Fluctuations of Charges at the Phase Boundary

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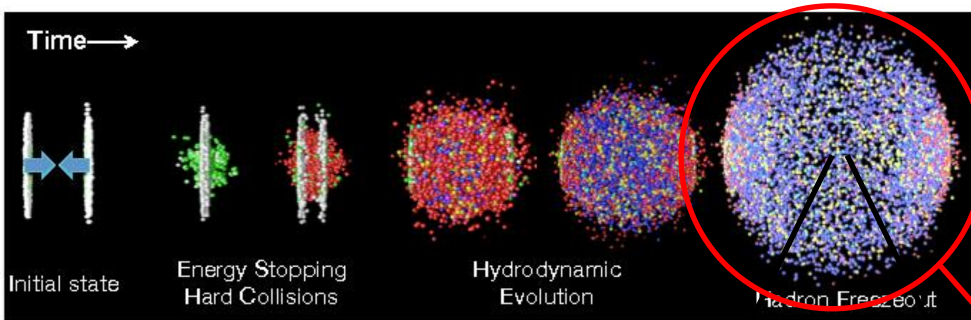
1. Fluctuations of Conserved Charges in Thermal Equilibrium
2. Critical Behavior : Baryon number fluctuations
   Consequence from O(4) and smearing by finite volume and quark mass
3. Electric Charge Fluctuations in π gas and Hadron Resonance Gas
   Importance of "reference" distribution and role of quantum statistics

"Thermodynamics" in Heavy Ion Collisions

~10fm/c, V=π(5fm)^2×(10-100fm)



Conserved Charges in QCD
- Baryon B (← Stopping)
- Electric Charge Q (← Baryon)
- Strangeness S (=0, pair creation)

Globally conserved:
Au+Au Collisions@RHIC
B=197×2
Q=79×2
S=0

Dynamical system
in local equilibrium
(Hydrodynamics)

$T(x), \mu(x), u^\mu(x)$

Exp. acceptance
$\Delta y, \Delta p_t, \Delta\varphi$

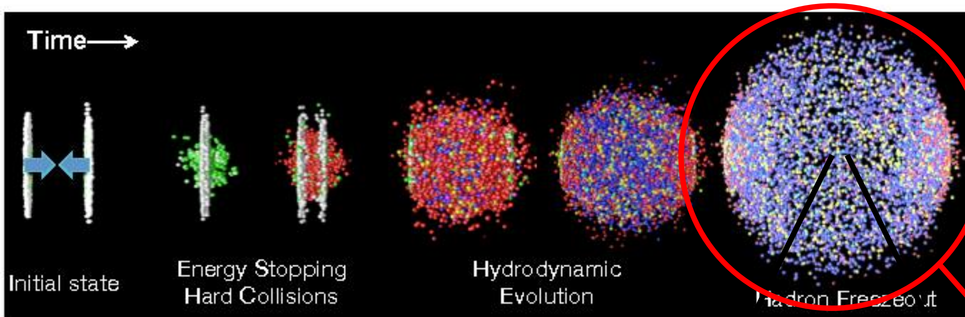
Cooper-Frye

$$N = \int \frac{d^3\boldsymbol{k}}{E_k} \int k_\mu d\sigma^\mu f(u_\nu k^\nu, T(x), \{\mu_i(x)\}) \simeq Vn(T, \{\mu_i\})$$ Small Δy, Low p_t

Subsystem in Grand-Canonical Ensemble $\mathcal{Z}(T,V,\mu_B, \mu_Q, \mu_S)$

Fluctuation Studies in Heavy Ion Collisions

~10fm/c, V=π(5fm)^2×(10-100fm)



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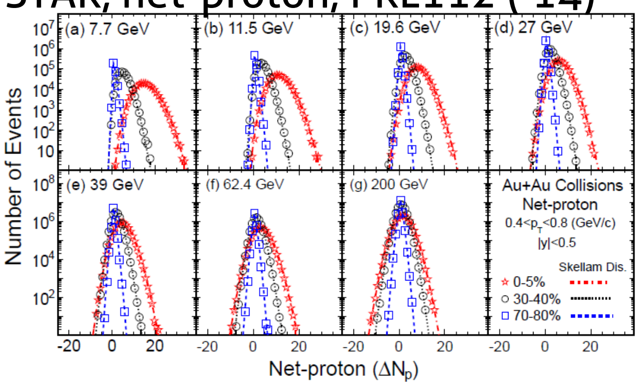
Counting # of particles on e-by-e basis

STAR, net-proton, PRL112 ('14)



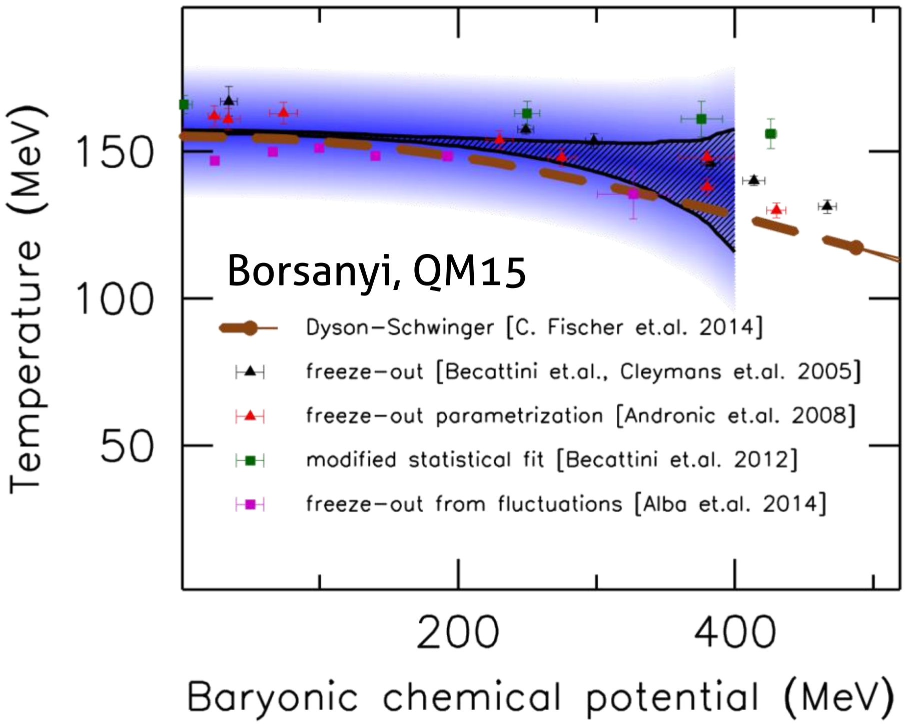
Exp. acceptance Δy, Δp_t, $\Delta\varphi$ Small Δy, Low p_t

Fluctuations of B, Q, S in GC Subsystem $\mathcal{Z}(T, V, \mu_B, \mu_Q, \mu_S)$

$$P(N) = Z_c(T, V, N)e^{\mu/T} / \mathcal{Z}$$

From Fluctuations to QCD Critical Property

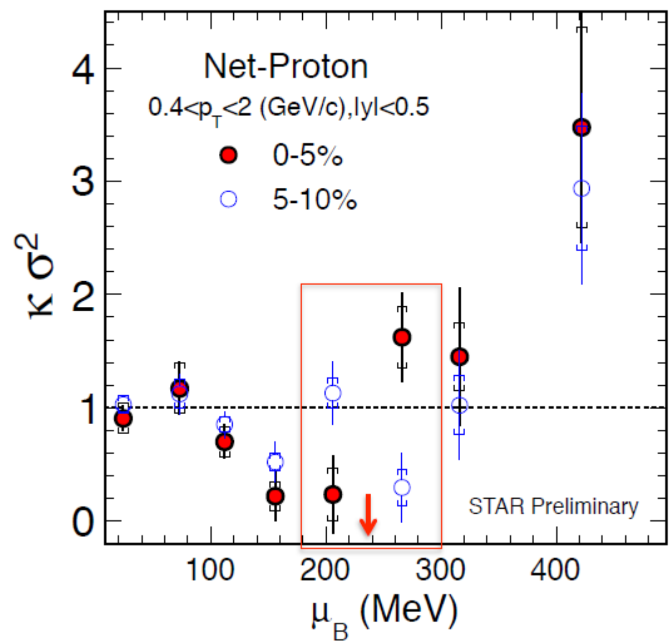
Hadronic observables - information at freeze-out : Determination of T and μ of a subsystem of the hot matter from particle yields / fluctuations



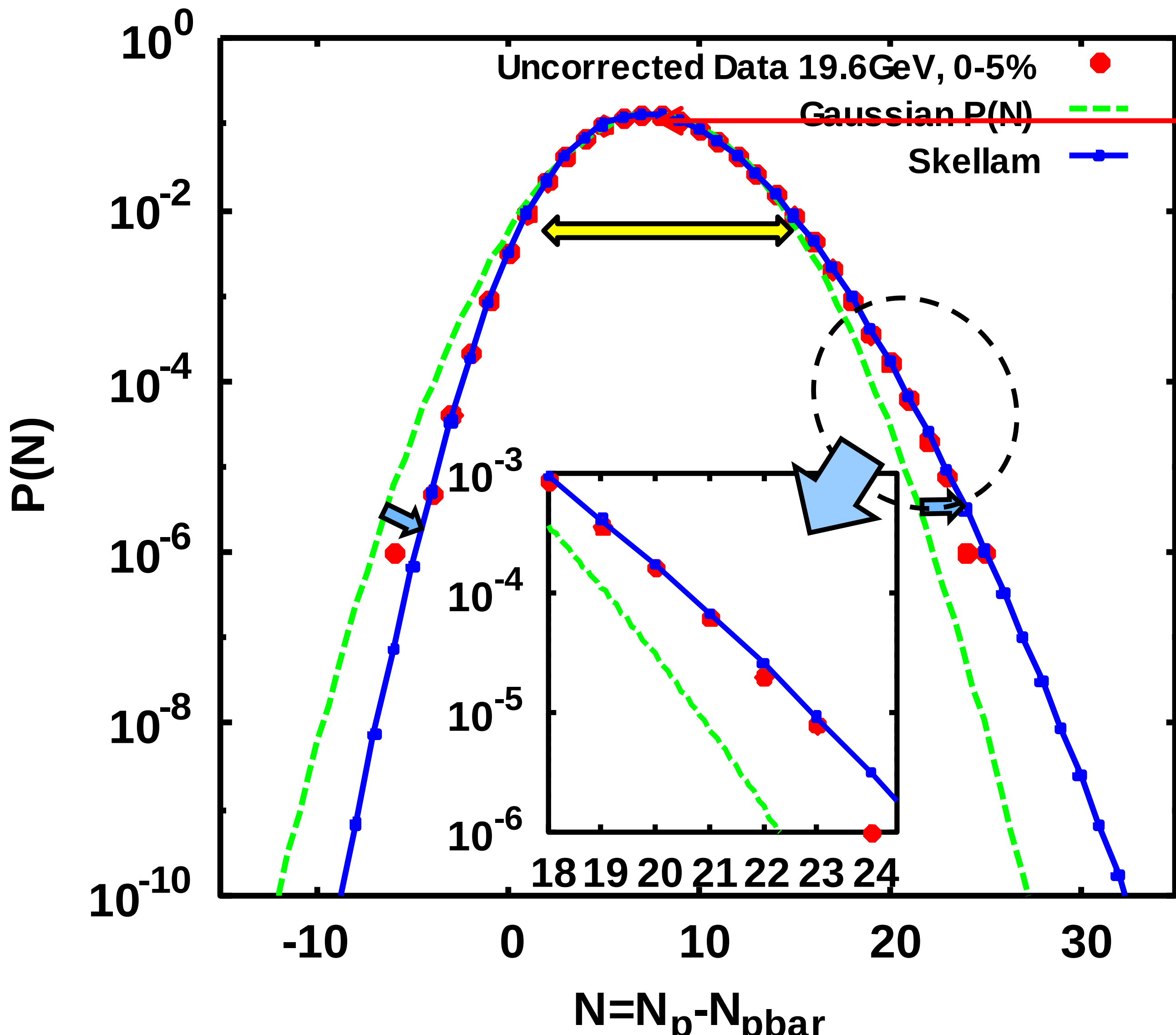
$$\chi_n \equiv T^{n-4}\frac{\partial^n p(T,\mu)}{\partial \mu^n} \propto \frac{\langle (\delta N)^n \rangle}{VT^3}$$

Extracted (T,μ) coincides with the Crossover transition region in QCD
Up to μ_B~ 400MeV

(Remnant) critical property of QCD from fluctuation of conserved charges



Characterizing Fluctuations



Cumulants

$M = \langle N \rangle$ (=7.61)

$\sigma^2 = \langle (\delta N)^2 \rangle$ (=9.16)

$S\sigma^3 = \langle (\delta N)^3 \rangle$ (=7.23)

$\kappa\sigma^4 = \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2$ (=7.36)

$\kappa\sigma^2 = 1$ Skellam

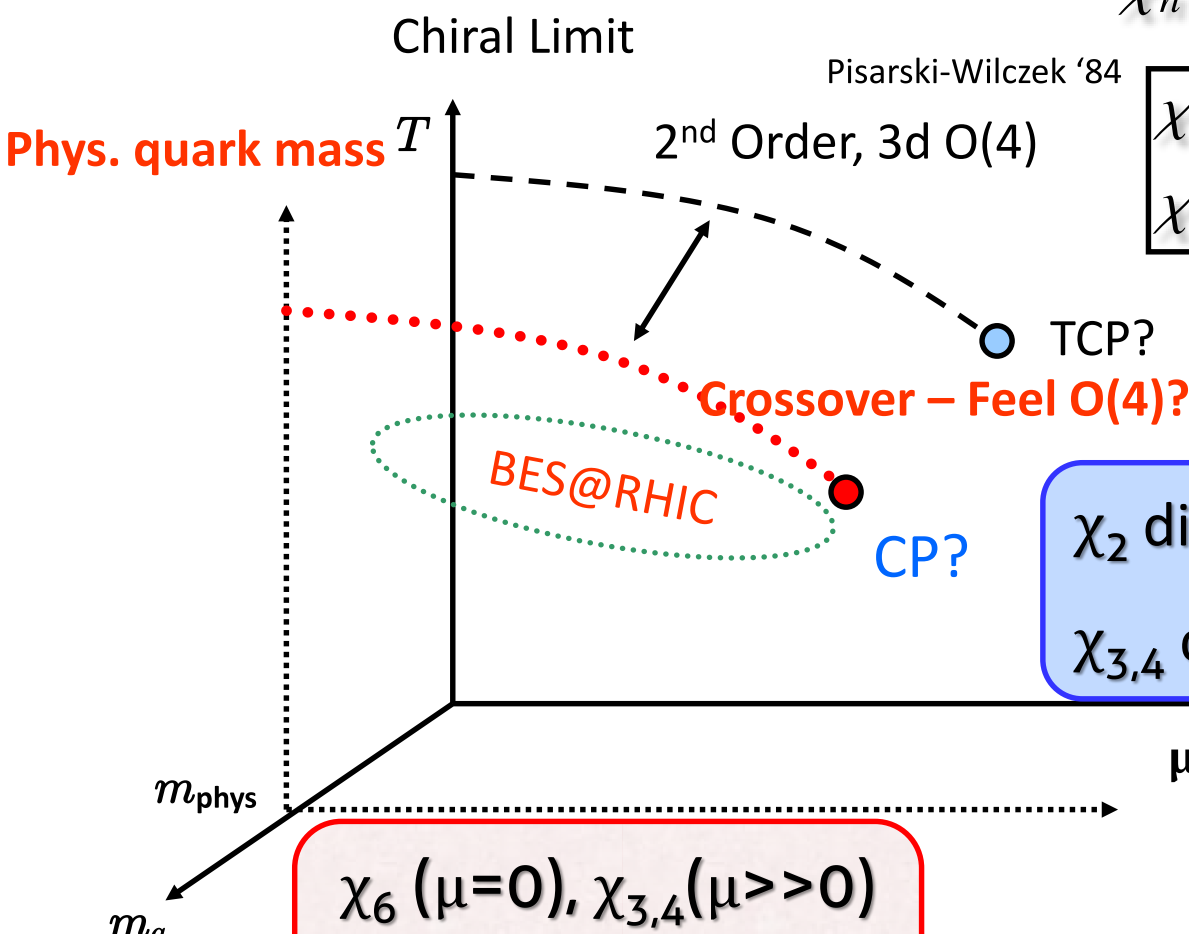
$= 0.8$ (Data)

$= 0$ Gauss

Ratios characterize the shape of P(N)

QCD Phase Transition / N_B Fluctuations

$$\chi_n \equiv T^{n-4}\frac{\partial^n p(T,\mu)}{\partial \mu^n} \propto \frac{\langle(\delta N)^n\rangle}{VT^3}$$



$$\boxed{\begin{array}{l}\chi_6(T \to T_c^{\mp}, \mu = 0) \to \pm\infty \\ \chi_3(T \to T_c^{\mp}, \mu \neq 0) \to \pm\infty\end{array}}$$

$$\chi_2 \to \begin{cases}\infty, & (T \to T_{\mathrm{TCP}}^{-}) \\ \text{Const.} & (T \to T_{\mathrm{TCP}}^{+})\end{cases}$$

χ_2 diverges

$\chi_{3,4}$ change the sign

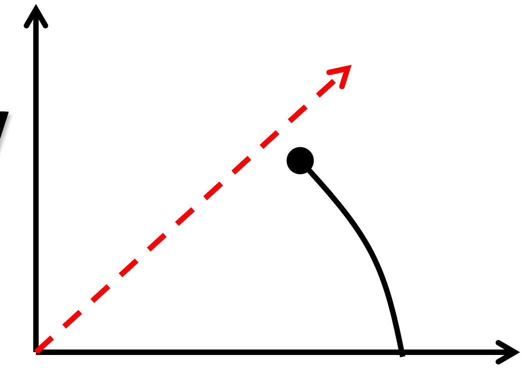
Hatta-Ikeda '03, Asakawa et al.,'09, Skokov et al., '11, Stephanov, '11

χ_6 (μ=0), $\chi_{3,4}$(μ>>0) changes the sign across crossover

General property from O(4) scaling function

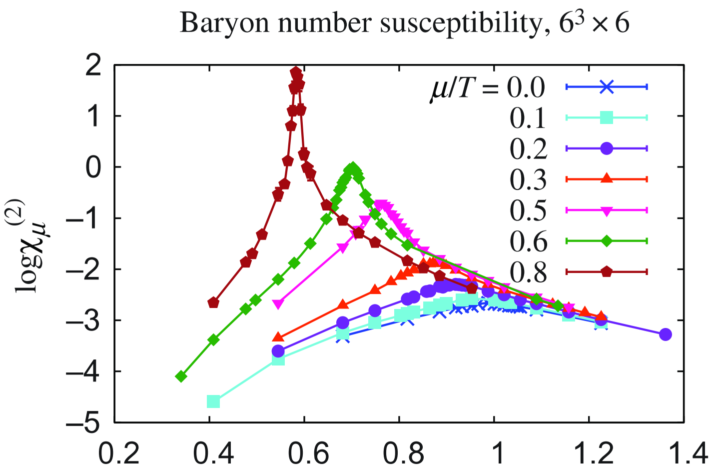
(Engels and Karsch '11, Friman et al., '11)

Sign Change of Cumulants

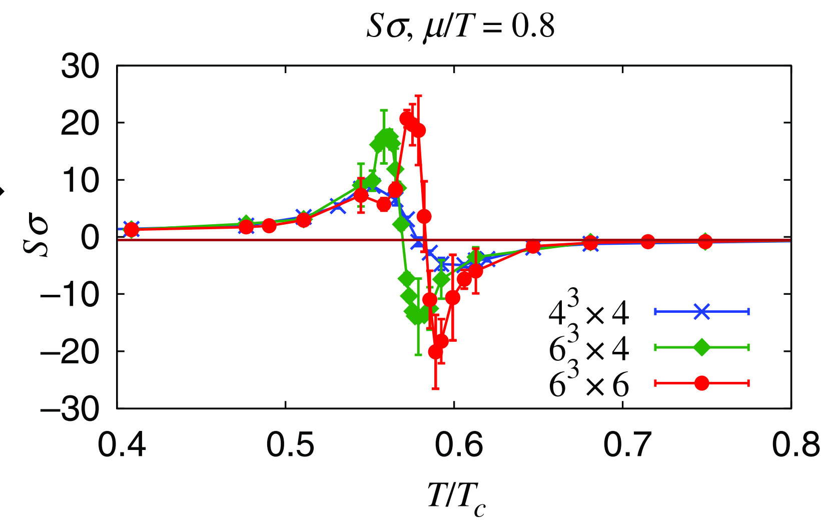


Near TCP, Chiral Limit, Finite V

(Lattice QCD in Strong Coupling Limit, Ichihara, KM, Ohnishi, '15)



$$\frac{\partial}{\partial\mu}$$



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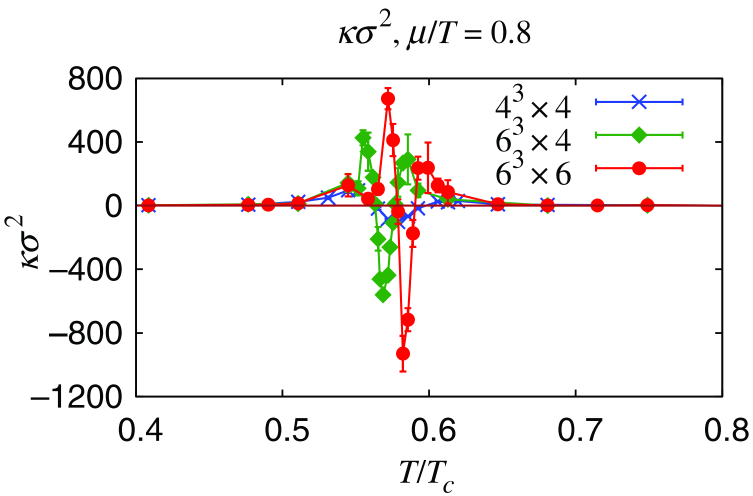
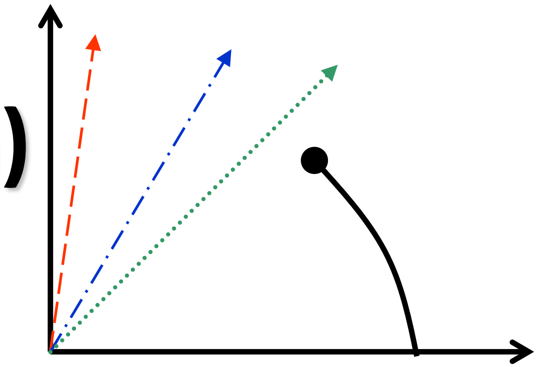


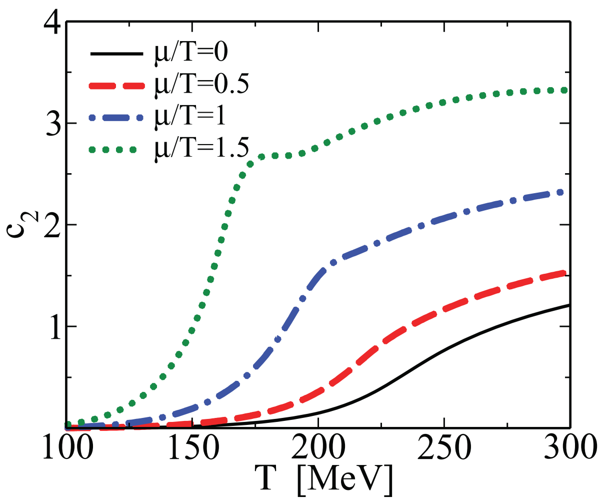
Divergence replaced by sign changes due to finite volume

Sign Change of Cumulants

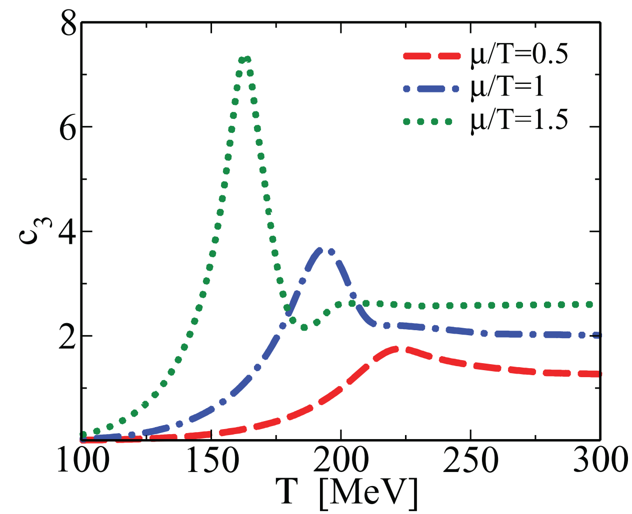


Crossover near O(4) (PQM+FRG)

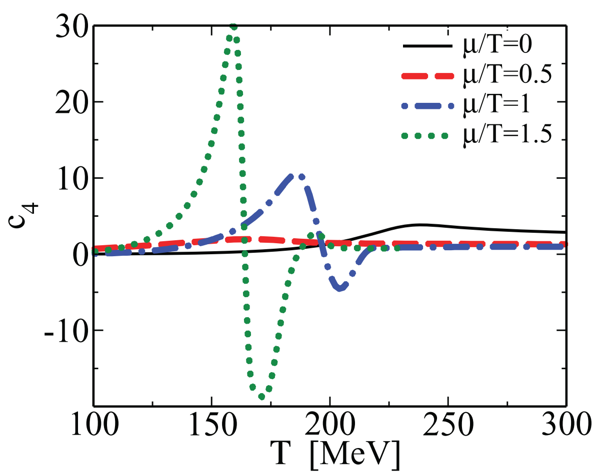
Skokov, Friman, Redlich, PRC'11



$$\frac{\partial}{\partial \mu}$$



$$\frac{\partial}{\partial \mu}$$



Weaker "Critical behavior"

No peak in χ_2, no negative χ_3

Smeared by finite m_q and fluctuations via FRG

Importance of Reference Distribution

$$\chi_n = \chi_n^{\text{regular}} + \chi_n^{\text{singular}}$$

Hadron Gas

Divergent : in χ limit or CP
Finite : cause sign changes in crossover
Suppression factor $(\mu_q/T)^n$

⇨ Obscured when $|\chi_n^{\text{regular}}| \gg |\chi_n^{\text{singular}}|$

Remnant of the divergence may only appear as <u>deviation</u> from the regular part contribution

- **Net-Baryon : *N*, Δ, Hyperon, etc...**
  - *m* >> *T*,μ – Boltzmann Gas — Skellam Distribution
- **Net-electric charge : *π*, *K*, *ρ*, *p*, ...**
  - **π and Δ^{++} cause substantial deviation from Skellam**
- **Net-strangeness : *K*, *K**, Λ, Σ, Ξ, ...**
  - **Heavy enough, but multi-charged (up to 3!)**

Baryon Gas : Skellam Distribution

$$P(N) = \left(\frac{b}{\bar{b}}\right)^{N/2} I_N(2\sqrt{b\bar{b}})e^{-(b+\bar{b})}$$

# of baryons（Poisson) # of antibaryon (Poisson)

- **Statistical mechanics : Boltzmann distribution**

$$b = d\int \frac{d^3p}{(2\pi)^3} e^{-(E_p-\mu)/T}, \quad \bar{b} = d\int \frac{d^3p}{(2\pi)^3} e^{-(E_p+\mu)/T}$$

- **2 parameters**

$$\chi_{2n+1} = b - \bar{b}$$

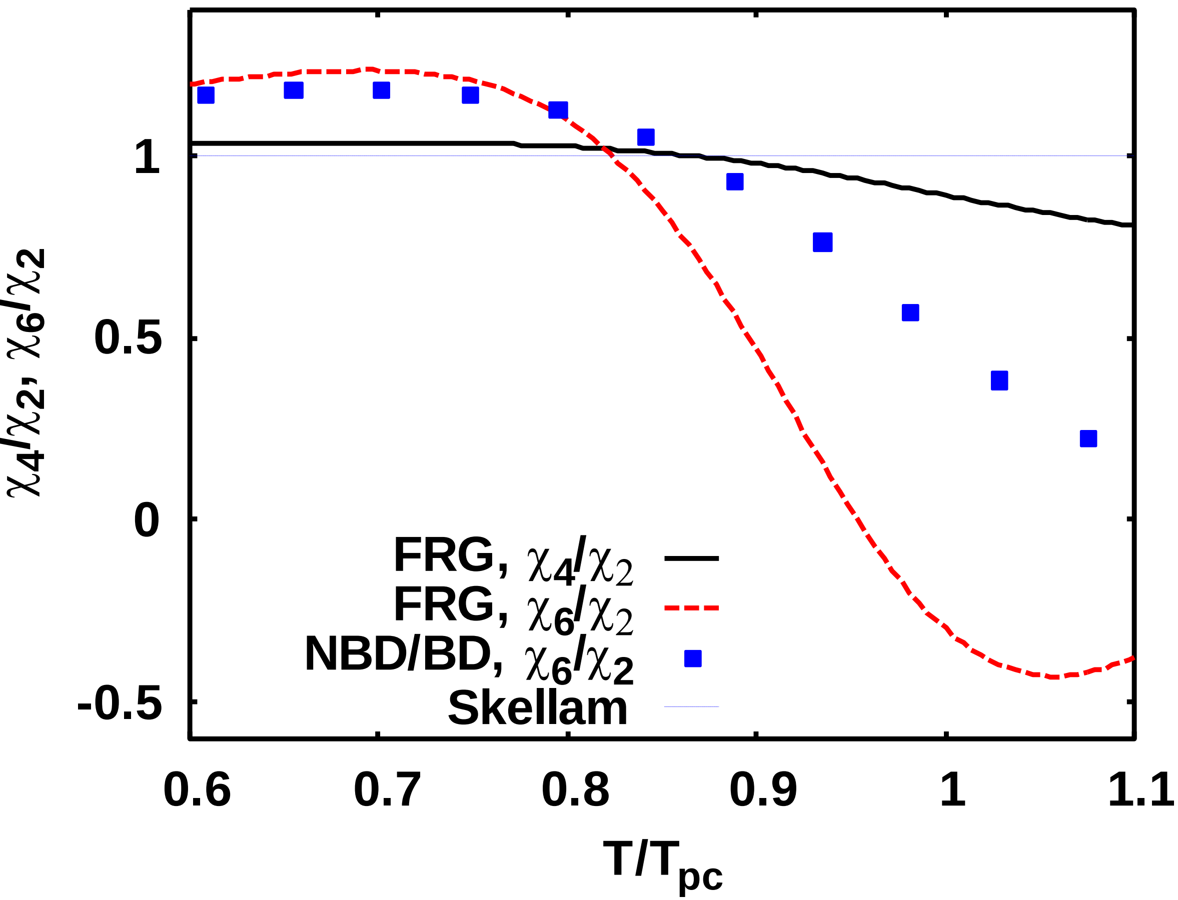
$$\chi_{2n} = b + \bar{b}$$

Independent of momentum integration range

Expectation (confirmed by lattice): χ_1 and χ_2 are well described by HRG

Deviation from Skellam in higher order χ_n can reflect the phase transition

(Negative) Binomial Distribution?



KM, Friman, Redlich, PLB '15

NB ($\kappa\sigma^2$ >1), BD($\kappa\sigma^2$<1) can fit the critical χ_2 and χ_4, but cannot reproduce χ_6

Higher (n>4) cumulants necessary to identify the critical fluctuations

Charge Fluctuations in π gas

Pion $m_\pi \sim T$: Bose statistics

P. Braun-Munzinger et al., NPA'12

$$\chi_n^Q = \begin{cases} \dfrac{m^2}{\pi^2 T^2} \displaystyle\sum_{k=1}^{\infty} k^{n-2} \hat{K}_2(km/T) \cosh(k\mu_Q/T), \ n = \text{even} \\ \dfrac{m^2}{\pi^2 T^2} \displaystyle\sum_{k=1}^{\infty} k^{n-2} \hat{K}_2(km/T) \sinh(k\mu_Q/T), \ n = \text{odd} \end{cases}$$

$$\hat{K}_2(km/T) = \frac{k}{2m^2 T} \int_{\eta_{\min}}^{\eta_{\max}} d\eta \int_{p_{t\min}}^{p_{t\max}} dp_t p_t |\boldsymbol{p}| e^{-kE_p/T}$$

$$= K_2(km/T) \quad (\eta_{\min} = -\infty, \eta_{\max} = \infty, p_{t_{\min}} = 0, \text{and } p_{t_{\max}} = \infty)$$

= Multicomponent Boltzmann gas with mass km, charge k, and degeneracy k^{n-4}

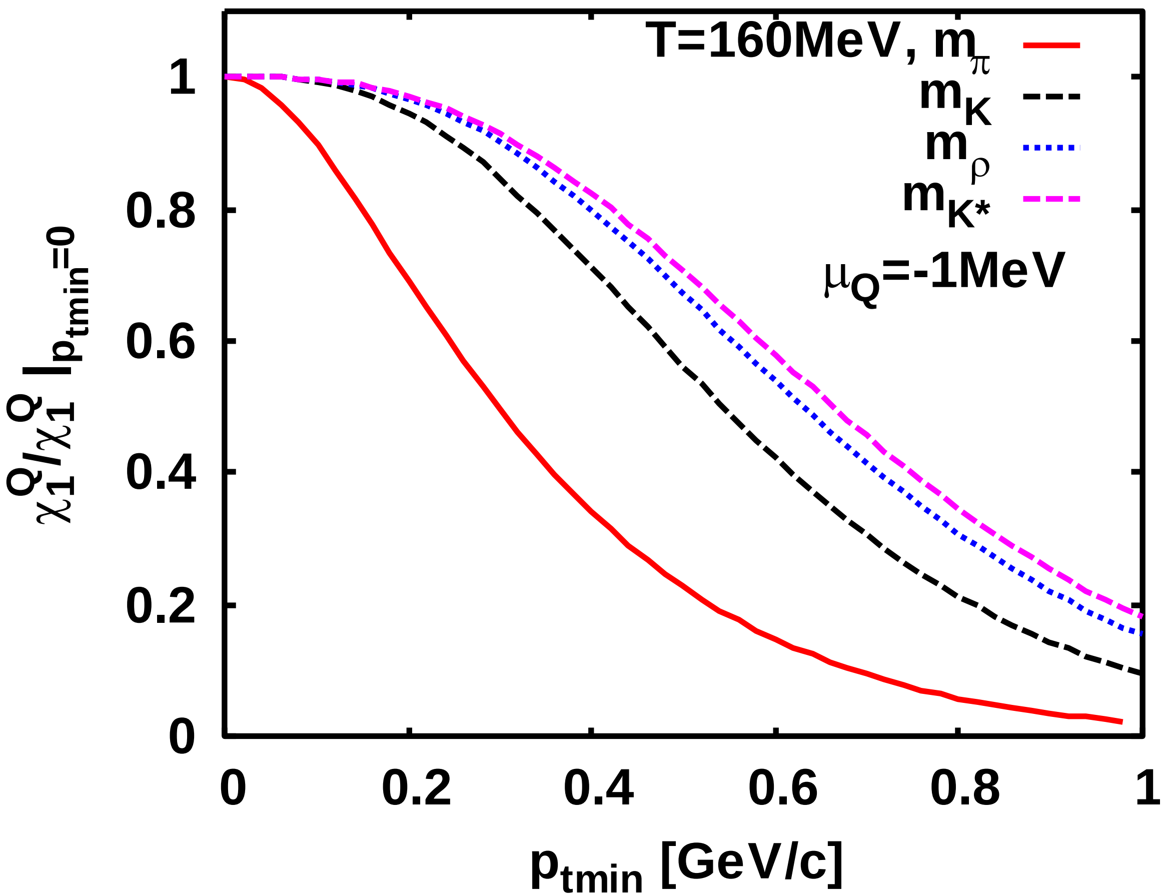
- **Leading order : Skellam**
- **all k >1 terms : positive**
- **p-integration range can change χ_n ratio**

$$\longrightarrow \frac{\chi_{n+2}^Q}{\chi_n^Q} > 1$$

Effect of Low p_t Cut

Karsch, KM, Redlich, PRC '16

Electric Charge Cumulants of a free gas



χ_1 – Density vs Mass:

Heavier particles less affected by p_t cut

π contribution suppressed by p_t cut

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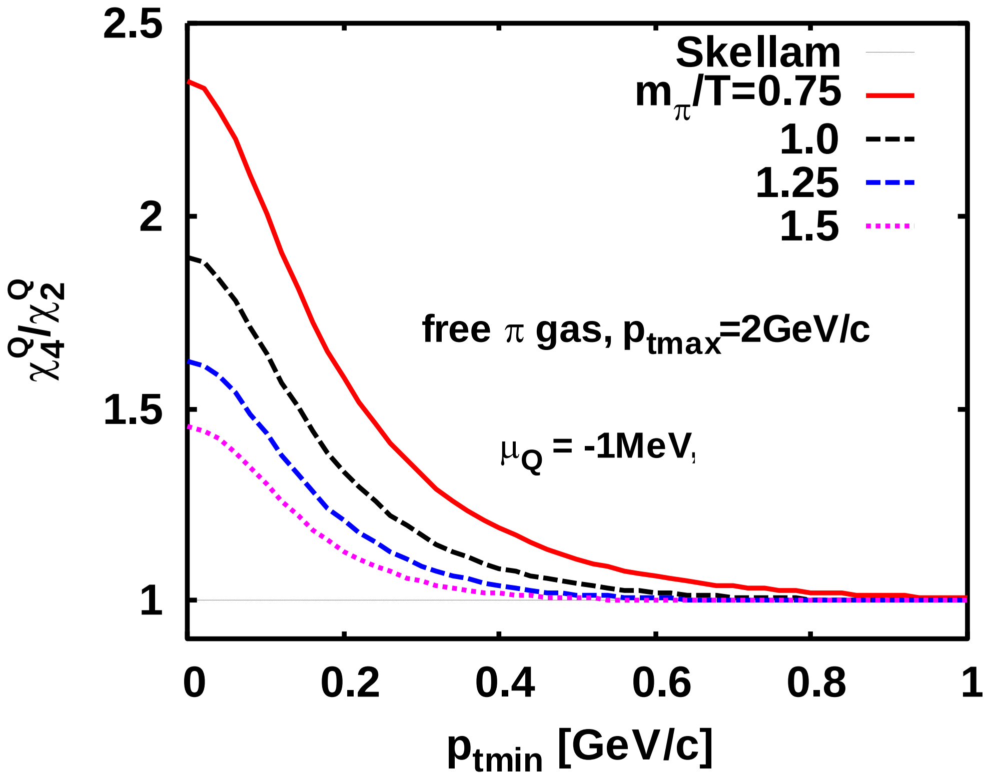


Low pt cut : closer χ_4/χ_2 to 1

$$m_{\mathrm{eff}}^2 = p_{t\min}^2 + m_\pi^2$$

For small μ_Q/T,

$$\frac{\chi_3^Q}{\chi_1^Q} \simeq \frac{\chi_4^Q}{\chi_2^Q}, \cdots, \frac{\chi_{2n+1}^Q}{\chi_{2n-1}^Q} \simeq \frac{\chi_{2n+2}^Q}{\chi_{2n}^Q}$$

$$\frac{\chi_{2n-1}^Q}{\chi_{2n}^Q} = \frac{\sum_k k^{2n-3}\hat{K}_2(km/T)k\mu_Q/T}{\sum_k k^{2n-2}\hat{K}_2(km/T)} = \frac{\mu_Q}{T}$$

$$\hat{K}_2(km/T) = \frac{1}{2}\left(\frac{T}{km}\right)^2 \int d\eta \int_{kp_{t_{\min}}/T}^{kp_{t_{\max}}/T} dx x^2 \cosh\eta\, e^{-\sqrt{x^2\cosh^2\eta + (km/T)^2}}$$

Substantial decrease from p_{tmin}=0 to p_{tmin} = 0.2 (STAR) or 0.3 (PHENIX) GeV

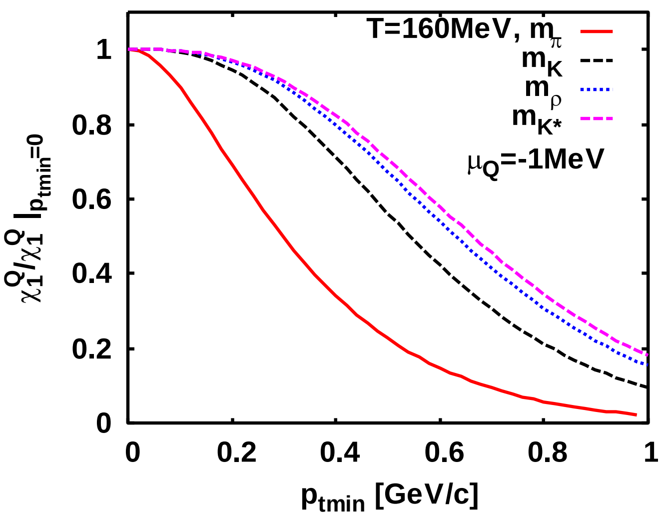
M/σ^2 in HRG

M = (π, ρ, etc) + (strange mesons) + (baryons) + (hyperons)

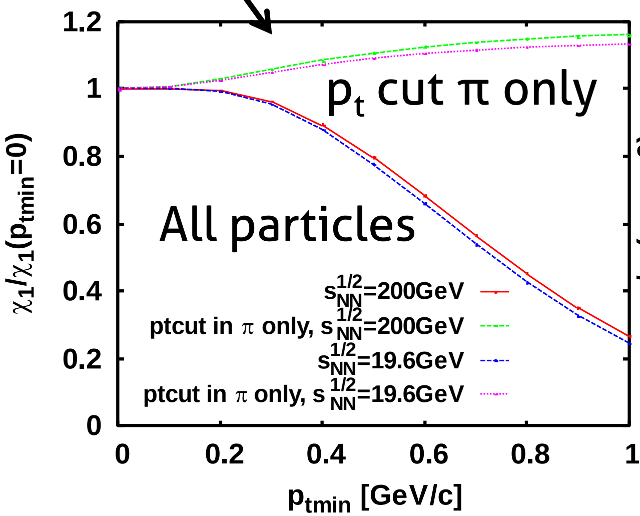
< 0 (μ_Q < 0) > 0 (μ_S > 0) > 0 (μ_B > 0) > 0 (μ_B-(1~3)μ_S > 0)

Negative meson contribution is reduced by pt cut

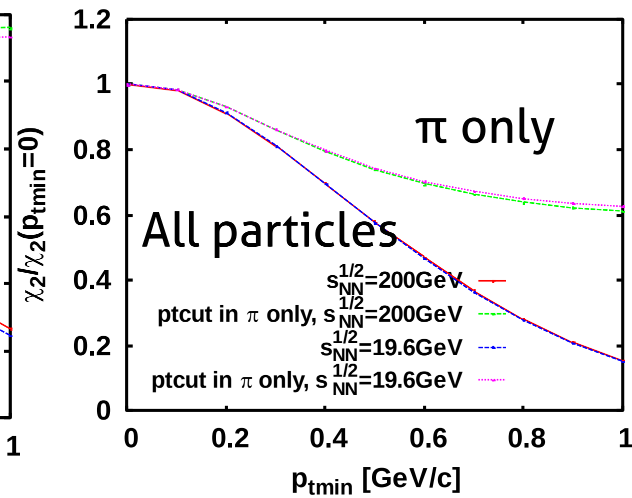
Total M can be non-monotonic in p_{tmin}
σ^2 simply decreases



Single component

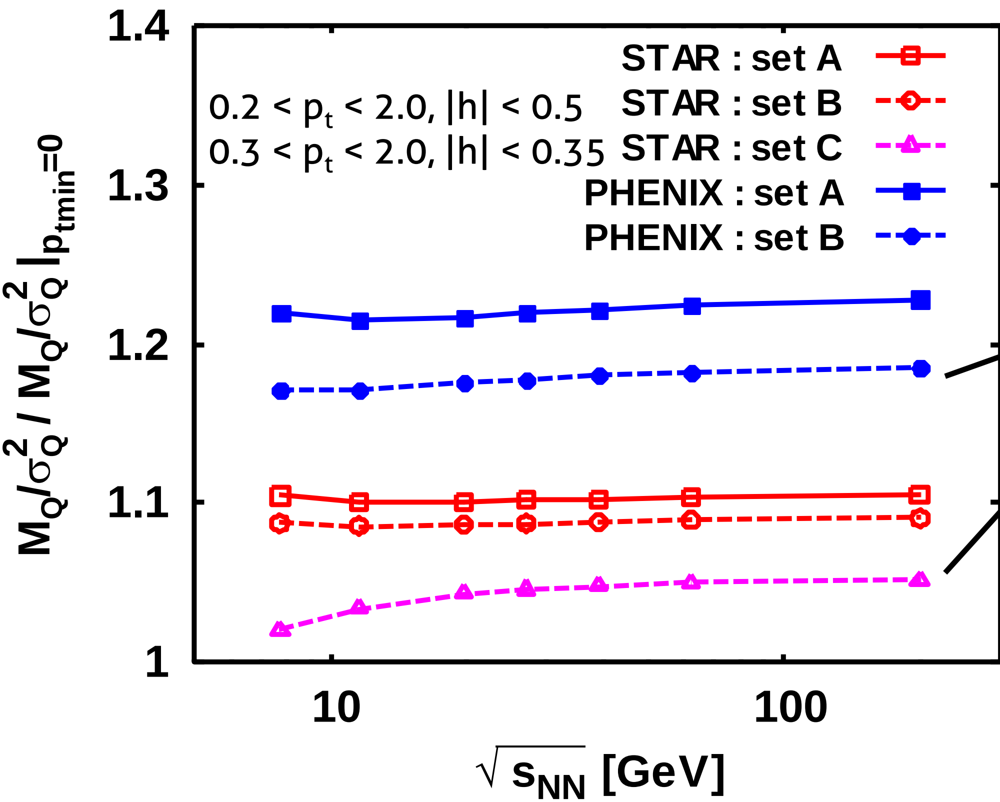


M in HRG



σ^2 in HRG

M/σ^2 in HRG : STAR vs PHENIX

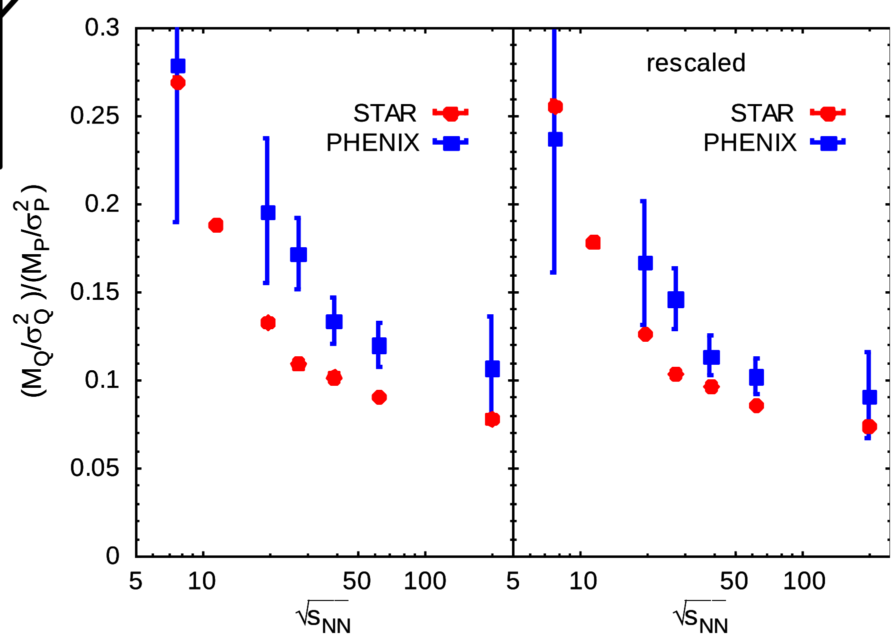


A : p_t cut for π only

B : Same p_t cut for all hadrons

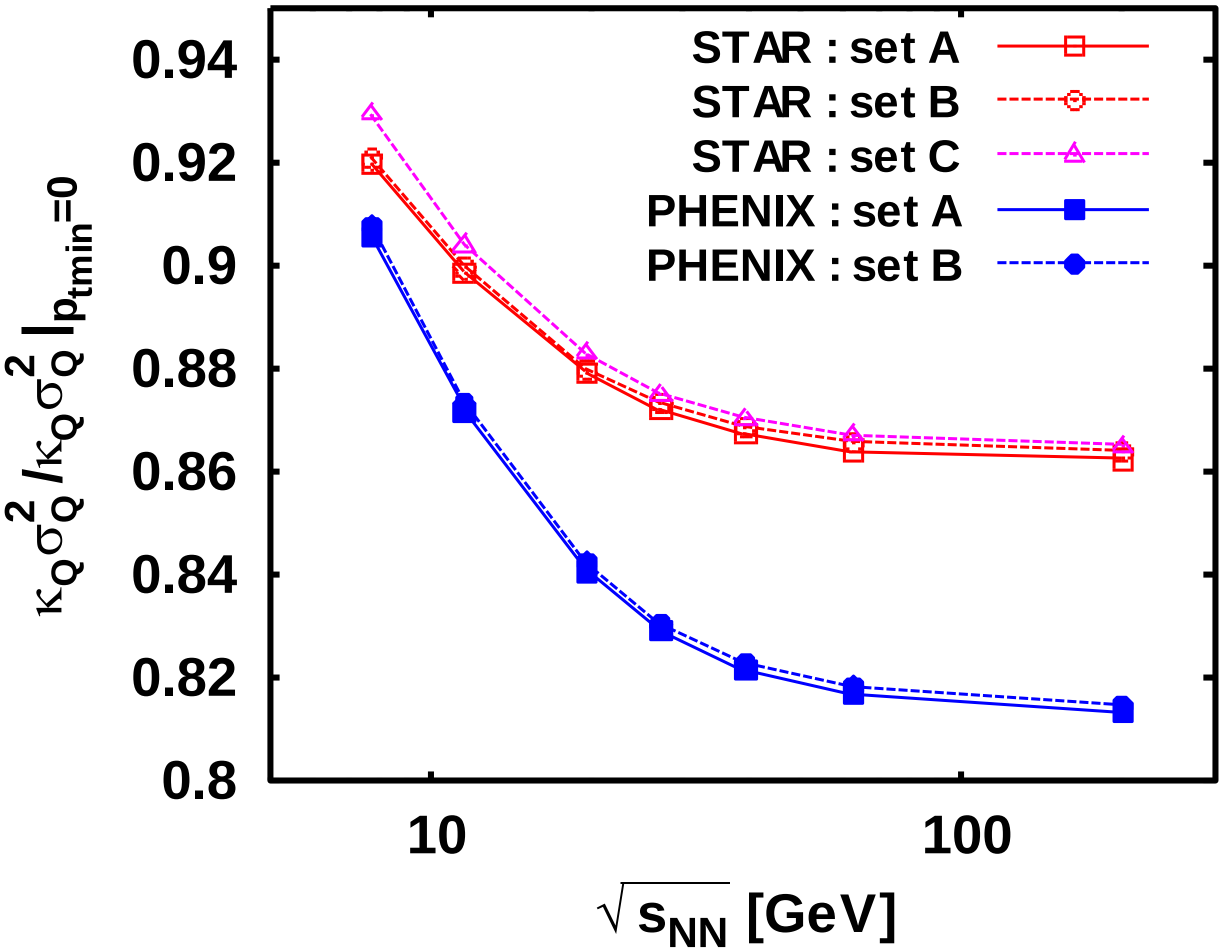
C : p_{tmin} = 0.4 GeV for protons (STAR)

10% larger M/σ^2 in PHENIX acceptance



Data : PHENIX/STAR ~ 1.35
After rescaling, data become closer

$\kappa\sigma^2$ in HRG (prediction)



$$\frac{\chi_4^Q}{\chi_2^Q} = \frac{\chi_4^\pi + \chi_4^K + \chi_4^\rho + \cdots}{\chi_2^\pi + \chi_2^K + \chi_2^\rho + \cdots}$$

6% difference btw. STAR and PHENIX expected

p_t cut reduces $\kappa\sigma^2$ < 10% at low energies, due to more proton contribution

p_t cut reduces $\kappa\sigma^2$ by 10-20% at high energies

Concluding Remarks

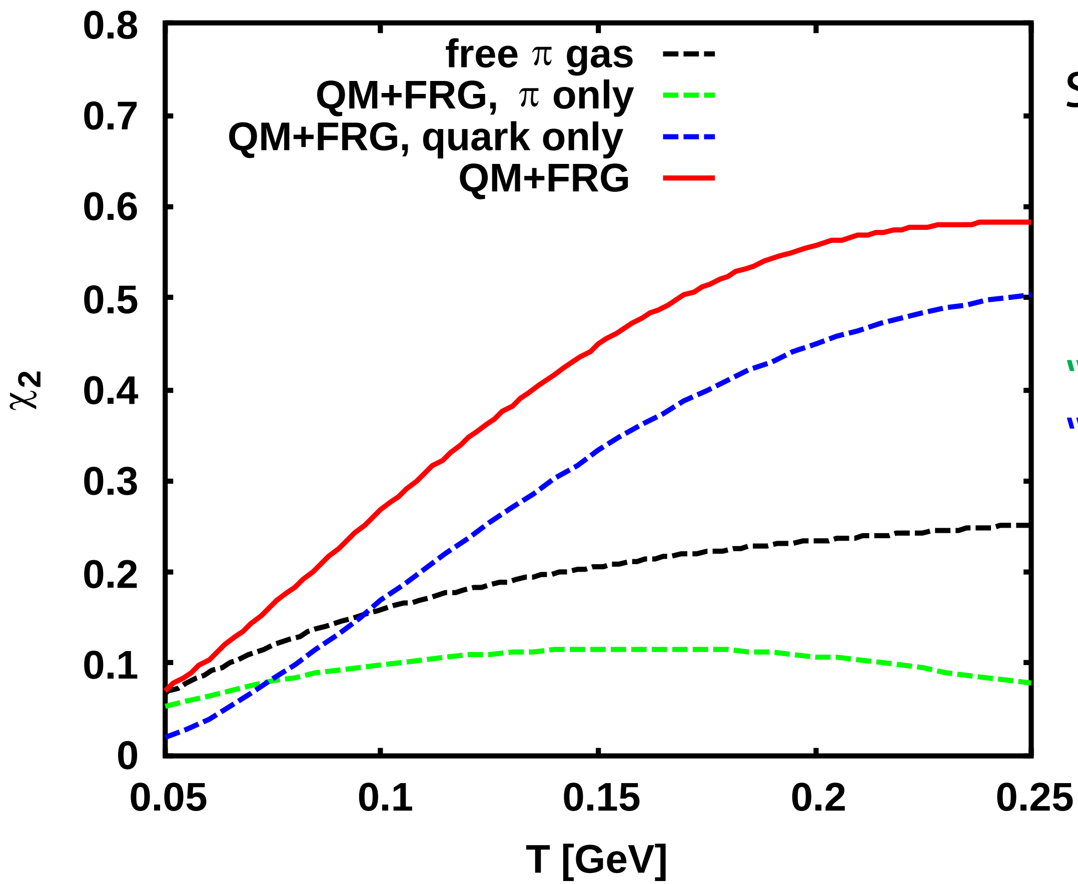
- **Higher-order fluctuations of net-B, Q around phase boundary**
  - **Critical behavior : Smeared by finite V, nonzero m_q**
    - Lattice QCD in Strong Coupling Limit
    - Chiral model calculations with FRG
  - **Deviation from the reference**
  - **net-B : Skellam distribution – momentum independent**
  - **net-Q : Bose statistics – affected by low pt cut**
    - Explaining part of difference btw. STAR and PHENIX

Backup

Effects of Interaction

Charge fluctuations in QM model (N_f=2)

$\mu_B = \mu_Q = 0$



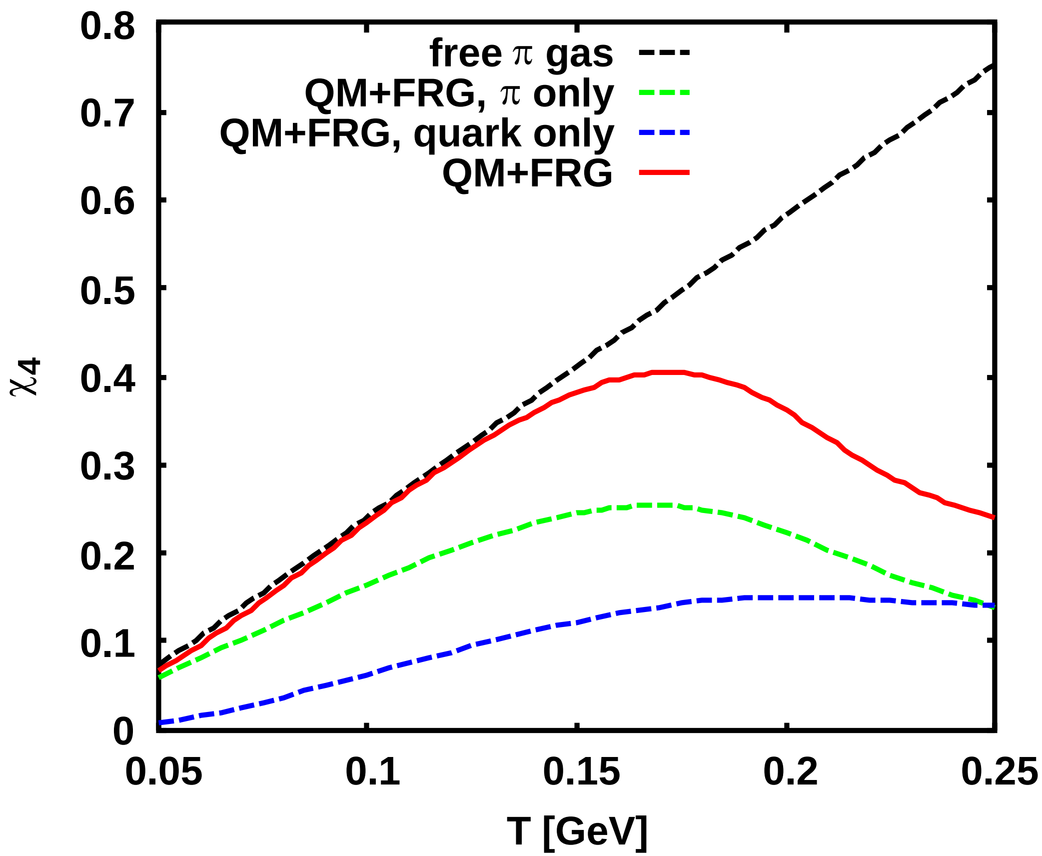
Susceptibility:
quark dominant
(Caveat: deconfined – quarks have Q=2/3, -1/3)

"π only" : $Q_{u,d}=0$
"quark only" : $Q_\pi=0$

Effects of Interaction

Charge fluctuations in QM model (N_f=2)

$\mu_B=\mu_Q=0$



4th cumulant:

π dominant

(Caveat: deconfined – quarks have Q=2/3, -1/3)

Quark contribution suppressed by Fermi statistics

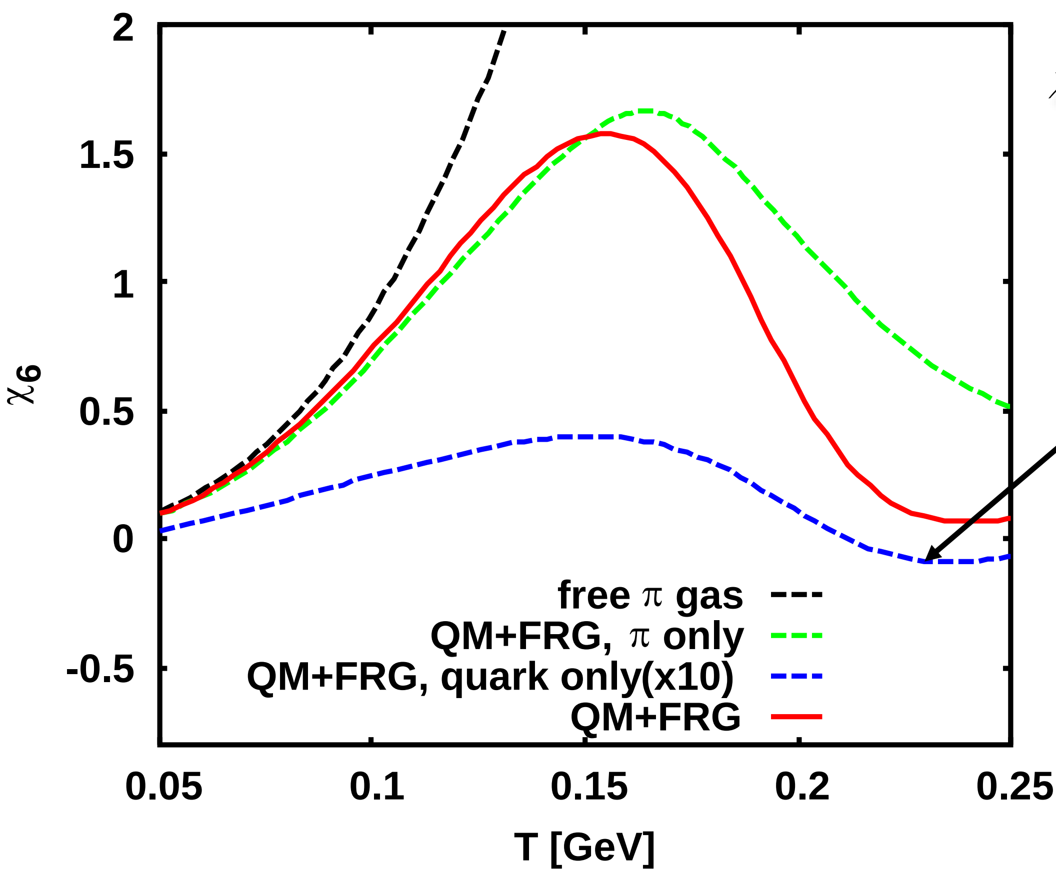
"π only" : $Q_{u,d}=0$

"quark only" : $Q_\pi=0$

Effects of Interaction

Charge fluctuations in QM model (N_f=2)

$\mu_B = \mu_Q = 0$



$$\chi_n^Q = \frac{1}{2^n}\left[\chi_n^B + \chi_n^I + \sum_{i=1}^{n-1} {}_nC_i \frac{\partial^n (p\beta^4)}{\partial(\beta\mu_I)^i \partial(\beta\mu_B)^{n-1}}\right]$$

6th cumulant:

Negative χ_6 from chiral crossover

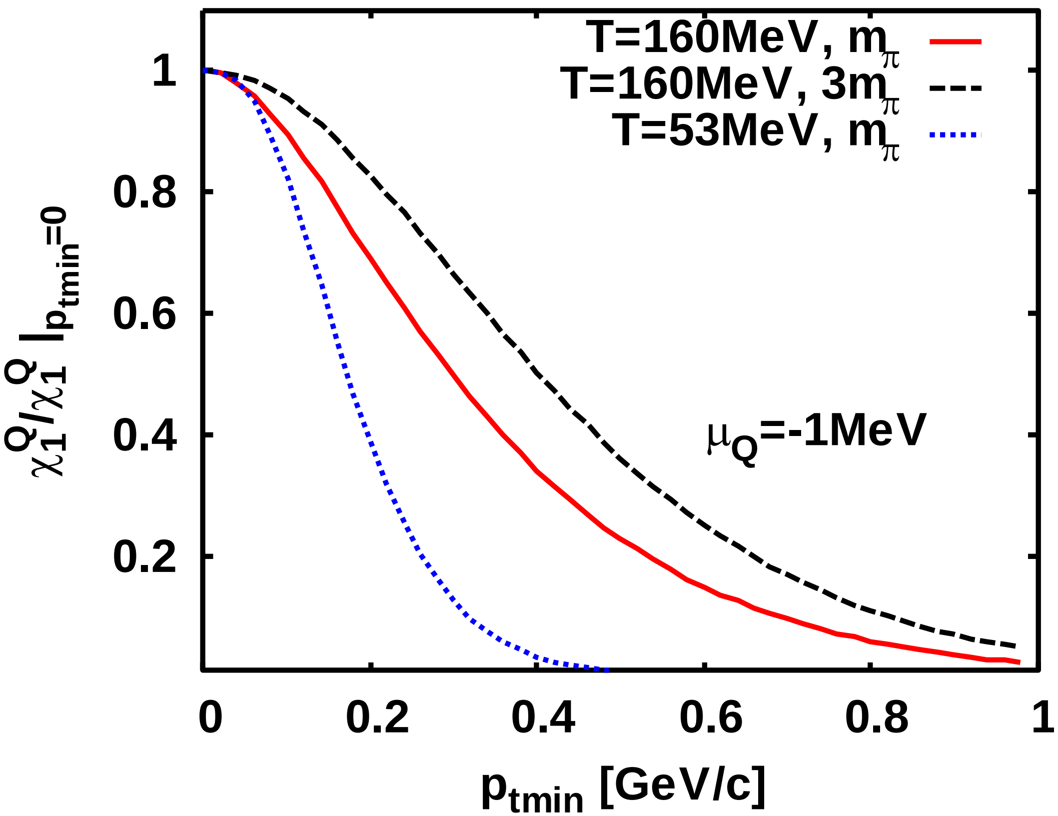
Obscured by π contribution

"π only" : $Q_{u,d}$=0

"quark only" : Q_π=0

Effect of Low p_t Cut

Electric Charge Cumulants of a free gas



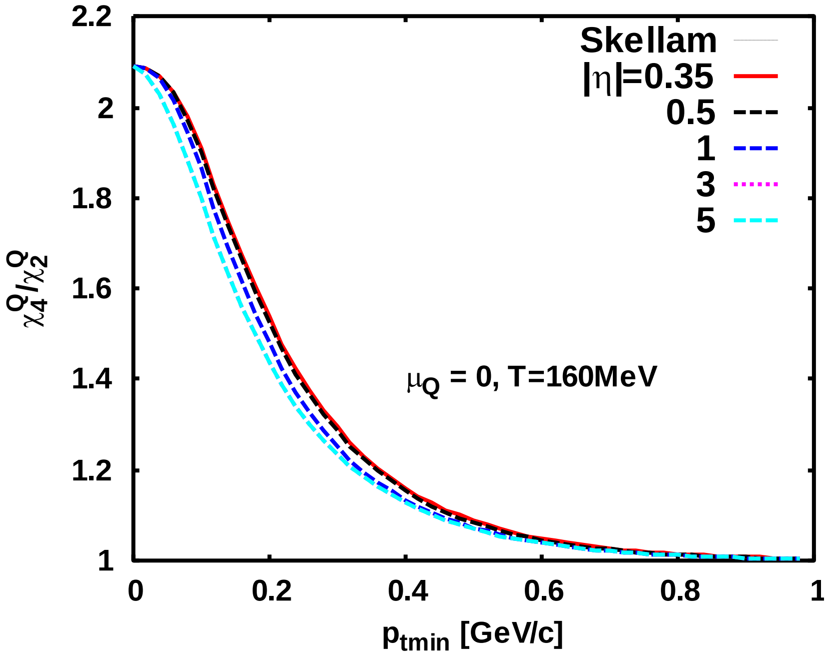
χ_1 – Density vs Mass:

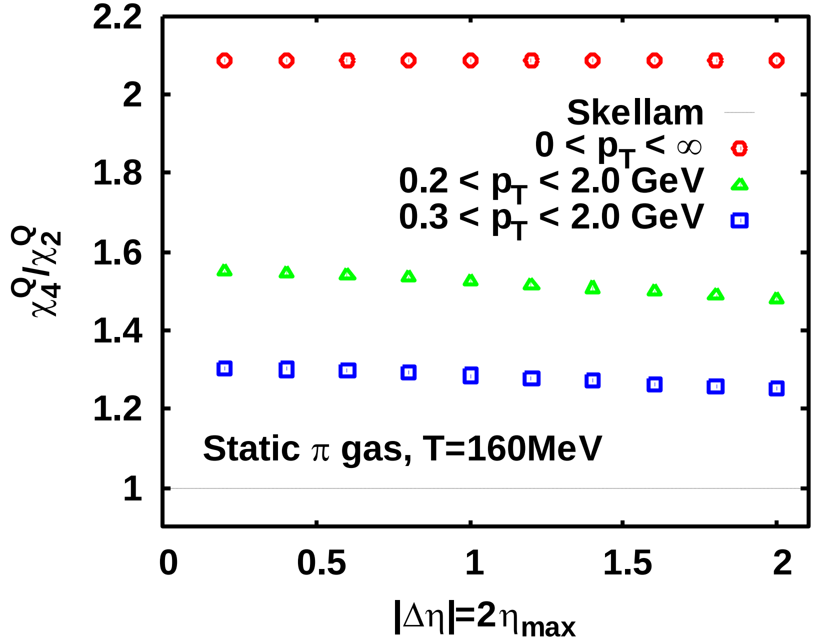
Heavier particles less affected by p_t cut

π contribution suppressed by p_t cut

At p_{tmin}=0, χ_n scale with m/T
Different p_t cut effect for same m/T!

Pseudorapidity Cut





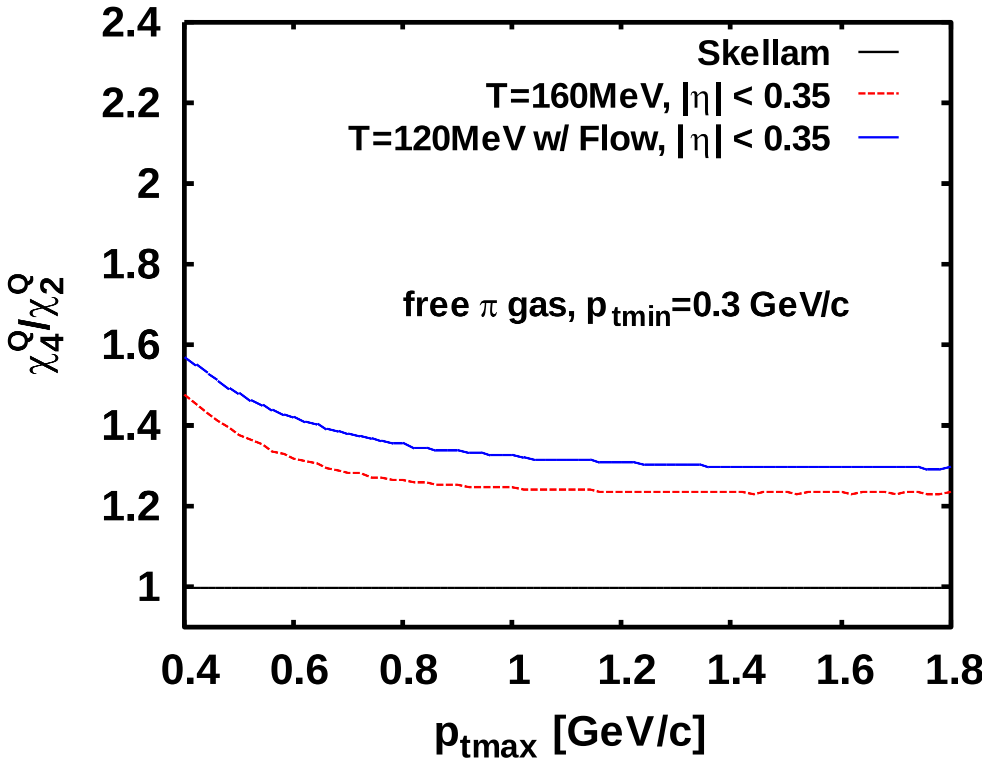
No significant dependence on η cut

Difference coming from lower p_t cut

$$E_{\min} = \sqrt{p_{t\min}^2 \cosh^2 \eta(=0) + m^2}$$

Lower cut induces effective mass heavier than m, thus approaching Skellam distribution by increasing $p_{t\min}$.
Cuts in η_{max} only affect high momentum particle contribution.

High p Cut Effects on Cumulant Ratios

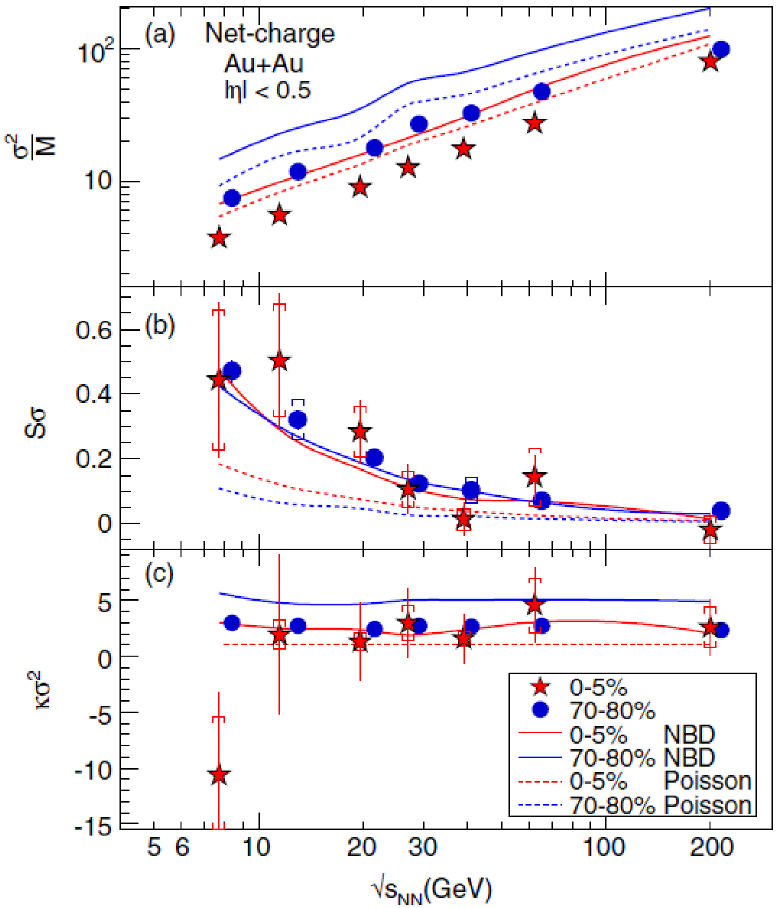


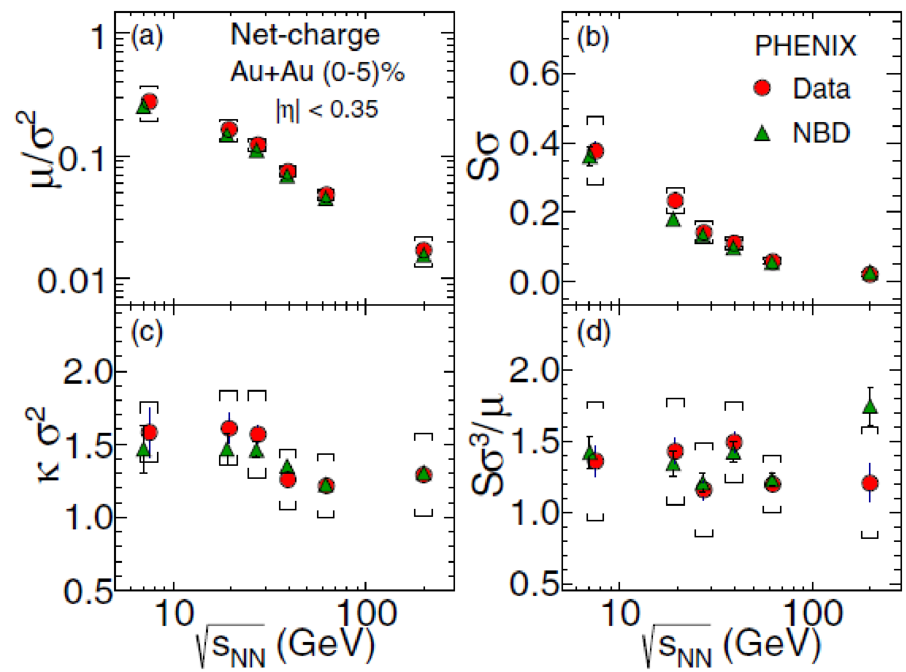
p_{tmax} < 1GeV : stronger influences from Bose statistics

Electric Charge Fluctuations

Recent measurements @RHIC

- STAR('14), PHENIX('15)



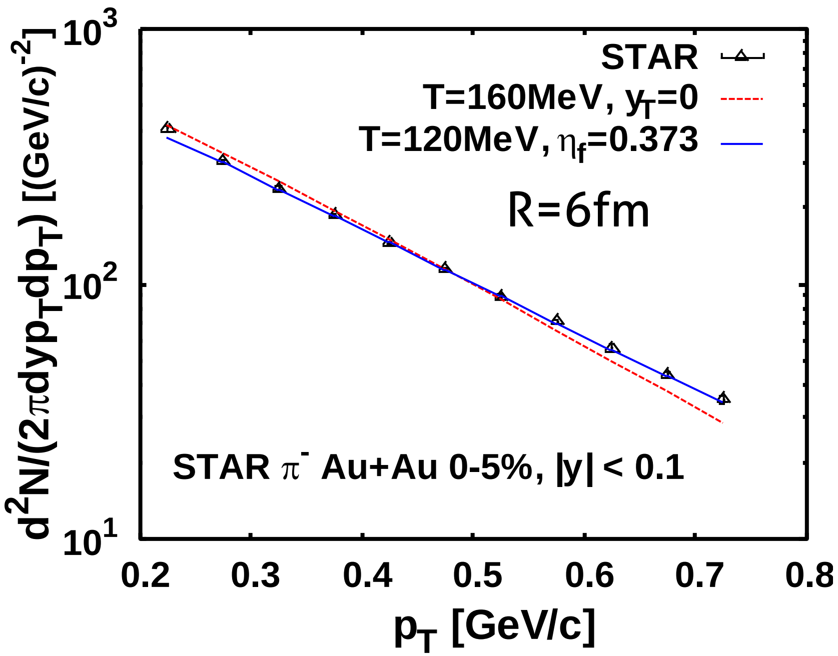


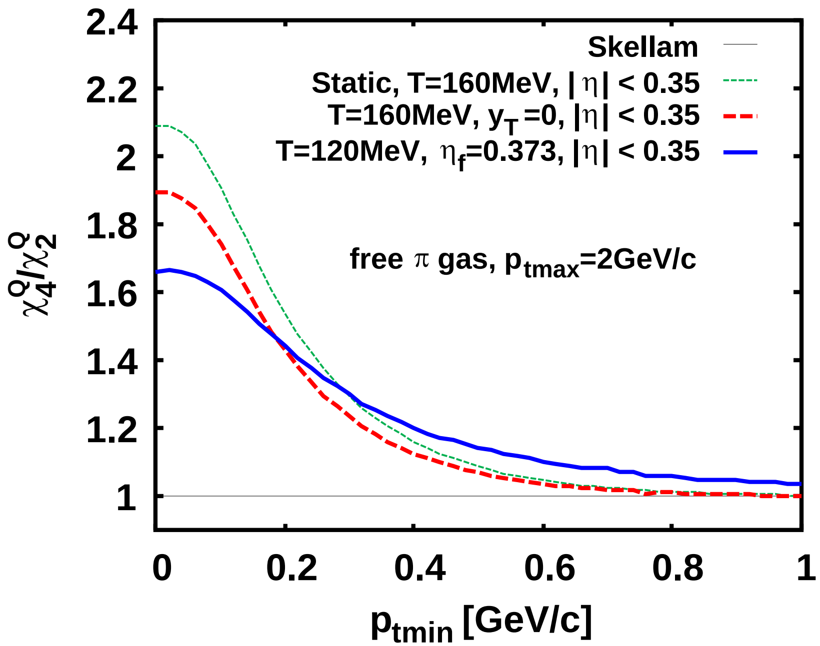
Statistics problem (particularly STAR)
PHENIX: above Skellam ?

Effects of expansion

$$\chi_n^Q \propto \frac{\partial^{n-1}N}{\partial^{n-1}\mu_Q} \qquad N = \int \frac{d^3p}{(2\pi)^3}\int d^4x \frac{m_T\cosh(y-\eta_s)}{E_p} n_B(u\cdot p, T, \mu_Q)\exp\left(-\frac{r^2}{2R^2}\right)\delta(\tau-\tau_0)$$

Boost-invariant + Transverse Gaussian + Linear flow $v_T = \tanh^{-1}\eta_f \frac{r}{R}$





Similar χ_4/χ_2 in T=120MeV w/ Flow and T=160MeV w/o flow for p_{tmin} ~ 0.2GeV

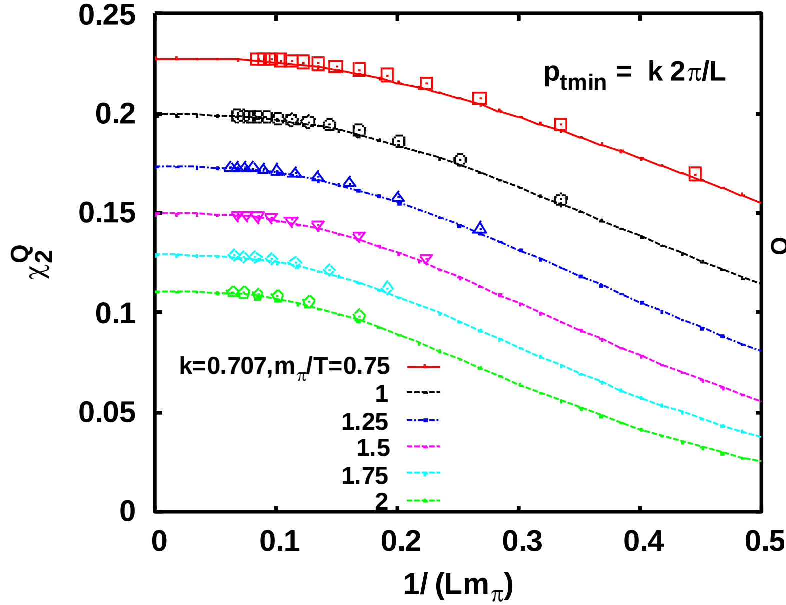
HRG results found in P. Garg et al., '13

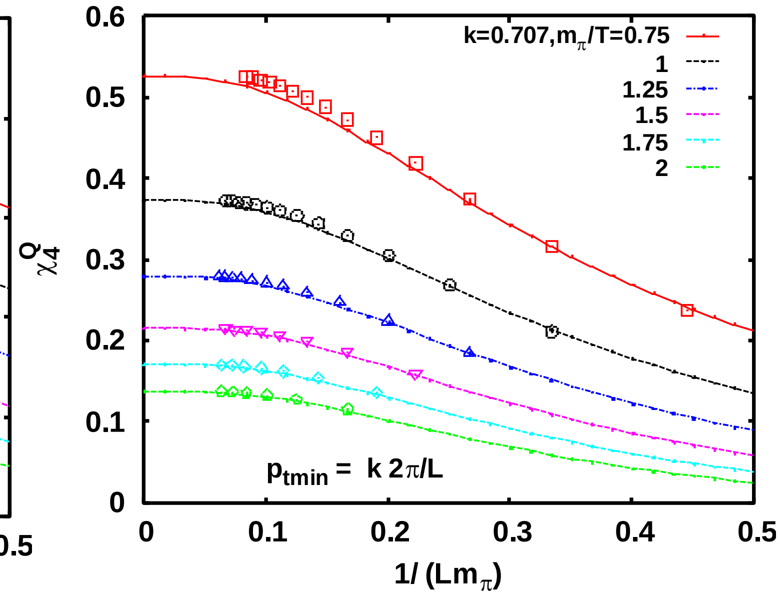
Finite Size and Low pt cut

Pion gas in a finite (L^3) box
Periodic boundary condition

?

$$p_{t\min} = k\frac{2\pi}{L}$$





|η| < 0.35

k = 0.707

[|η| < 5 : k=0.5]

$$p_{t\min} = 0.2 - 0.3, \quad T = 0.16\,\mathrm{GeV}$$

LQCD $LT \simeq 4$

$$LT \simeq 2.4 - 3.6$$

Electric Charge Fluctuations

- **Complementary to net-baryon**

$$\chi_n^Q = \frac{1}{2^n}\left[\chi_n^B + \chi_n^I + \sum_{i=1}^{n-1} {}_nC_i \frac{\partial^n(p\beta^4)}{\partial(\beta\mu_I)^i \partial(\beta\mu_B)^{n-1}}\right]$$

Leading singularity from chiral transition

- **No "proton≠baryon" problem**
- **Larger multiplicity (as π dominates) than net-baryon**
  - Less efficiency, however.