Chiral Thermodynamics
in a box

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in collaboration with **Ana Juricic**

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HIC for FAIR Helmholtz International Center — **Germany**

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Critical Point and Onset of Deconfinement 2016

and

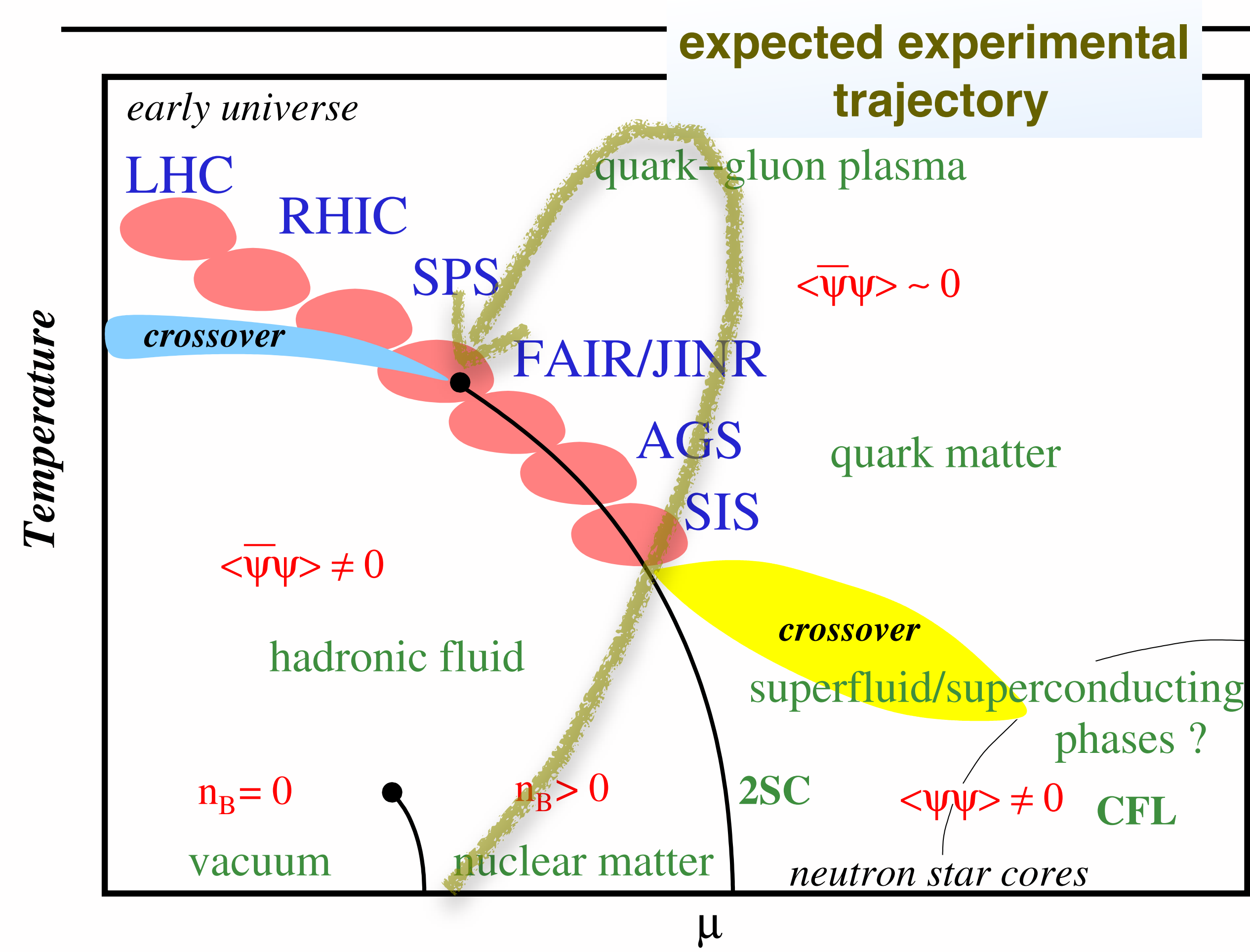
Working Group Meeting of COST Action MP1304

CPOD

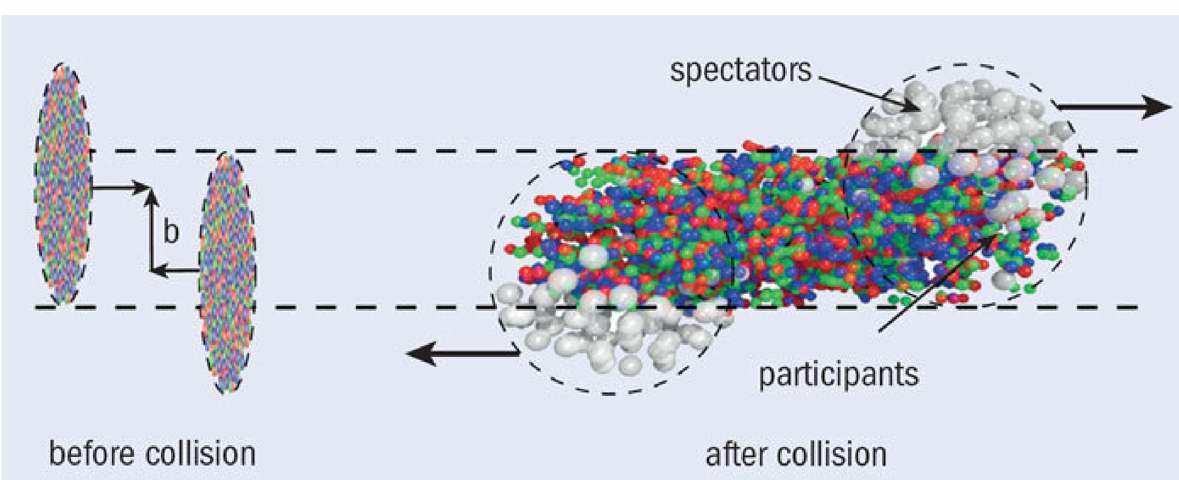
Wrocław, Poland

May 30th - June 4th, 2016

Conjectured QC3D phase diagram



Experiment:



Theory:

➜ Lattice: but simulations restricted to small μ

➜ Functional QFT methods: FRG,DSE, nPI

➜ Models: effective theories parameter dependency

Experiment: (non-equilibrium)

➜ in a finite box (HBT radii: freeze-out vol. ~ 2000-3000 fm³)

(UrQMD ($\sqrt{s}$): system vol. ~ 50 - 250 fm³)

Theoretical aim:

deeper understanding & more realistic HIC description

➜ existence of critical end point(s)?

Non-trivial physical issues!

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Agenda

- **Motivation: physics in a finite volume**

- **Generalized susceptibilities**

  ➜ towards chiral phase transition

- **Role of Fluctuations:**
  from mean-field approximations to RG

- **Comparison:**
  Finite/infinite volume effects

FAIR ***Aug. 2014***

Agenda

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FAIR ***Sept. 2015***

Lattice simulations

[Colangelo 2004]

Lattice simulations:

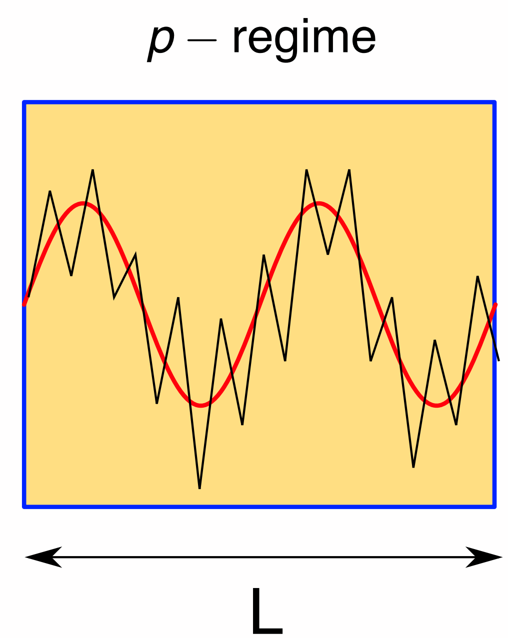
QCD lattice simulations

- finite volume
- finite lattice spacing
- finite quark masses

  ➜ three (expensive) extrapolations are necessary

  ➜ analytical methods are needed

  e.g. chiral perturbation theory



pion Compton wavelength

➜ fits into box

and close to chiral limit:

➜ expensive

Physical system in a finite box

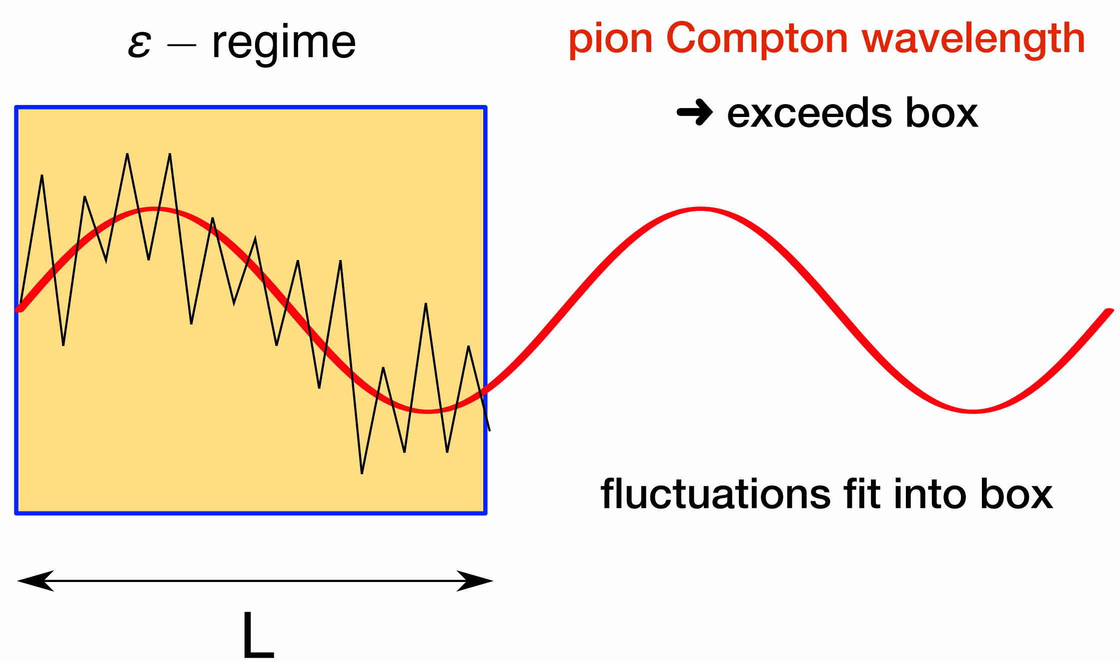
periodic boundary: $\vec{p} \sim \frac{2\pi}{L}\vec{n} \quad n_i \in \mathbb{Z}$

Interest: soft, non-perturbative dynamics of QCD

➜ region of soft momenta should look like a continuum

size scale separation (chiral Goldstone bosons):

$$\frac{2\pi}{L} \ll 4\pi f_\pi \quad \Rightarrow \quad L \gg \frac{1}{2 f_\pi} \sim 1 \text{ fm}$$



pion Compton wavelength

➜ exceeds box

fluctuations fit into box

Physics in a finite Volume

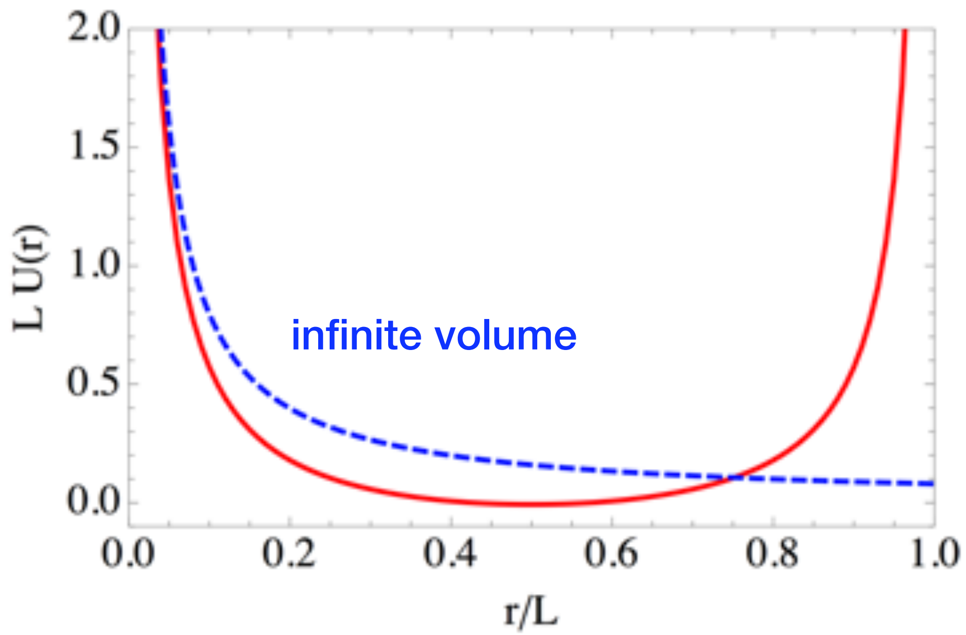
[Davoudi, Savage 2014]

Lattice simulations:

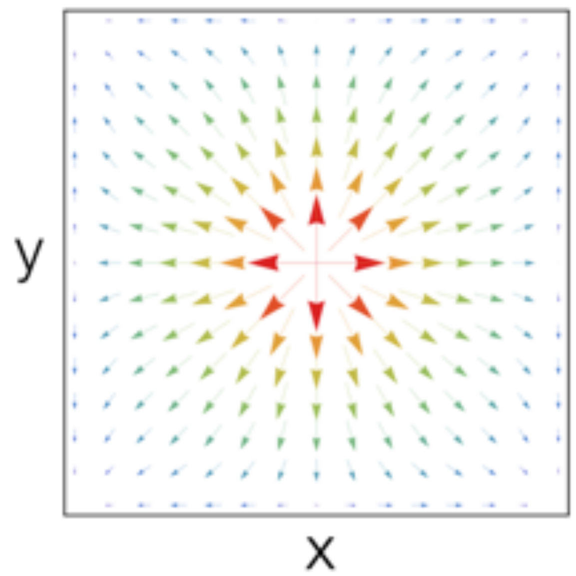
QCD (short-ranged) with QED (long-ranged ➜ truncated) corrections

➜ violation Gauss's & Ampere's law

if EM gauge field subject to periodic boundary condition



finite volume Coulomb potential between two charges



point charge at the center

circumvent this problem:

introduce uniform background charge density

similar to many-body physics

➜ equivalent to

removing zero modes of the gauge field

Physics in a finite Volume

Quantum Field Theory in a finite volume:

➜ no spontaneous symmetry breaking

if only finite number of degrees of freedom

QCD: [Gasser, Leutwyler 1988]

chiral condensate: non-perturbative phenomenon

e.g.

chiral symmetry

$$N_f = 2 : SU(2) \times SU(2) \cong O(4)$$

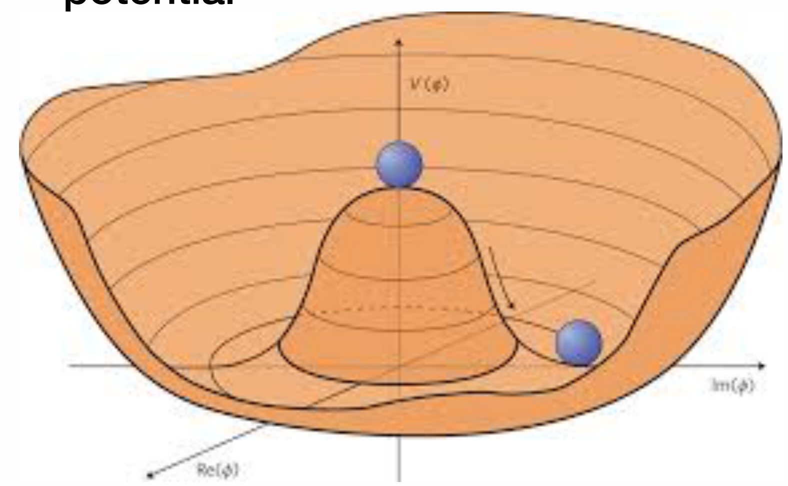
$$O(4) \to O(3)$$ infinite volume

massless Goldstone Bosons

finite volume:

fluctuations of Goldstone bosons always restore symmetry

potential



minimum: zero-momentum mode of the field

$$Z_2 : \varphi \to -\varphi$$

probability of tunneling: $P_{\text{tunnel}} \sim e^{-L}$

exponentially suppressed with volume

$O(N)$ - case: rotation ➜ averaging to zero (no breaking)

infinite volume ➜ no tunneling ➜ symmetry broken

Physics in a finite Volume

result so far:

long-range correlations are necessary to obtain spontaneous SB (for a continuous symmetry)

chiral limit: massless Goldstone boson fluctuations in a finite box avoid symmetry breaking

but

symmetry breaking in mean-field approximations are possible:

Goldstone-fluctuations are absent

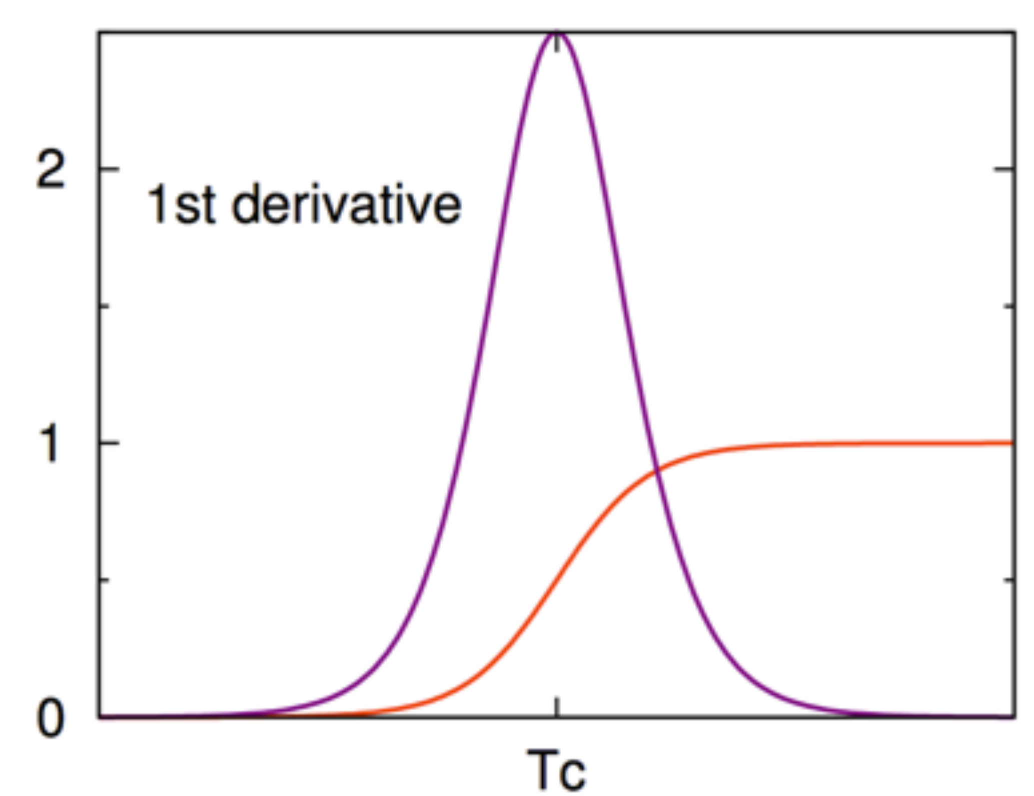
Thermodynamics on a torrus:

correlation length always finite ➜ no real 2nd order phase transition

criterion for phase transition: (generalized) susceptibilities

➜ derivatives of order parameter reveal more details

derivatives of thermodynamic quantities ↔ fluctuations



Fluctuation observables

- generalized susceptibilities:

$$\chi_n = \left. \frac{\partial^n p(T,\mu)/T^4}{\partial(\mu/T)^n} \right|_T$$

- Fluctuations of conserved charges

$$\delta Q_X = Q_X - \langle Q_X \rangle \qquad X = Q, B, S, \ldots$$

mean value: $\chi_1 \sim \langle Q \rangle$

$\chi_2 \sim \langle (\delta Q)^2 \rangle$

$\chi_3 \sim \langle (\delta Q)^3 \rangle$

- Measured in event-by-event multiplicity distributions

variance: $\sigma^2 \sim \frac{\chi_2}{\chi_1}$

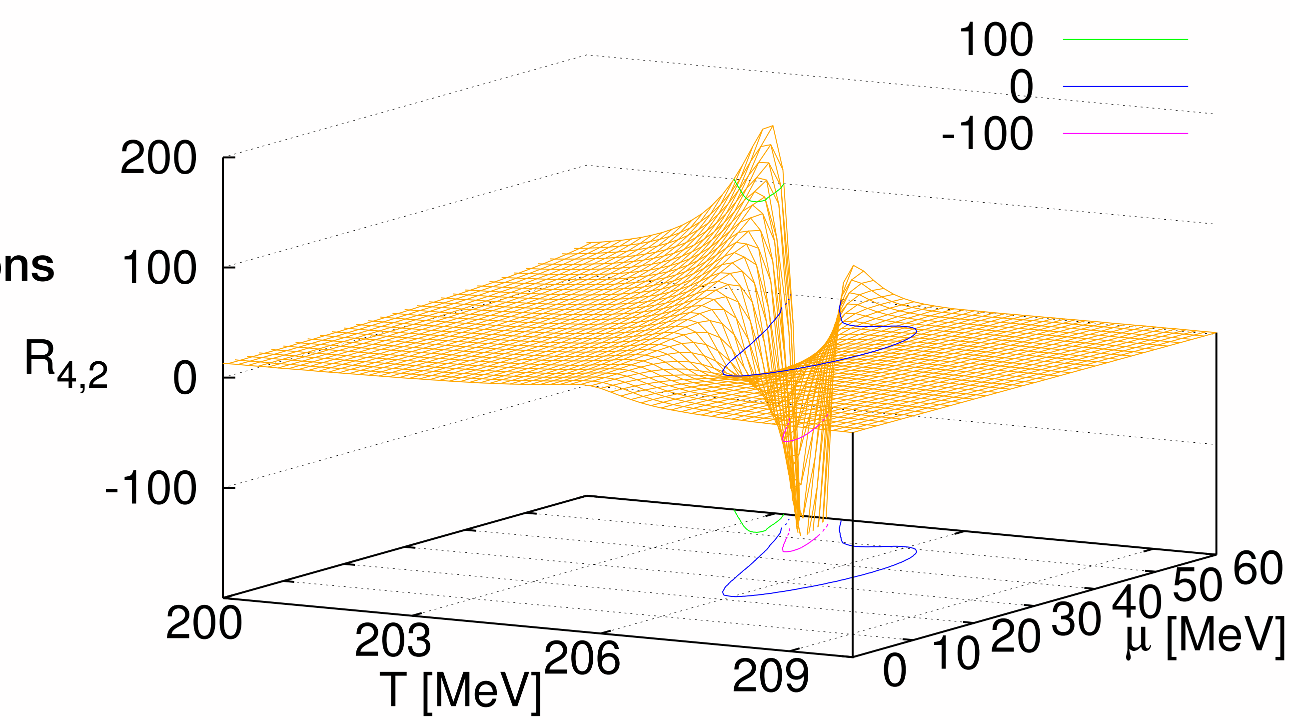
skewness: $S\sigma = \frac{\chi_3}{\chi_2}$

kurtosis: $\kappa\sigma^2 = \frac{\chi_4}{\chi_2}$

[STAR Coll. 2014; PHENIX Coll 1506.07834]

strong temperature & density dependence of ratios

[BJS, M. Wagner 2012]

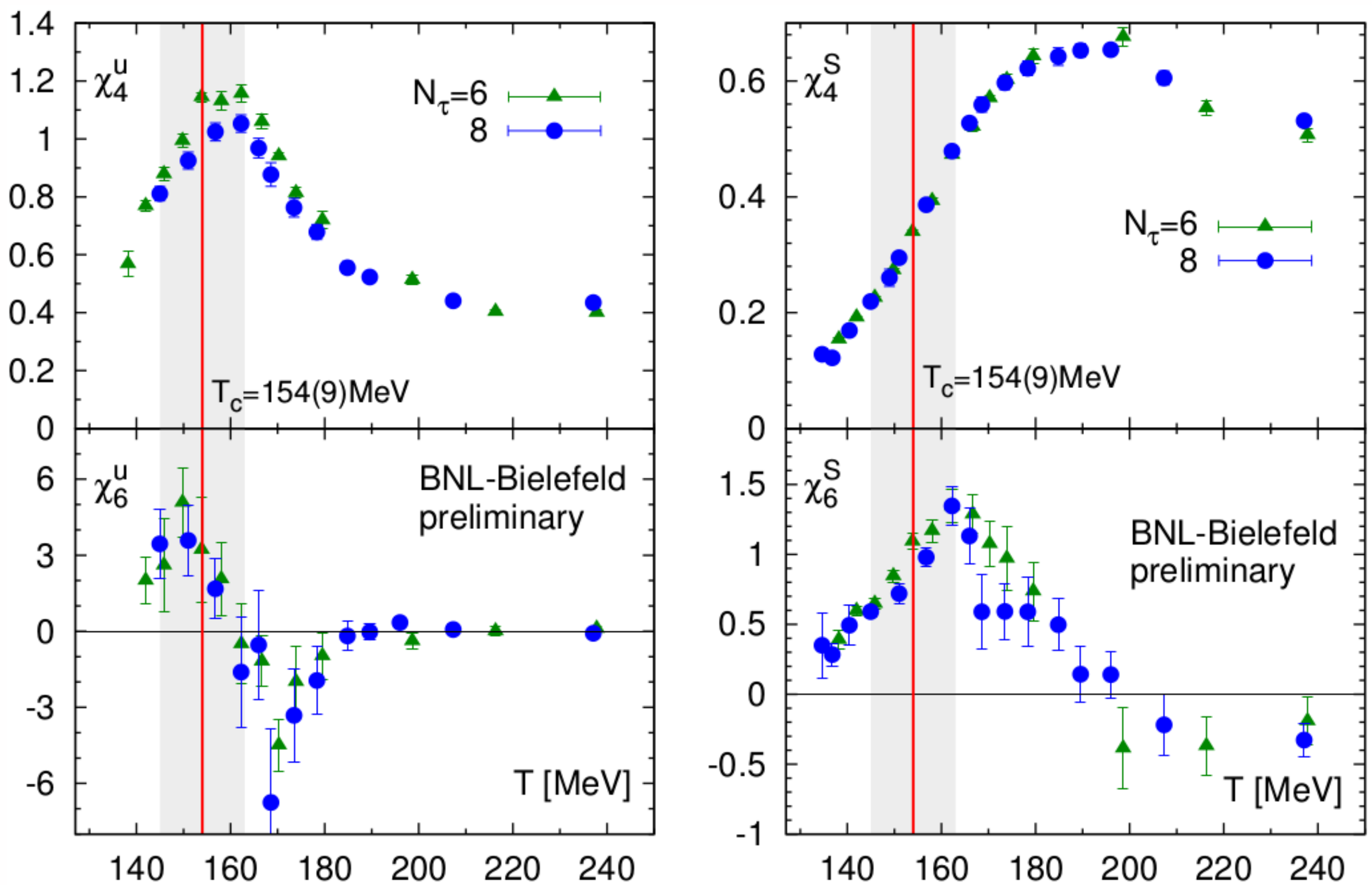


Fluctuation observables

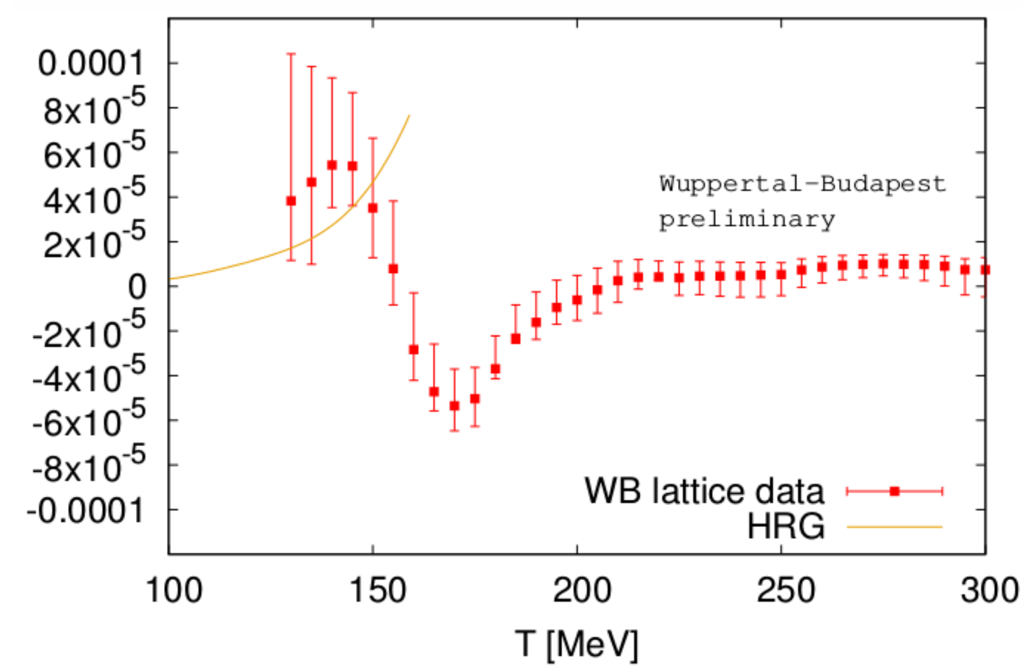
$$\chi_n = \left.\frac{\partial^n p(T,\mu)/T^4}{\partial(\mu/T)^n}\right|_T$$

change of sign ➜ deviations from HRG ➜ criticality

[Schmidt et al. 2015]



[Bellwied et al. 2016]



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Grand potential

Low energy QCD model: $\phi = (\sigma, \vec{\pi})$

$$\mathcal{L} = \bar{\psi}\left(\not{\partial} + g(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)\right)\psi + \frac{1}{2}(\partial_\mu\phi)^2 + V_{\text{Meson}}(\phi)$$

Partition function:

$$\mathcal{Z}(T,\mu) = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\phi\, e^{-\int d^4x\, \mathcal{L}(\bar{\psi},\psi,\phi)}$$

replace with (const.) condensate σ

Integration of quarks, neglect bosonic fluctuations: ➜ mean-field approximations MFA

Integration of quarks and bosonic fluctuations: ➜ renormalization group treatment FRG

Grand potential (Polyakov-)quark-meson model (quark loop)

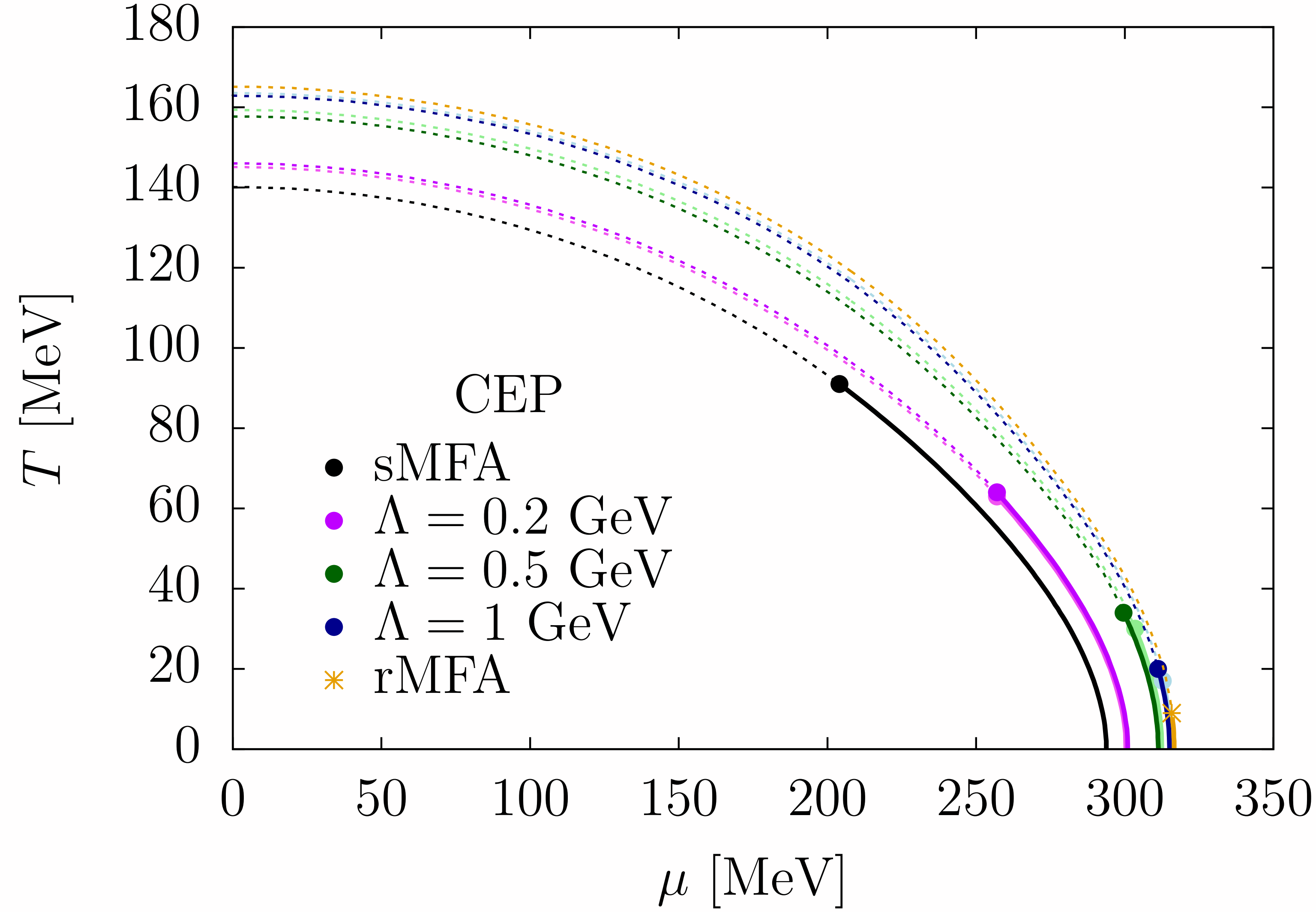
$$\Omega(T,\mu;\sigma) = \Omega_{\text{vac}} + \Omega_{\text{T}} + V_{\text{MF}}(\sigma) \qquad (+\mathcal{U}_{\text{Poly}}(\Phi)\ )$$

$$\Omega_{\text{vac}} = -4\int^{\Lambda} \frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^2 + m_q^2}$$

Vacuum term: regularize e.g. with sharp O(3)-momentum cutoff

Infinite volume

[A Juricic, BJS to be published]



sMFA: no vacuum fluctuations

rMFA: renormalized MFA

e.g.

Pauli-Villars regularization

sharp O(3)-momentum cutoff

proper-time regularization

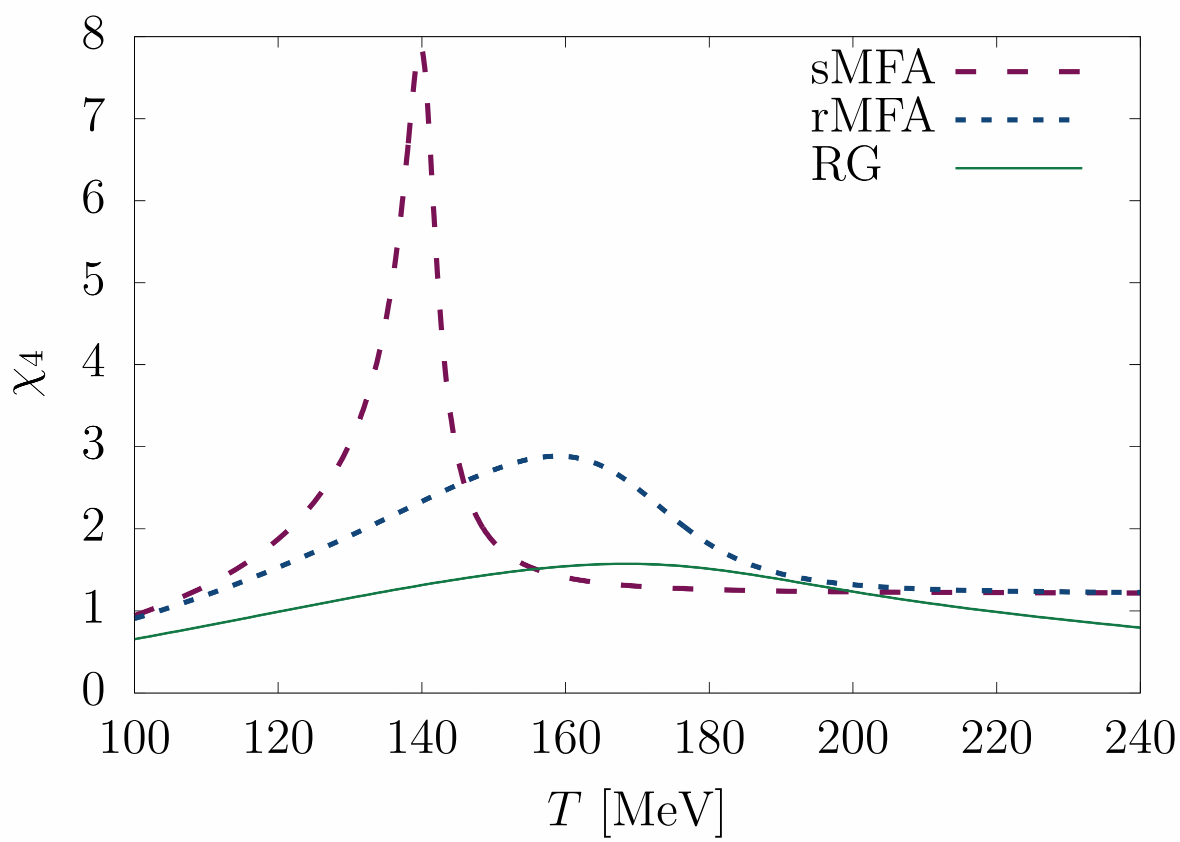
Infinite volume

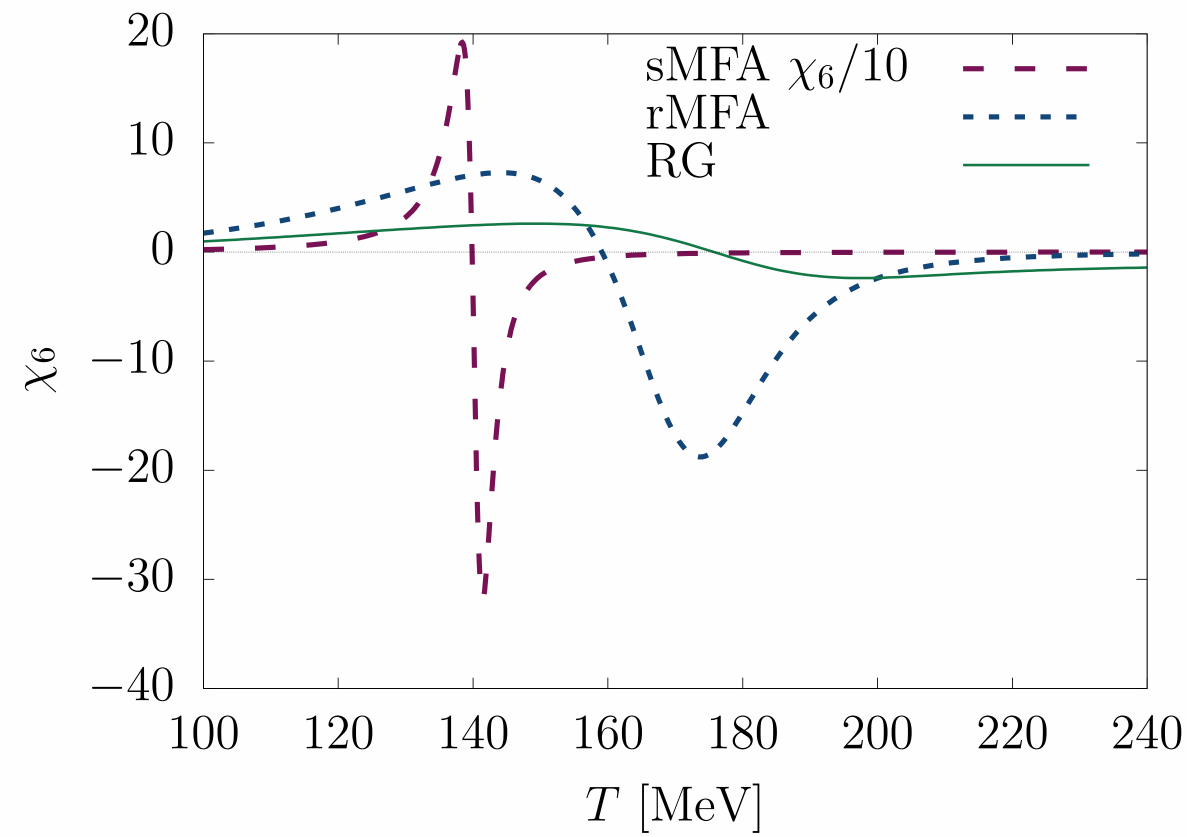
generalized susceptibilities

standard MFA:
no quark vacuum fluctuations

renormalized MFA:
including quark vacuum fluctuations

RG:
quark + meson fluctuations





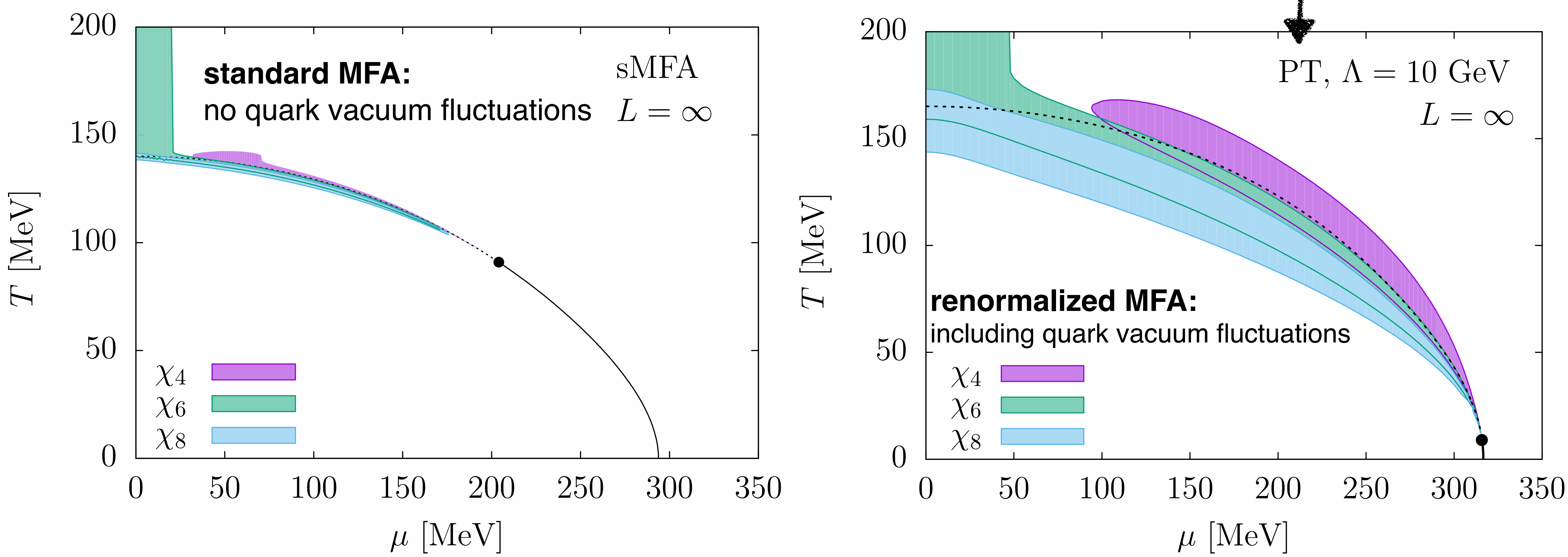
[A Juricic, BJS to be published]

Higher cumulants

[A Juricic, BJS to be published]

proper time regularization

infinite volume: influence of fluctuations



findings:

(quark) fluctuations pushes CEP to smaller T and bigger µ

Fluctuations wash out phase transition ➜ broader negativer regions

Higher cumulants

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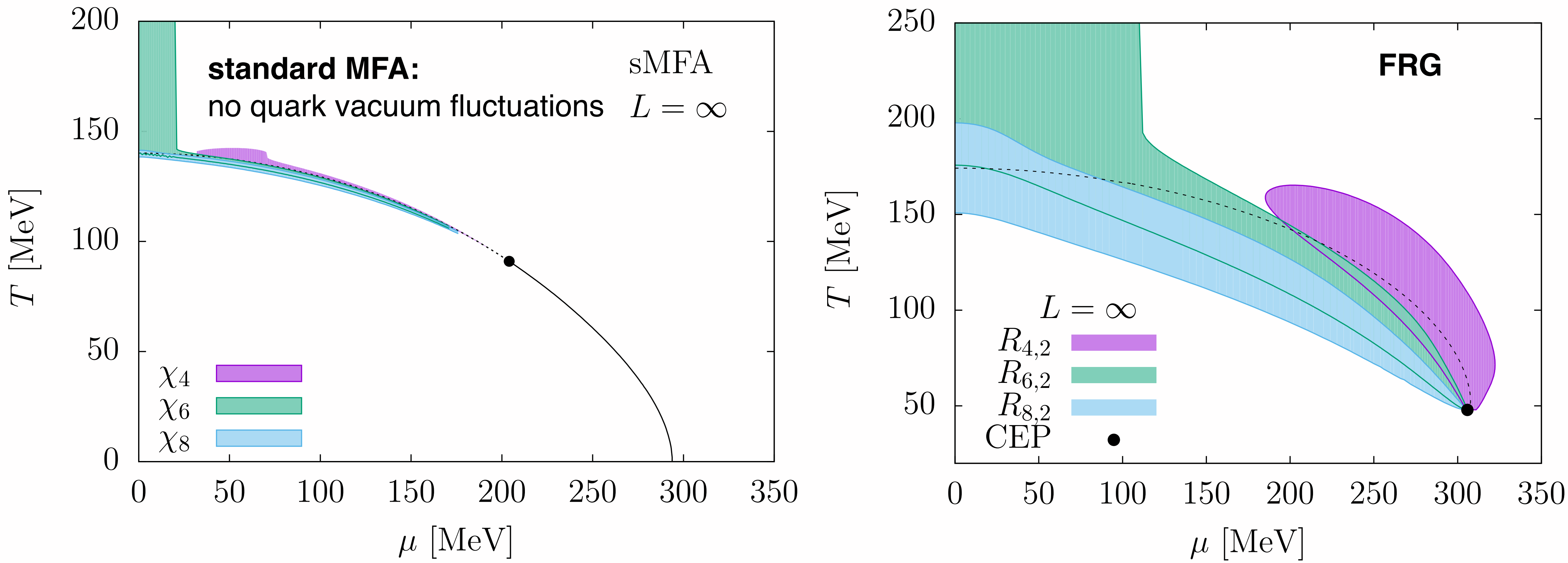
infinite volume: influence of fluctuations



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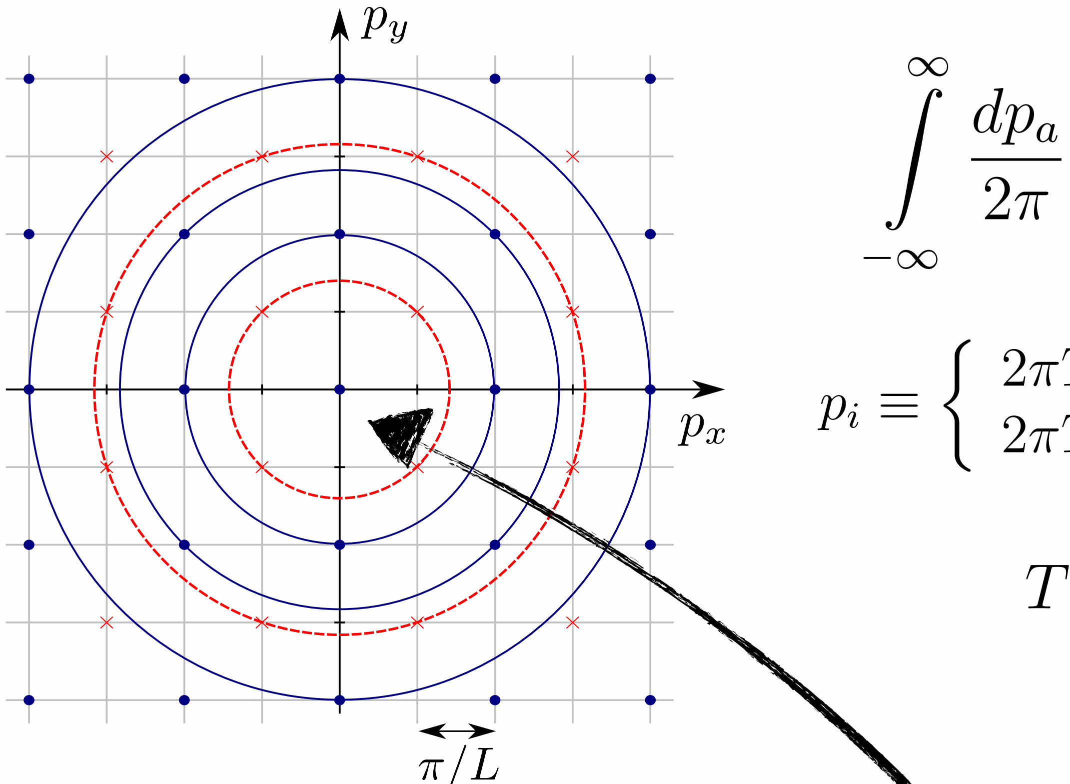
Finite volume



• periodic

× antiperiodic boundary conditions

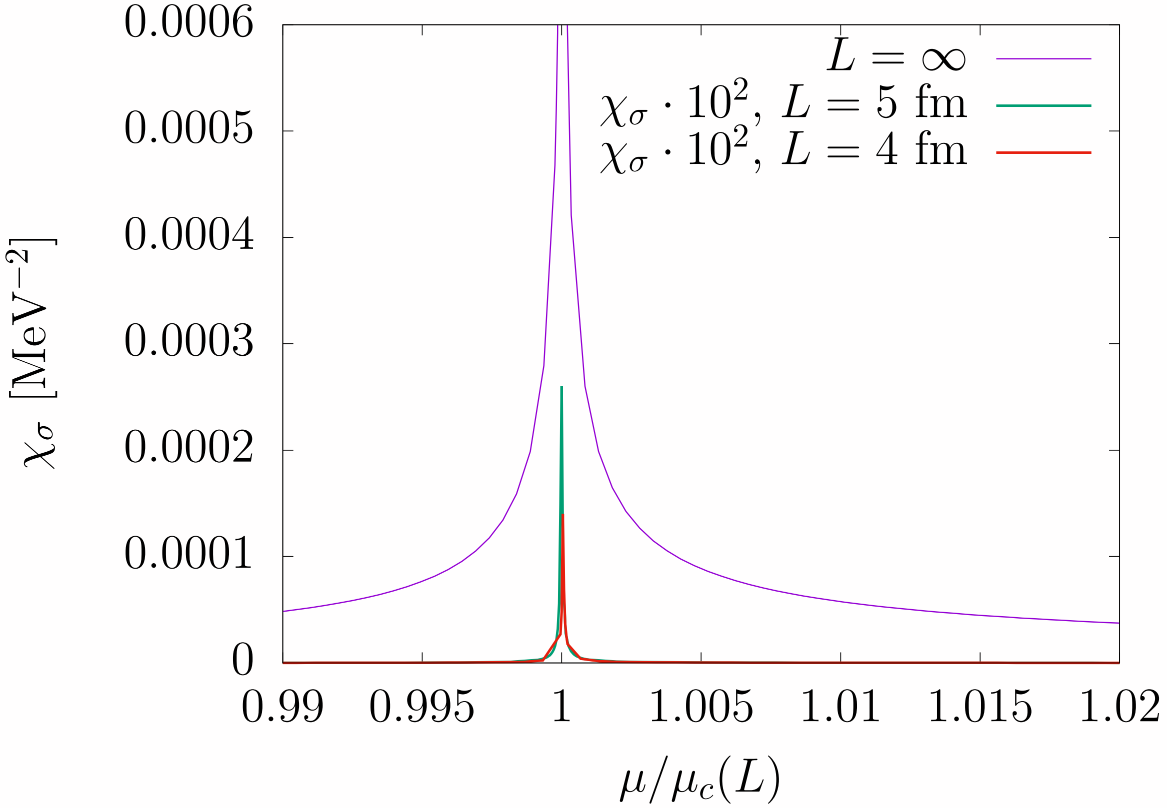
$$\int_{-\infty}^{\infty} \frac{dp_a}{2\pi} \ldots \to \frac{1}{L} \sum_{n_a}$$

$$p_i \equiv \begin{cases} 2\pi T n_i \\ 2\pi T (n_i + \frac{1}{2}) + i\mu \end{cases}$$

$$T \leftrightarrow 1/L$$

Longitudinal susceptibility:

$$\chi_\sigma = \frac{1}{m_\sigma^2} \sim \frac{\partial \langle \bar{q} q \rangle}{\partial m_q}$$



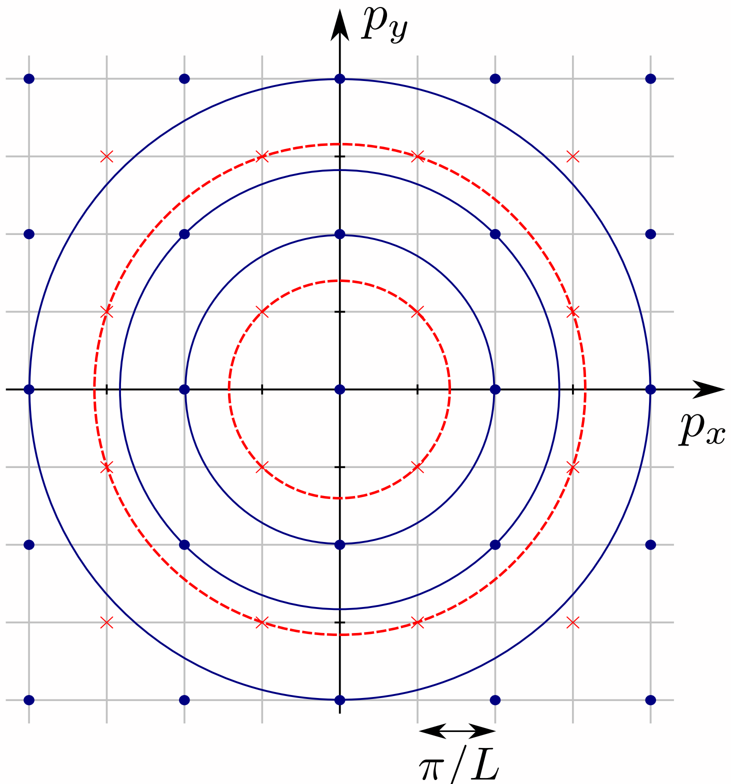
<u>Boundary conditions (BC)</u>

PBC: periodic including zero mode

PBC*: star means without zero mode

ABC: antiperiodic

Finite volume



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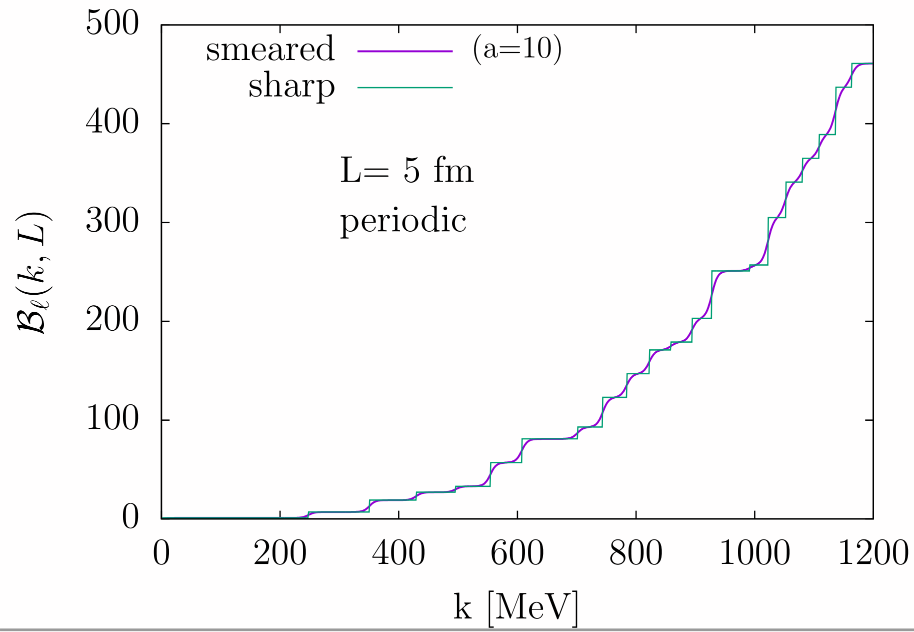

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Longitudinal susceptibility:

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Flow for sharp Litim regulator (not suitable for finite volume)

$$\partial_k U_k(T, L) \sim \mathcal{B}_\ell \cdot \partial_k U_k(T, \infty)$$

$$\mathcal{B}_\ell(k, L) = \frac{6\pi^2}{(kL)^3} \sum_{\vec{n}} \Theta(k^2 - \vec{p}_\ell^{\,2})$$

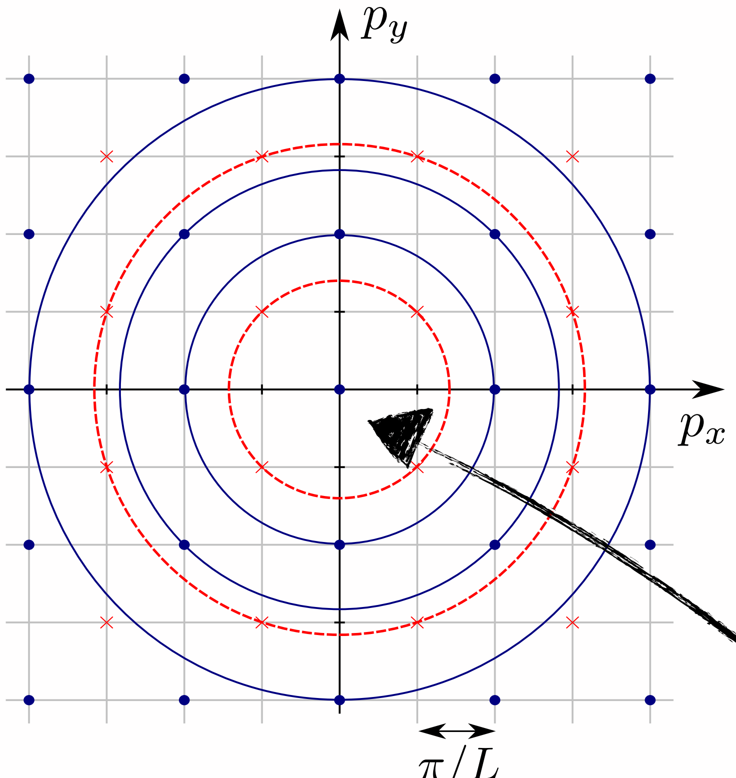
➜ use smeared regulator

[Juricic, BJS in preparation]

[Fister, Pawlowski 2015]

[Tripolt, Braun, Klein, BJS 2012, 2014]

Finite volume



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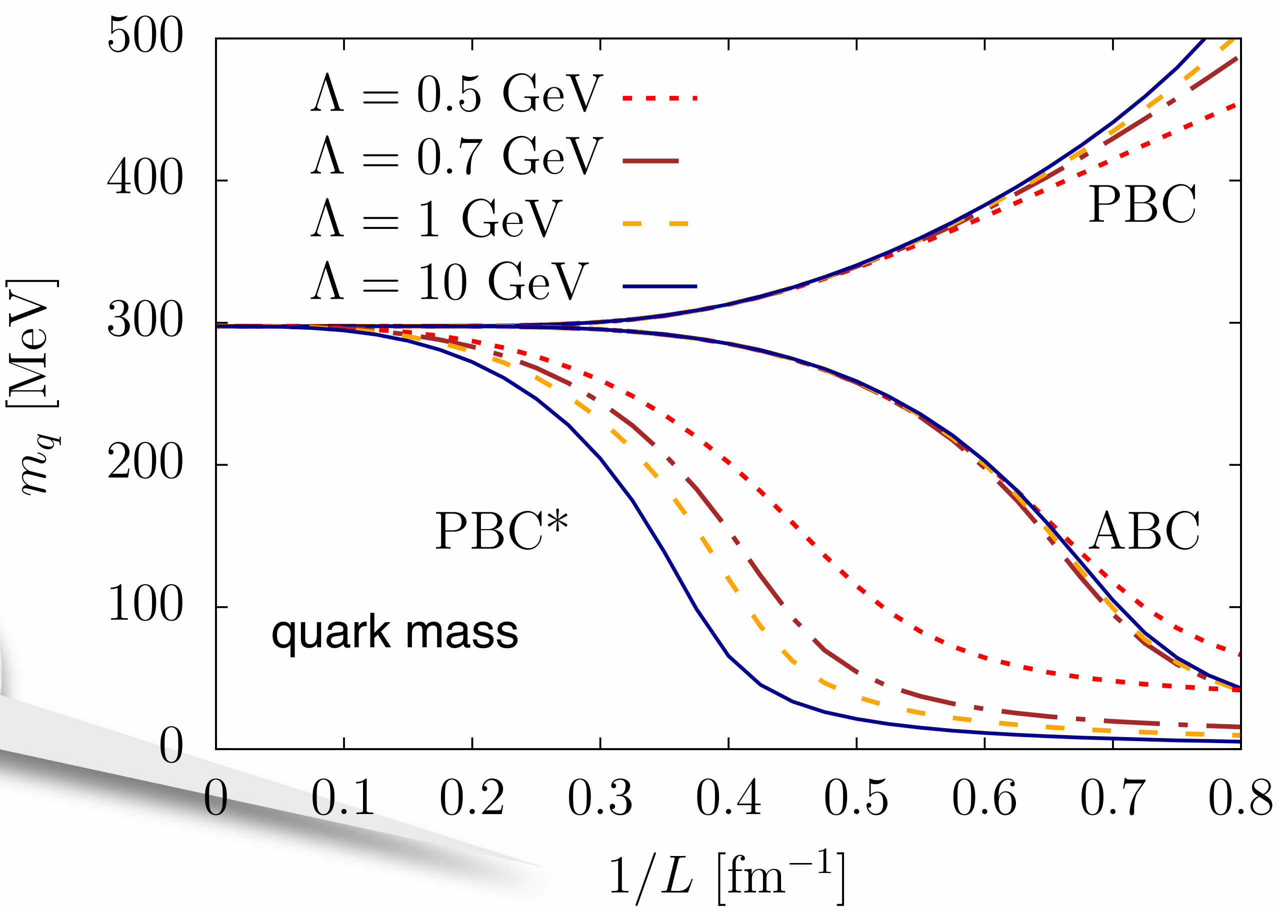

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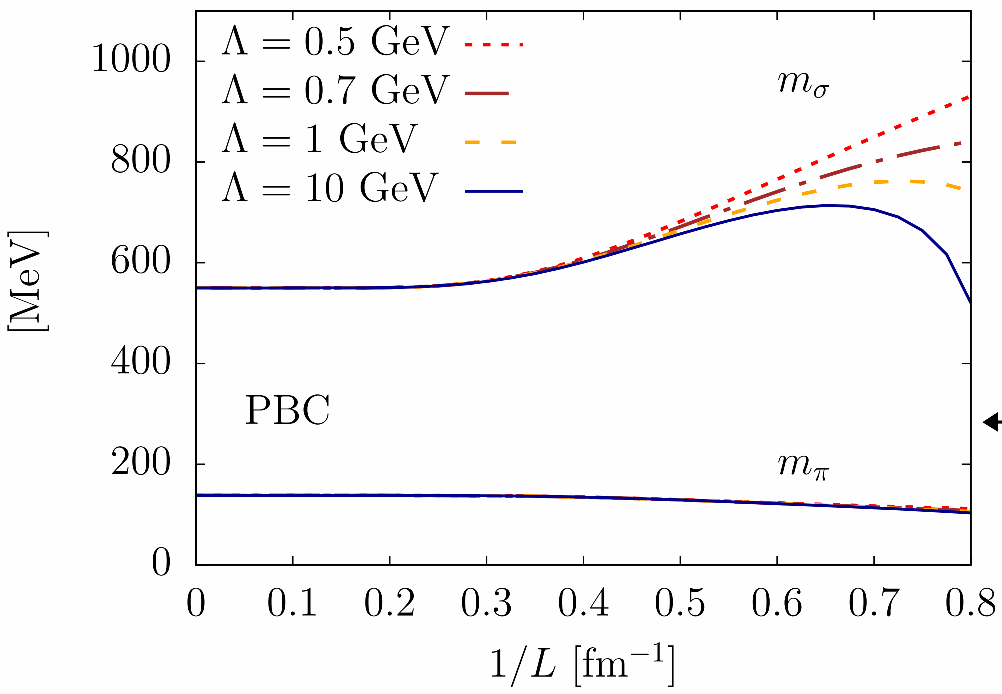
Vacuum meson masses

[A Juricic, BJS in preparation]

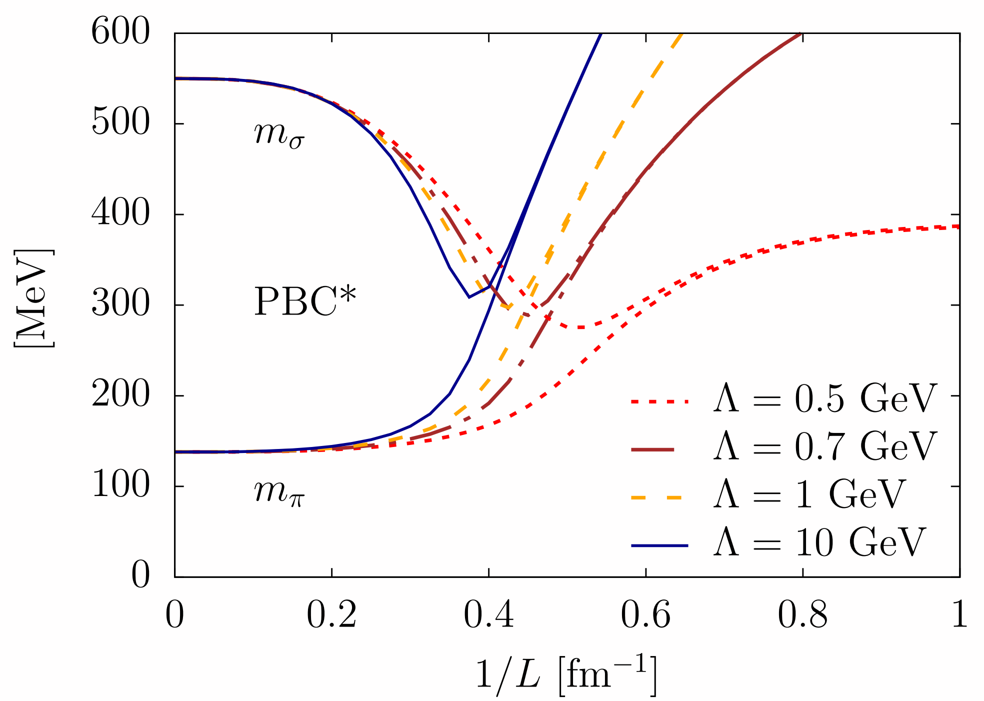


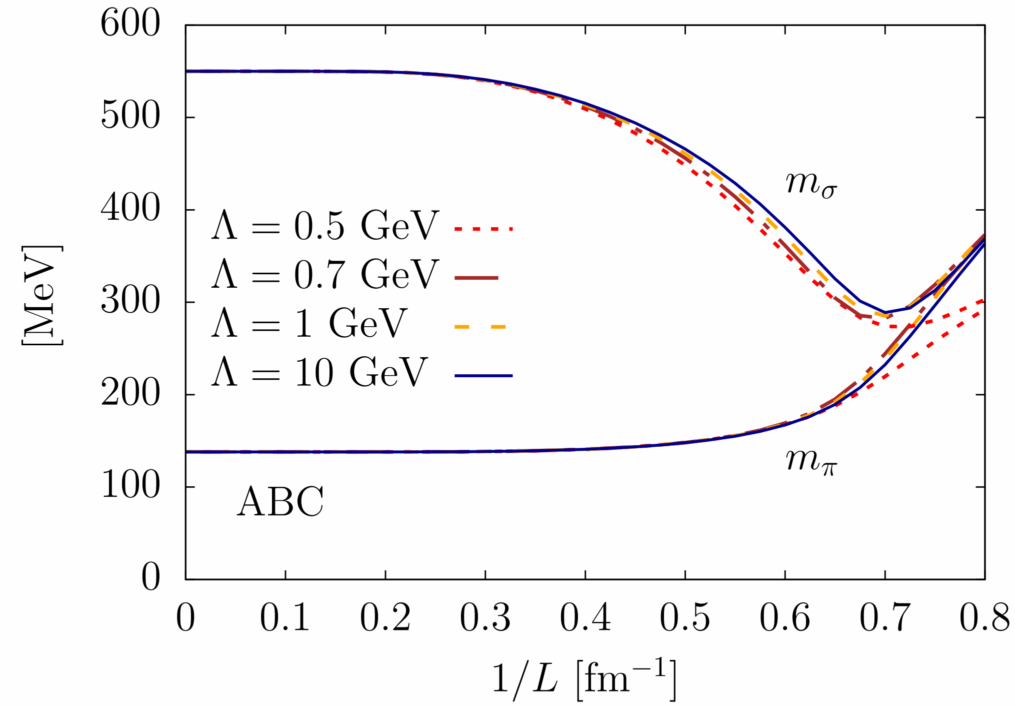
curvature masses (not pole masses)

← periodic including zero-mode (PBC)

periodic without zero-mode (PBC*)
↙

antiperiodic (ABC)





Thermodynamics on a torus

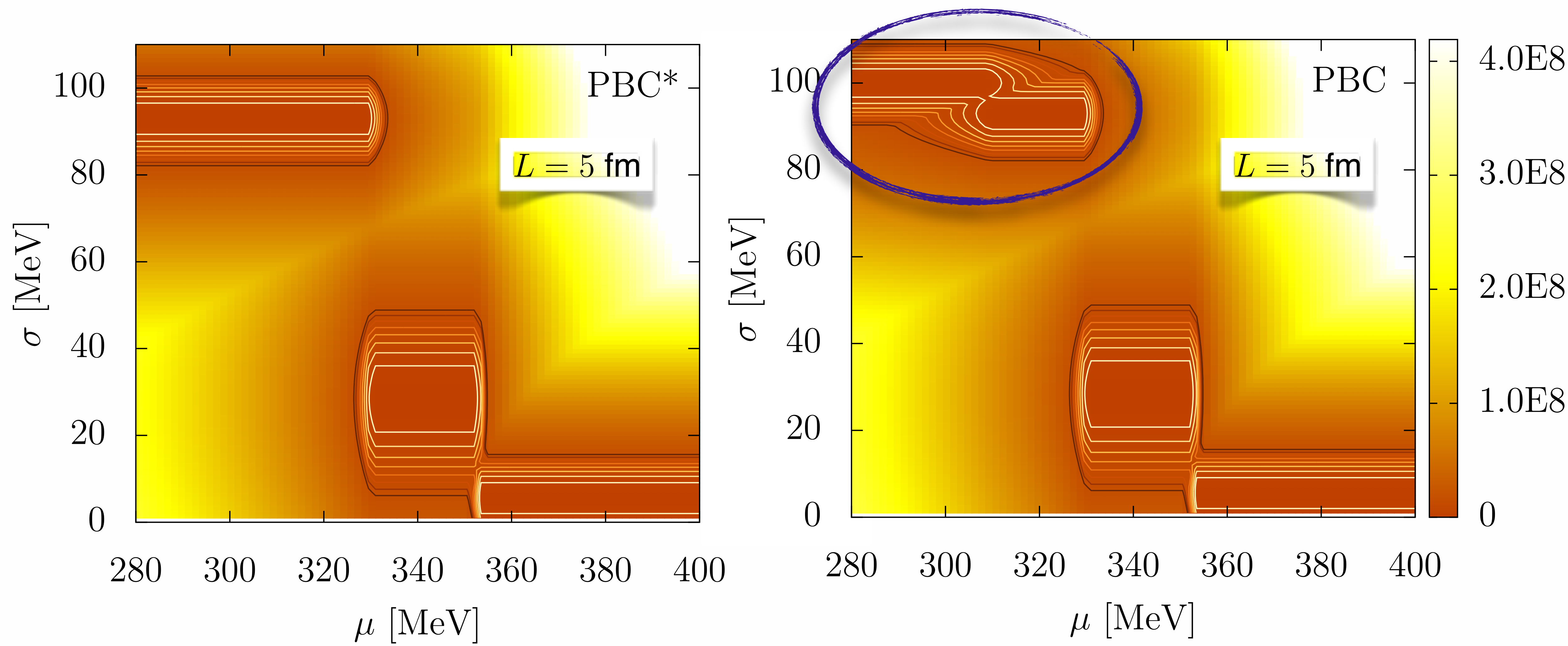
grand potential T=0 & μ>0:

[A Juricic, BJS in preparation]

$$U^{\text{therm}} = 2N_c N_f \frac{1}{L^3} \sum_{\vec{n}\in\mathbb{Z}^3} (\mu - E_{q,\ell})\Theta(\mu - E_{q,\ell})$$

for each mode: discontinuous jumps in potential

contour plot of potential



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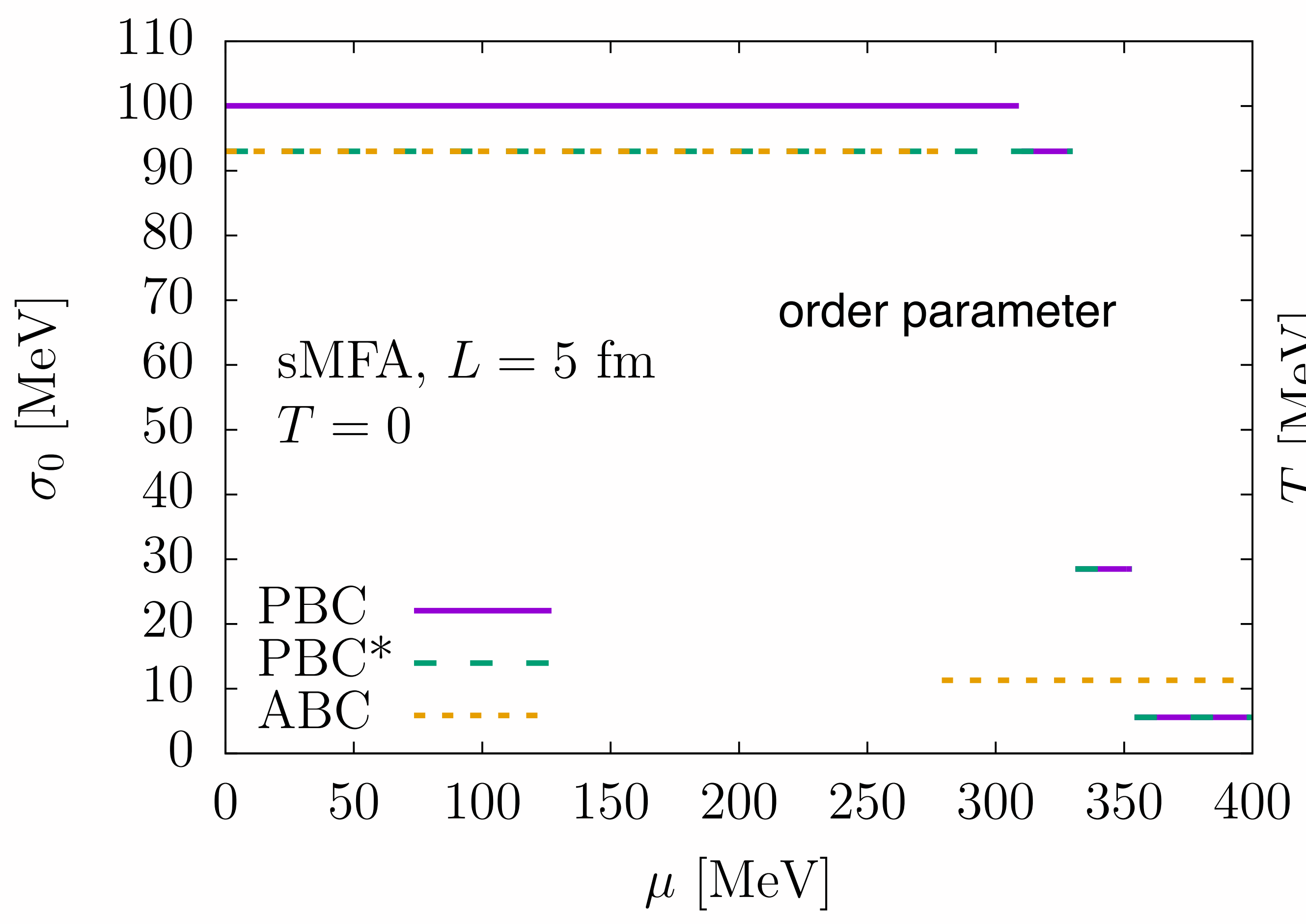
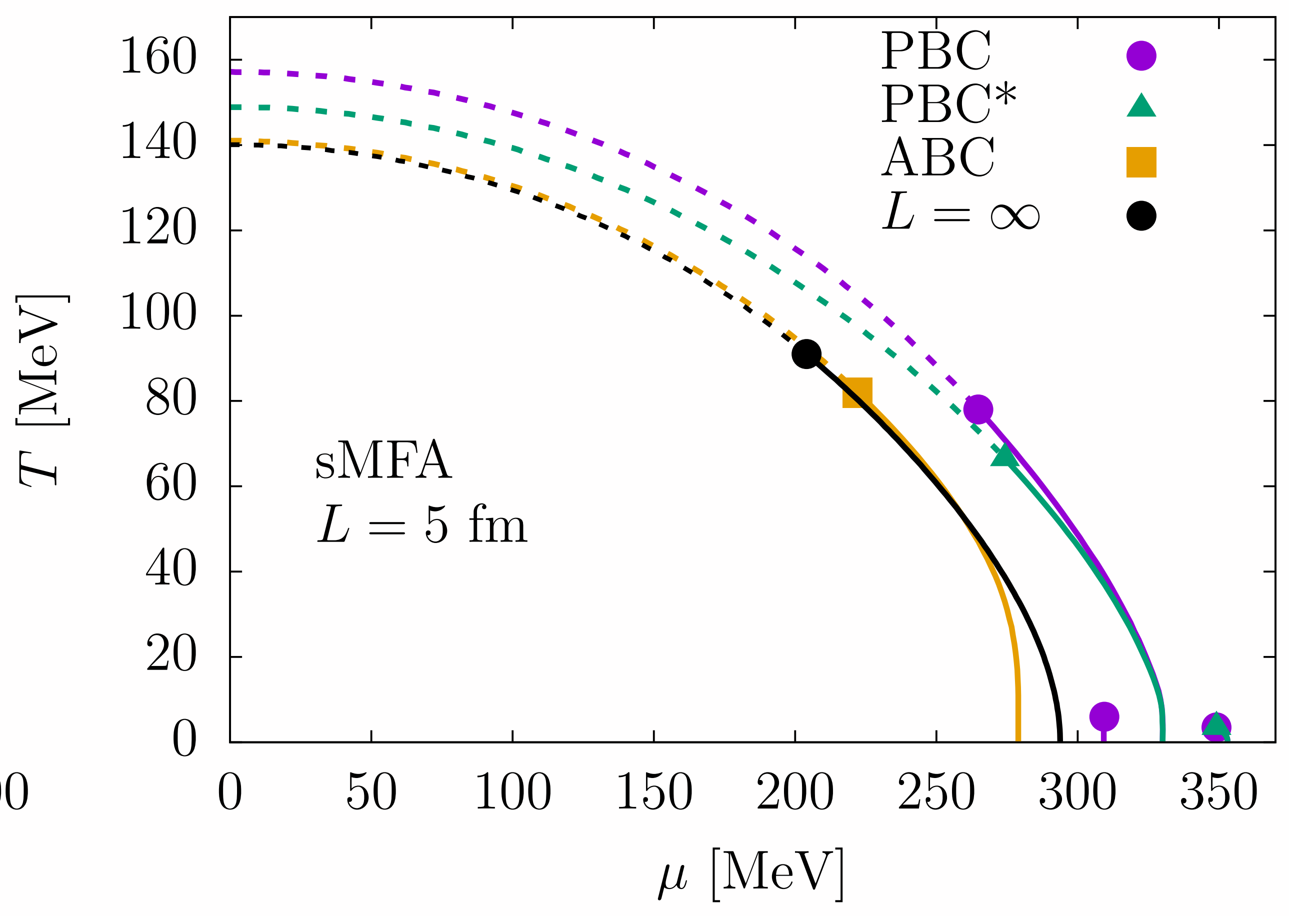
for each mode: discontinuous jumps in potential

discontinuous jumps washed out at T>0



phase diagram without vacuum fluctuations



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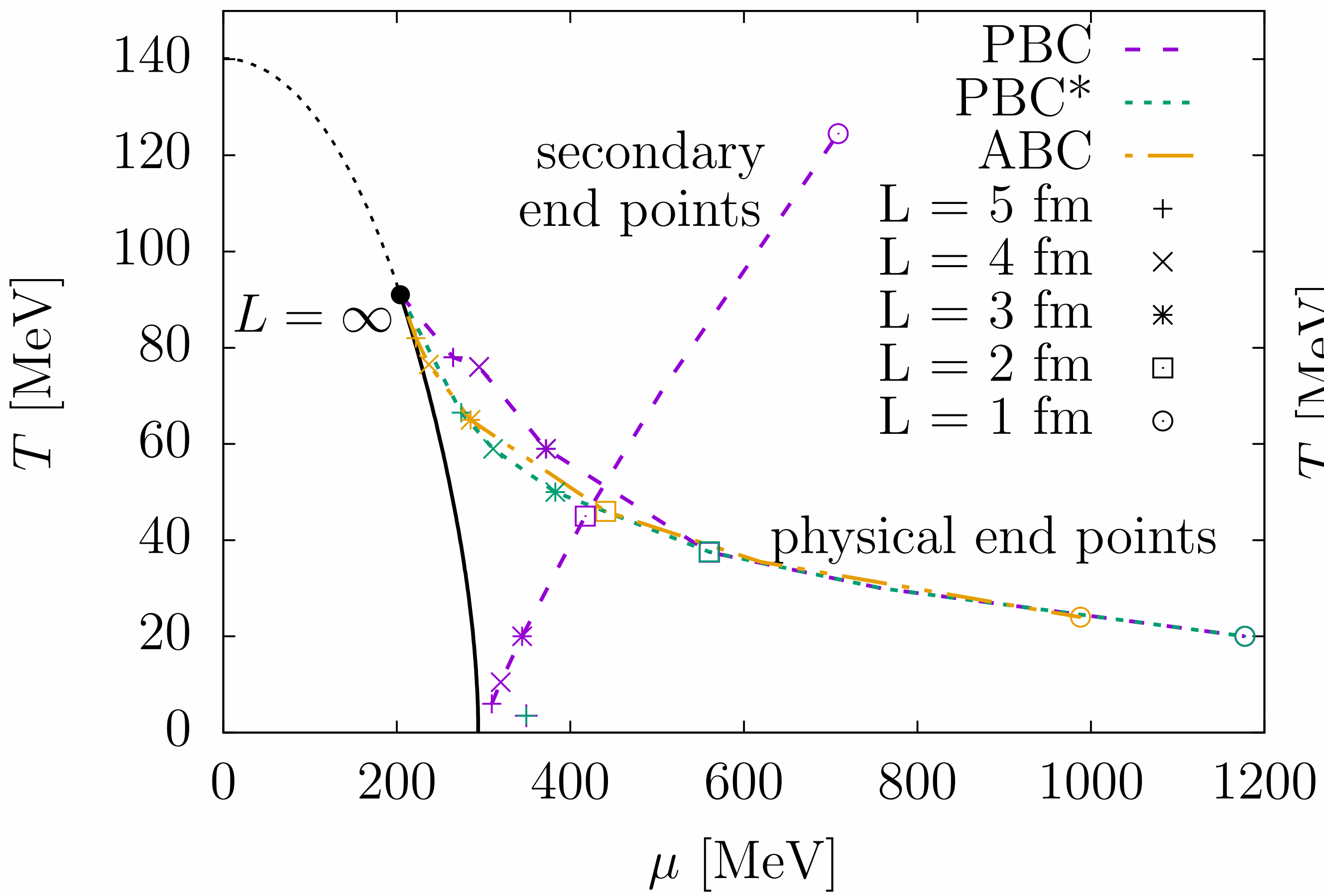
for each mode: discontinuous jumps in potential

movement of the CEP's

standard MFA (no vacuum fluctuations)



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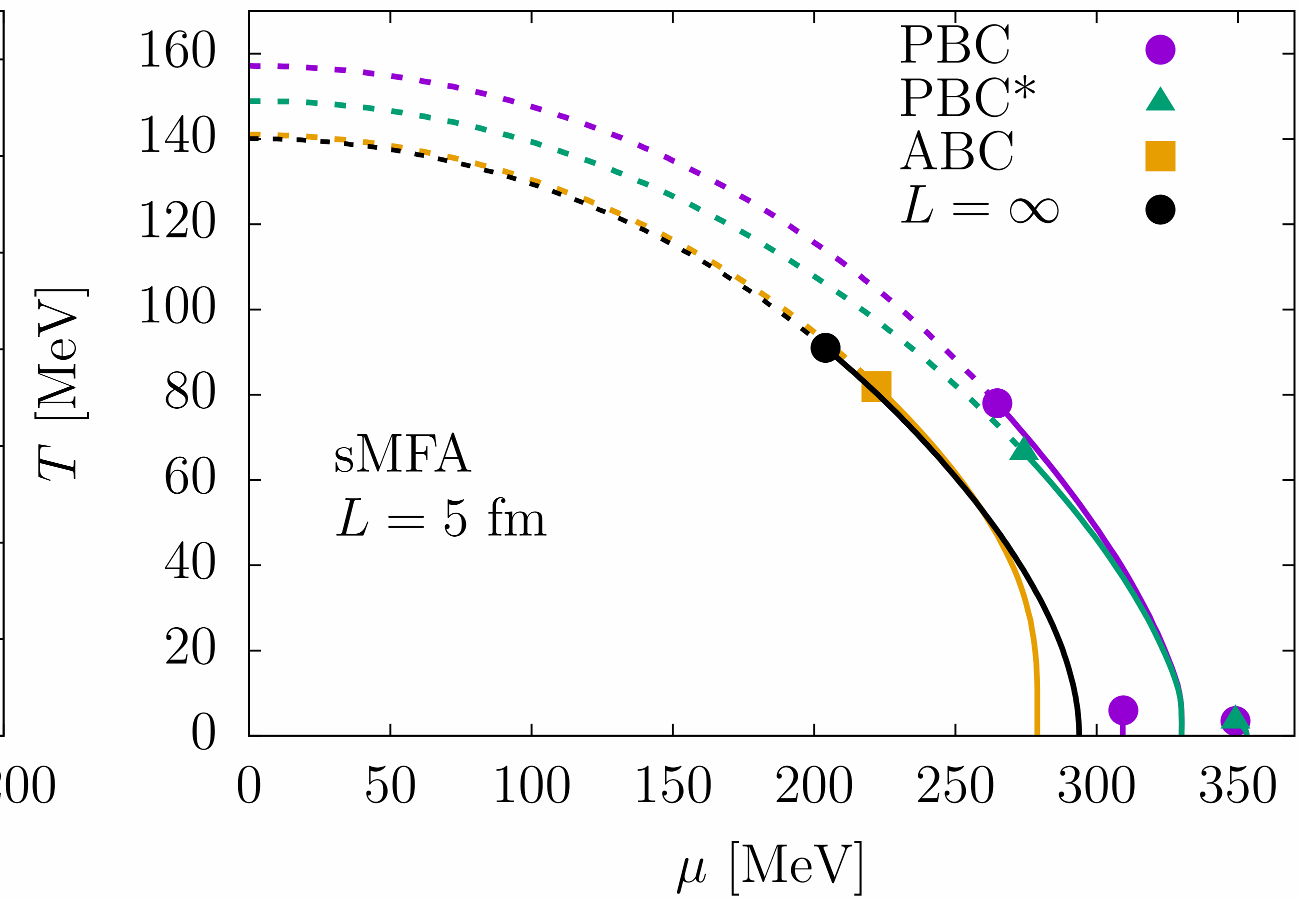

Vacuum fluctuations

movement of the CEP’s

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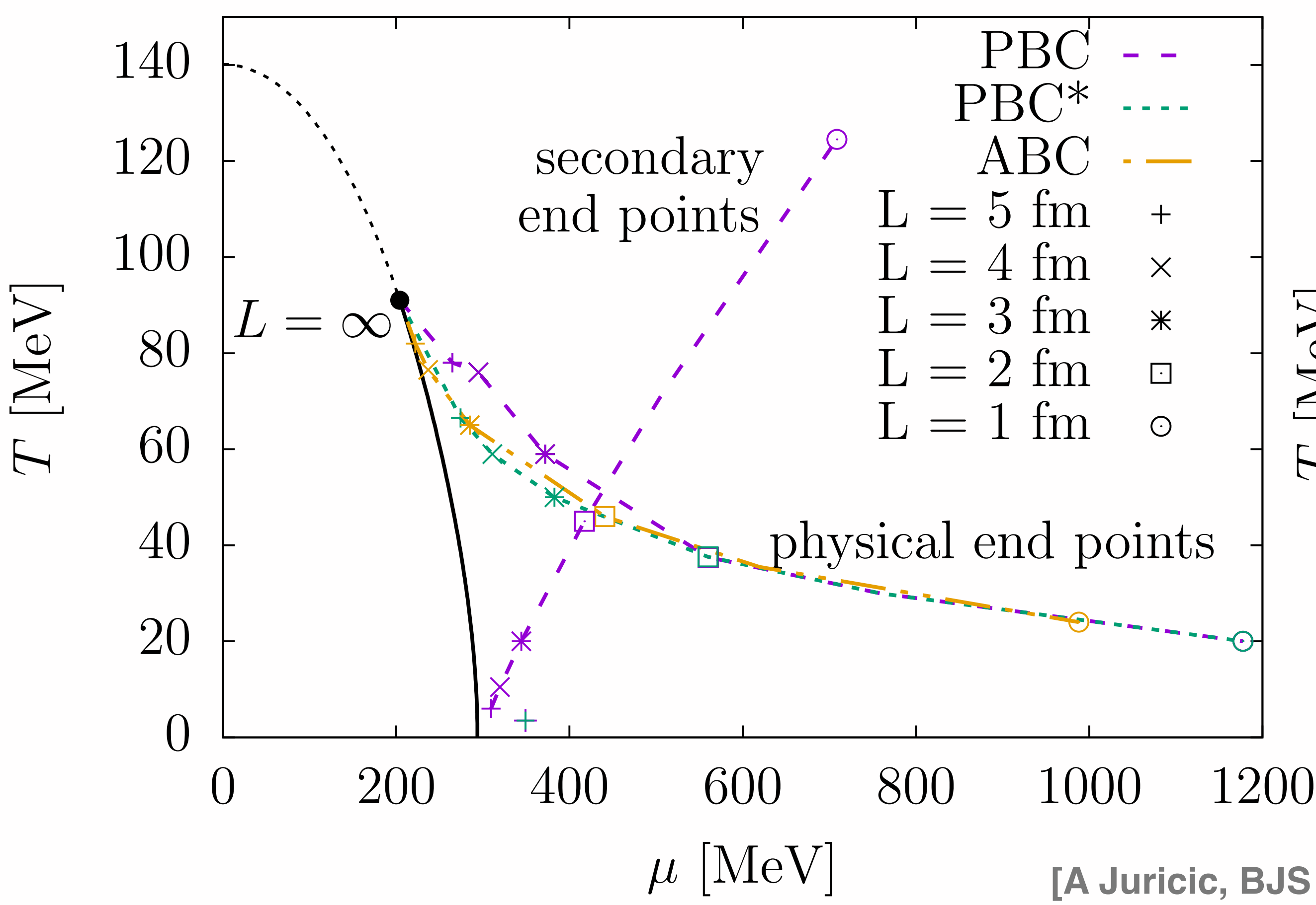
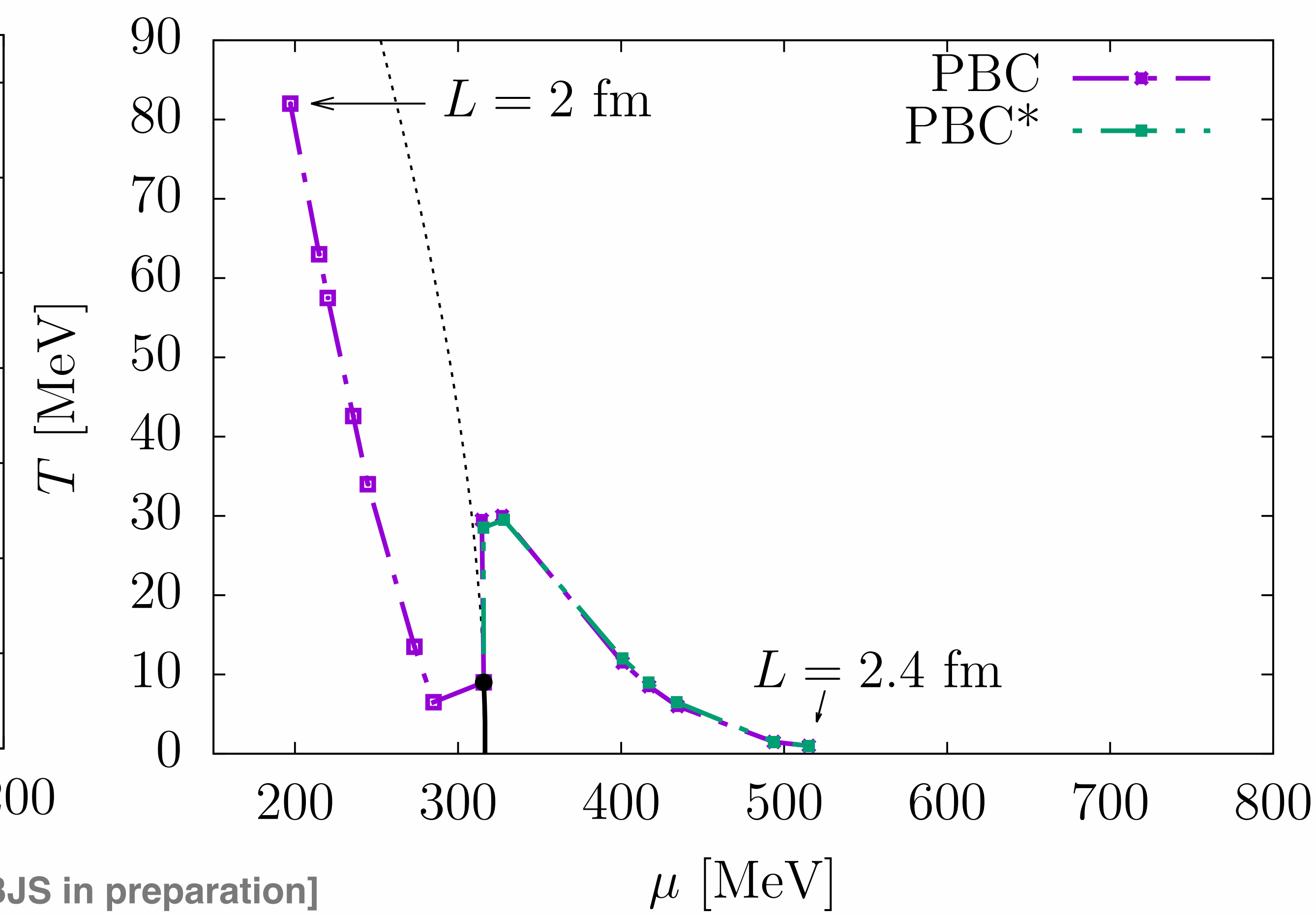


renormalizes MFA (with vacuum fluctuations)



[A Juricic, BJS in preparation]

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Vacuum fluctuations

movement of the CEP's

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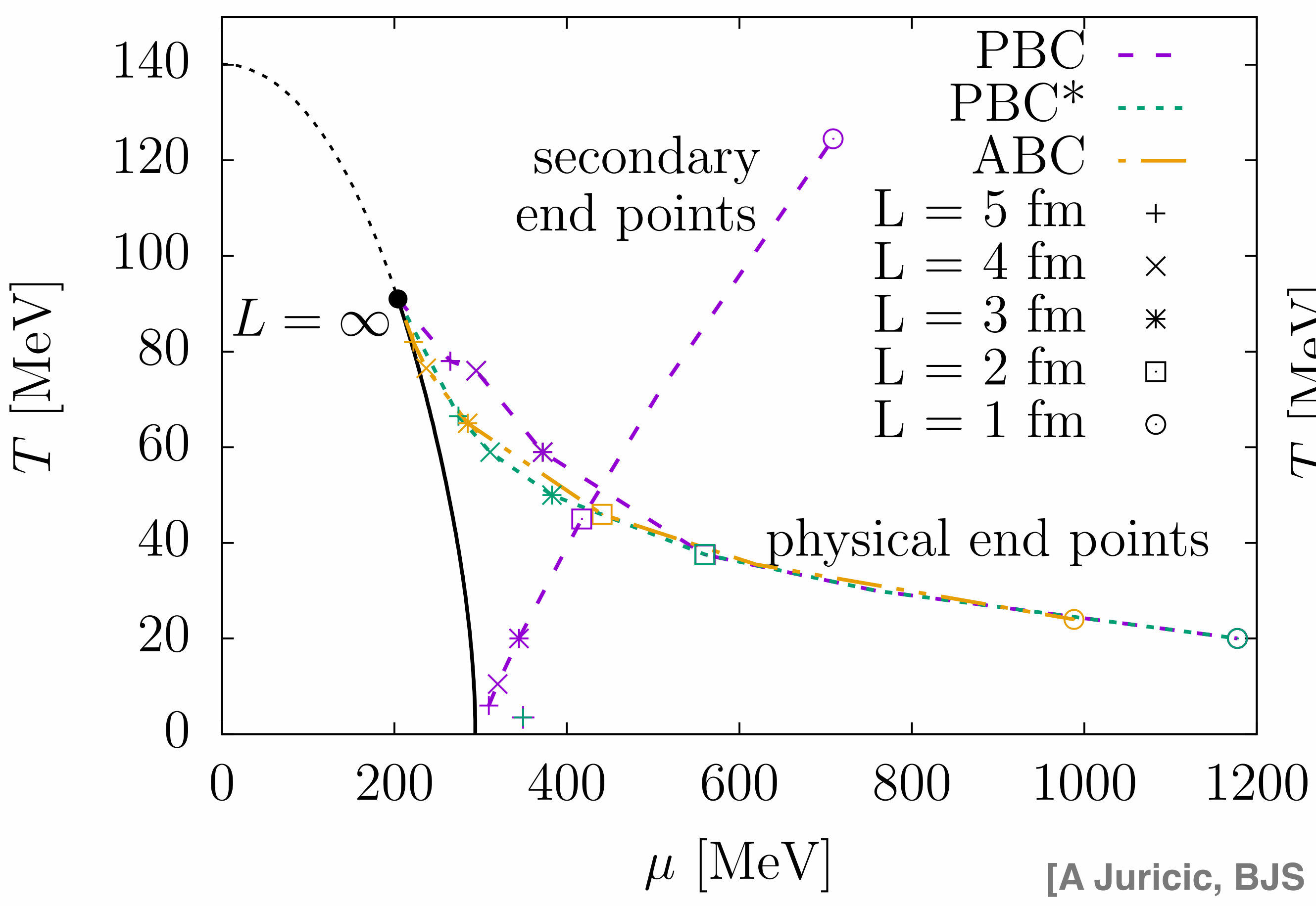

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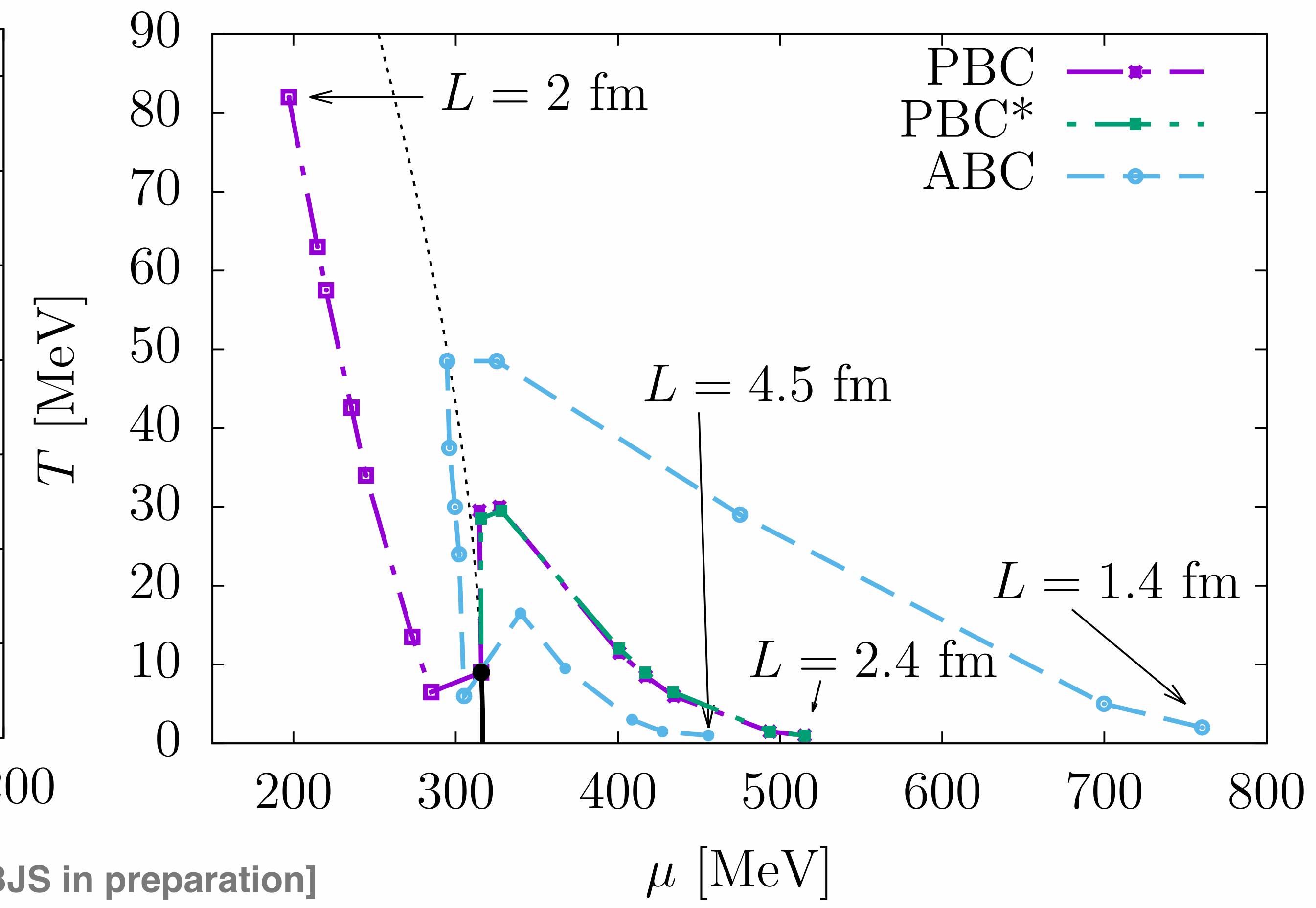

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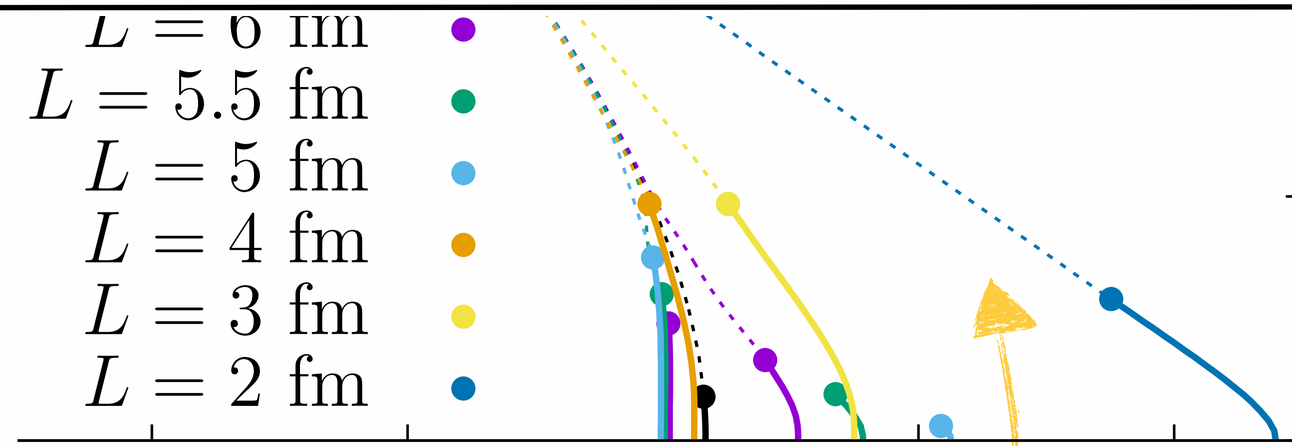

movement of the CEP's

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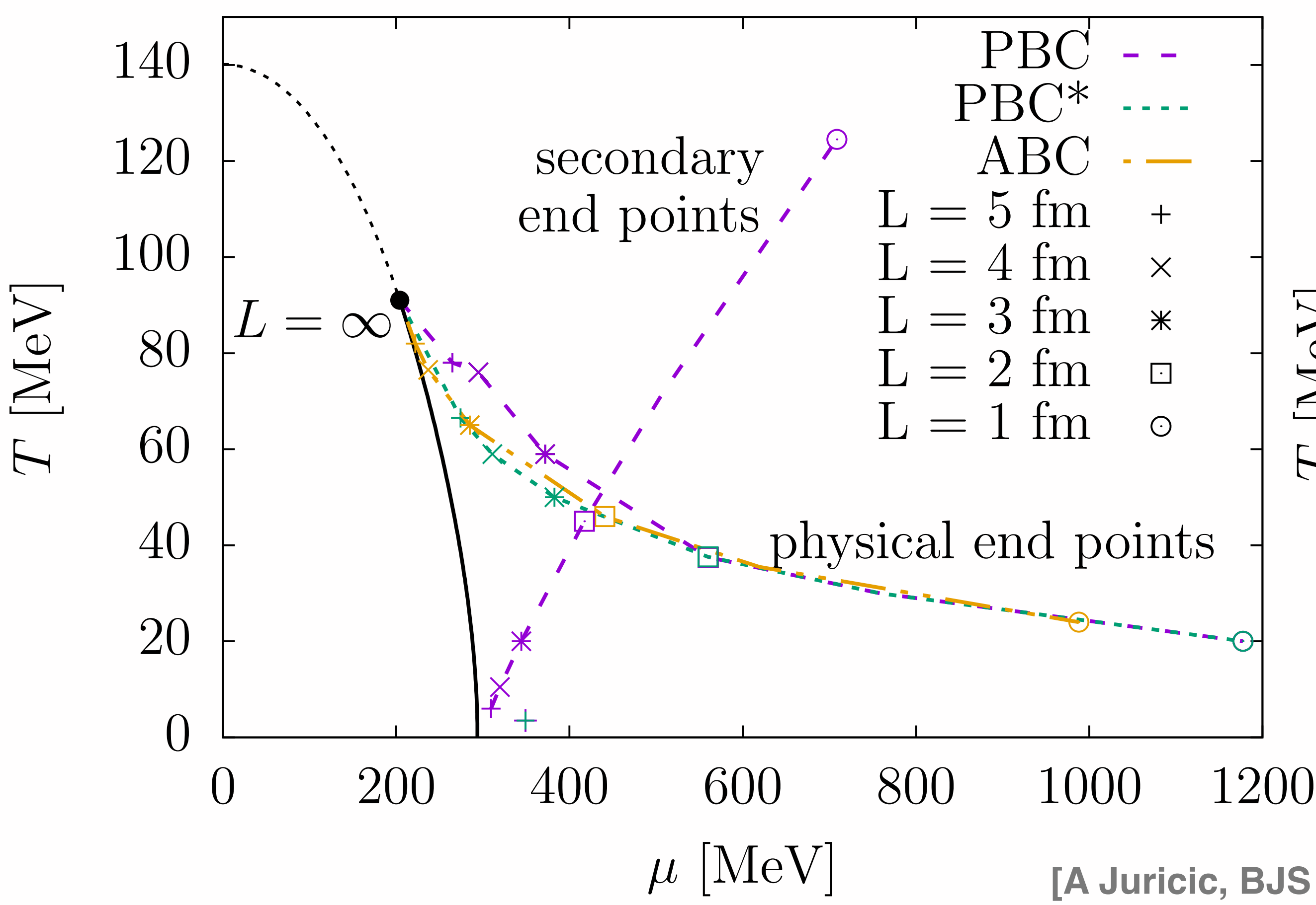

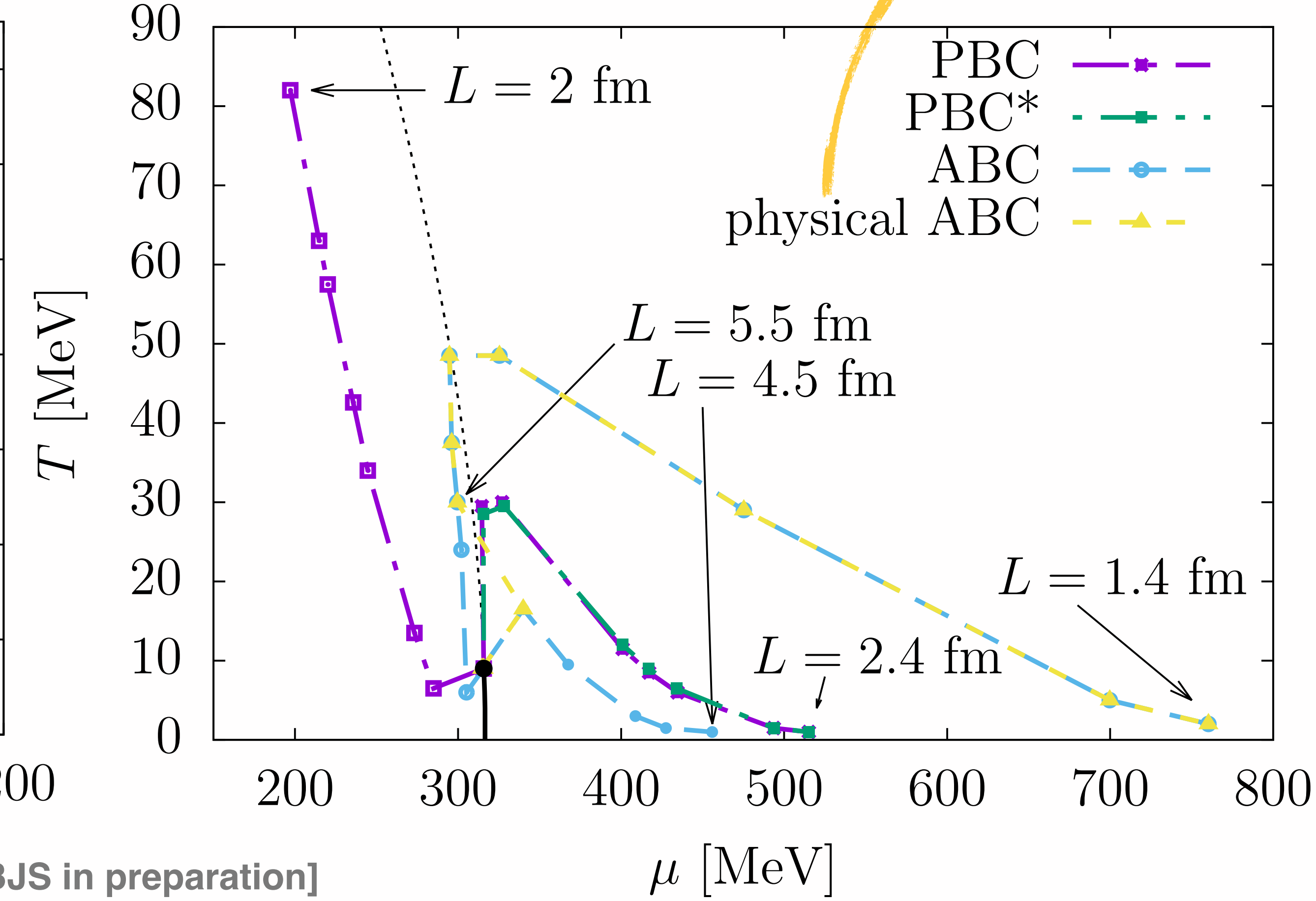

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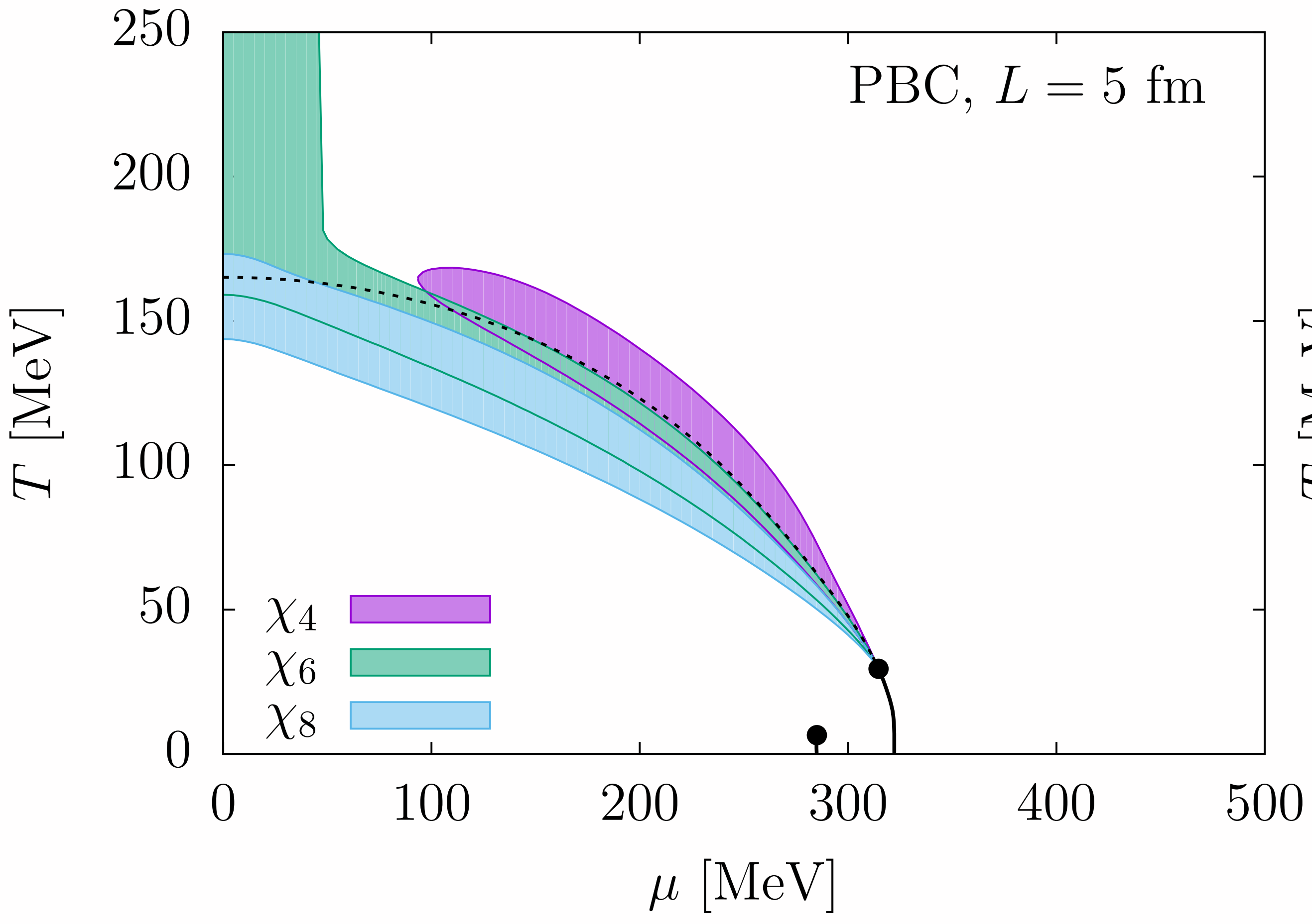
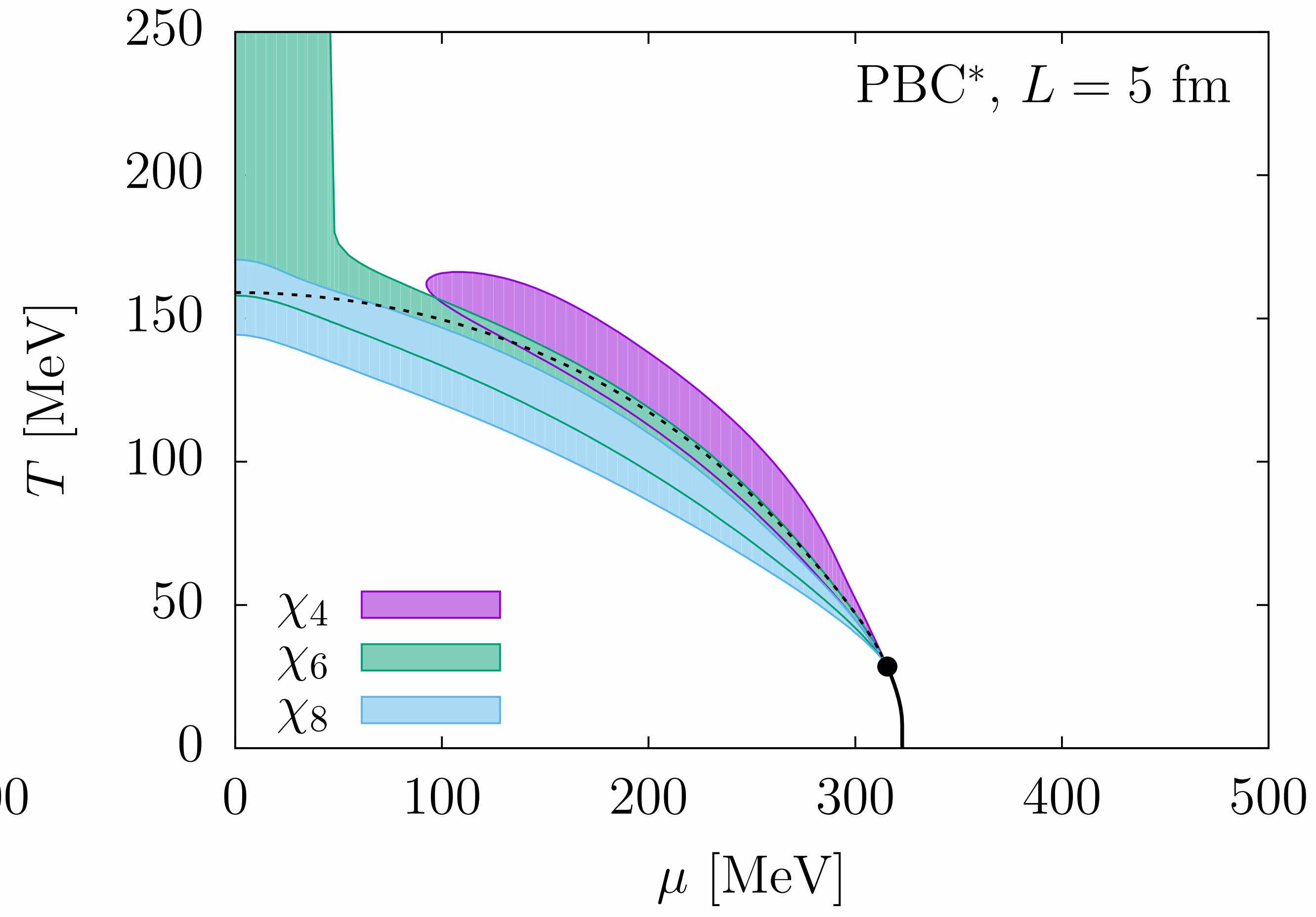




T_c = const at μ = 0 for decreasing volumes

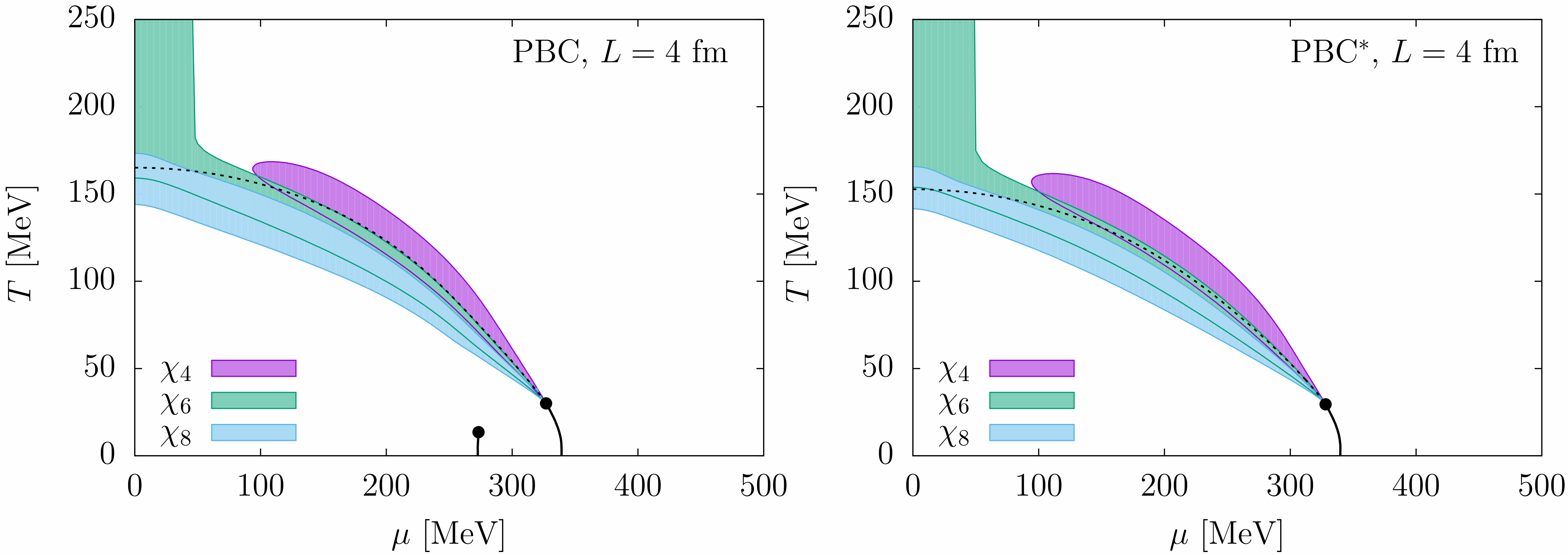
T_c shrinks at μ = 0 for decreasing volumes

CEP vanishes for small volumes

[A Juricic, BJS in preparation]

Vacuum fluctuations

renormalizes MFA (with vacuum fluctuations)



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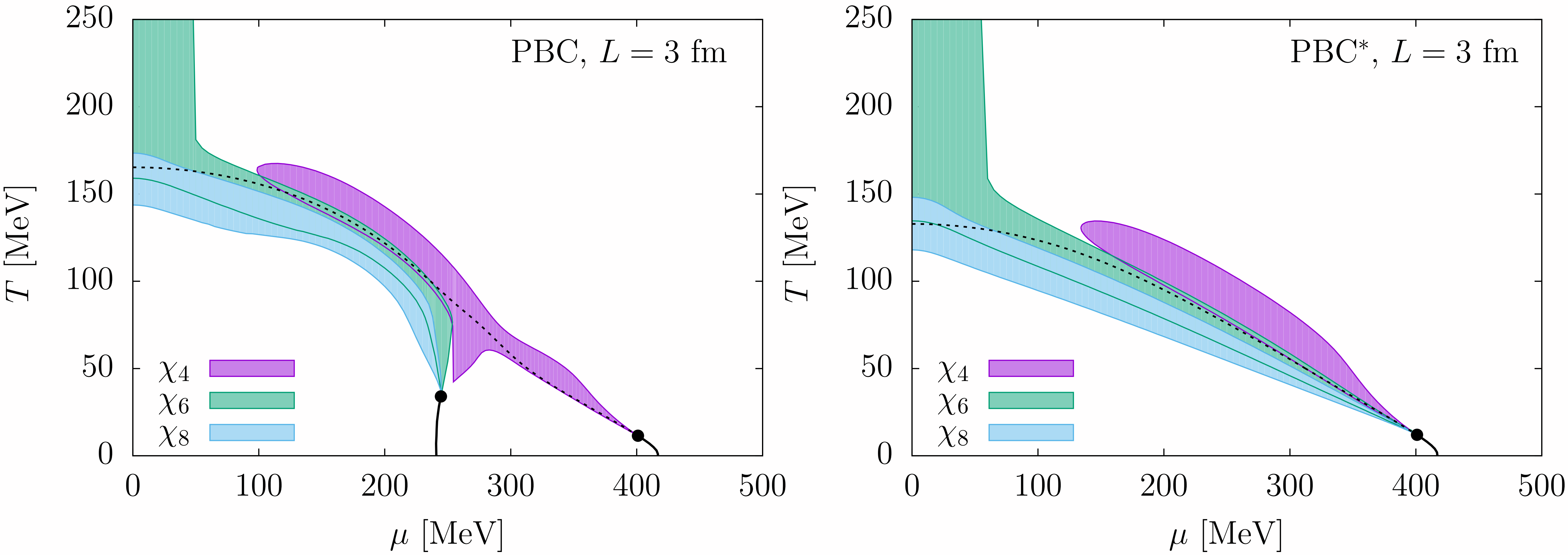

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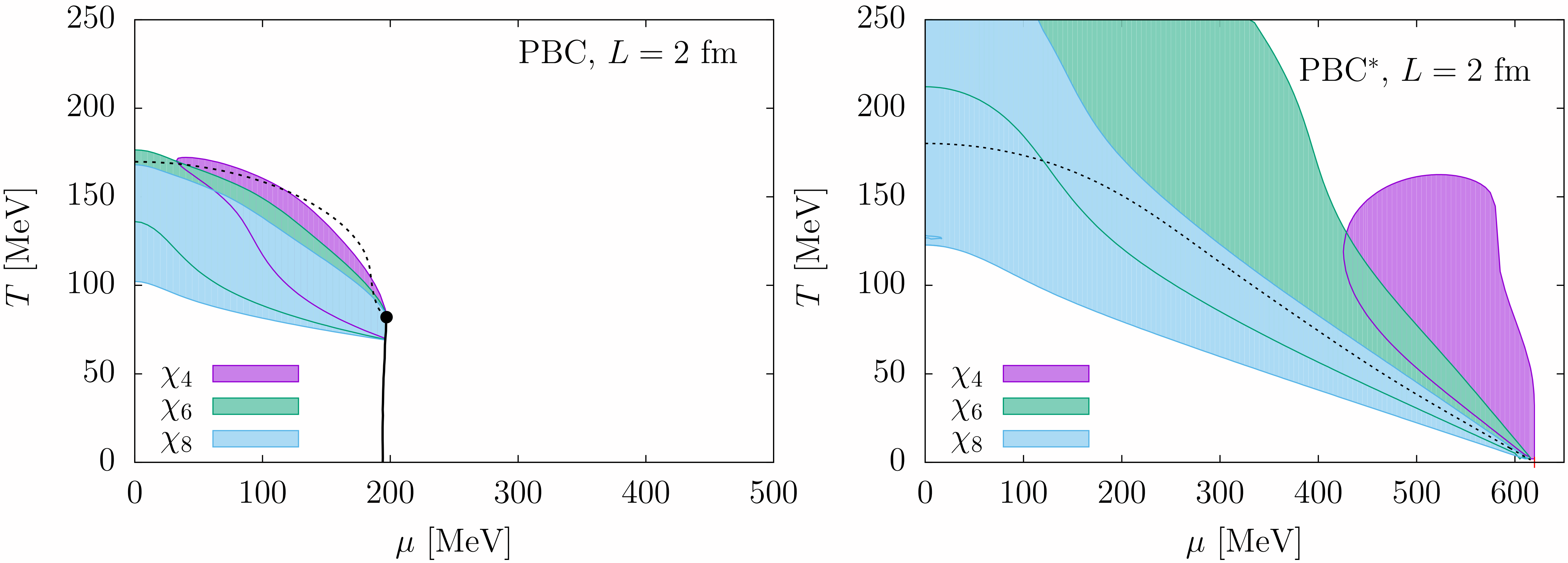


[A Juricic, BJS in preparation]

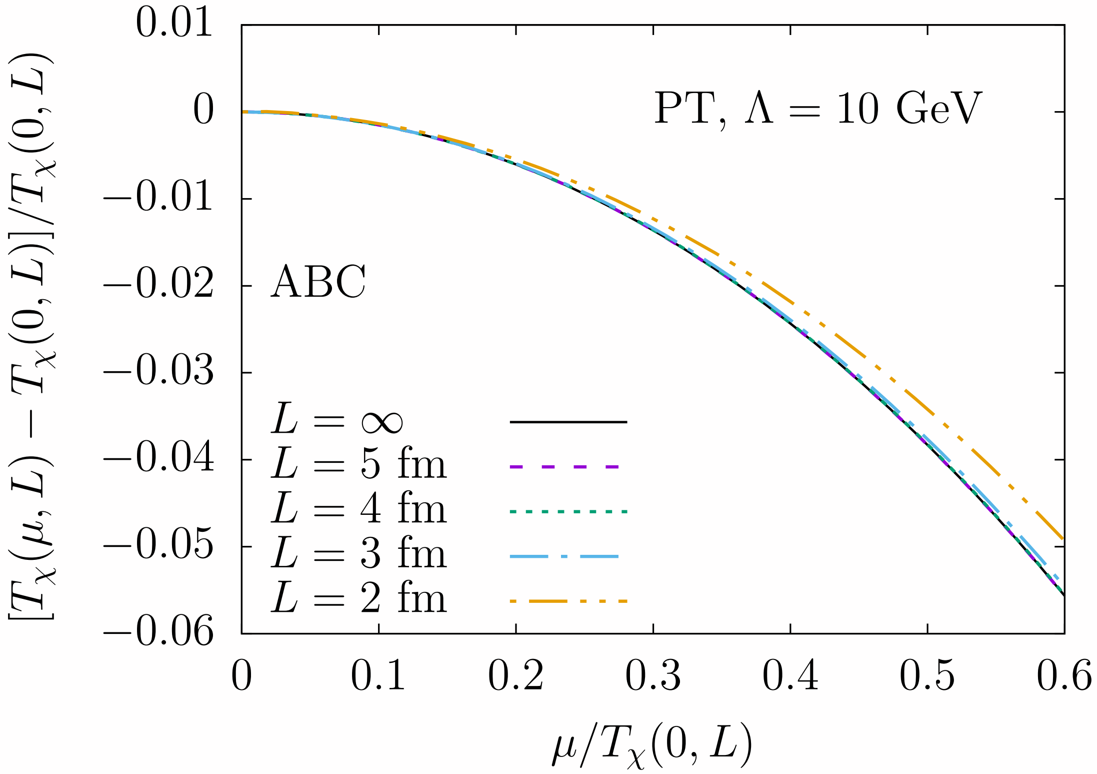
Crossover curvature κ(L)

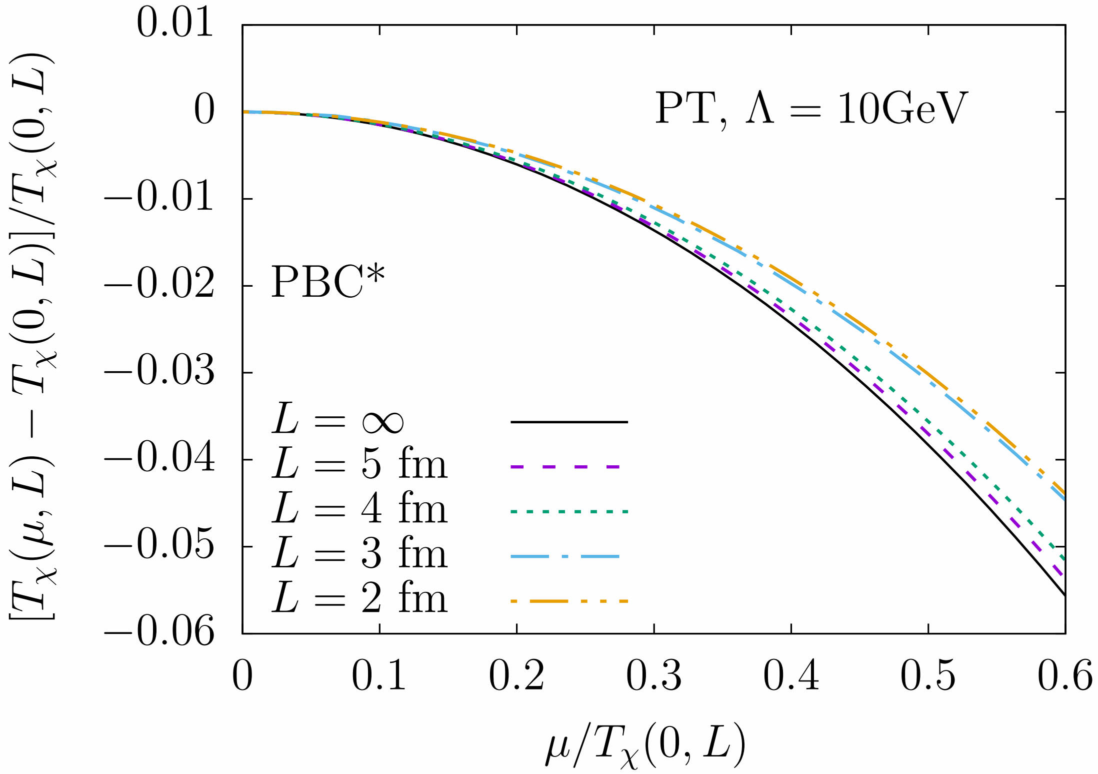
crossover temperature

$$\frac{T_\chi(\mu,L)}{T_\chi(0,L)} = 1 - \kappa(L)\frac{\mu^2}{\pi^2 T_\chi^2(0,L)} + \mathcal{O}(\mu^4)$$

curvature κ(L) decreases with decreasing L

(in rMFA for PBC* and ABC)





[A Juricic, BJS in preparation]

Crossover curvature κ(L)

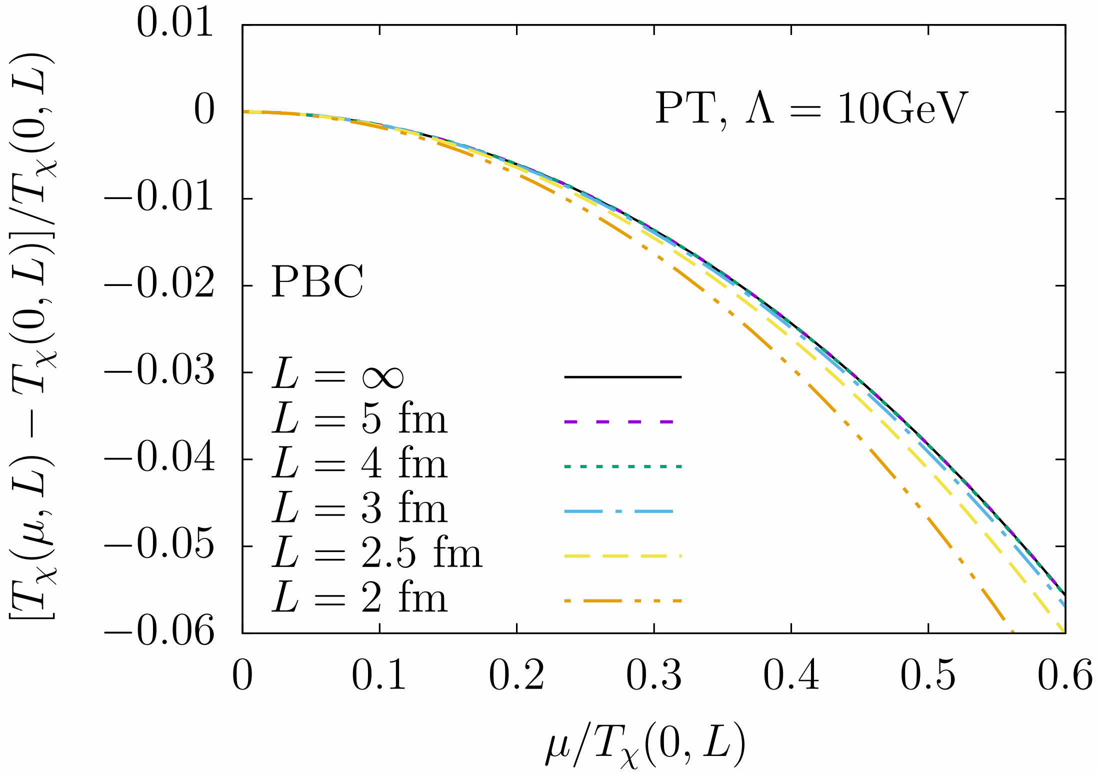
crossover temperature

$$\frac{T_\chi(\mu, L)}{T_\chi(0, L)} = 1 - \kappa(L)\frac{\mu^2}{\pi^2 T_\chi^2(0, L)} + \mathcal{O}(\mu^4)$$

curvature κ(L) decreases with decreasing L

(in rMFA for PBC* and ABC)

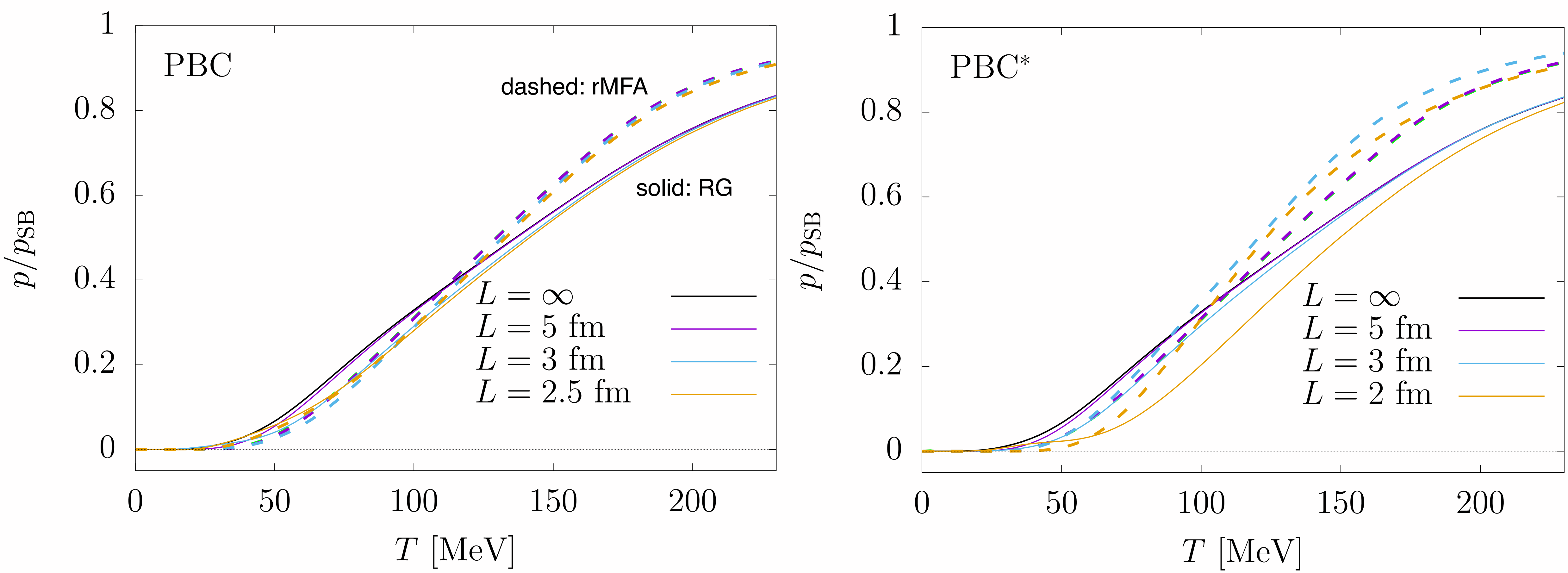
but for PBC: curvature κ(L) first decreases L > Lc and then increases for L < Lc



[A Juricic, BJS in preparation]

Pressure: RG vs. rMFA

[A Juricic, BJS in preparation]



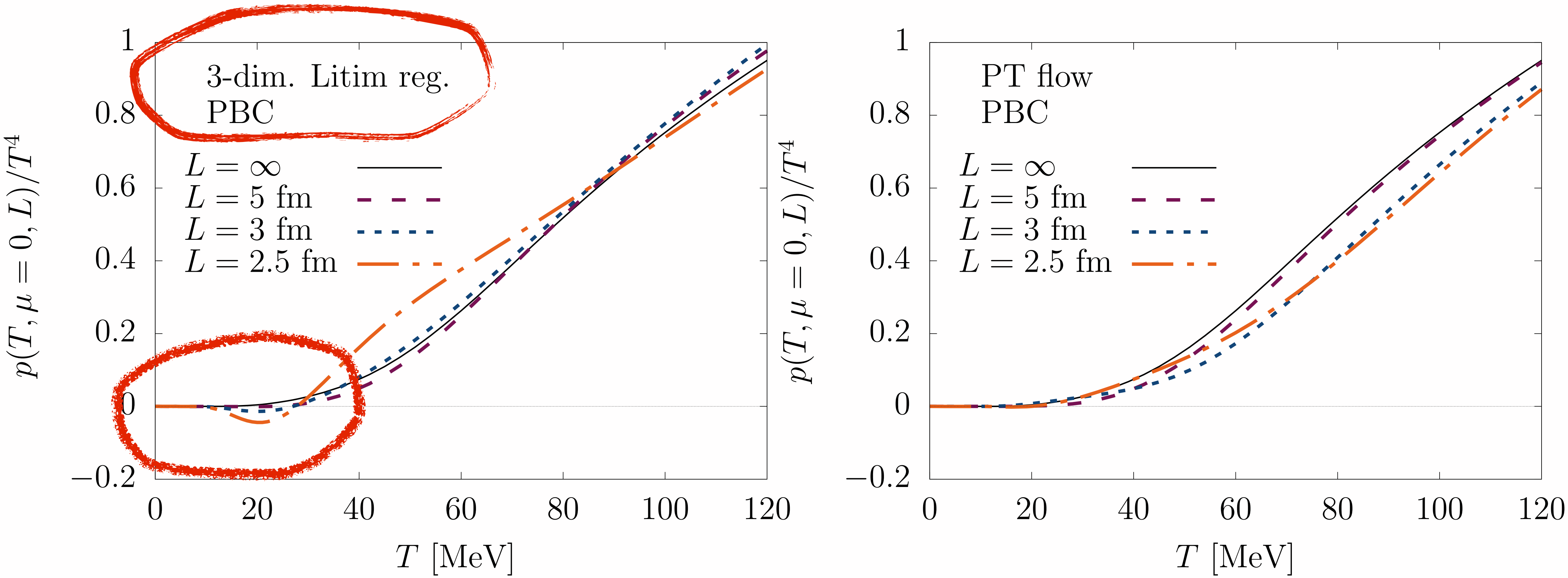
rMFA: pressure grows faster to Stefan-Boltzmann limit

PBC (with zero-mode): size dependence smaller

PBC* (without zero-mode): pressure decreases with smaller volumes

Pressure: scheme dependence

[A Juricic, BJS in preparation]



infinite volume: pressure regulator independent

3dim Litim regulator: negative pressure (avoided if zero-modes excluded)

smeared regulator: pressure always positive [Fister,Pawlowski 2015]

Summary & Conclusions

- effects of quantum and thermal fluctuations in a box

  comparison: sMFA, rMFA, RG

  ➜ fluctuations wash out the phase transition

- existence of critical points in phase diagram in finite volume

- crossover curvature changes in a box