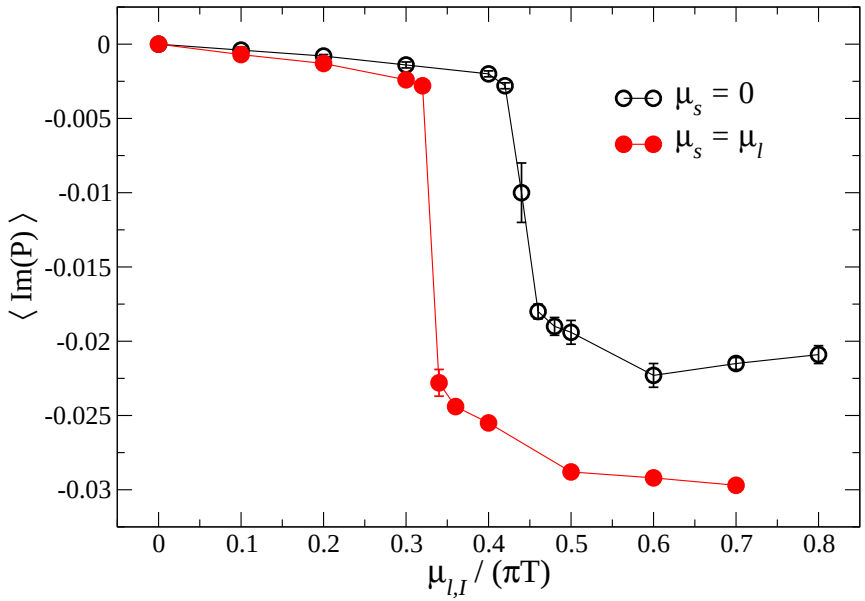


$\mu_s = 0,\ \theta_\ell = \mu_\ell^I / T$

Phase diagram at imaginary chemical potential



One loop effective potential for the Polyakov loop, $\theta_\ell = \mu_\ell^I / T$



8×32^3 lattice at $T \simeq 208\,\mathrm{MeV}$