

# Chiral symmetry breaking in continuum QCD

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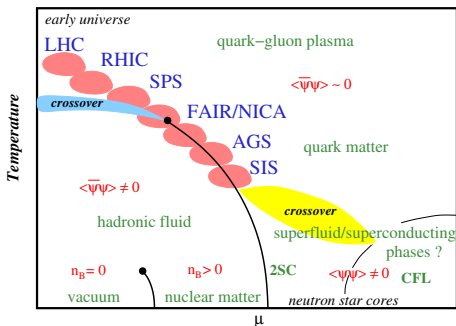


GEFÖRDERT VOM

Bundesministerium  
für Bildung  
und Forschung

# fQCD collaboration - QCD (phase diagram) with FRG:

J. Braun, A. K. Cyrol, L. Fister, W. J. Fu, T. K. Herbst, MM  
N. Müller, J. M. Pawłowski, S. Rechenberger, F. Rennecke, N. Strodthoff

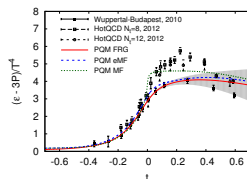


# Outline

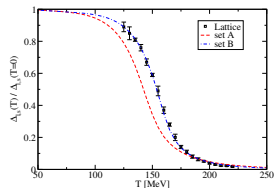
- 1 Motivation: Why the vacuum?
- 2 Vertex Expansion of effective action
- 3 “Quenched” Landau gauge QCD
- 4 Outlook: unquenching and finite temperature
- 5 QCD-enhanced models:  $\eta'$ -meson mass at chiral crossover
- 6 Conclusion

# QCD phase diagram with functional methods

- works well at  $\mu = 0$ : agreement with lattice



[Herbst, MM, Pawłowski, Schaefer, Stiele, '13]

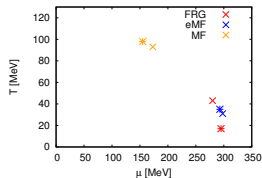


[Luecker, Fischer, Welzbacher, 2014]

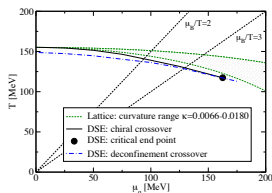
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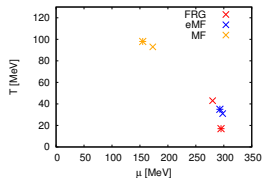
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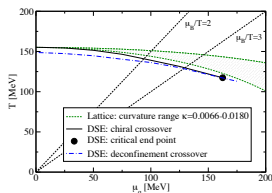
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  - ▶ Polyakov-quark-meson model with FRG:
    - ★ initial values at  $\Lambda \approx \mathcal{O}(\Lambda_{\text{QCD}})$
    - ★ input for Polyakov loop potential
  - ▶ quark propagator DSE:
    - ★ IR quark-gluon vertex



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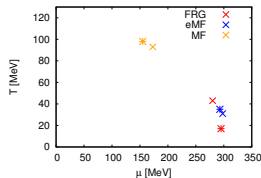
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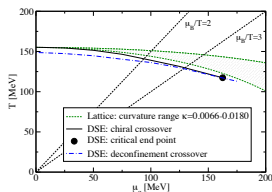
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possible explanation for disagreement:

- $\mu \neq 0$ : relative importance of diagrams changes  
 $\Rightarrow$  summed contributions vs. individual contributions



[MM, Schaefer, 2013]



[Luecker, Fischer, Fister, Pawłowski, '13]

## Back to QCD in the vacuum (Wetterich equation)

- use only perturbative QCD input
  - ▶  $\alpha_S(\Lambda = \mathcal{O}(10) \text{ GeV})$
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$$\partial_k \Gamma_k[A, \bar{c}, c, \bar{q}, q] = \frac{1}{2} \left( \text{Diagram 1} - \text{Diagram 2} - \text{Diagram 3} \right)$$

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- $\partial_k$ : integration of momentum shells controlled by regulator
- full field-dependent equation with  $(\Gamma^{(2)}[\Phi])^{-1}$  on rhs
- gauge-fixed approach (Landau gauge): ghosts appear

# Vertex Expansion

- approximation necessary - vertex expansion

$$\Gamma[\Phi] = \sum_n \int_{p_1, \dots, p_{n-1}} \Gamma_{\Phi_1 \dots \Phi_n}^{(n)}(p_1, \dots, p_{n-1}) \Phi^1(p_1) \cdots \Phi^n(-p_1 - \cdots - p_{n-1})$$

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- functional derivatives with respect to  $\Phi_i = A, \bar{c}, c, \bar{q}, q$ :  
⇒ equations for 1PI  $n$ -point functions, e.g. gluon propagator:

The diagrammatic equation shows the derivative of the inverse gluon propagator with respect to the coupling  $t$ . On the left, a wavy line with a minus sign is followed by an equals sign. To the right, there are three terms: 1) a gluon loop diagram (a circle of wavy lines with two external wavy lines) multiplied by 1; 2) a ghost loop diagram (a circle of dashed lines with a cross on top and two external wavy lines) multiplied by -2; 3) a ghost-gluon loop diagram (a circle with a cross on top and a wavy line at the bottom, and two external wavy lines) multiplied by +1/2.

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- want “apparent convergence” of  $\Gamma[\Phi] = \lim_{k \rightarrow 0} \Gamma_k[\Phi]$

# “Quenched” Landau gauge QCD

- two crucial phenomena:  $S\chi$ SB and confinement
- similar scales - hard to disentangle
- quenched QCD: allows separate investigation:

see e.g. [Williams, Fischer, Heupel, 2015]

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- quenched QCD: allows separate investigation:
  - matter part [MM, Strodthoff, Pawłowski, 2014]
  - pure YM-theory [Cyrol, Fister, MM, Pawłowski, Strodthoff, 2016]

# Chiral symmetry breaking

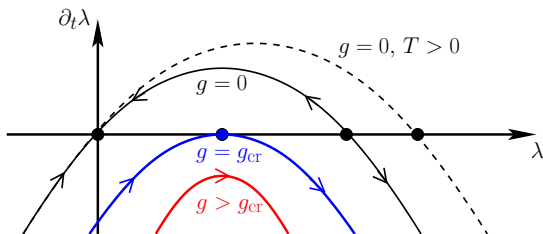
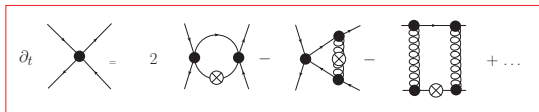
- $\chi$ SB  $\Leftrightarrow$  resonance in 4-Fermi interaction  $\lambda$  (pion pole):



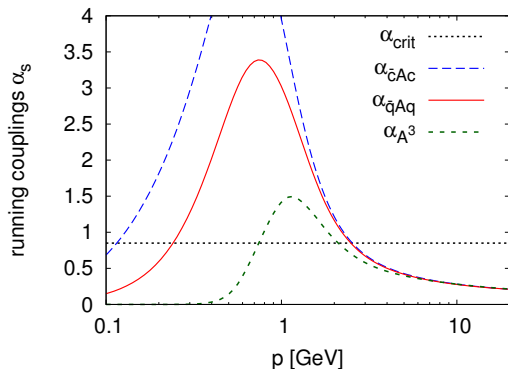
# Chiral symmetry breaking

- $\chi$ SB  $\Leftrightarrow$  resonance in 4-Fermi interaction  $\lambda$  (pion pole):
- resonance  $\Rightarrow$  singularity without momentum dependency

$$\partial_t \lambda = a \lambda^2 + b \lambda \alpha + c \alpha^2, \quad b > 0, \quad a, c \leq 0$$



[Braun, 2011]



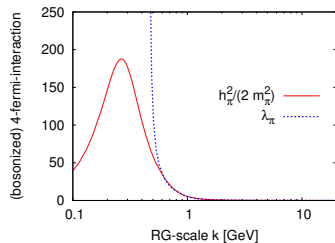
- agreement in perturbative regime required by gauge symmetry
- non-degenerate in nonperturbative regime: reflects gluon mass gap
- $\alpha_{\bar{q}Aq} > \alpha_{cr}$ : necessary for chiral symmetry breaking
- area above  $\alpha_{cr}$  very sensitive to errors

## 4-Fermi vertex via dynamical hadronization

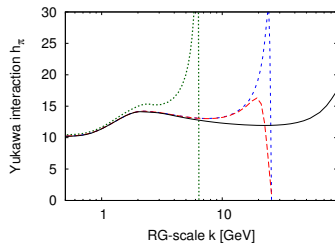
[Gies, Wetterich, 2002]

- change of variables: particular 4-Fermi channels  $\rightarrow$  meson exchange
- efficient inclusion of momentum dependence  $\Rightarrow$  no singularities
- calculation of model parameters from QCD

$$\partial_k \Gamma_k = \frac{1}{2} \left( \text{Diagram 1} - \text{Diagram 2} - \text{Diagram 3} + \frac{1}{2} \text{Diagram 4} \right)$$



[MM, Strodthoff, Pawłowski, 2014]



[Braun, Fister, Haas, Pawłowski, Rennecke, 2014]

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# Vertex Expansion

[MM, Strodthoff, Pawłowski, 2014],

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$$\partial_t \text{---}^{-1} = \text{---}^{\text{---}} + \text{---}^{\text{---}} + \frac{1}{2} \text{---}^{\text{---}} + \text{---}^{\text{---}} + \text{---}^{\text{---}} - \text{---}^{\text{---}}$$

$$\partial_t \text{---}^{\text{---}} = - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \frac{1}{2} \text{---}^{\text{---}} - \text{---}^{\text{---}} + 2 \text{---}^{\text{---}} - \text{---}^{\text{---}} + \text{perm.}$$

$$\partial_t \text{---}^{\text{---}} = 2 \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} + \text{perm.}$$

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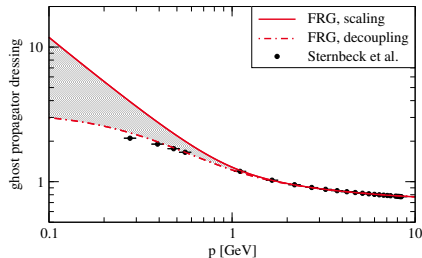
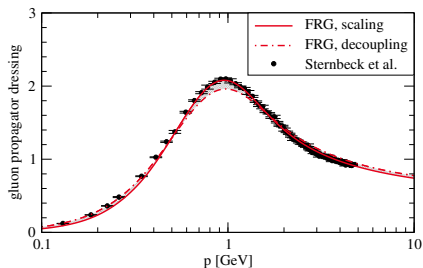
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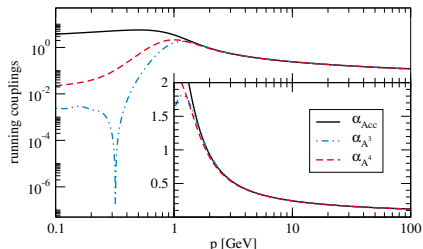
- $\Gamma_{AA}^{(2)}(p) \propto Z_A(p) p^2 (\delta^{\mu\nu} - p^\mu p^\nu / p^2)$

- $\Gamma_{\bar{c}c}^{(2)}(p) \propto Z_c(p) p^2$



- band: family of decoupling solutions bounded by scaling solution

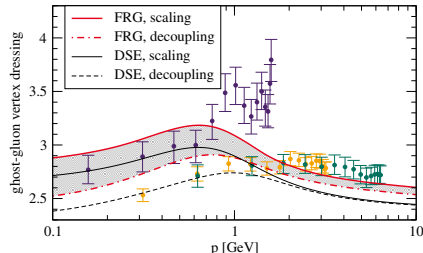
- running couplings from different vertices



- ghost-gluon vertex: comparison to lattice SU(2) DSE

[Maas, unpublished]

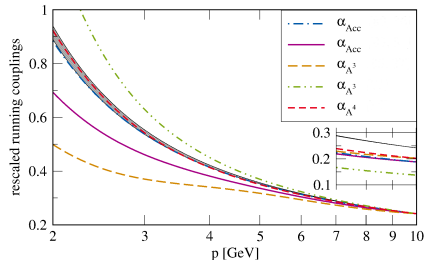
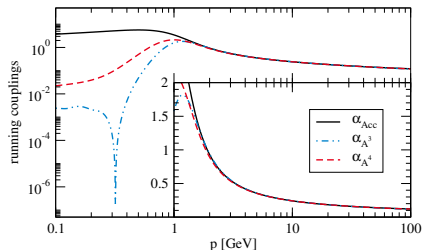
[Huber, Smekal, 2012]



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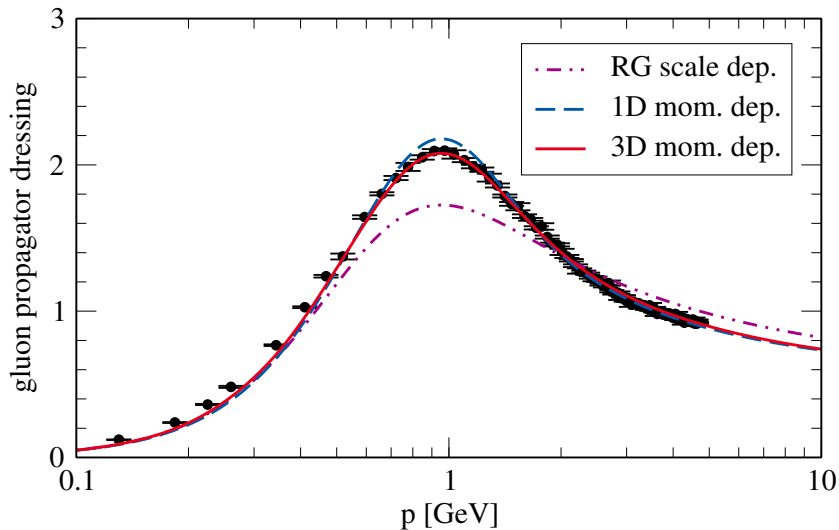
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# Apparent Convergence

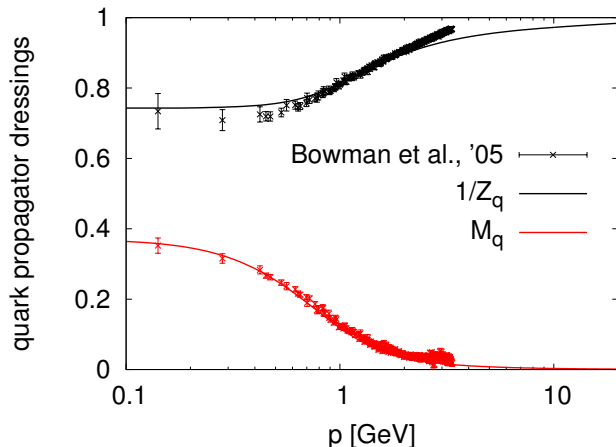
[Cyrol, Fister, MM, Pawłowski, Strodthoff, 2016]



# Quark propagator

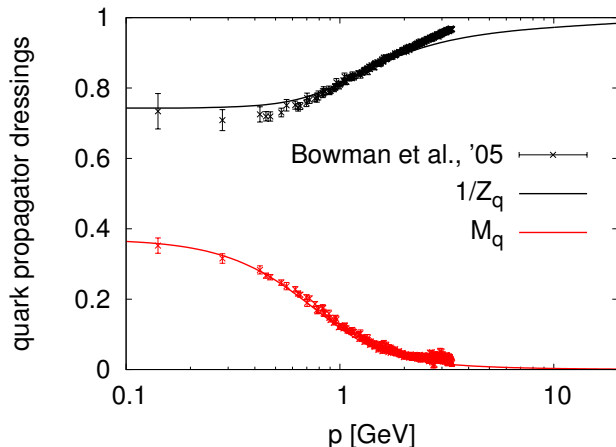
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- $\Gamma_{\bar{q}q}^{(2)}(p) \propto Z_q(p) \not{p} + M(p)$



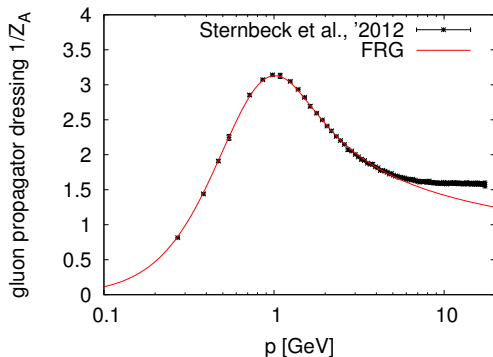
- FRG vs. lattice: bare mass, quenched, scale set via gluon propagator

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- FRG vs. lattice: bare mass, quenched, scale set via gluon propagator
- agreement not sufficient: need apparent convergence at  $\mu \neq 0$

# Outlook: unquenched gluon propagator



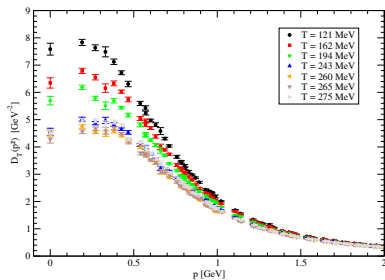
- self-consistent solution of classical propagators and vertices (1D)
- massless quarks

[Cyrol, MM, Pawłowski, Strodthoff, in preparation]

# Outlook: (transversal) gluon propagator at $T \neq 0$

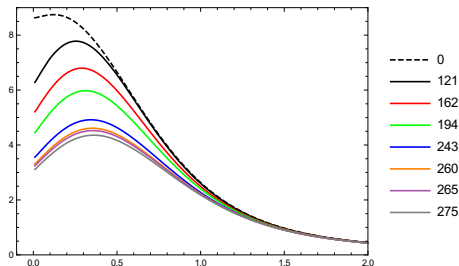
## ● Lattice

[Silva, Oliveira, Bicudo, Cardoso, 2013]



## ● FRG

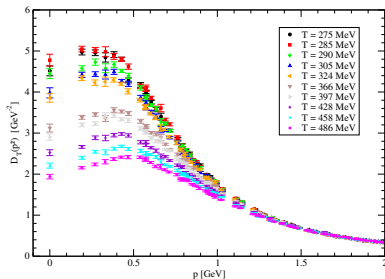
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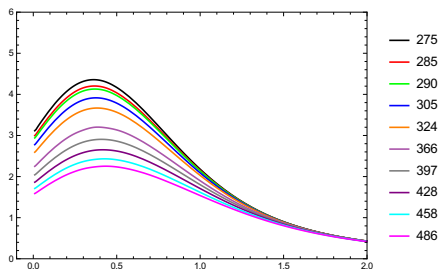
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## $\eta'$ -meson (screening) mass at chiral crossover

- small  $\eta'$ -meson mass above chiral crossover?
- drop in  $\eta'$  mass at chiral crossover?

[Kapusta, Kharzeev, McLerran, 1998]

[Csörgo et al., 2010]



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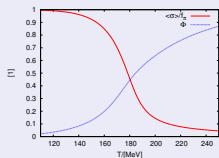
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## chiral crossover: Polyakov-Quark-Meson model (extended mean-field)



- $N_f = 2$  quark and meson degrees of freedom
- describes chiral crossover
- (de-)confinement via Polyakov loop potential
- $U(1)_A$ -anomaly: mesonic 't Hooft determinant

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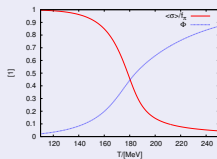
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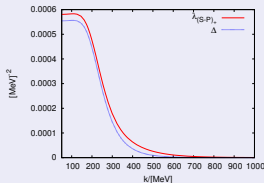
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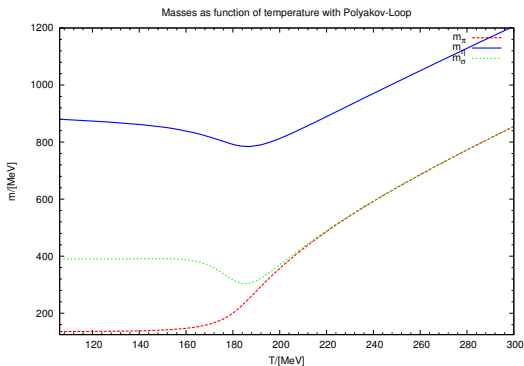
## 't Hooft determinant

[Heller, MM, 2015]



- RG-scale dependence from fQCD
- temperature dependence  $k(T)$ :
  - ▶  $\lambda_{(S-P)_+, fQCD}(k) \equiv \lambda_{(S-P)_+, PQM}(T)$

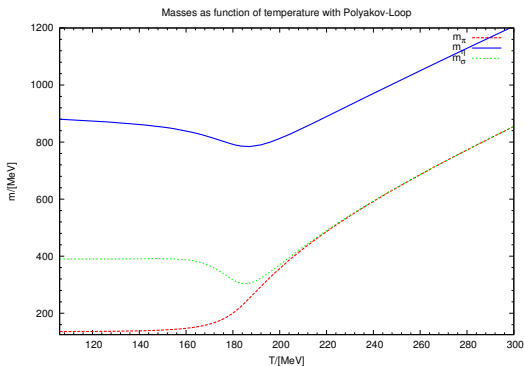
# $\eta'$ -meson (screening) mass at chiral crossover: result



- screening masses!

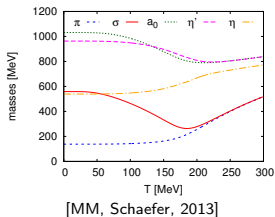
[Heller, MM, 2015]

# $\eta'$ -meson (screening) mass at chiral crossover: result



[Heller, MM, 2015]

- screening masses!
- QM-Model  $N_f = 2 + 1$ :



- chiral symmetry restoration:  
 $\Rightarrow$  drop in  $m_{\eta'}$

# Summary and Outlook

## (quenched) QCD with functional RG

- QCD phase diagram: need for quantitative precision
- quenched QCD in vacuum:
  - ▶ sole input  $\alpha_S(\Lambda = \mathcal{O}(10) \text{ GeV})$  and  $m_q(\Lambda = \mathcal{O}(10) \text{ GeV})$
  - ▶ good agreement with lattice simulations (sufficient?)
- phenomenology:  $\eta'$ -meson and pion mass splitting

# Summary and Outlook

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- 
- unquenching (first results)
  - finite temperature (first results)
  - finite chemical potential
  - order parameters, equation of state and fluct. of cons. charges
  - more checks on convergence of vertex expansion
  - bound-state properties (form factor, PDA. . .)