Directed flow in heavy-ion collisions and softening of equation of state

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in collaboration with

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CPOD 2016 (Critical Point and Onset of Deconfinement 2016)
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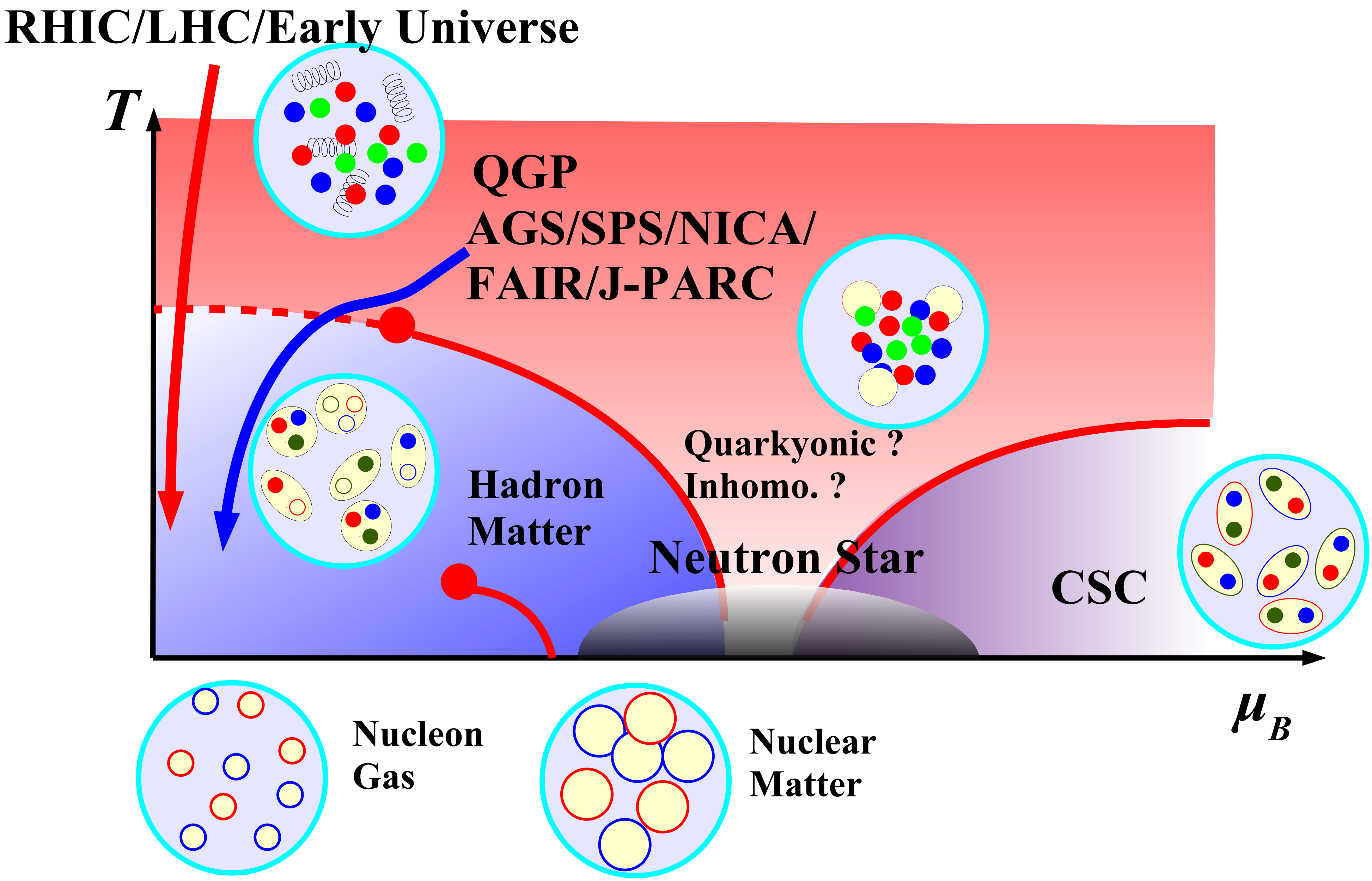
Y. Nara, A. Ohnishi, arXiv:1512.06299 [nucl-th] (QM2015 proc.)
Y. Nara, A. Ohnishi, H. Stoecker, arXiv:1601.07692 [hep-ph]

QCD Phase Diagram

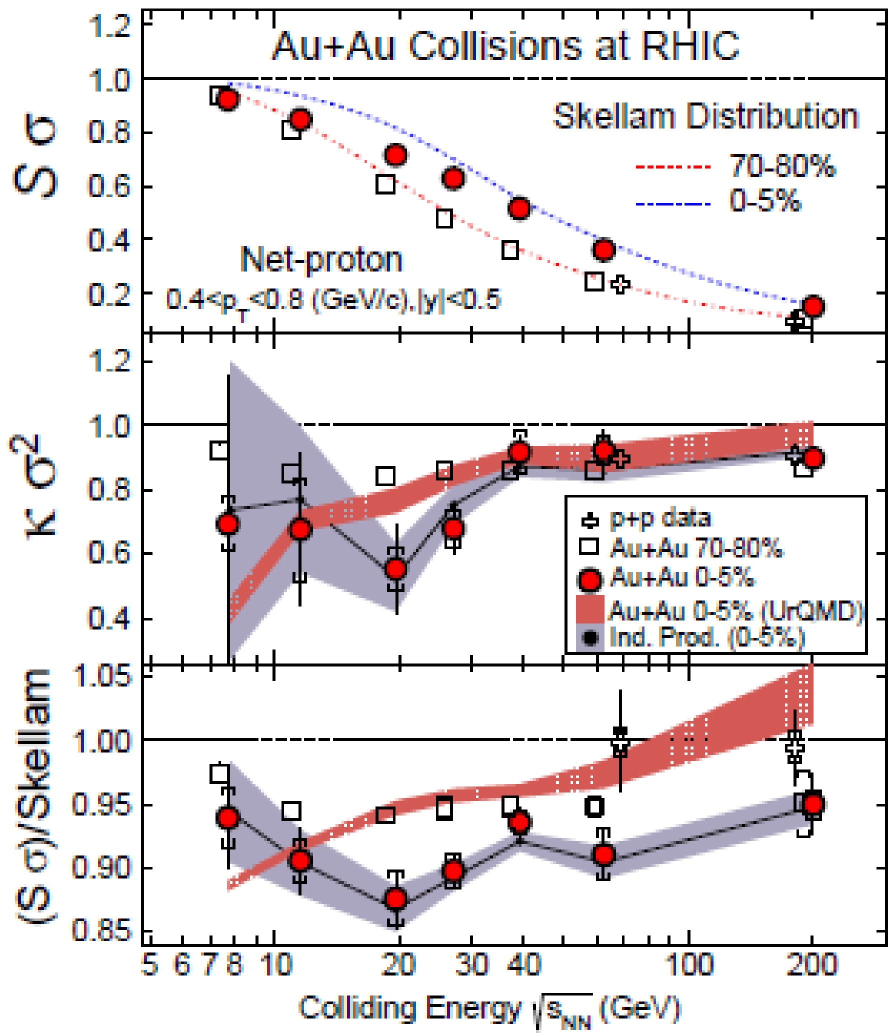


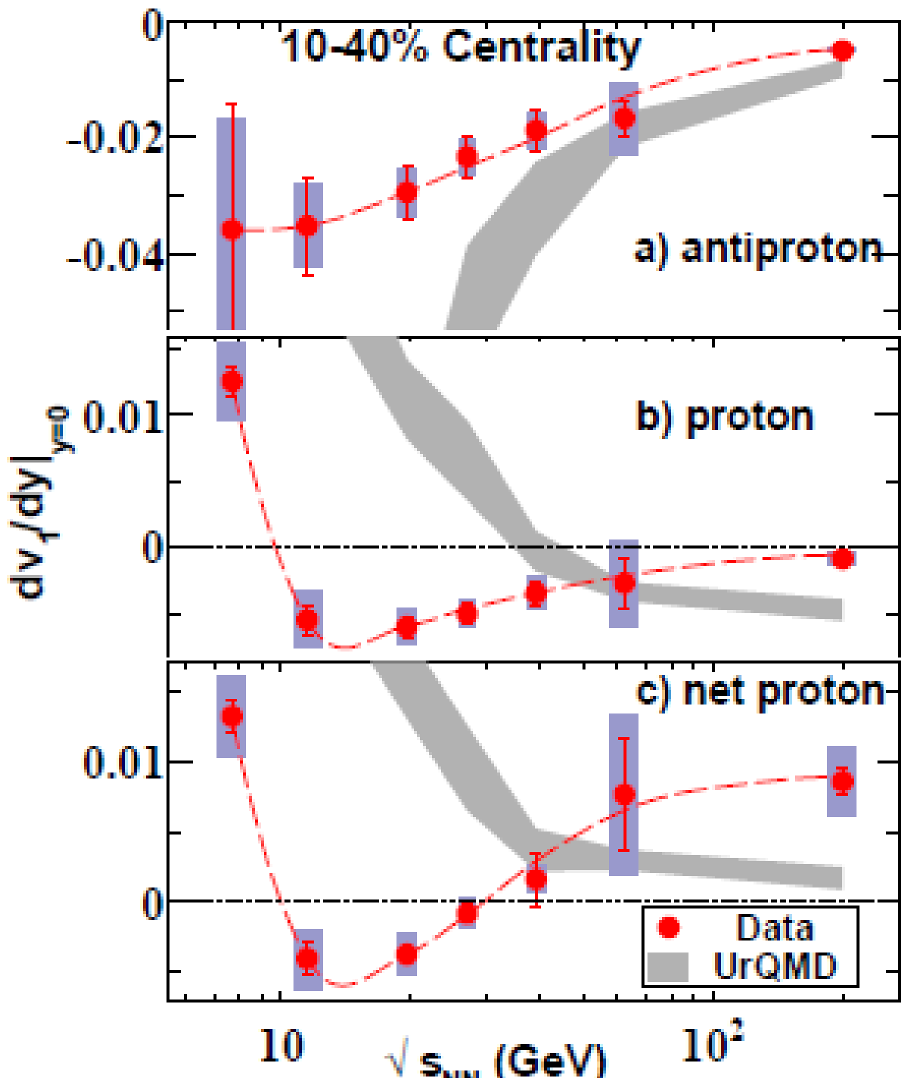
Signals of QGP formation & QCD phase transition

- Signals of QGP formation at top RHIC & LHC energies
  - Jet quenching in AA collisions (not in dA)
  - Large elliptic flow (success of hydrodynamics)
  - Quark number scaling (coalescence of quarks)
- Next challenges
  - Puzzles: Early thermalization, Photon v2, Small QGP, …
    → Complete understanding from initial to final states
  - Discovery of QCD phase transition
- Signals of QCD phase transition at BES energies ?
  - Critical Point → Large fluctuation of conserved charges
  - First-order phase transition → Softening of EOS

→ Non-monotonic behavior of
proton number moment ($\kappa\sigma^2$) and collective flow (dv_1/dy)

Net-Proton Number Cumulants & Directed Flow



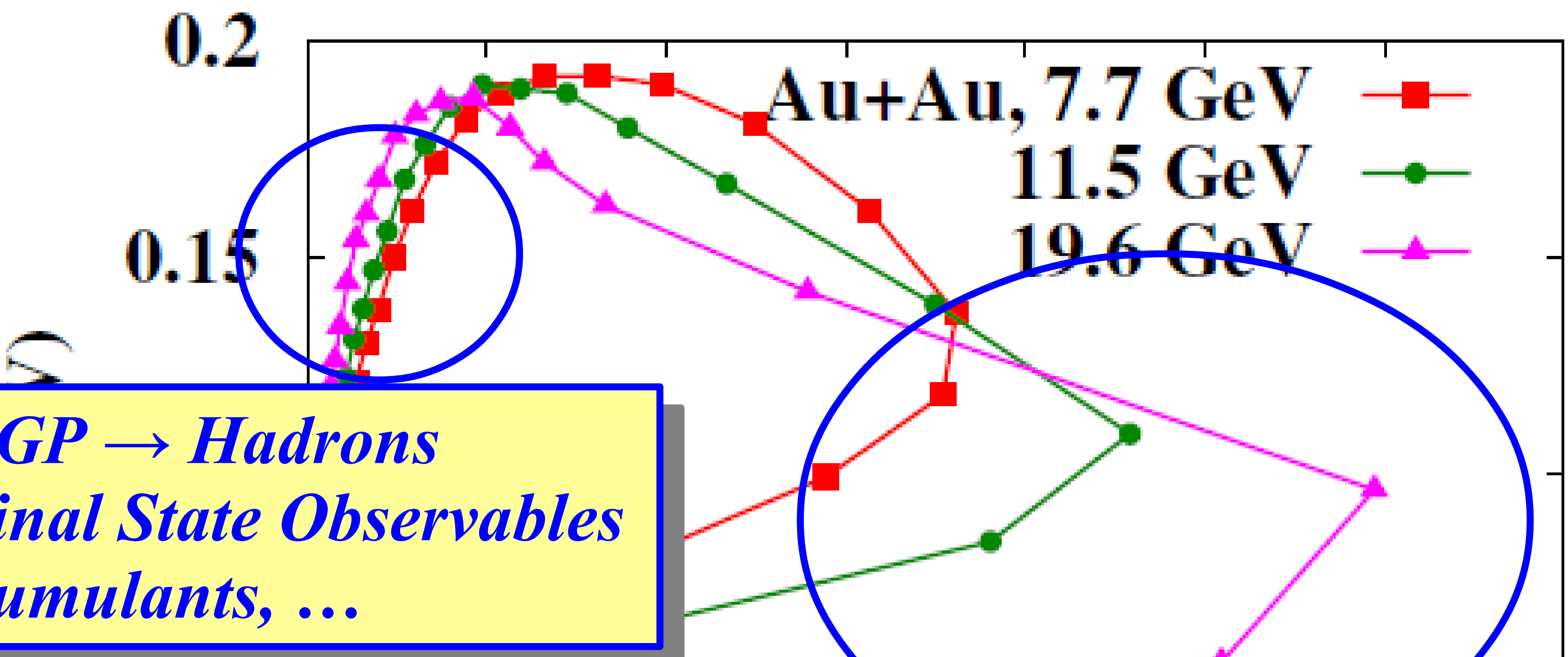


STAR Collab. PRL 112('14)032302

STAR Collab., PRL 112('14)162301.

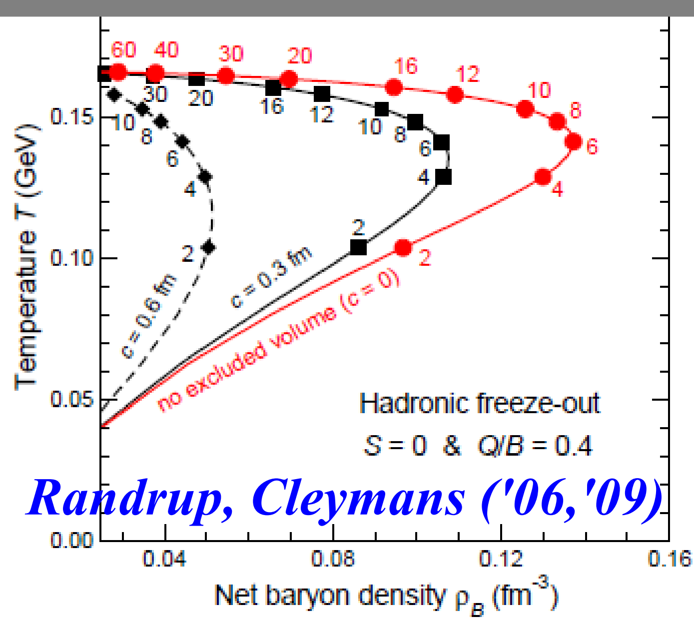
CPOD 2016: Nu Xu, Morita, Friman, Schaefer, Koch, Kitazawa, ...

Two ways to probe QCD phase transition



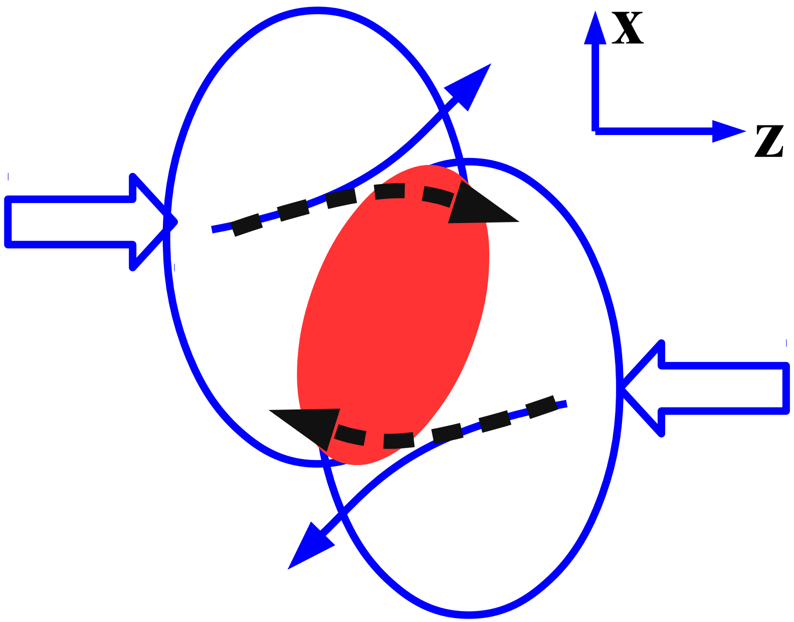
QGP → Hadrons
Final State Observables
Cumulants, …

Hadrons → QGP
Early Stage Observables
Caution: (Partial) Equilibration is necessary !

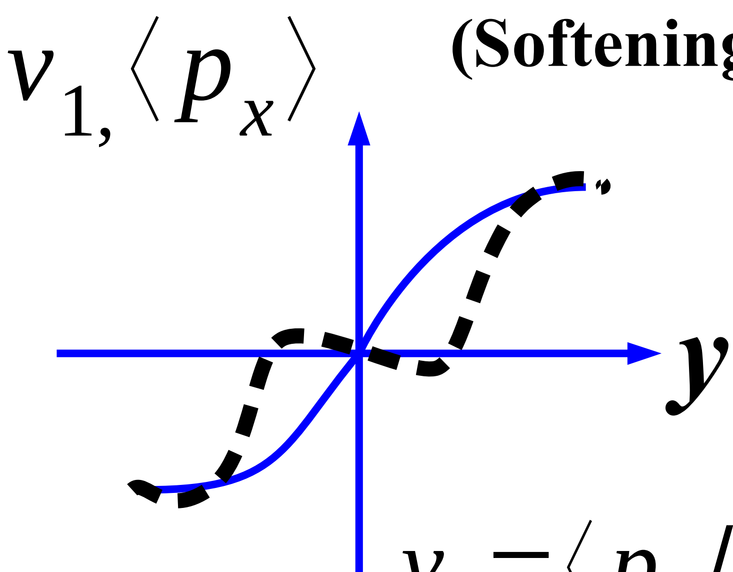


Randrup, Cleymans ('06,'09)

What is directed flow ?



Attraction (Softening)



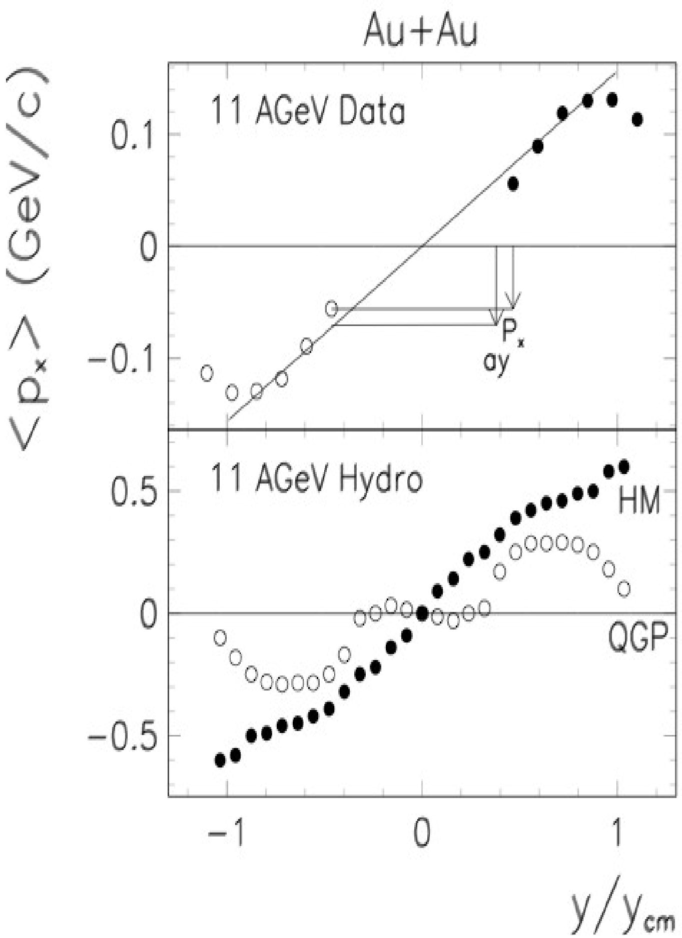
$$v_1 = \langle p_x / p \rangle = \langle \cos\varphi \rangle$$

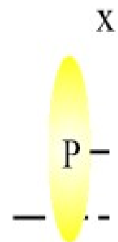
- **v_1 or $\langle p_x \rangle$ as a function of y is called directed flow.**
- **Created in the overlapping stage of two nuclei**
  → Sensitive to the EOS in the early stage.
- **Becomes smaller at higher energies.**

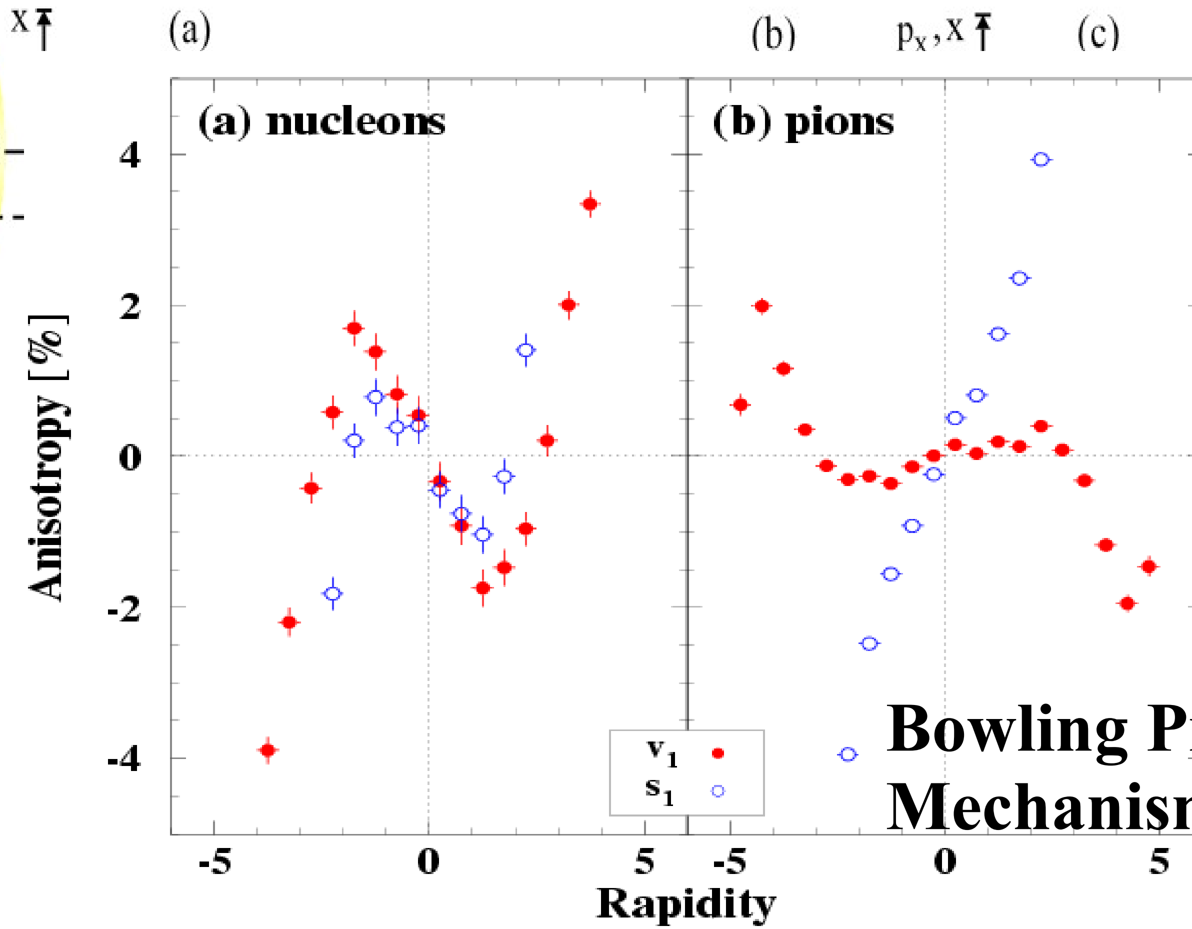
> ***How can we explain non-monotonic dependence of dv_1/dy ?***
> ***→ Softening or Geometry***

Does the "Wiggle" signal the QGP ?

- **Hydro predicts wiggle with QGP EOS.**
- **Baryon stopping + Positive space-momentum correlation leads wiggle (w/o QGP)**







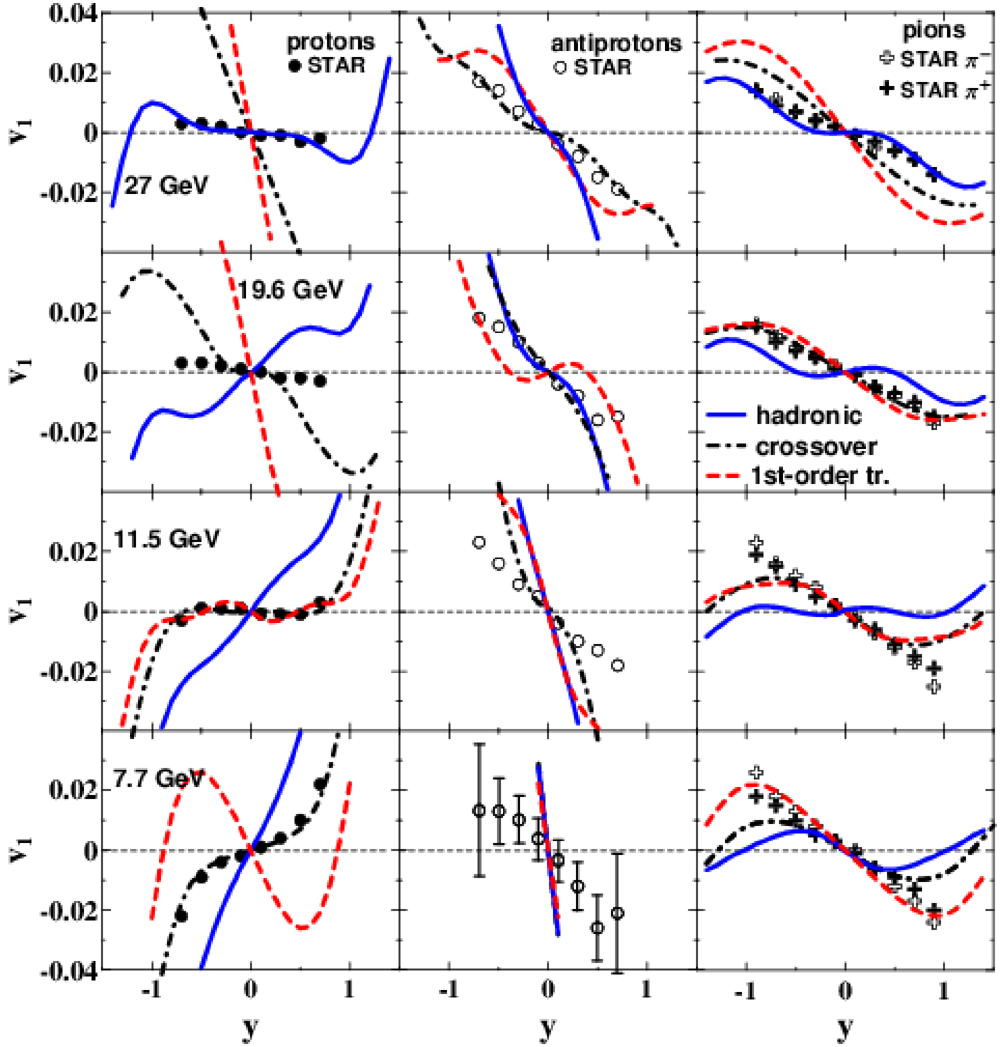
Bowling Pin Mechanism

L. P. Csernai, D. Röhrich, PLB 45 (1999), 454.

R.Snellings, H.Sorge, S.Voloshin, F.Wang, N. Xu, PRL (84) 2803(2000)

Negative dv_1/dy around $\sqrt{s_{NN}}$~ 10 GeV

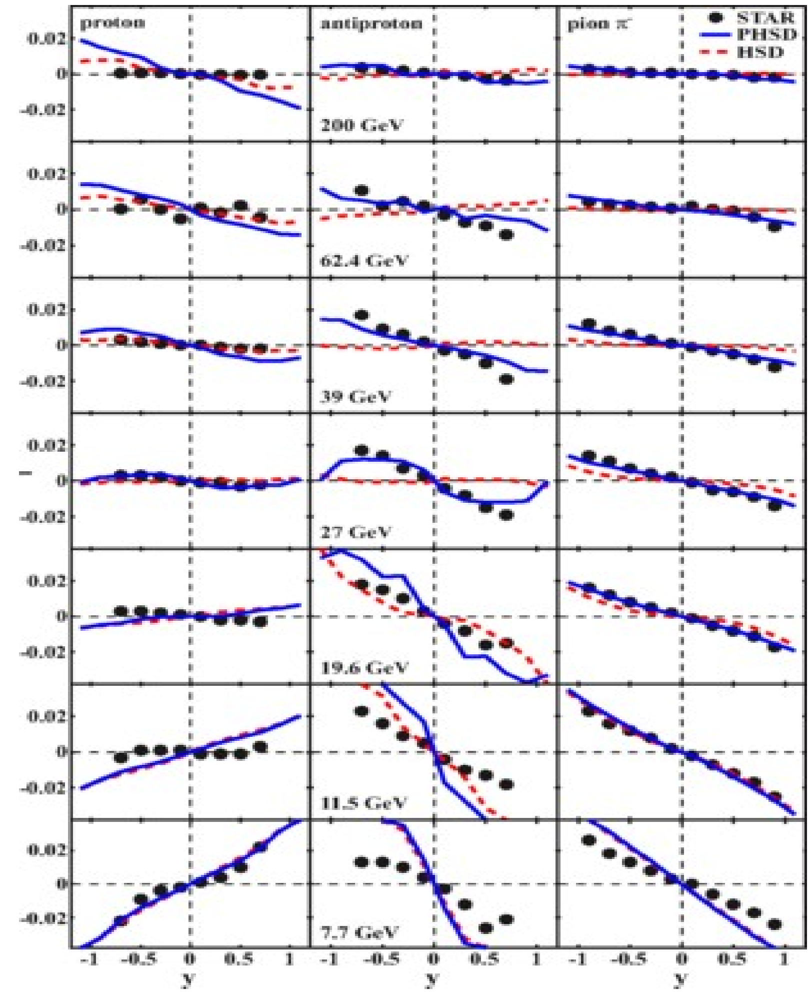
- **Yes in Hydrodynamics**



Black: Crossover, Red: 1st

Y. B. Ivanov and A. A. Soldatov, PRC91 (2015)024915

- **No at around $\sqrt{s_{NN}}$~10 GeV in transport models.**

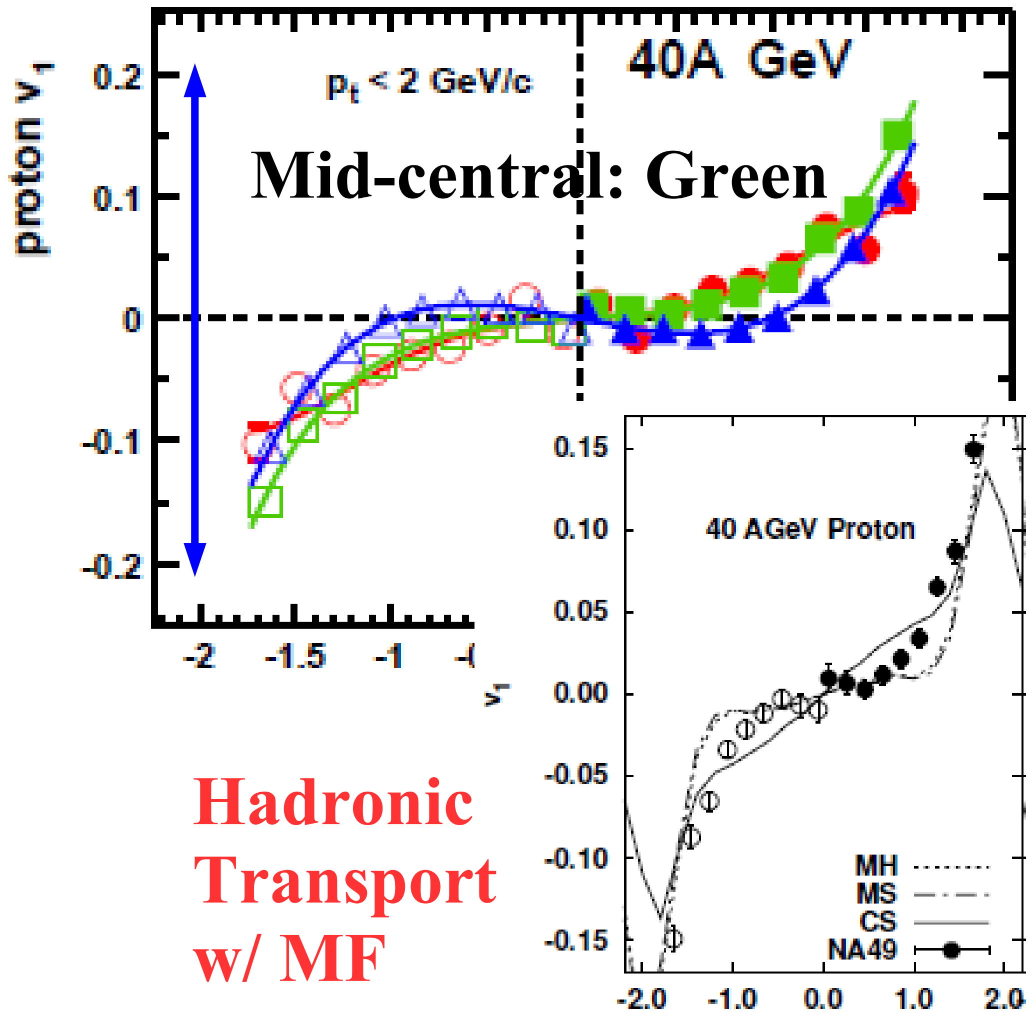


V. P. Konchakovski, W. Cassing, Y. B. Ivanov, V. D. Toneev, PRC90('14)014903

SPS(NA49) vs RHIC(STAR)

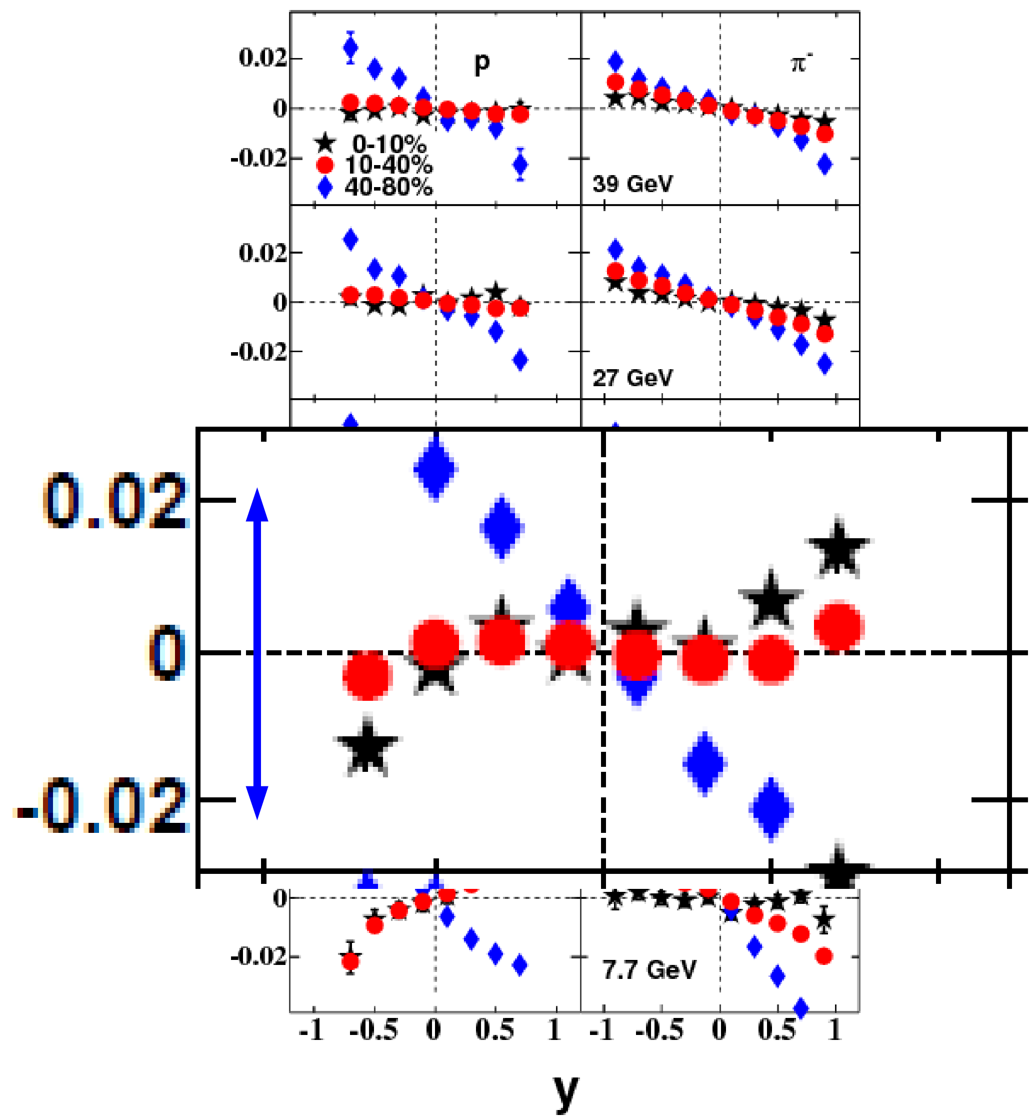
- SPS (NA49), $\sqrt{s_{NN}}$ = 8.9 GeV
- RHIC(STAR), 7.7-39 GeV

C. Alt et al. (NA49), PRC68 ('03) 034903



Hadronic Transport w/ MF

M.Isse,AO,N.Otuka,P.K.Sahu,Y.Nara, PRC72 ('05)064908



L. Adamczyk et al. (STAR), PRL 112(2014)162301

Does Directed Flow Collapse Signal Phase Tr. ?

- **Negative dv_1/dy at high-energy ($\sqrt{s_{NN}} > 20$ GeV)**
  - **Geometric origin (bowling pin mechanism), not related to FOPT**
    R.Snellings, H.Sorge, S.Voloshin, F.Wang, N. Xu, PRL84,2803('00)
- **Negative dv_1/dy at $\sqrt{s_{NN}} \sim 10$ GeV**
  - **Yes, in three-fluid simulations. → Thermalization ?**
    Y. B. Ivanov and A. A. Soldatov, PRC91('15)024915
  - **No, in transport models incl. hybrid.**
    E.g. J. Steinheimer, J. Auvinen, H. Petersen, M. Bleicher, H. Stoecker, PRC89('14)054913.
    Exception: *B.A.Li, C.M.Ko ('98) with FOPT EOS*

We investigate the directed flow at BES energies
in hadronic transport model
with / without mean field effects
with / without softening effects via attractive orbit.

Contents

- Introduction
  - Two ways to probe QCD phase transition
  - Collapse of Directed Flow at $\sqrt{s_{NN}} \sim 10$ GeV
- Hadronic Transport Model Approaches
  - Boltzmann equation with potential effects
  - Jet AA Microscopic transport model (JAM)
- Additional Softening Effects
  - Attractive Orbit Scattering
  - Transition Density and Pressure (conjecture)
- Summary

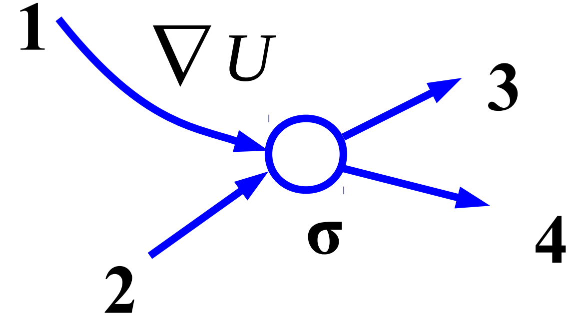
Hadronic Transport Approaches
Cascade / Cascade + Mean Field

Microscopic Transport Models

- **UrQMD 3.4 Frankfurt public**
  resonance model N*,D*, string pQCD, PYTHIA6.4
- **PHSD Giessen (Cassing) upon request**
  D(1232),N(1440),N(1530), string, pQCD, FRITIOF7.02
- **GiBUU 1.6 Giessen (Mosel) public**
  resonance model N*,D*, string, pQCD,PYTHIA6.4
- **AMPT public**
  HIJING+ZPC+ART
- **JAM (Y. Nara) public**
  resonance model N*,D*, string, pQCD, PYTHIA6.1

Transport Model

- **Boltzmann equation with (optional) potential effects**
  E.g. Bertsch, Das Gupta, Phys. Rept. 160(88), 190



$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f - \nabla U\cdot\nabla_p f = I_{\rm coll}$$

$$I_{\rm coll}(\mathbf{r},\mathbf{p}) = -\frac{1}{2}\int \frac{d\mathbf{p}_2}{(2\pi)^3} d\Omega\ v_{12}\frac{d\sigma}{d\Omega}\left[f f_2(1-f_3)(1-f_4)) - (12\leftrightarrow 34)\right]$$

(NN elastic scattering case)

- **Hadron-string transport model JAM**
  - **Collision term → Hadronic cascade with resonance and string excitation**
    Nara, Otuka, AO, Niita, Chiba, Phys. Rev. C61 (2000), 024901.
  - **Potential term → Mean field effects in the framework of RQMD/S**
    Sorge, Stocker, Greiner, Ann. of Phys. 192 (1989), 266.
    Tomoyuki Maruyama et al., Prog. Theor. Phys. 96(1996), 263.
    Isse, AO, Otuka, Sahu, Nara, Phys.Rev. C 72 (2005), 064908.

Relativistic QMD/Simplified (RQMD/S)

- **RQMD is developed based on constraint Hamiltonian dynamics**
  H. Sorge, H. Stoecker, W. Greiner, Ann. Phys. 192 (1989), 266.
  - **8N dof → 2N constraints → 6N (phase space)**
  - **Constraints = on-mass-shell constraints + time fixation**
- **RQMD/S uses simplified time-fixation**
  Tomoyuki Maruyama, et al. Prog. Theor. Phys. 96(1996),263.
  - **Single particle energy (on-mass-shell constraint)**

$$p_i^0 = \sqrt{\boldsymbol{p}_i^2 + m_i^2 + 2 m_i V_i}$$

  - **EOM after solving constraints**

$$\dot{\boldsymbol{r}}_i = \frac{\boldsymbol{p}_i}{p_i^0} + \sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \boldsymbol{p}_i} \qquad \dot{\boldsymbol{p}}_i = -\sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \boldsymbol{r}_i}$$

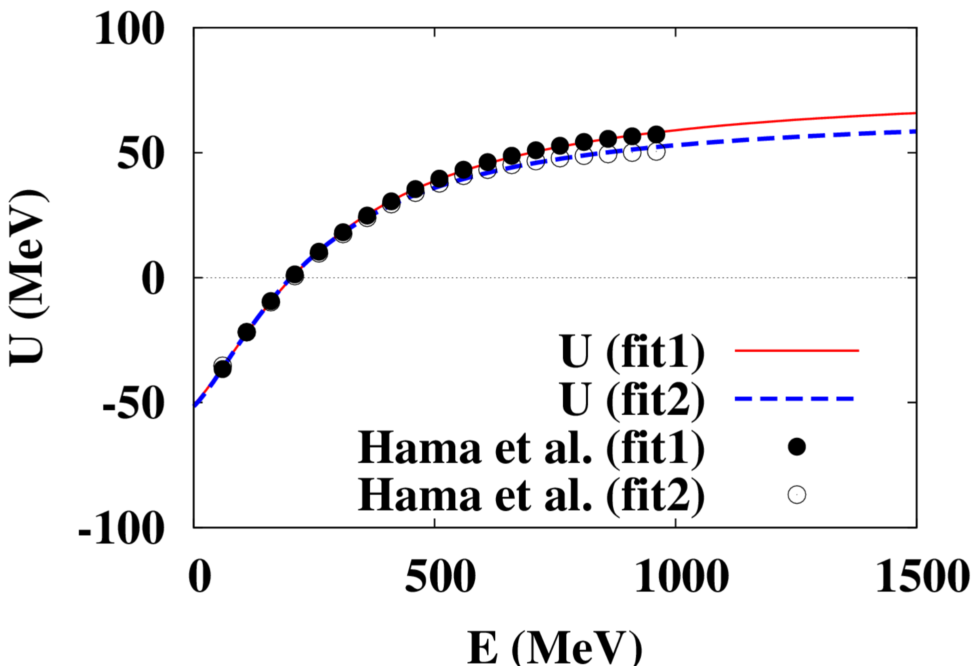
  - **Relative distances $(r_i - r_j)^2$ are replaced with those in the two-body c.m.**
    → Potential becomes Lorentz scalar

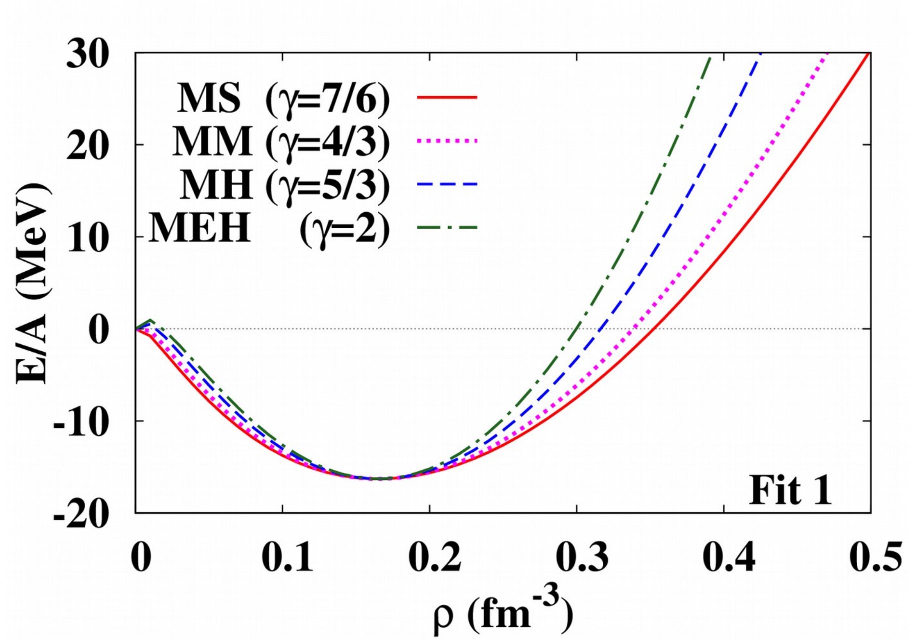
Mean Field Potential

- **Skyrme type density dependent + momentum dependent potential**

$$V = \sum_i V_i = \int d^3r \left[\frac{\alpha}{2} \left(\frac{\rho}{\rho_0} \right)^2 + \frac{\beta}{\gamma+1} \left(\frac{\rho}{\rho_0} \right)^{\gamma+1} \right] + \sum_k \int d^3\boldsymbol{r} d^3\boldsymbol{p} d^3\boldsymbol{p}' \frac{C_{ex}^{(k)}}{2\rho_0} \frac{f(\boldsymbol{r},\boldsymbol{p}) f(\boldsymbol{r},\boldsymbol{p}')}{1+(\boldsymbol{p}-\boldsymbol{p}')^2/\mu_k^2}$$

Type	α (MeV)	β (MeV)	γ	$C_{\rm ex}^{(1)}$ (MeV)	$C_{\rm ex}^{(2)}$ (MeV)	μ_1 (fm^{-1})	μ_2 (fm^{-1})	K (MeV)
MH1	-12.25	87.40	5/3	-383.14	337.41	2.02	1.0	371.92
MS1	-208.89	284.04	7/6	-383.14	337.41	2.02	1.0	272.6



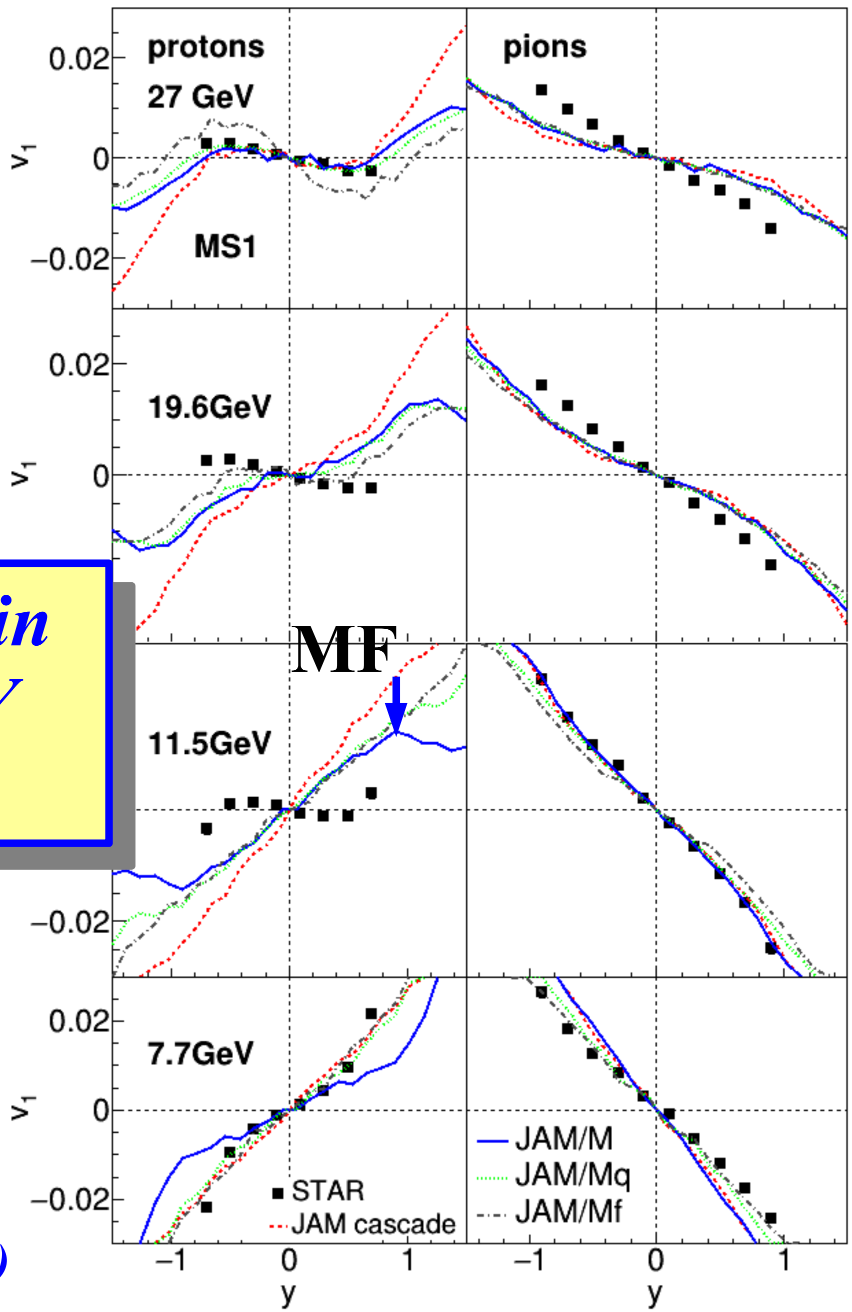


Y. Nara, AO, arXiv:1512.06299 [nucl-th] (QM2015 proc.)
Isse, AO, Otuka, Sahu, Nara, PRC 72 (2005), 064908.

Comparison with RHIC data on v_1

- **Pot. Eff. on the v_1 is significant, but dv1/dy becomes negative only at $\sqrt{s_{NN}}$ > 20 GeV.**

Hadronic approach does not explain directed flow collapse at 10-20 GeV even with potential effects.



JAM/M: only formed baryons feel potential forces
JAM/Mq: pre-formed hadron feel potential with factor 2/3 for diquark, and 1/3 for quark
JAM/Mf: both formed and pre-formed hadrons feel potential forces.

Y. Nara, AO, arXiv:1512.06299 [nucl-th] (QM2015 proc.)

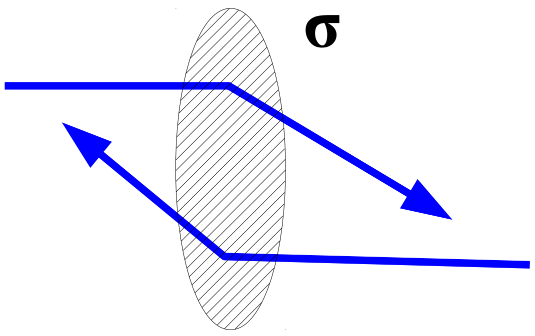
Additional Softening Effects

Softening Effects via Attractive Orbit Scattering

- **Attractive orbit scattering simulates softening of EOS**
  P. Danielewicz, S. Pratt, PRC 53, 249 (1996)
  H. Sorge, PRL 82, 2048 (1999).

$$P = P_f + \frac{1}{3TV} \sum_{(i,j)} (q_i \cdot r_i + q_j \cdot r_j)$$

(Virial theorem)



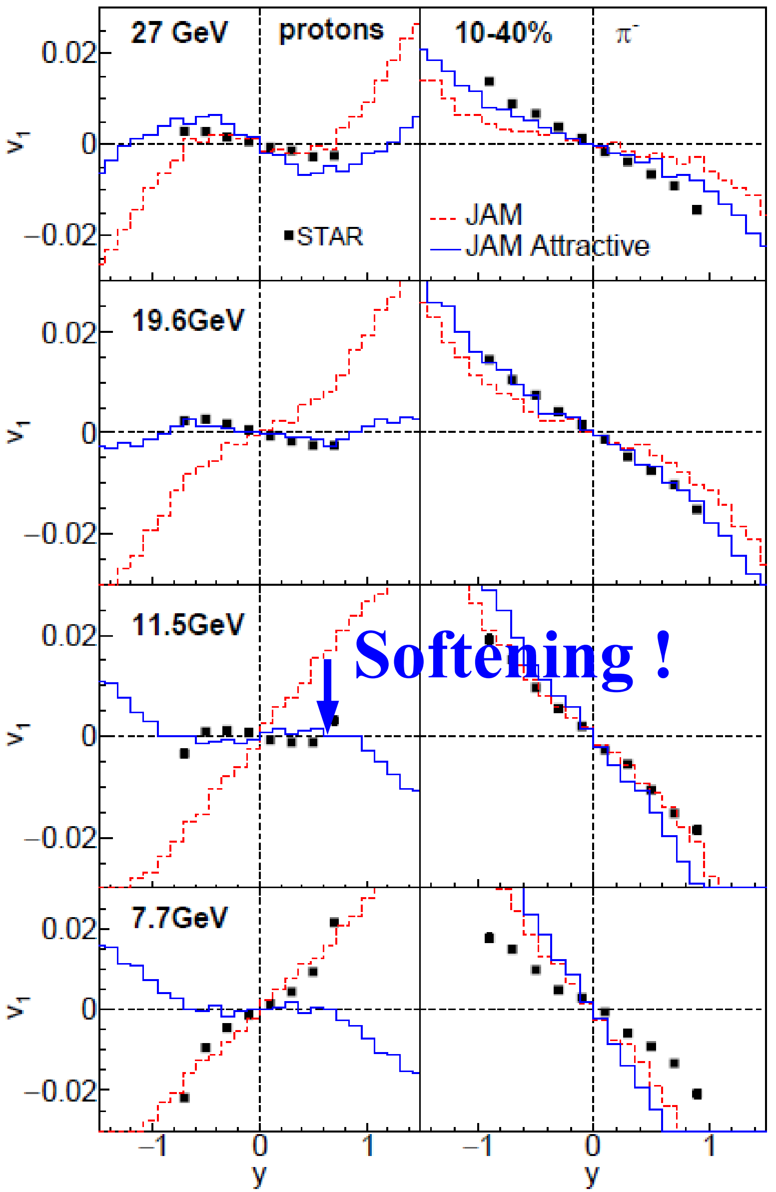
  - **Attractive orbit → particle trajectory are bended in denser region**

Let us examine the EOS softening effects,
which cannot be explained in hadronic mean field potential,
by using attractive orbit scatterings !

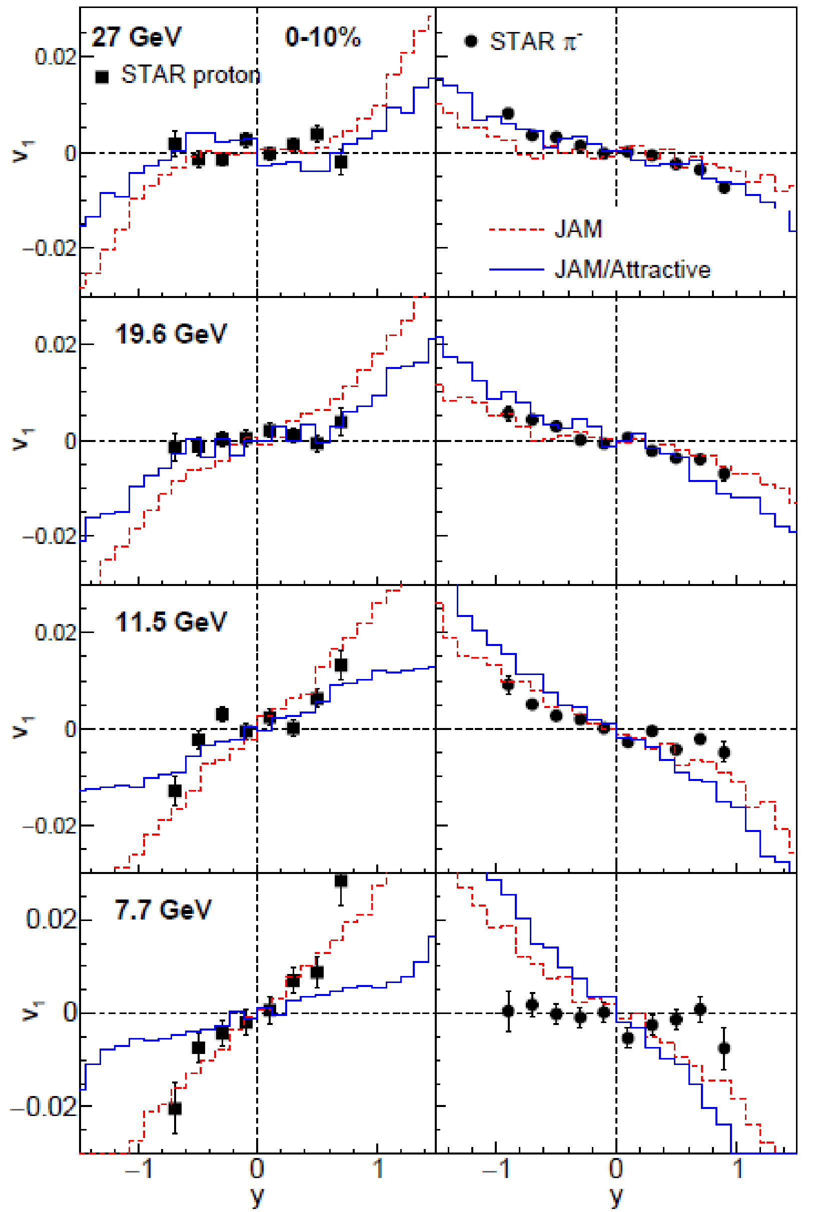
Y. Nara, AO, H. Stöcker, arXiv:1601.07692 [hep-ph]

Directed Flow with Attractive Orbits

Nara, AO, Stöcker ('16)



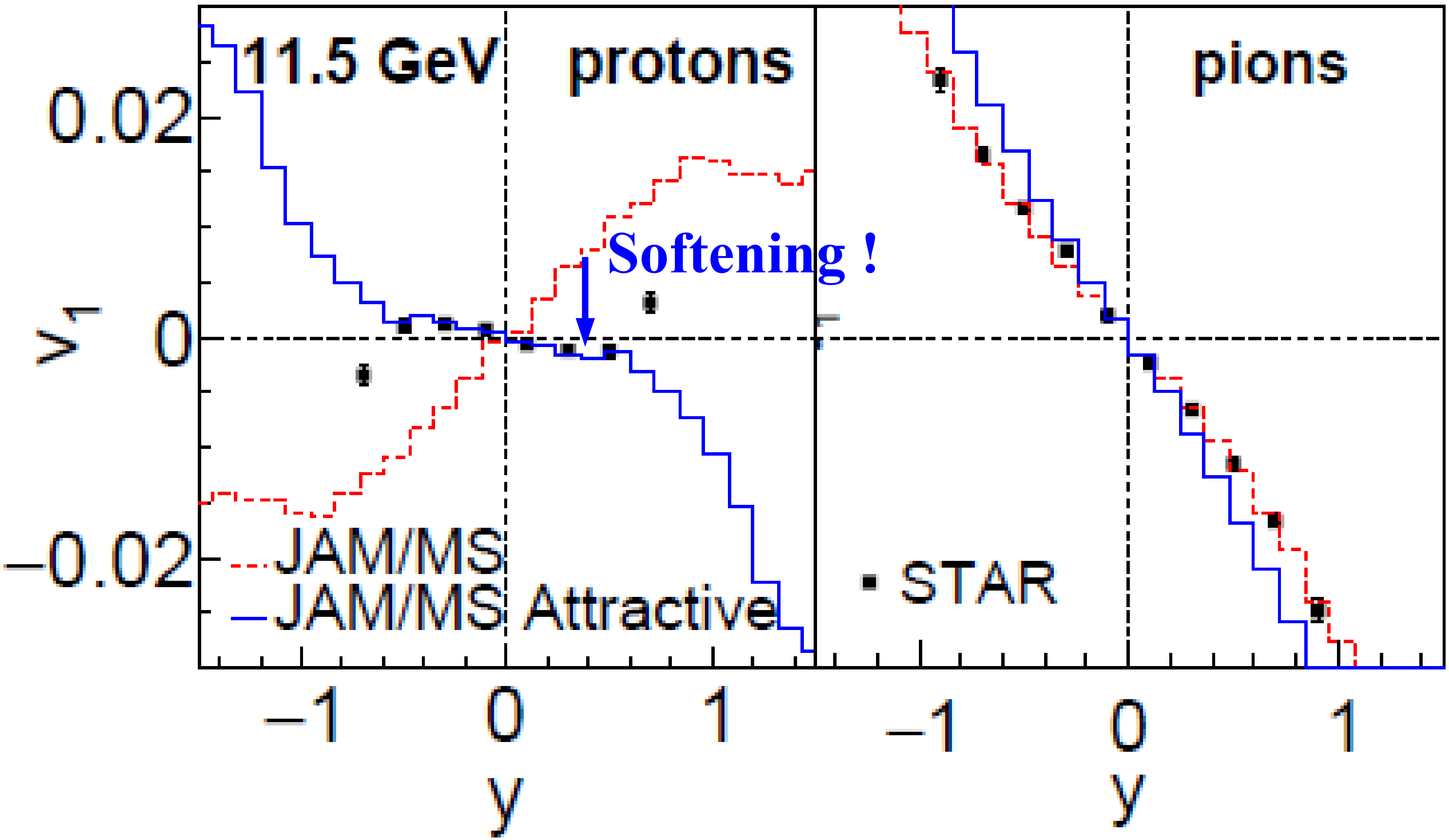
mid-central (10-40 %)



central (0-10 %)

Mean Field + Attractive Orbit

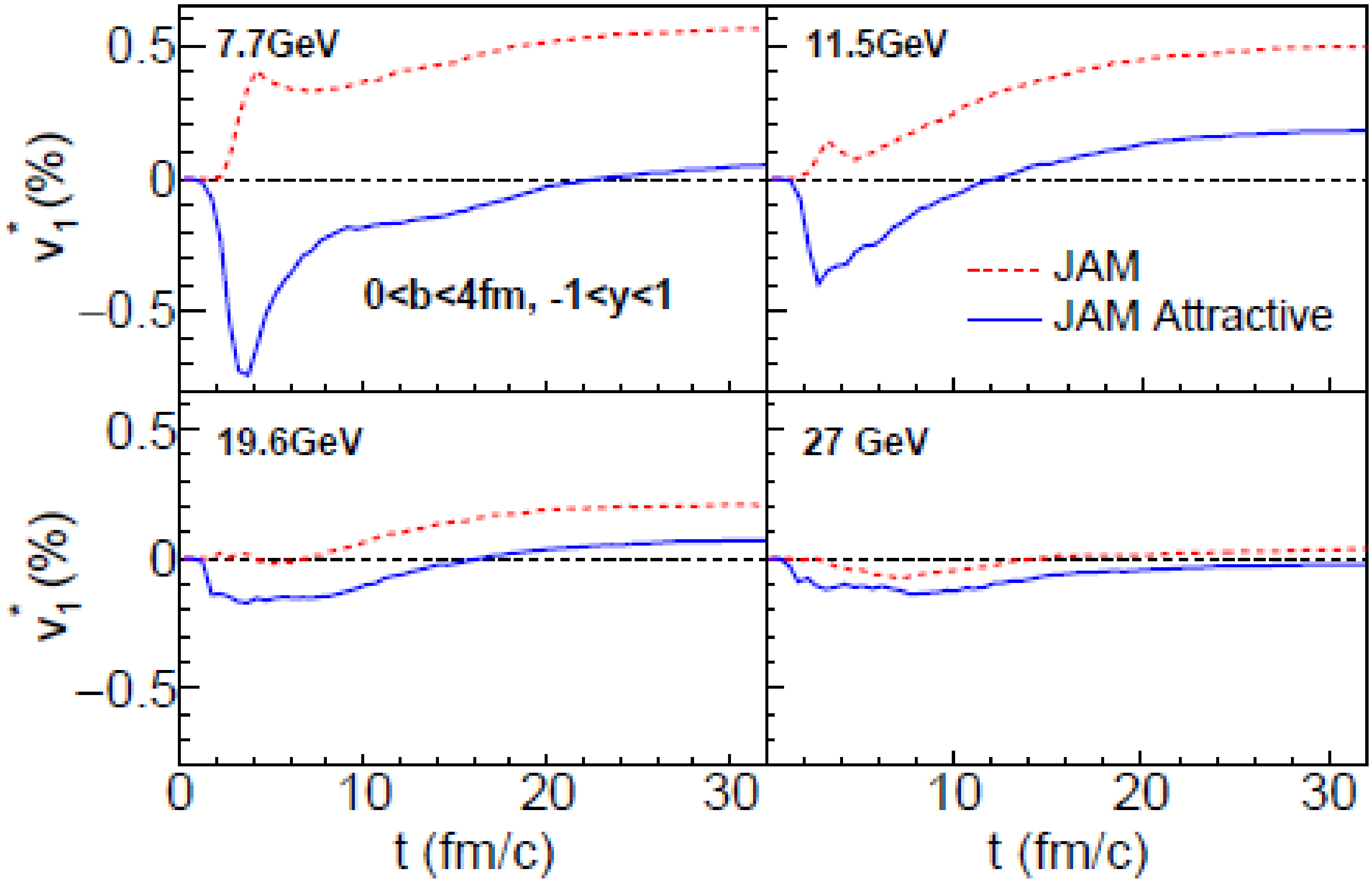
Nara, AO, Stöcker ('16)



MF+Attractive Orbit make dv_1/dy negative at $\sqrt{s_{NN}}$ ~ 10 GeV

When is negative v_1 slope generated ?

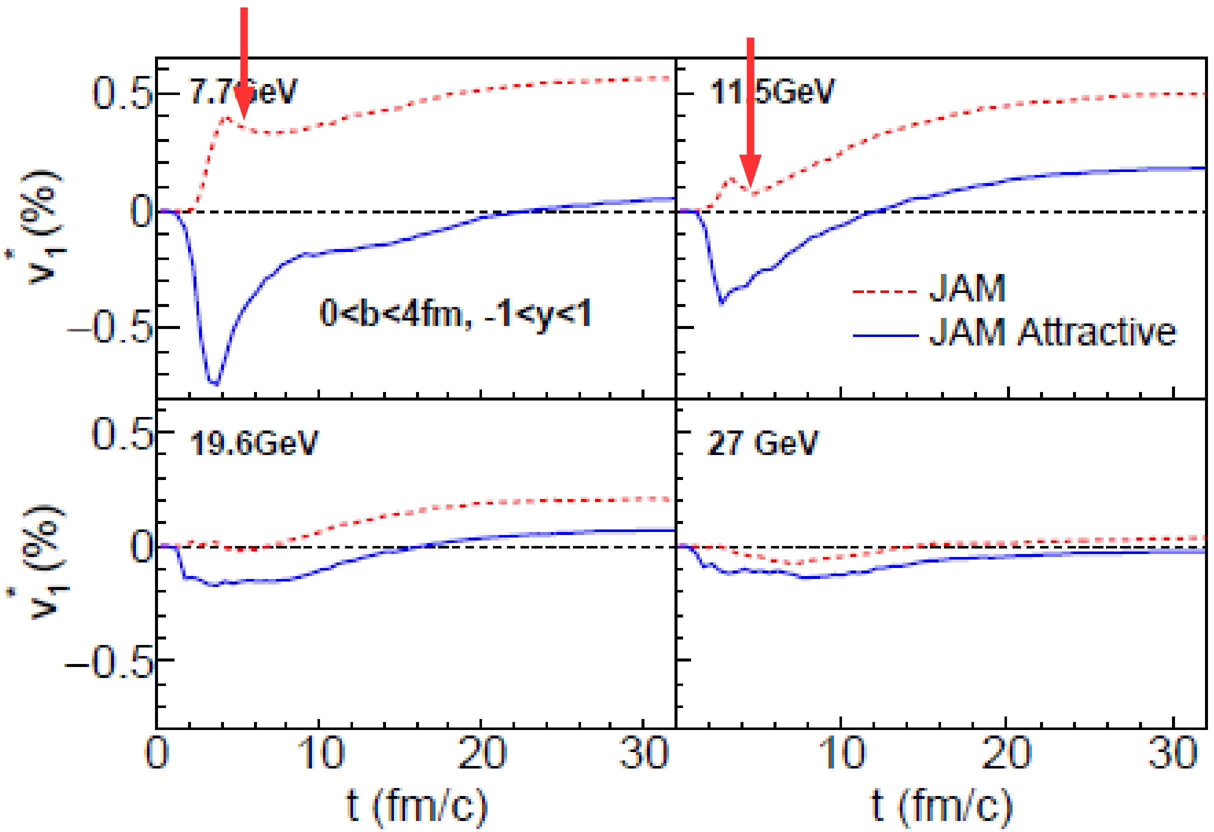
Nara, AO, Stöcker ('16)

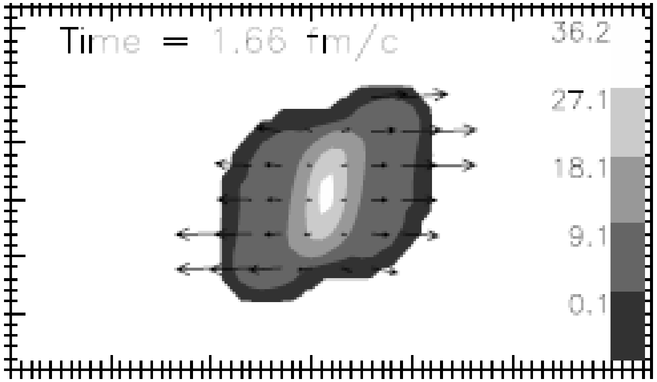


We need to make v_1 slope negative in the compressing stage.

Tilted Ellipsoid ?

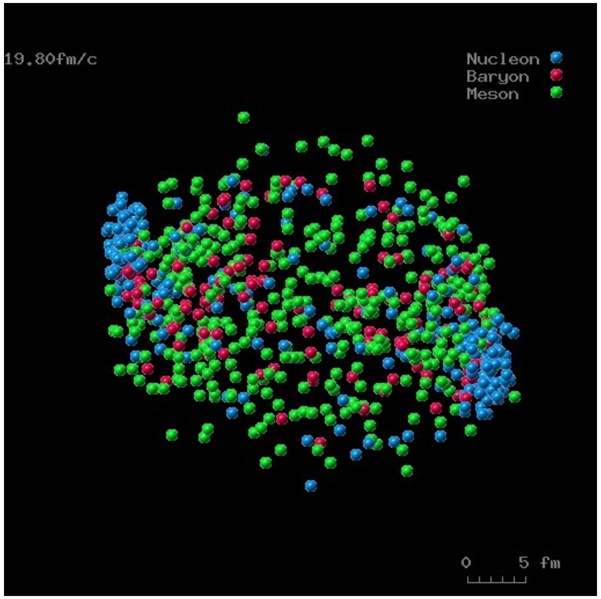
Nara, AO, Stöcker ('16)





18 GeV, 3-fluid

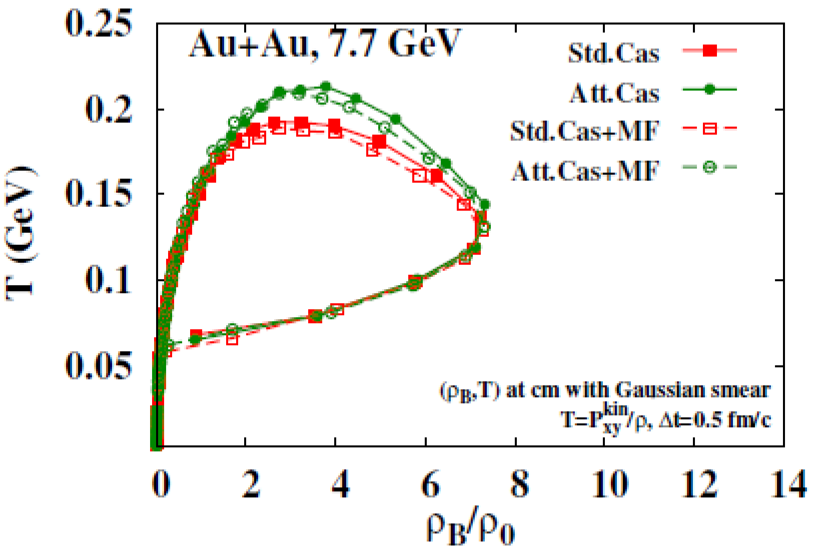
Toneev et al. ('03)

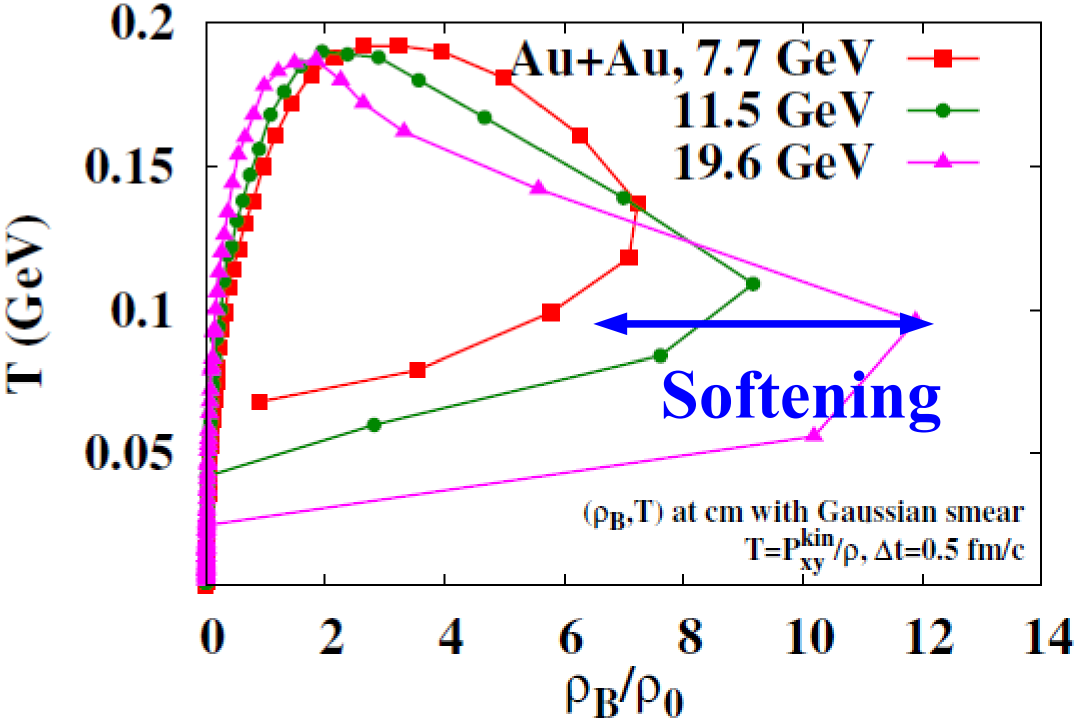


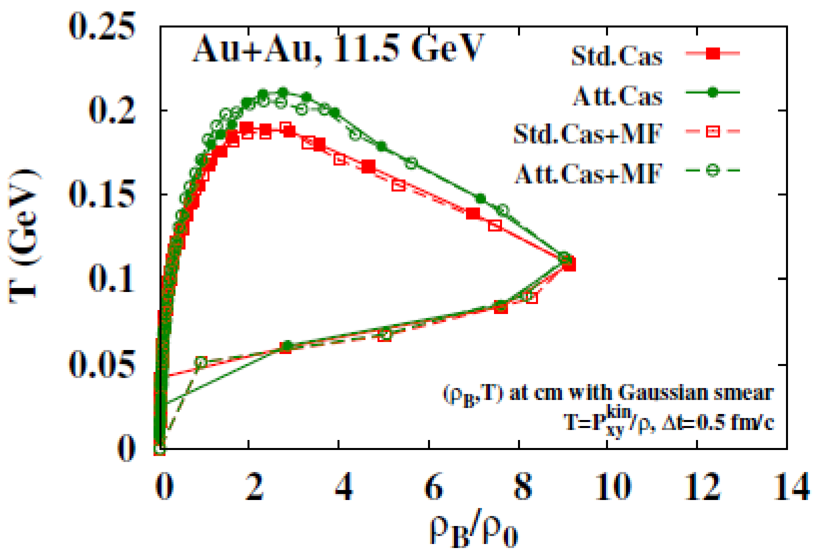
Transport model results also show tilted-ellipsoid-like behavior, but it is not enough.

Softening of EOS: Where and How much ?

- "Softening" should take place at $\sqrt{s_{NN}}$=11.5 GeV → $\rho/\rho_B \sim$ (6-10)
- Attractive orbit
  → Larger interactions
  & Higher T at later times







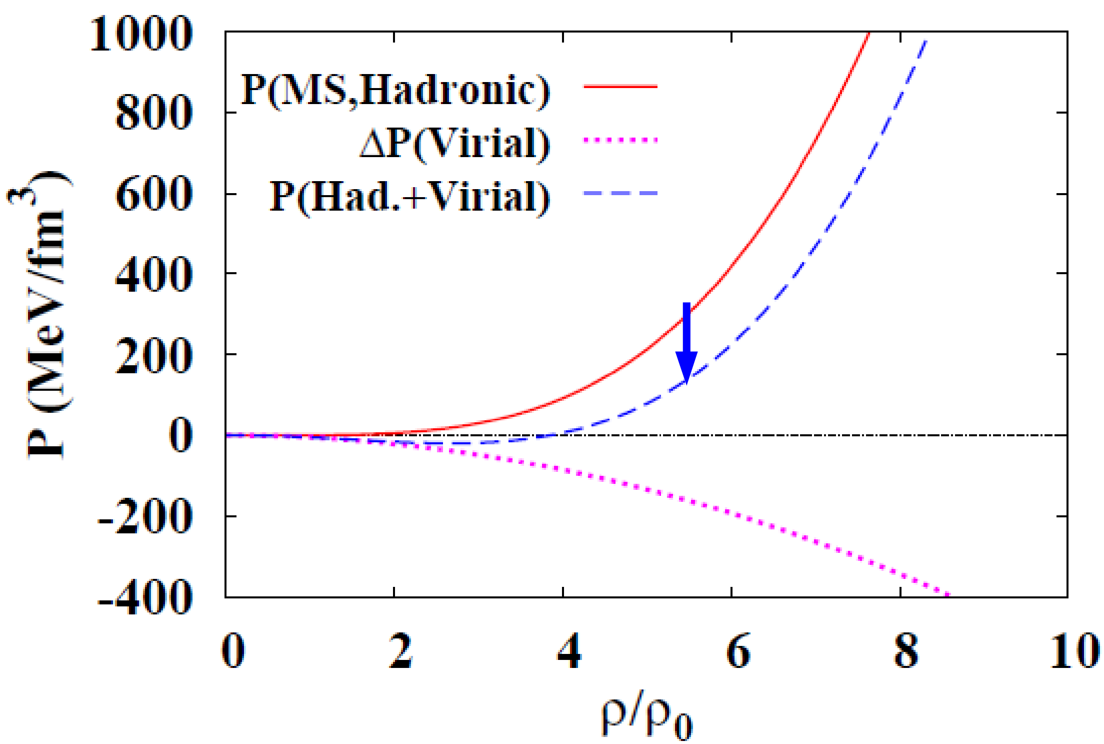
How much softening do we need ?

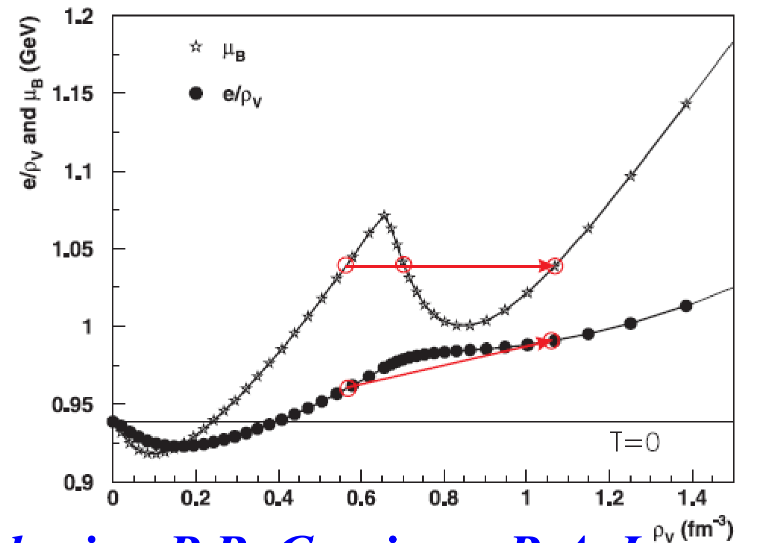
- **Virial theorem**

$$\Delta P = \frac{1}{3}\langle v \rho^2 \sigma \mathbf{q} \cdot \mathbf{\Delta r} \rangle$$

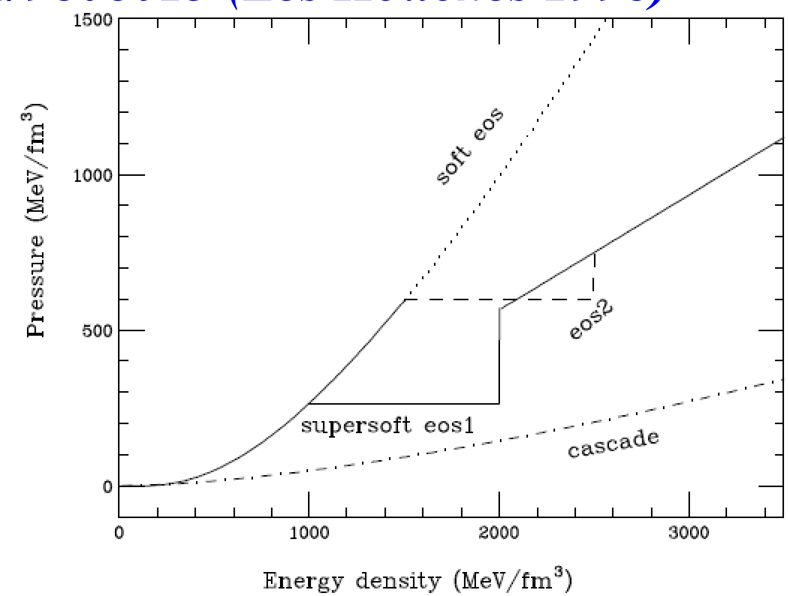
Simple esitmate:

σ= 30 mb, <q ΔR> ~ -1





Danielewicz, P.B. Gossiaux, R.A. Lacey, ucl-th/9808013 (Les Houches 1998)



B. A. Li, C. M. Ko, PRC58 ('98) 1382

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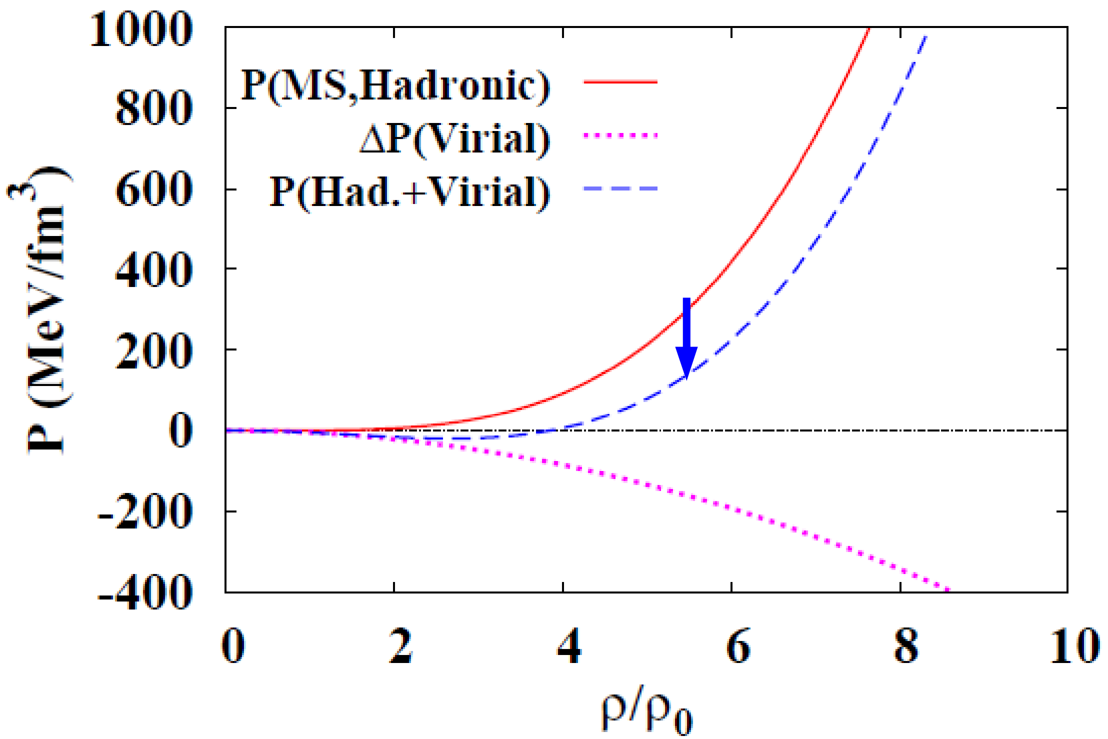
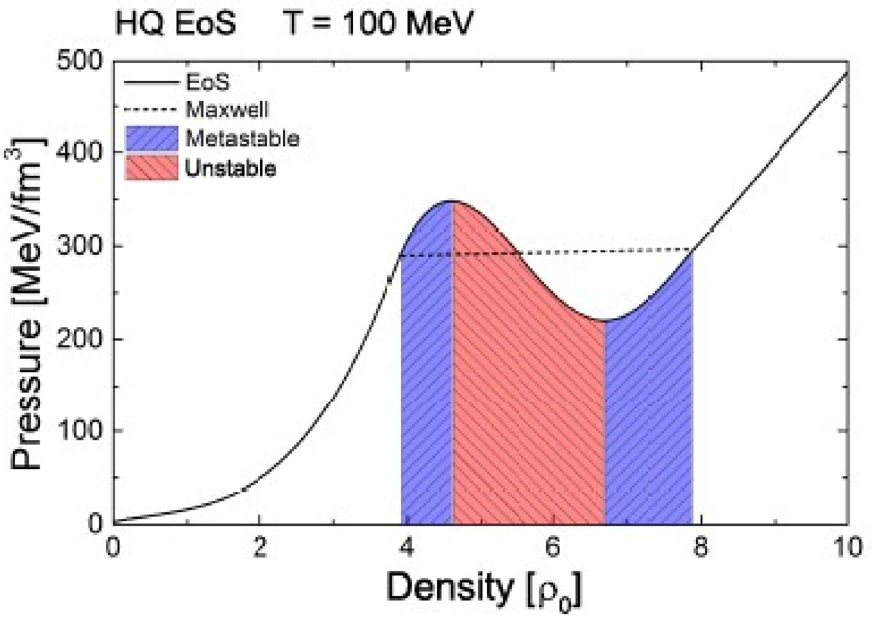
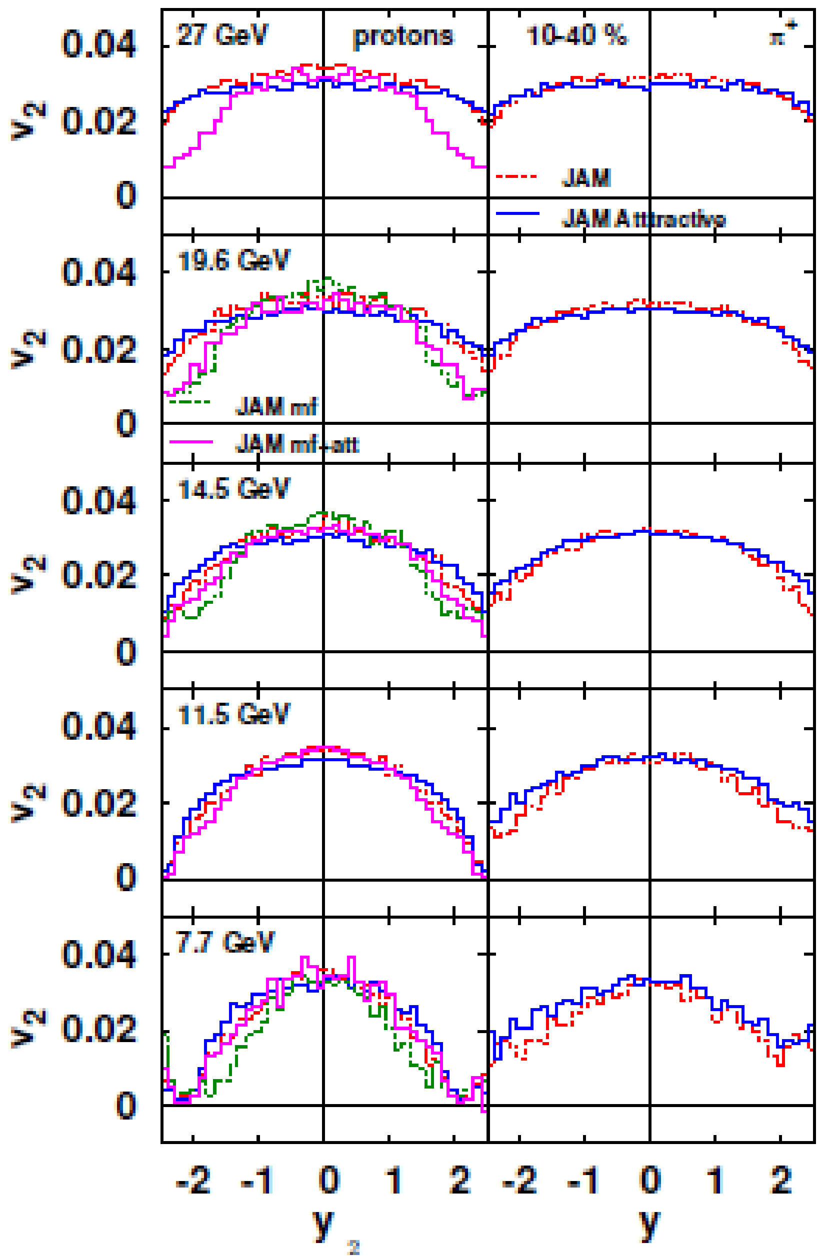




J. Steinheimer, J. Randrup, V. Koch, PRC89('14)034901.

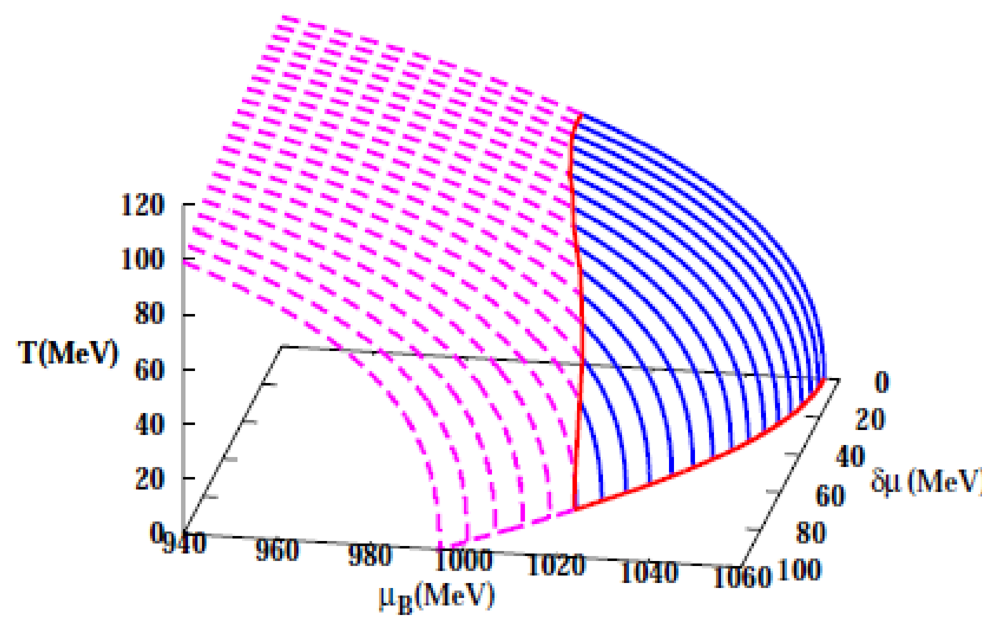
How about v_2 ?

- **Do we see softening effects in other observables, e.g. v_2 ?**
- **Yes, attractive orbits reduces proton v_2 by ~ 0.2 %.**
  (but there is no qualitative change.)



Relation to Neutron Star Matter

- **We may need early transition (2-5 ρ_0) to quark matter to solve the hyperon puzzle. Contradicting ?**
  → Temperature effects (T ~ 0 MeV & 100 MeV)
  Isospin chem. pot. (Weaker transition with finite $\delta\mu$)
  Hyperon repulsion may push up the transition density.



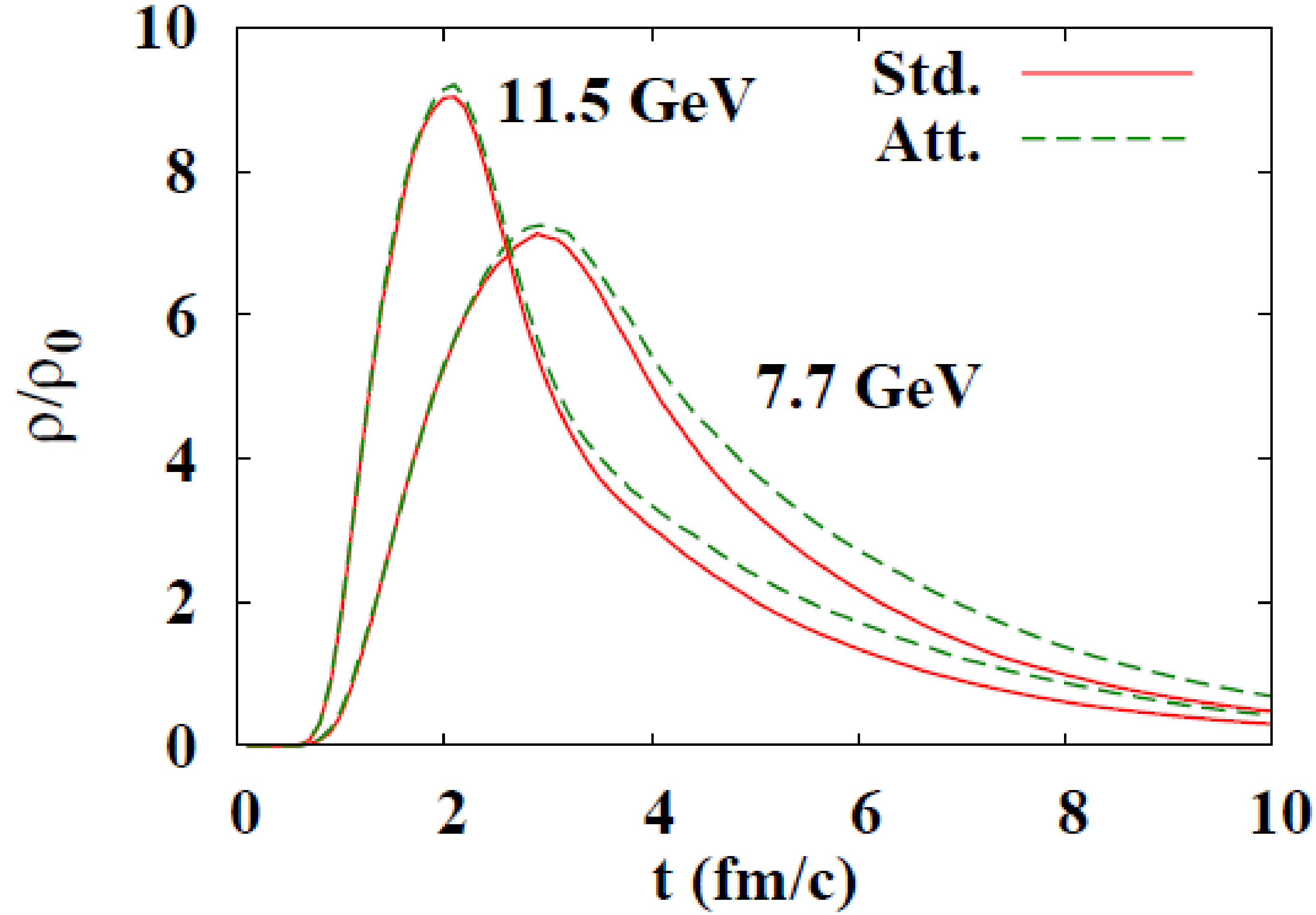
AO, Ueda, Nakano, Ruggieri, Sumiyoshi, PLB704('11),284
H. Ueda, T. Z. Nakano, AO, M. Ruggieri, K. Sumiyoshi, PRD88('13),074006

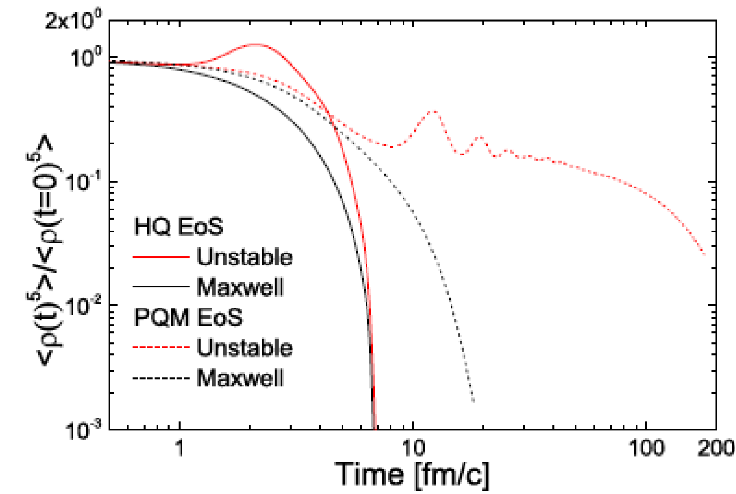
Summary

- **We may see QCD phase transition (1^{st} or 2^{nd}) signals at BES (or J-PARC) energies in baryon number cumulants and v_1 slope.**
- **Hadronic transport models cannot explain negative v_1 slope below $\sqrt{s_{NN}}$ = 20 GeV.**
  - **Geometric (bowling pin) mechanism becomes manifest at higher energies (JAM, JAM-MF, HSD, PHSD, UrQMD, ….).**
- **Hadronic transport with EOS softening can describe negative v_1 slope below $\sqrt{s_{NN}}$ = 20 GeV.**
  Y. Nara, A. Ohnishi, H. Stoecker, arXiv:1601.07692 [hep-ph]
  - **Attractive orbit scattering simulates EOS softening (virial theorem).**
  - **We need more studies to confirm its nature.**
    First-order phase transition ? Crossover ? Forward-backward rapidities ? MF leading to softer EOS ?
- ***We need "re-hardening" at higher energies, e.g. $\sqrt{s_{NN}}$ = 27 GeV.***

Thank you !

Time-dependence of density



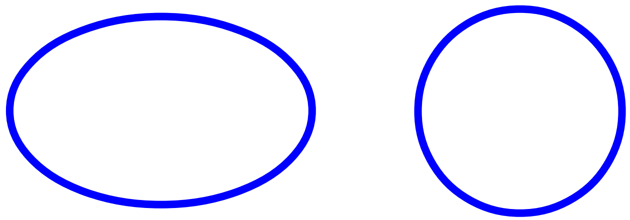


Steinheimer et al.('13)

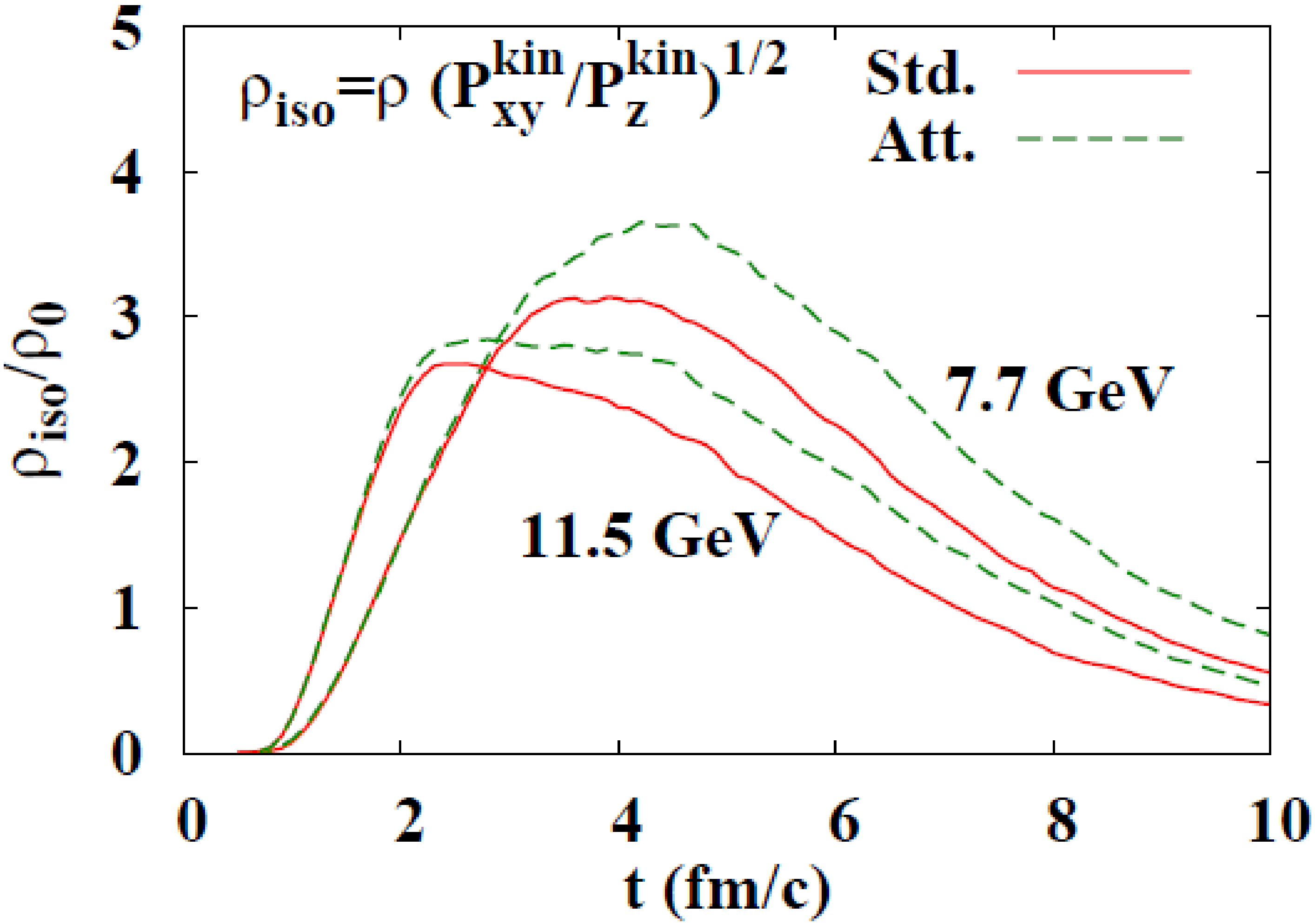
Gaussian smeared,
CM (x,y,z=0),
Collided/created hadrons,
incl. those in formation time

Attractive orbits keeps matter to be high density for a longer time.

Time-dependence of density

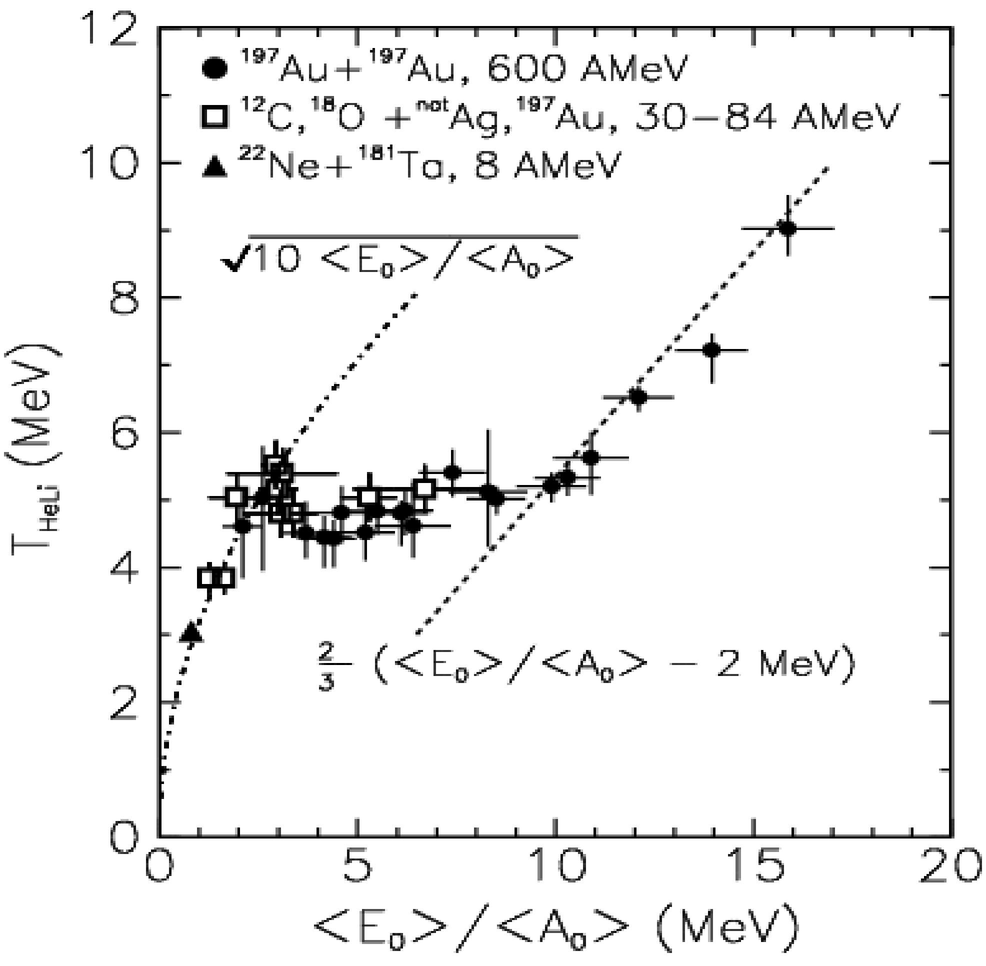


Anisotropy correction of the Fermi Sphere

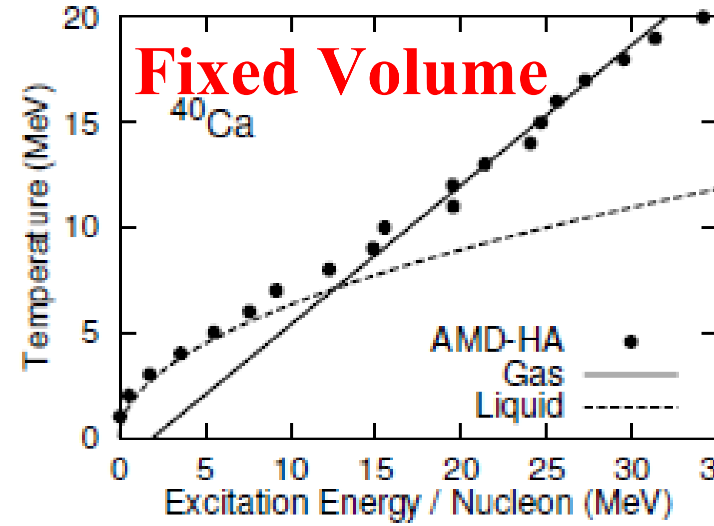


Nuclear Liquid-Gas Phase Transition

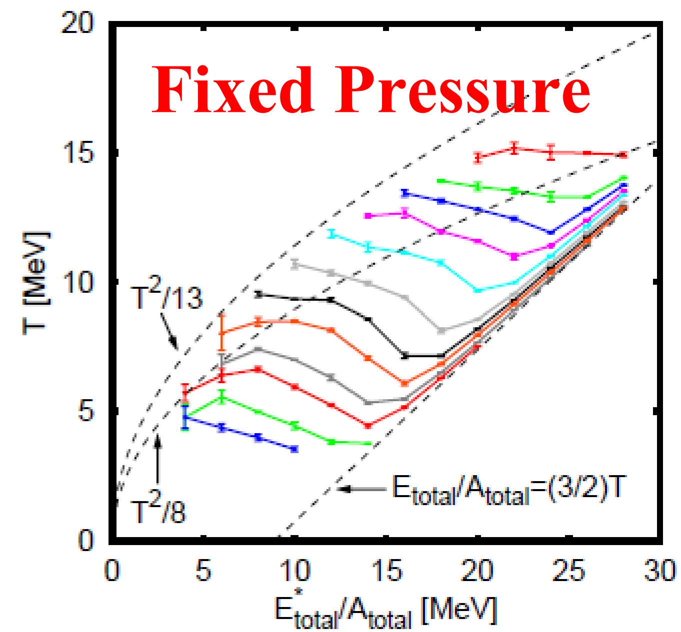
- **Caloric curve → LG phase transition (Smoking gun)**



J. Pochadzalla et al. (GSI-ALLADIN collab.), PRL 75 (1995) 1040.



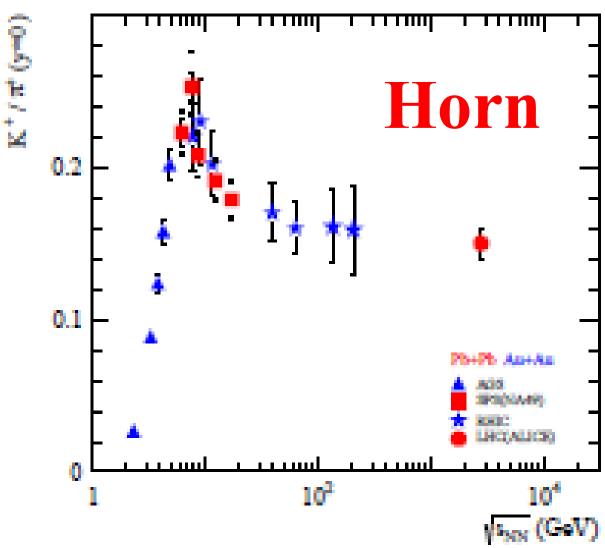
AO, Randrup ('98)

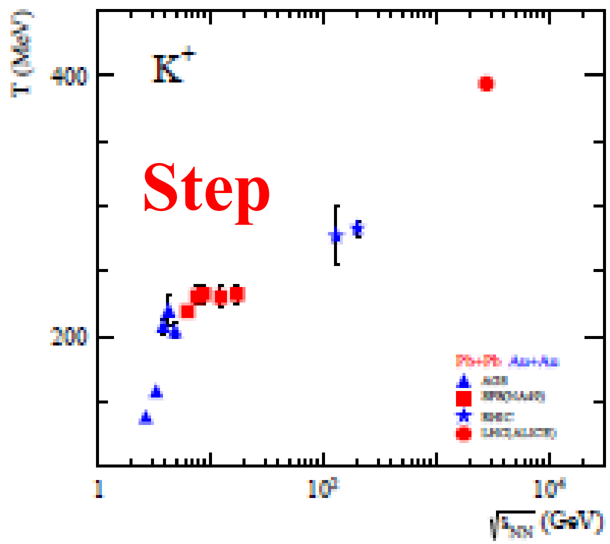


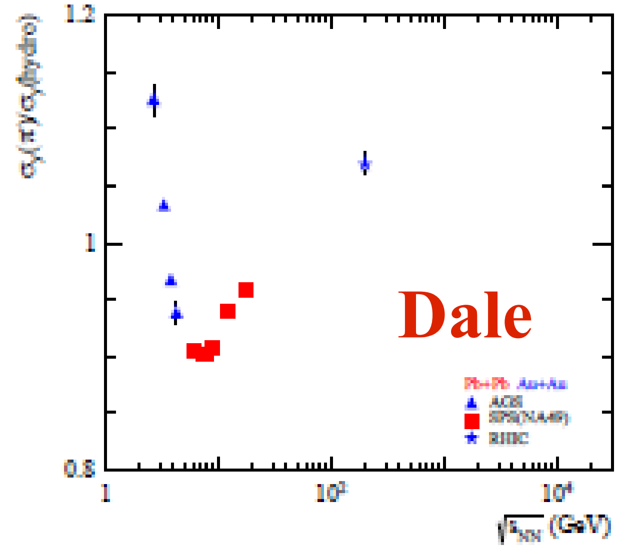
T. Furuta, A.Ono ('09)

Horn, Step and Dale

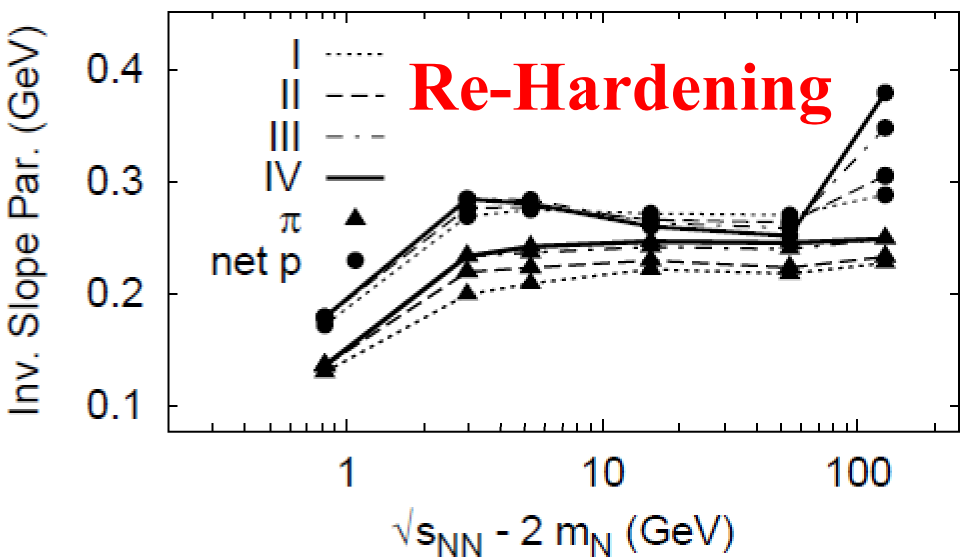
- **Non-monotonic behavior in K^+/π^+ ratio (Horn),**
  m slope par. (Step or re-hardening), rapidity dist. width of π







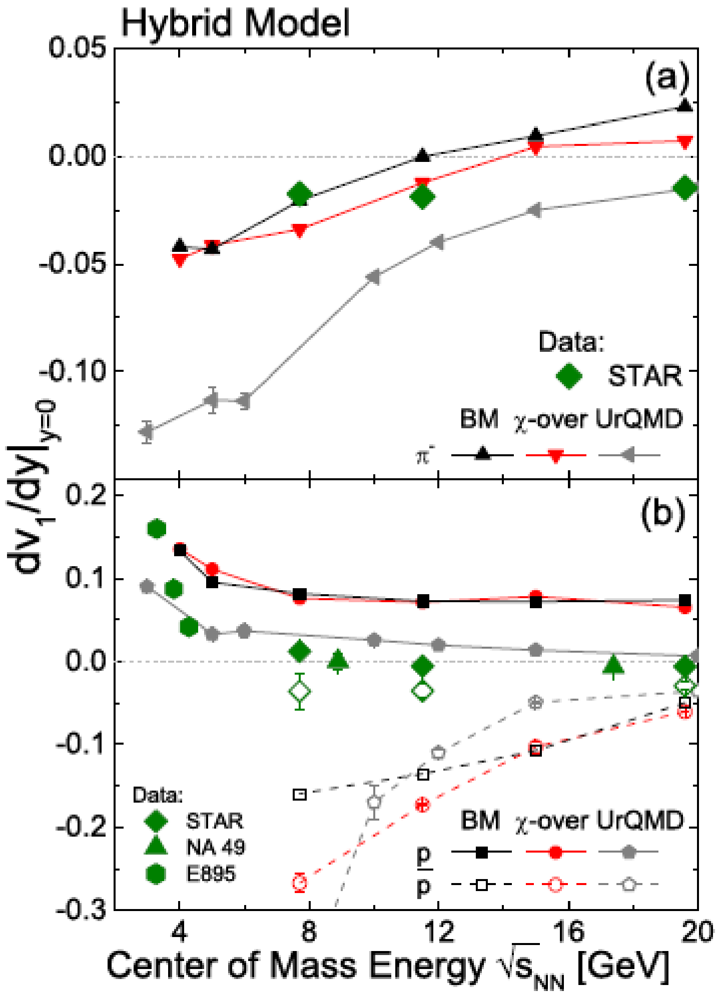
E.g. A. Rustamov (2012)

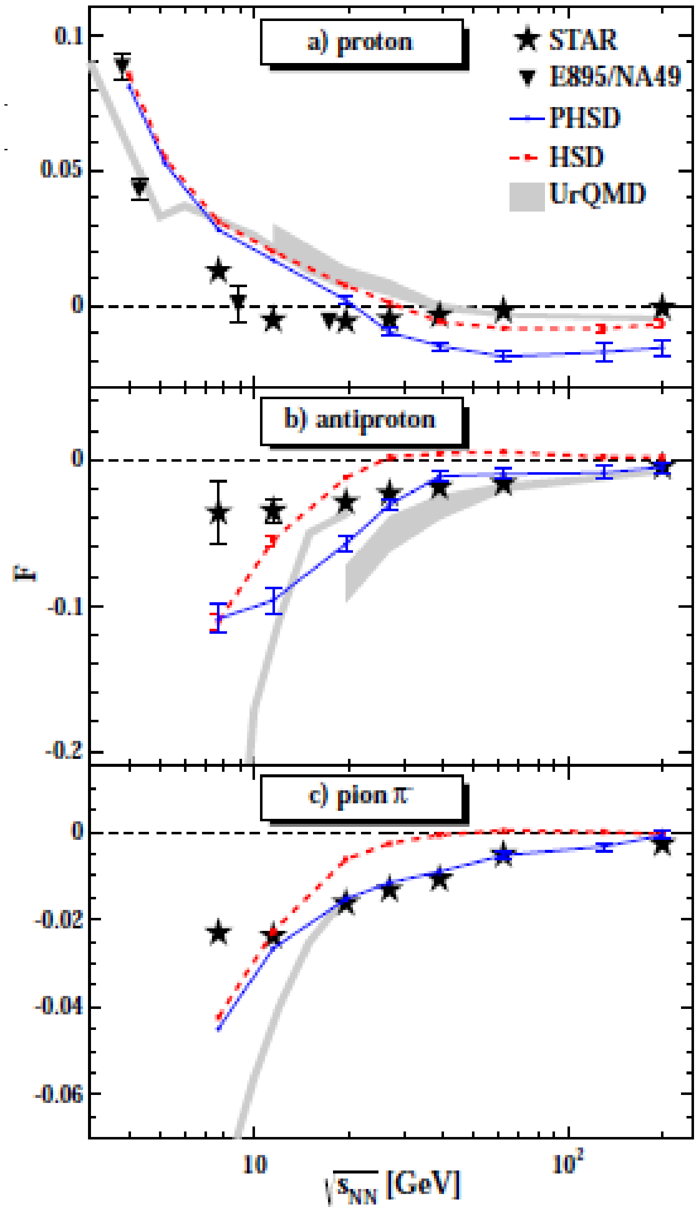


N. Otuka, P.K.Sahu, M. Isse,
Y. Nara, AO, nucl-th/010205

Hybrid Approaches

- **Both Hybrid model (Frankfurt) and PHSD (Giessen) show higher balance**





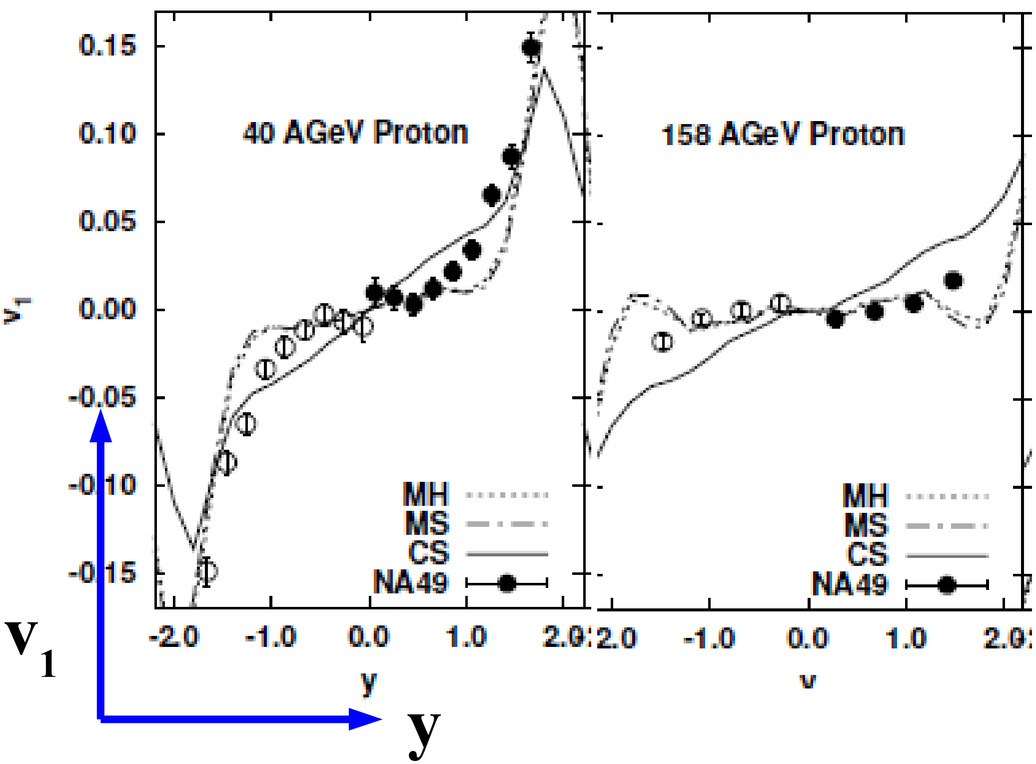
J. Steinheimer, J. Auvinen, H. Petersen, M. Bleicher, H. Stöcker, PRC89 ('14) 054913

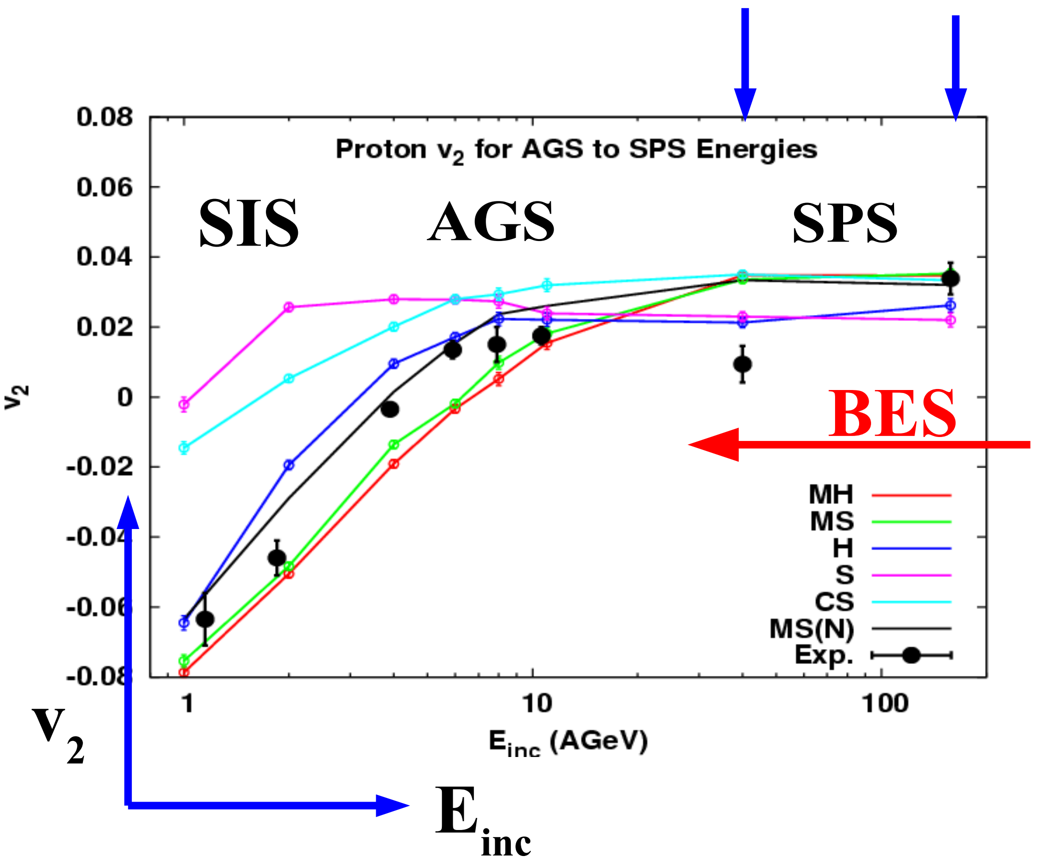
V. P. Konchakovski, W. Cassing, Yu. B. Ivanov, V. D. Toneev, PRC90('14)014903

JAM results at AGS and SPS Energies

- **JAM w/ Mean-Field effects roughly explains v_1 and v_2 at AGS & SPS**
  (1-158 A GeV → $\sqrt{s_{NN}}$ = 2.5-20 GeV)

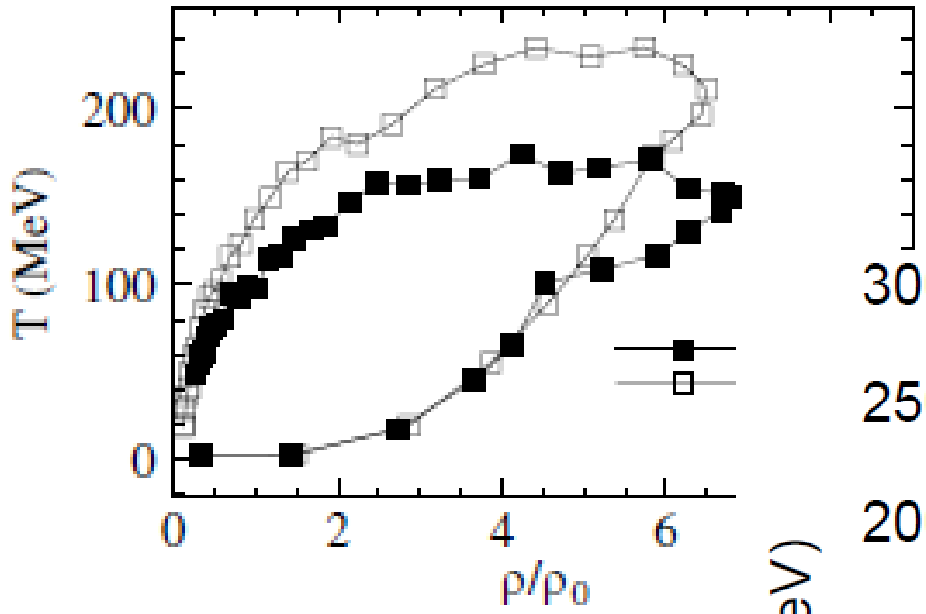
$\sqrt{s_{NN}}$=8.9 GeV **$\sqrt{s_{NN}}$=17.3 GeV**



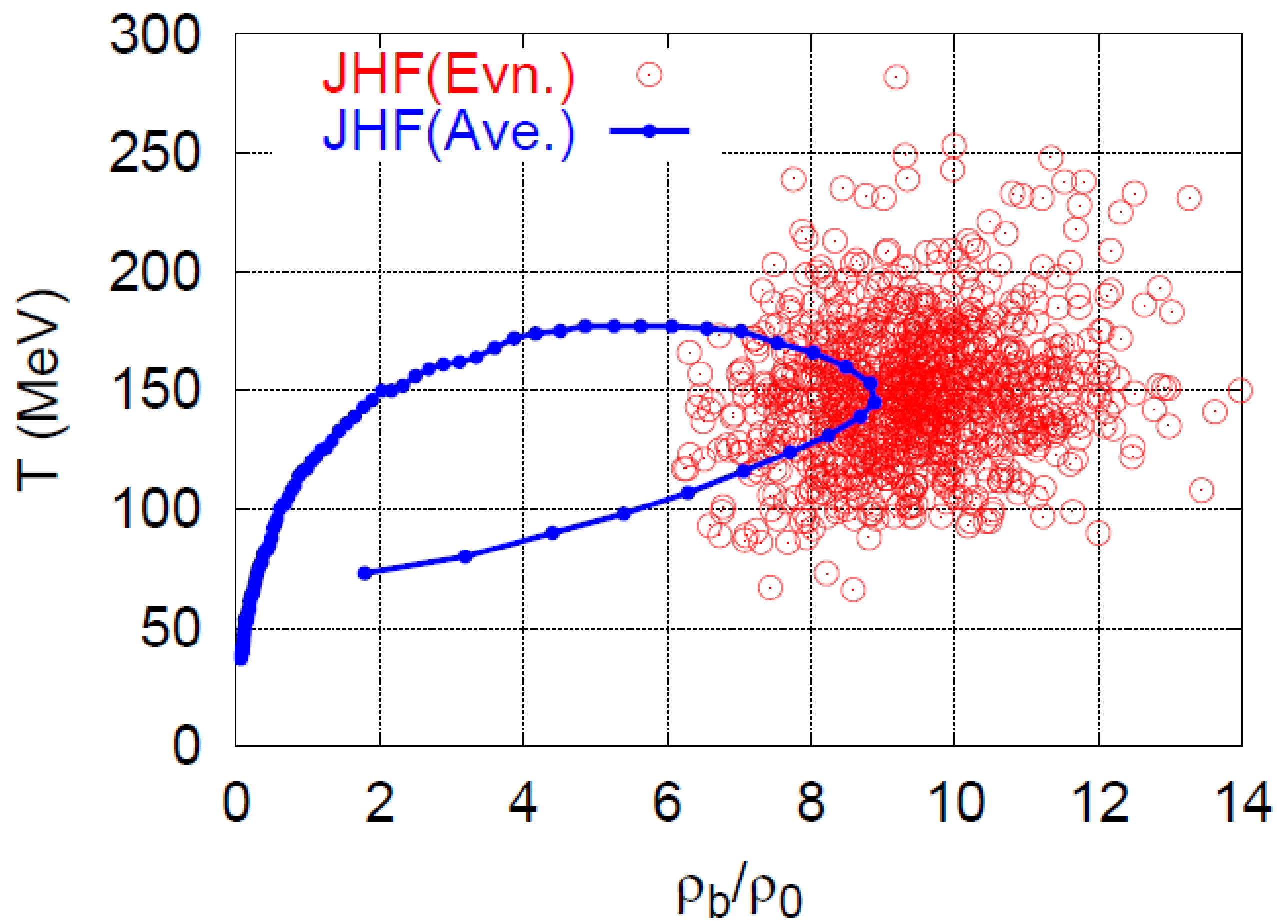


M. Isse, AO, N. Otuka, P. K. Sahu, Y. Nara, PRC72('05)064908

Highest Density Matter at J-PARC ?



Nara, Otuka, AO, Maruyama ('97)

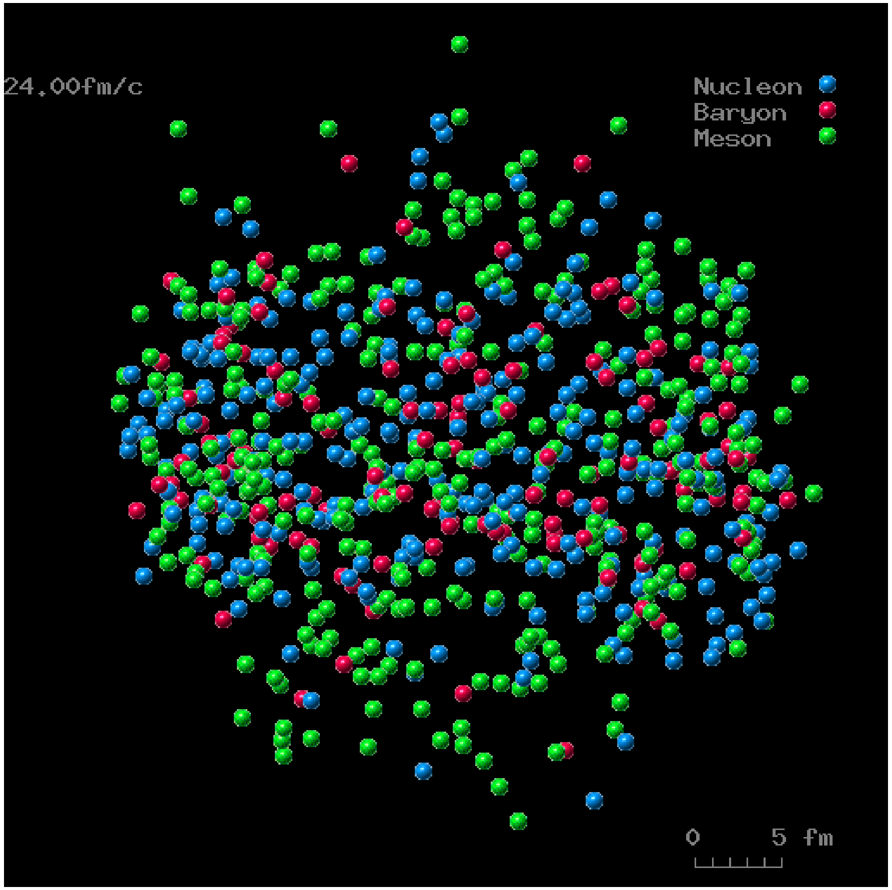


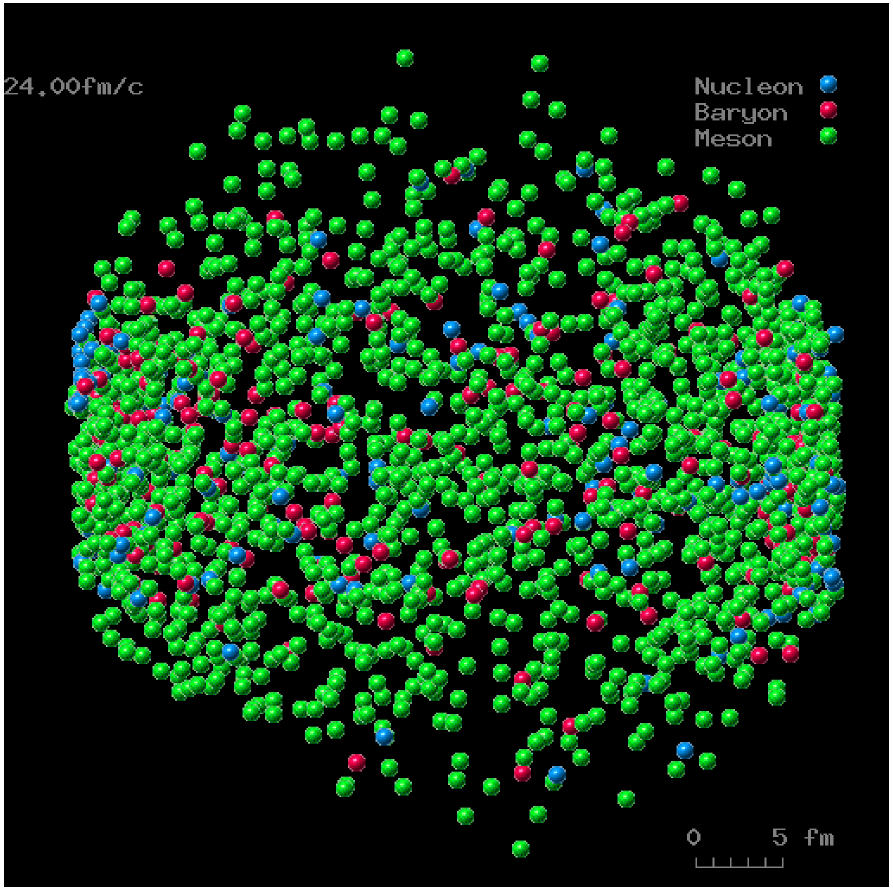
Central 1 fm^3 cube.

AO, JHF workshop (2002)

How do heavy-ion collisions look like ?

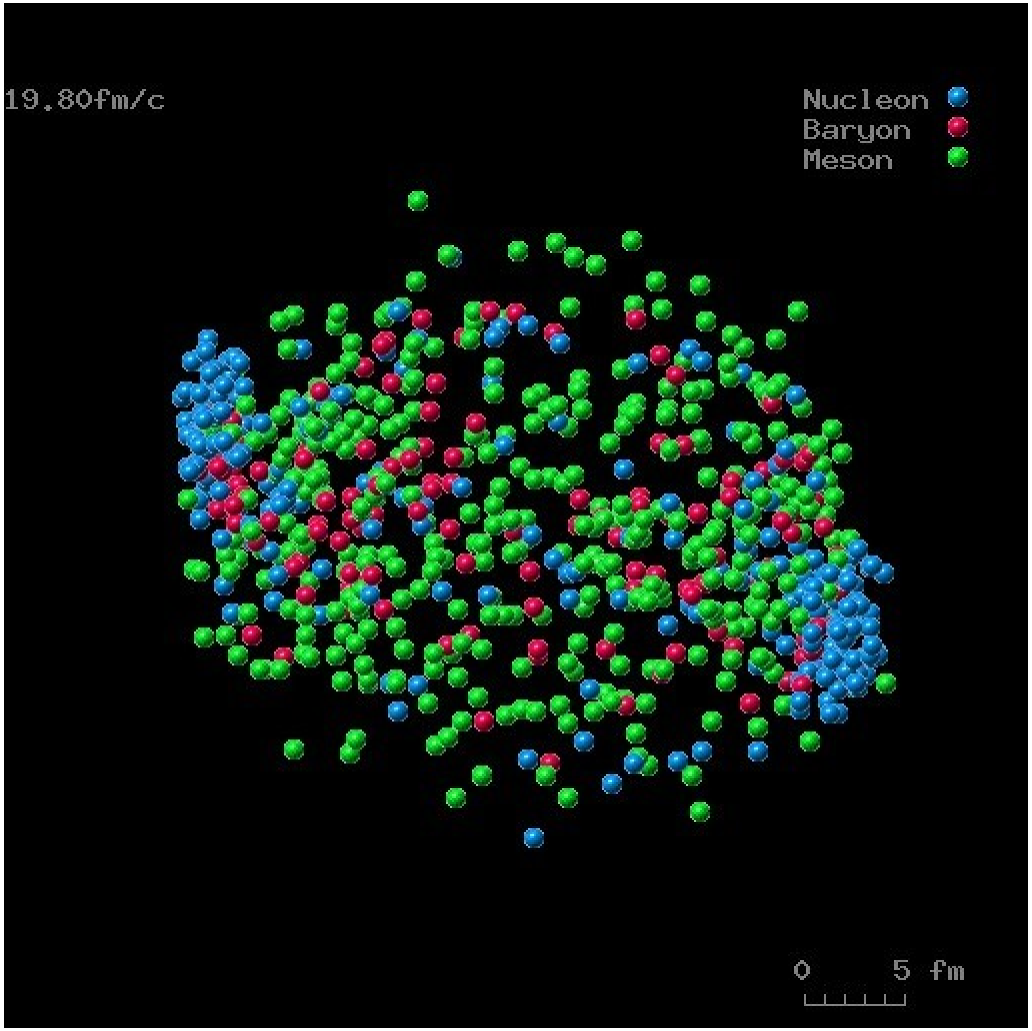
Au+Au, 10.6 A GeV | **Pb+Pb, 158 A GeV**





JAMming on the Web **http://www.jcprg.org/jow/**

J-PARC energy



Au+Au, 25 AGeV, b=5 fm (JOW)

BES (J-PARC) Energies ?

- ***J-PARC Energies***: $\sqrt{s_{NN}}$ = 4-40 GeV (or $\sqrt{s_{NN}}$ = 1.9-6.2 GeV)
  - E(p)=30 GeV → E(Au) ~ 12 AGeV (full strip, $\sqrt{s_{NN}}$ = 5.1 GeV for Au+Au)
  - E(p)=50 GeV → E(Au) ~ 20 AGeV ($\sqrt{s_{NN}}$= 6.4 GeV)
  - E(p)=30 GeV (50 GeV) Collider → $\sqrt{s_{NN}}$= 26 GeV (42 GeV)
- **Two Aspects of J-PARC energies**
  - Formation of highest baryon density matter
  - Various non-monotonic behaviors → Onset of deconfinement

Question

Do these Non-mono. behaviors signal the onset of QCD phase transition and/or QCD critical point ?

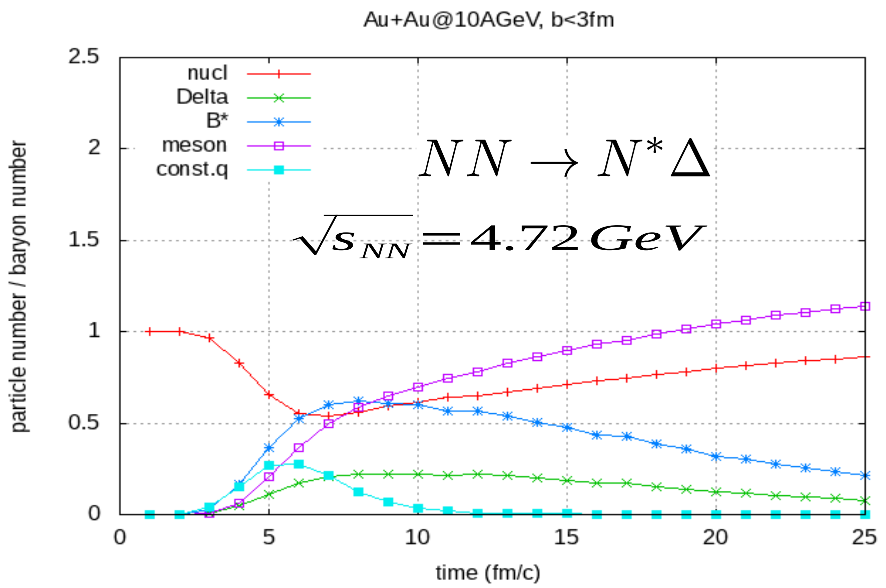
or Do they show some properties of hadronic matter ?

→ Let's examine in hadronic transport models !

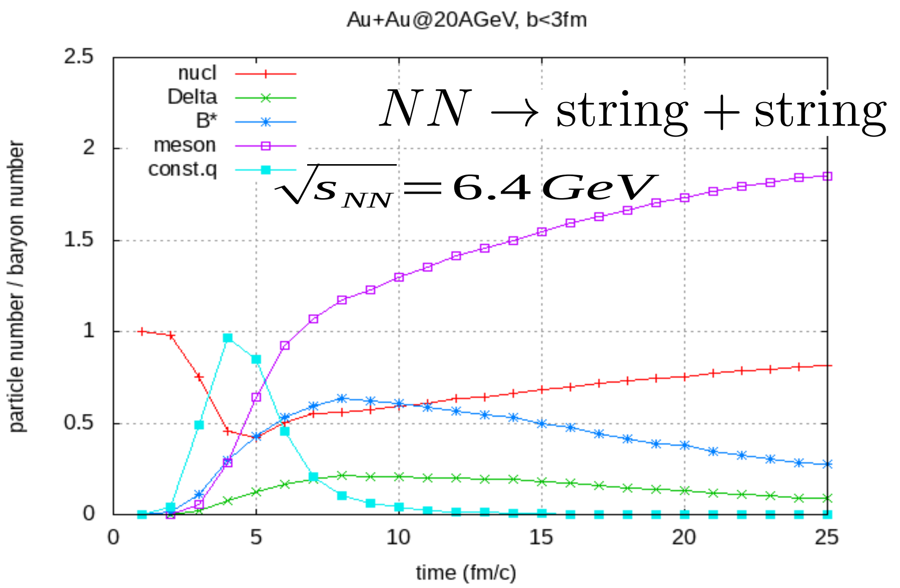
© Y. Nara

How to treat mean-field for excited matter?

Hadronic resonance dominant



constituent quark dominant due to string



Model 1 JAM/M: potential for all formed baryons

Model 2 JAM/Mq: potentials for quarks inside the pre-formed hadrons

Model 3: JAM/Mf: both formed and pre-formed baryons

Hadronic transport Approach

Purpose： Effects of hadron mean field potential on the directed flow v1

JAM hadronic cascade model : resonance and string excitation

Mean field by the framework of the Relativistic Quantum Molecular Dynamics

Nuclear cluster formation by phase space coalescence.

Statistical decay of nuclear fragment

Relativistic QMD/Simplified (RQMD/S)

RQMD based on Constraint Hamiltonian Dynamcis

Sorge, Stoecker, Greiner, Ann. Phys. 192 (1989), 266.

RQMD/S: Tomoyuki Maruyama, et al. Prog. Theor. Phys. 96(1996),263.

Single particle energy: $$p_i^0 = \sqrt{\boldsymbol{p}_i^2 + m_i^2 + 2m_i V_i}$$

$$\dot{\boldsymbol{r}}_i = \frac{\boldsymbol{p}_i}{p_i^0} + \sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \boldsymbol{p}_i} \qquad \dot{\boldsymbol{p}}_i = -\sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \boldsymbol{r}_i}$$

Arguments of potential $\boldsymbol{r}_i - \boldsymbol{r}_j$ and $\boldsymbol{p}_i - \boldsymbol{p}_j$ are replaced by the distances in the two-body c.m.

Relativistic QMD/Simplified (RQMD/S)

- **RQMD = Constraint Hamiltonian Dynamics**
  (Sorge, Stocker, Greiner, Ann. of Phys. 192 (1989), 266.)
- **Constraints: φ ≈ 0 (Satisfied on the realized trajectory, by Dirac)**
  - **Variables in Covariant Dynamics = 8N phase space: (q_μ, p_μ)**
  - **Variables in EOM = 6N phase space**
    → We need 2N constraints to get EOM
- **On Mass-Shell Constraints**

$$H_i \equiv p_i^2 - m_i^2 - 2m_i V_i \approx 0$$

- **Time-Fixation in RQMD/S**

$$\chi_i \equiv \hat{a}\cdot(q_i - q_N) \approx 0 \; (i = 1, \sim N-1) \quad , \quad \chi_N \equiv \hat{a}\cdot q_N - \tau \approx 0$$

$$\hat{a} = \text{Time-like unit vector in the Calculation Frame}$$

(Tomoyuki Maruyama et al., Prog. Theor. Phys. 96(1996), 263.)

RQMD/S (cont.)

- **Hamiltonian is made of constraints**

$$H=\sum_i u_i\phi_i \quad (\phi_i = H_i (i=1\sim N), \chi_{i-N}(i=N+1\sim 2N))$$

- **Time Development** $\dfrac{df}{d\tau}=\dfrac{\partial f}{\partial\tau}+\{f, H\}$, $\{q_\mu, p_\nu\}=g_{\mu\nu}$

- **Lagrange multipliers are determined to keep constraints**
  → ***We can obtain the multipliers analytically in RQMD/S***

$$\frac{d\phi_i}{d\tau}\approx 0 \rightarrow \delta_{i,2\mathrm{N}}+\sum_j u_j\{\phi_i, \phi_j\}\approx 0$$

- **Equations of Motion**

$$H=\sum_i (p_i^2-m_i^2-2m_i V_i)/2p_i^0 \quad , \quad p_i^0=E_i=\sqrt{\vec{p}_i^{\,2}+m_i^2+2m_i V_i}$$

$$\frac{d\vec{r}_i}{d\tau}\approx -\frac{\partial H}{\partial\vec{p}_i}=\frac{\vec{p}}{p_i^0}+\sum_j \frac{m_j}{p_j^0}\frac{\partial V_j}{\partial\vec{p}_i} \quad , \quad \frac{d\vec{p}_i}{d\tau}\approx\frac{\partial H}{\partial\vec{r}_i}=-\sum_j\frac{m_j}{p_j^0}\frac{\partial V_j}{\partial\vec{r}_i}$$

We can include MF in an almost covariant way in molecular dynamics

Particle "DISTANCE"

$$r_{Tij}{}^2 \equiv r_\mu r^\mu - \left(r_\mu P_{ij}^\mu\right)^2 / P_{ij}^2 = \vec{r}^{\,2} \qquad (in\ CM)$$

$$P_{ij} \equiv p_i + p_j \quad , \qquad r \equiv r_i - r_j$$

Particle "Momentum Difference"

$$p_{Tij}{}^2 \equiv p_\mu p^\mu - \left(p_\mu P_{ij}^\mu\right)^2 / P_{ij}^2 = \vec{p}^{\,2} \qquad (in\ CM)$$

$$p \equiv p_i - p_j$$

Lorentz Invariant, and Becomes Normal Distance in CM !