From cold Fermi fluids to the hot QGP

Toward better understanding strongly coupled quantum fluids

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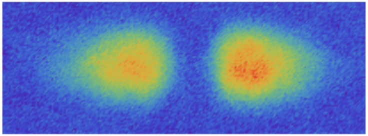
Prelude

- ultracold Fermi gases share features with other strongly coupled quantum systems
- unique experimental playground to study fermionic many-body problem: widely variable densities, temperatures and interatomic interaction strengths
- Feshbach-resonant scattering $\rightarrow$ investigate crossover between BCS-type superfluid and BEC of strongly bound molecules
- ^{40}K, ^{6}**Li**: fermionic atoms with electron spin $S = 1/2$, nuclear spin $I = 1$
- cooled to μK-regime: s-wave scattering (length a) dominant

Prelude

Fermi gas cloud collision:



evolution within first 12 ms

- repulsive optical potential divides trapped gas into two clouds
- clouds collide once repulsive potential switched off

$\Rightarrow$ formation of propagating shock fronts (fluid dynamical behavior)

J. Joseph et al., PRL **106** (2011) 150401

Prelude

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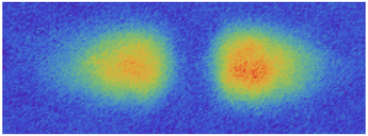

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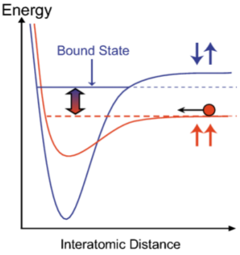
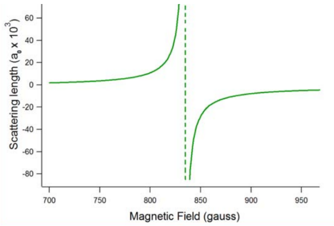
Tuning the interaction strength in Fermi gases

consider spin-balanced system of two (lowest) macroscopically occupied hyperfine eigenstates $|m_f = \pm\frac{1}{2}\rangle$ of ^{6}Li





- scattering between two atoms predominantly in **triplet state** (Open Channel) of interaction potential
- at low incident energy: **singlet** (**bound**) **state** (Closed Channel) with a <u>different</u> magnetic moment

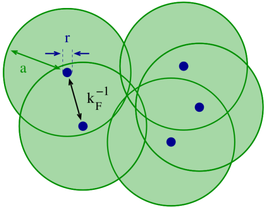
tune B-field $\rightarrow$ CC couples resonantly with OC scattering continuum

$\Rightarrow$ temporary capture in quasi-bound states: Fano-Feshbach resonance

Unitary Fermi gas

unitary limit $|a| \to \infty$:

- scale invariance, conformal invariance
- universal (s-wave) collision cross-section: $\sigma_{\rm coll} = 4\pi/q^2$
- properties of the gas are universal functions of n and T, e.g. $\eta = \hbar n \cdot \alpha(z)$ with fugacity $z \sim n\lambda^3 \sim n/(mT)^{3/2}$



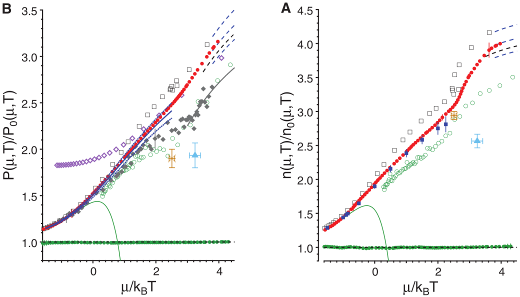
$\to$ dilute regime: $r \cdot n^{1/3} \ll 1$

- extremely strong coupling/correlations reachable if a exceeds average interparticle spacing, i.e. $a \cdot n^{1/3} \gg 1$
- in high-T limit: $z \ll 1$, $\sigma_{\rm coll} \sim 1/T$ $\to$ kinetic theory applies

Equilibrium properties of unitary Fermi gases

universal thermodynamics:

- $P = \frac{2}{3}\mathcal{E}_0$
- $P = T\lambda^{-3} f_P(z)$, $n = \lambda^{-3} f_n(z)$ with universal functions f_P and f_n

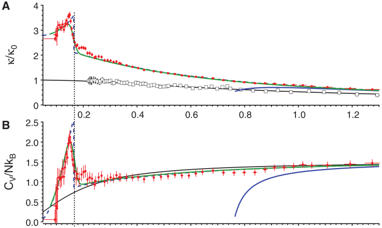


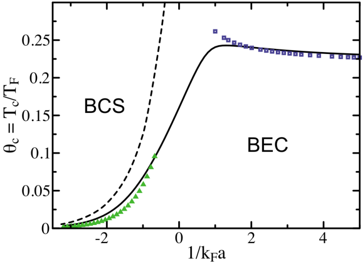
measurement across superfluid transition

- change in behavior of $n(T,\mu)$ around $\mu/T \gtrsim 2.5$

T.-L. Ho, PRL **92** (2004) 090402; M.J.H. Ku et al., Science **335** (2012) 563

Equilibrium properties of unitary Fermi gases





- rise in compressibility κ
- λ-shape structure in specific heat c_V as functions of T/T_F

$\Rightarrow$ 2^{nd}-order phase transition with $T_c/T_F = 0.167(13)$

- self-consistent T-matrix, FRG and QMC approaches: $\rightarrow T_c/T_F \simeq 0.16$

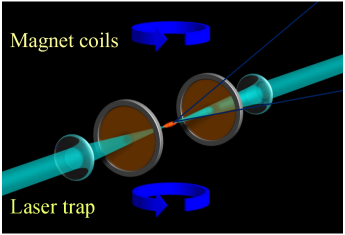
M.J.H. Ku et al., Science **335** (2012) 563; R. Haussmann et al., PRA **75** (2007) 023610; E. Burovski et al., PRL **96** (2006) 160402; A. Bulgac et al., PRA **78** (2008) 023625; O. Goulko, M. Wingate, PRA **82** (2010) 053621; I. Boettcher et al., PRA **89** (2014) 053630

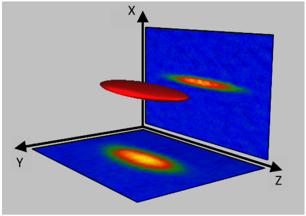
Transport properties of unitary Fermi gases

1) How can they be measured?

- analysis of the gas cloud *flow dynamics* after release from a deformed trap or of the *damping* of the transverse breathing mode after recapturing the gas

→ **elliptic flow measurement**:





study expansion after release

K.M. O'Hara et al., Science **298** (2002) 2179; C. Cao et al., Science **331** (2011) 58; J. Kinast et al., PRA **70** (2004) 051401 (R); M. Bartenstein et al., PRL **92** (2004) 203201; A. Turlapov et al., J. Low Temp. Phys. **150** (2008) 567; C. Cao et al., New J. Phys. **13** (2011) 075007; E. Elliott et al., PRL **112** (2014) 040405

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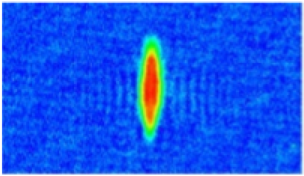

100 $\mu s \ldots 2$ *ms*

- shows fluid dynamical behavior
- analyze time-evolution of gas cloud geometry

K.M. O'Hara et al., Science **298** (2002) 2179; C. Cao et al., Science **331** (2011) 58; J. Kinast et al., PRA **70** (2004) 051401 (R); M. Bartenstein et al., PRL **92** (2004) 203201; A. Turlapov et al., J. Low Temp. Phys. **150** (2008) 567; C. Cao et al., New J. Phys. **13** (2011) 075007; E. Elliott et al., PRL **112** (2014) 040405

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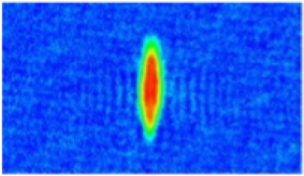

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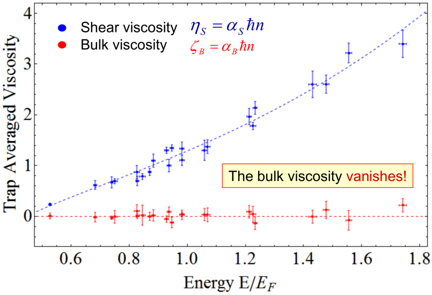
Transport properties of unitary Fermi gases

2) Experimental results:

- dependence on initial energy per particle E



in normal phase
$\rightarrow \langle\eta\rangle / \langle s\rangle \geq 0.4$

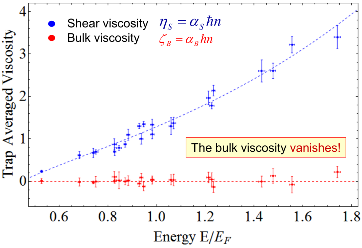
$\Rightarrow$ nearly perfect fluidity

$\rightarrow$ only cloud-averaged quantities determined: $\alpha_S \sim \int \eta dV$

E. Elliott et al., PRL **112** (2014) 040405

Transport properties of unitary Fermi gases

3) Theory results:



bulk viscosity $\zeta = 0$

D.T. Son, PRL **98** (2007) 020604; P. Massignan et al., PRA **71** (2005) 033607; G.M. Bruun, H. Smith, PRA **72** (2005) 043605; *ibid.* PRA **76** (2007) 045602; G. Rupak, T. Schäfer, PRA **76** (2007) 053607; T. Enss et al., Ann. Phys. **326** (2011) 770; H. Guo et al., New J. Phys. **13** (2011) 075011; *ibid.* PRL **107** (2011) 020403; G. Wlazłowski et al., PRL **109** (2012) 020406; *ibid.* PRA **88** (2013) 013639; Y. He, K. Levin, PRB **89** (2014) 035106; T. Schäfer, PRA **90** (2014) 043633

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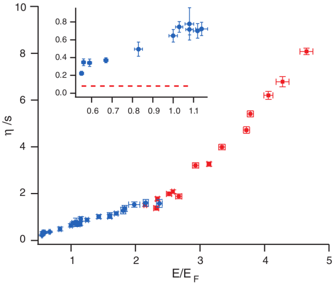


shear viscosity η

- in high-T limit
  $\rightarrow \eta = \frac{15}{32\sqrt{\pi}}(mT)^{3/2}$
- at $z \simeq 12$ superfluid transition
  $\rightarrow \eta \sim 1/T^5$ for phonons

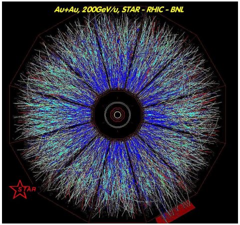
$\Rightarrow$ suggests minimum near T_c

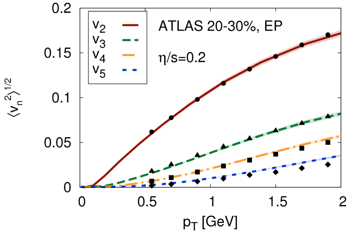
- for $z \gtrsim 1$
  $\rightarrow$ T-matrix: $\eta/s \simeq 0.5$
  $\rightarrow$ QMC: $\eta/s \simeq 0.2$

D.T. Son, PRL **98** (2007) 020604; P. Massignan et al., PRA **71** (2005) 033607; G.M. Bruun, H. Smith, PRA **72** (2005) 043605; *ibid.* PRA **76** (2007) 045602; G. Rupak, T. Schäfer, PRA **76** (2007) 053607; T. Enss et al., Ann. Phys. **326** (2011) 770; H. Guo et al., New J. Phys. **13** (2011) 075011; *ibid.* PRL **107** (2011) 020403; G. Wlazłowski et al., PRL **109** (2012) 020406; *ibid.* PRA **88** (2013) 013639; Y. He, K. Levin, PRB **89** (2014) 035106; T. Schäfer, PRA **90** (2014) 043633

Transport properties of the QGP

information on QGP transport properties come from observables associated with fluid dynamical flow in HIC





- in non-central HIC find anisotropies in azimuthal particle distribution

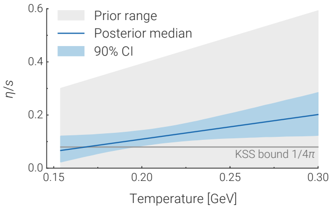
- analysis of flow harmonics: $\eta/s \simeq 0.12 \ldots 0.24$

$\Rightarrow$ nearly perfect fluidity

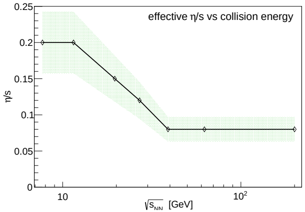
K. Aamodt et al., PRL **105** (2010) 252302; G. Aad et al., PRC **86** (2012) 014907; P. Romatschke, U. Romatschke, PRL **99** (2007) 172301; K. Dusling, D. Teaney, PRC **77** (2008) 034905; H. Song, U.W. Heinz, PRC **77** (2008) 064901; H. Song et al., PRC **83** (2011) 054912; C. Gale et al., PRL **110** (2013) 012302

Local determination of η in the QGP

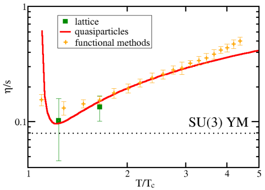
recent efforts aim at determining dependence of η / s on T and n



see talk by S. Bass



see talk by Iu. Karpenko



MB, B. Kämpfer, K. Redlich, PRC **84** (2011) 025201

see talk by J.M. Pawlowski

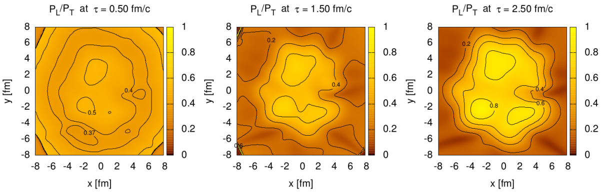
H. Niemi et al., PRC **86** (2012) 014909; Iu.A. Karpenko et al., PRC **91** (2015) 064901; J.E. Bernhard et al., 1605.03954; MB et al., PLB **709** (2012) 77; N. Christiansen et al., PRL **115** (2015) 112002; H. Berrehrah et al., Int. J. Mod. Phys. E **25** (2016) 1642003

Recent development in fluid dynamics for HICs

at early times large momentum-space anisotropies within fireball from pre-equilibrium evolution

- viscous hydro and AdS/CFT estimates give $P_L/P_T \lesssim 0.3$:
  $\rightarrow$ anisotropy stronger for larger η/s and lower T

$\Rightarrow$ development of (viscous) anisotropic hydrodynamics for relativistic fluids



see talk by W. Florkowski

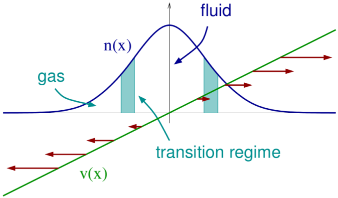
- reproduces known exact solutions of Boltzmann equation accurately

M.P. Heller et al., PRL **108** (2012) 201602; W. van der Schee et al., PRL **111** (2013) 222302; M. Martinez, M. Strickland, NPA **848** (2010) 183; W. Florkowski, R. Ryblewski, PRC **83** (2011) 034907; M. Martinez et al., PRC **85** (2012) 064913; W. Florkowski et al., NPA **916** (2013) 249; *ibid.* PRC **88** (2013) 024903; D. Bazow et al., PRC **90** (2014) 054910; L. Tinti, W. Florkowski, PRC **89** (2014) 034907; W. Florkowski et al., PRC **89** (2014) 054908; *ibid.* PRC **89** (2014) 054909; G.S. Denicol et al., PRL **113** (2014) 202301; *ibid.* PRD **90** (2014) 125026; M. Nopoush et al., PRD **91** (2015) 045007; W. Florkowski et al., PRC **92** (2015) 054912; M. Alqahtani et al., PRC **92** (2015) 054910; *ibid.* 1605.02101; E. Molnár et al., 1602.00573

Local η determination in unitary Fermi gases

Navier-Stokes theory breaks down in dilute corona $\rightarrow$ paradoxical fluid dynamical behavior

- must either assume $\eta \sim n$ or introduce cut-off radius



- reliable treatment of low density regime:

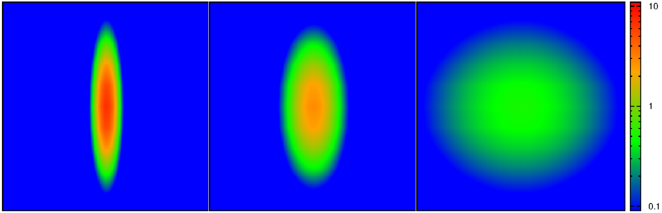
  solve Boltzmann equation numerically; Lattice Boltzmann simulation; Quantum Monte Carlo simulation; combine fluid dynamics with kinetic transport approach; Anisotropic fluid dynamics

J.A. Joseph et al., PRL **115** (2015) 020401; P.-A. Pantel et al., PRA **91** (2015) 013627; MB, T. Schäfer, PRA **92** (2015) 043602; J. Brewer et al., PRA **93** (2016) 013618; MB, T. Schäfer, PRL **116** (2016) 115301

Anisotropic fluid dynamics

based on guided choice for f_p^{an} (exact solution of Boltzmann equation for harmonic potentials in ballistic limit)

- conservation laws + evolution equations for *non-hydrodynamic* d.o.f. $\mathcal{E}_a$ with $\sum_a \mathcal{E}_a = \mathcal{E}$: depend on relaxation time $\tau = \eta / P$



MB, T. Schäfer, PRA **92** (2015) 043602

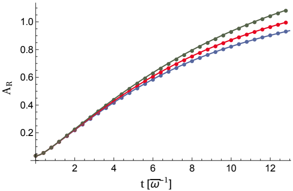
- small τ: fast relaxation to Navier-Stokes theory
- large τ: additional conservation laws $\rightarrow$ ballistic expansion

$\Rightarrow$ provides graceful exit for fluid dynamics in dilute regime

D. Guery-Odelin et al., PRA **60** (1999) 4851; C. Menotti et al., PRL **89** (2002) 250402; P. Pedri et al., PRA **68** (2003) 043608

Application of anisotropic fluid dynamics

baseline test:



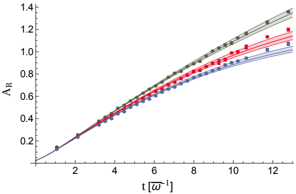
$T\,/\,T_F$

0.79

1.11

1.54

expansion data analysis:



MB, T. Schäfer, PRL **116** (2016) 115301

compare aspect ratio $A_R(t)$ with Boltzmann equation results (symbols)

- essentially perfect agreement (for $\tau = \eta^{\mathrm{CE}}/P$ with η^{CE} for unitary σ_{coll})
- accurate representation of kinetic theory with 2-body collisions

- fit result: $\eta = \eta_0 (mT)^{3/2}$ with $\eta_0 = 0.282$ (compare $\eta_0^{\mathrm{CE}} = 0.269$)
- **model-independent** benchmark result

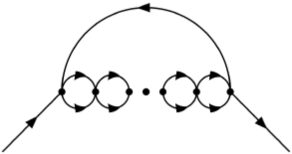
C. Cao et al., Science **331** (2011) 58; P.-A. Pantel et al., PRA **91** (2015) 013627

Bulk viscosity and conformal symmetry breaking

- in thermodynamics: $1 - \frac{2}{3}\frac{\mathcal{E}_0}{P} = \frac{\langle \mathcal{O}_C \rangle}{12\pi m a P} \sim z\frac{\lambda}{a}$

  $\rightarrow$ conformal symmetry breaking parameter
- impact on ζ: in high-T limit $\zeta = \frac{\lambda^{-3}}{24\sqrt{2}\pi}\left(z\frac{\lambda}{a}\right)^2$

$\rightarrow$ physical origin:



$$\mathcal{L}_{eff} = \psi^\dagger\left(\imath\partial_t + \frac{\nabla^2}{2m}\right)\psi - \frac{C_0}{2}(\psi^\dagger\psi)^2\,;\; C_0 = 4\pi a/m$$

$\rightarrow$ well-defined fermionic quasiparticles with **scale-invariance breaking self-energy**

$$Re\,\Sigma(p) \sim \left(z\frac{\lambda}{a}\right)\sqrt{\frac{T^3}{E_p}}$$

$\rightarrow$ ratio $\zeta/\eta_\infty \sim \left(1 - \frac{2}{3}\frac{\mathcal{E}_0}{P}\right)^2$

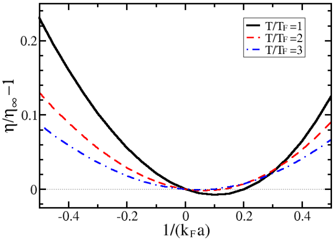
S. Tan, Ann. Phys. **323** (2008) 2971; *ibid.* 2987; Z. Yu et al., PRA **80** (2009) 023615; K. Dusling, T. Schäfer, PRL **111** (2013) 120603;
S. Weinberg, Astrophys. J. **168** (1971) 175

Shear viscosity and conformal symmetry breaking

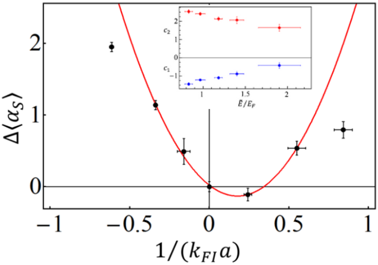
in high-T limit $\rightarrow$ fugacity expansion:

$$\eta = \eta_\infty \left(1 + \hat{c}_2 \left(\frac{\lambda}{a} \right)^2 + \hat{c}_1 \left(\frac{z\lambda}{a} \right) + \ldots \right) \; ; \; \hat{c}_1 < 0$$

$\mathcal{O}\left(z\lambda/a\right)$ originates from in-medium effects in scattering amplitude $\mathcal{A} = C_0/(1 - \Pi C_0)$ and in quasiparticle properties



MB, T. Schäfer, PRA **90** (2014) 063615



$\Rightarrow$ Pauli-blocking more efficient on BCS-side

$\rightarrow$ minimum on BEC-side

$\rightarrow$ with increasing T minimum shifted to unitarity

E. Elliott et al., PRL **113** (2014) 020406; P. Massignan et al., PRA **71** (2005) 033607; G.M. Bruun, H. Smith, PRA **72** (2005) 043605; *ibid.* PRA **75** (2007) 043612; G. Wlazłowski et al., PRA **92** (2015) 063628

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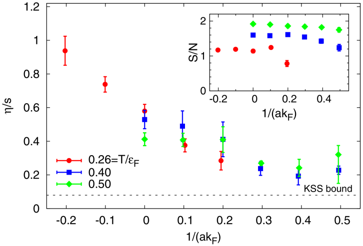
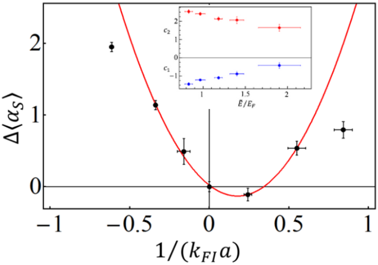


QMC also qualitative agreement

$\rightarrow$ $(\eta/s)_{\min} \simeq 0.2$ at $1/(k_F a) \simeq 0.4$



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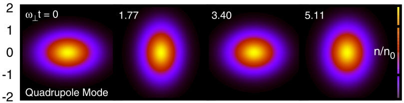
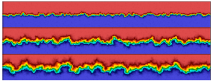
E. Elliott et al., PRL **113** (2014) 020406; P. Massignan et al., PRA **71** (2005) 033607; G.M. Bruun, H. Smith, PRA **72** (2005) 043605; *ibid.* PRA **75** (2007) 043612; G. Wlazłowski et al., PRA **92** (2015) 063628

Outlook and Conclusions

- extend anisotropic fluid dynamics studies to low-T to unfold full T and n dependence of η; requires inclusion of superfluid phase
- study spin diffusion in comparison to experiments; test quantum limit near T_c, connection with momentum-diffusion, ...
- study emergence of non-hydrodynamic modes



- study fluid dynamical fluctuations

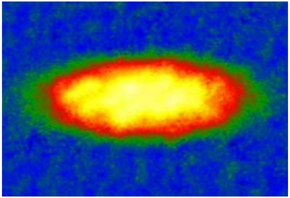


⇔ BEST COLLABORATION

see talk by M. Nahrgang

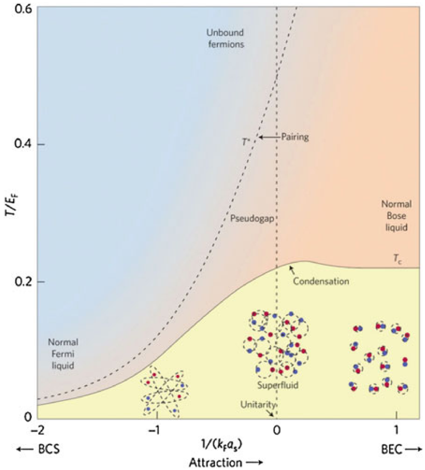
- ...

Outlook and Conclusions

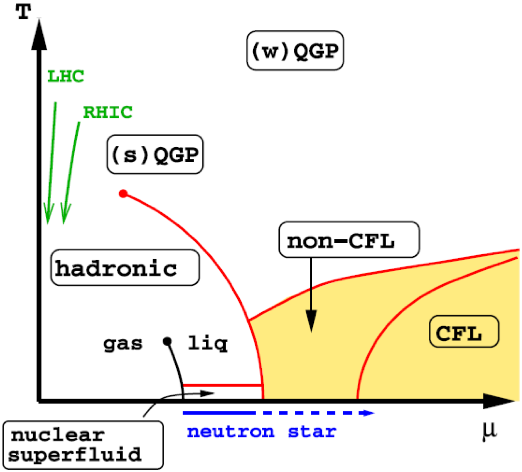


- experimental and theoretical study of cold atoms spans a brigde across different areas in physics

BCS–BEC crossover in cold atoms

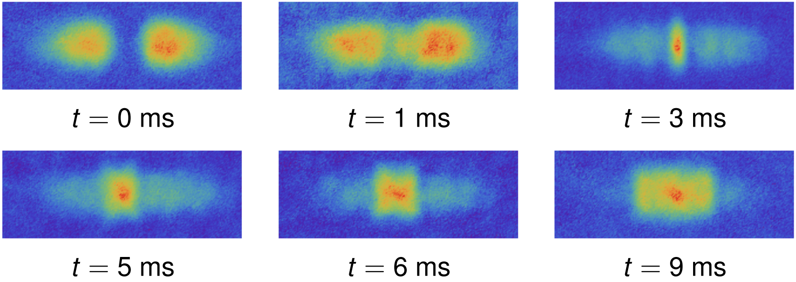


QCD phase diagram



Fermi gas cloud collision

Fermi gas cloud collision:



- repulsive optical potential divides trapped gas into two clouds
- clouds collide once repulsive potential switched off

$\Rightarrow$ formation of propagating shock fronts (fluid dynamical behavior)

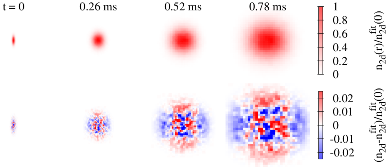
J. Joseph et al., PRL **106** (2011) 150401

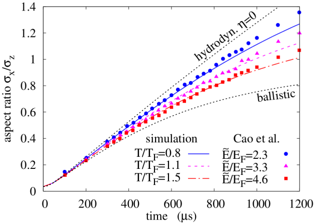
Numerical solution of the Boltzmann-equation

solve Boltzmann-equation numerically within *test-particle method*

$$\left(\partial_t + \vec{v} \cdot \vec{\nabla}_x - \vec{F} \cdot \vec{\nabla}_p\right) f_p(\vec{x}, t) = \mathcal{C}[f_p]$$

L. Boltzmann (1844-1906)





$\rightarrow$ not straightforward to extract dependence of η on n and T

C. Cao et al., Science **331** (2011) 58; P.-A. Pantel et al., PRA **91** (2015) 013627