Symmetry breaking effect on the inhomogeneous chiral phase in the external magnetic field

PRD 92, 116009 (2015)

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CPOD 2016

6/3

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- Construction of the thermodynamic potential and anomaly
- Numerical result
- Discussion and summary

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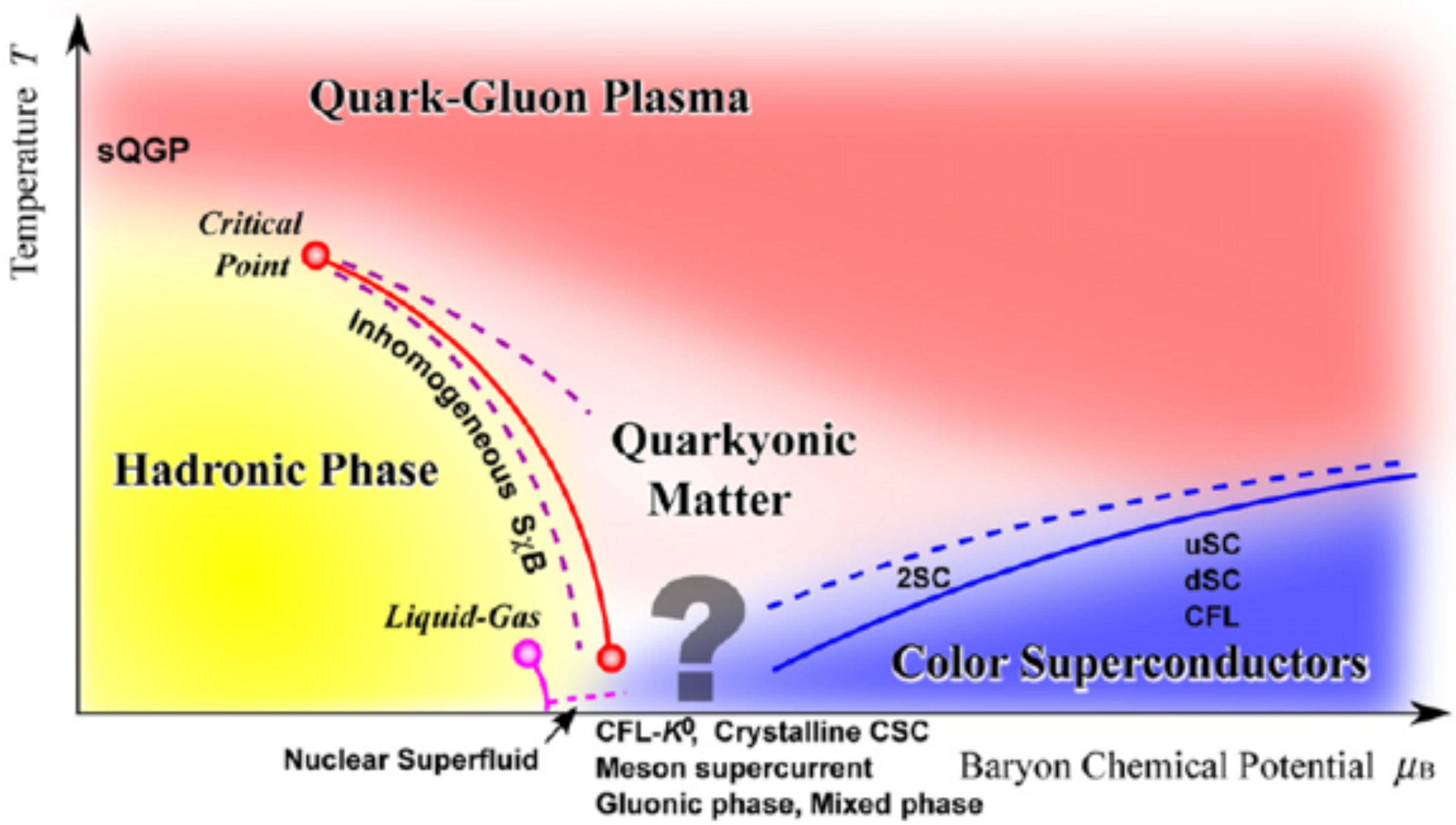
QCD phase diagram and chiral symmetry

Phase structure of quark matter



- Hadron phase
- Quark gluon plasma
- Colar super conductivity
- Inhomogeneous chiral phase

K. Fukushima, T. Hatsuda,
Rep. Prog. Phys. 74, 014001 (2011)

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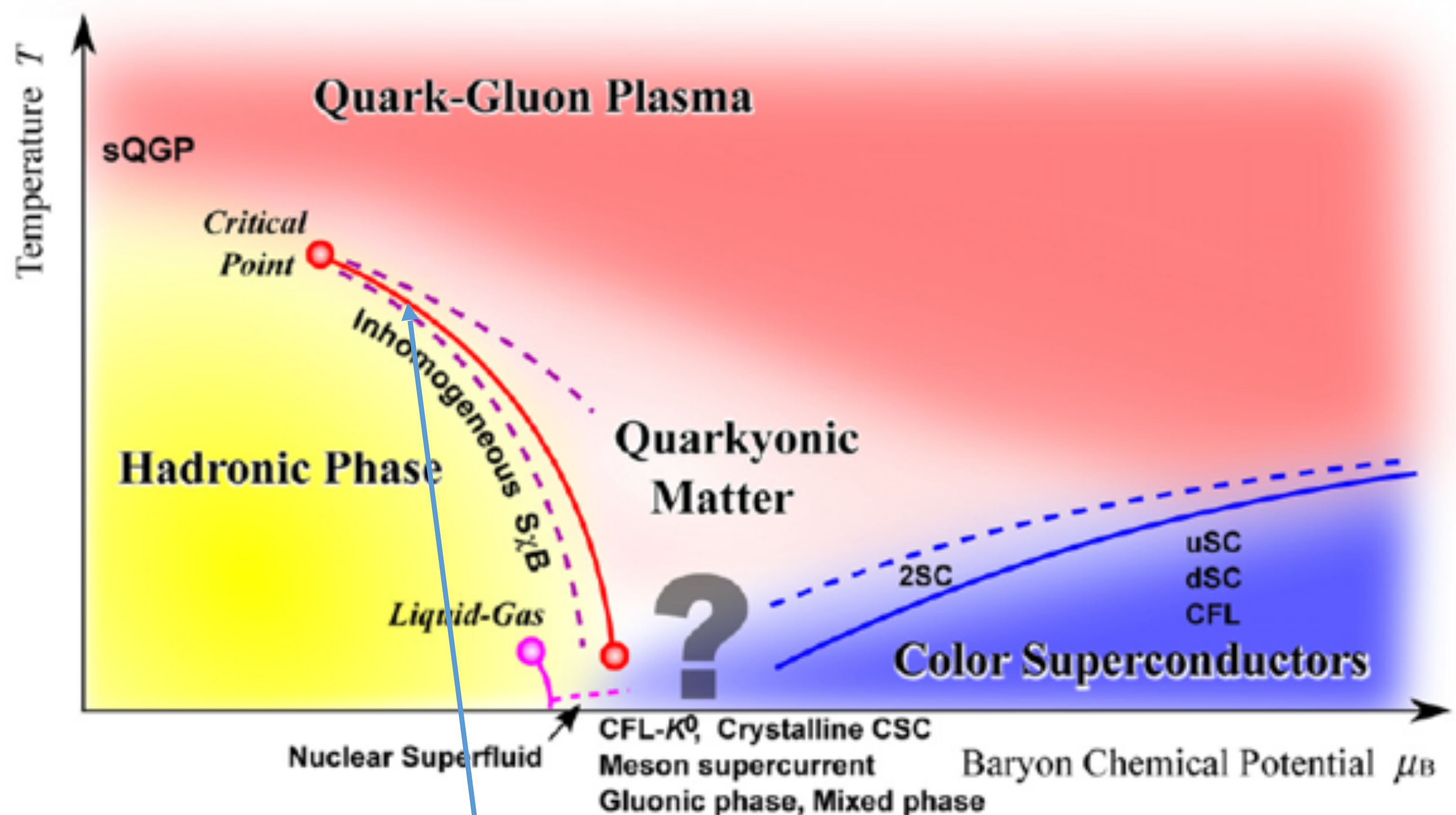

chiral phase transition line

- Hadron phase
- Quark gluon plasma
- Colar super conductivity
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K. Fukushima, T. Hatsuda, Rep. Prog. Phys. 74, 014001 (2011)

Chiral symmetry

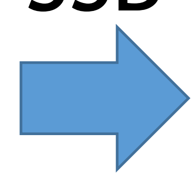
(chiral limit)

restored phase

$\langle \bar{\psi}\psi \rangle = 0$

chiral condensate

SSB



broken phase

$\langle \bar{\psi}\psi \rangle \neq 0$

spatially modulated in the inhomogeneous chiral phase

cf. FFLO superconductivity, CDW, SDW

QCD phase diagram and chiral symmetry

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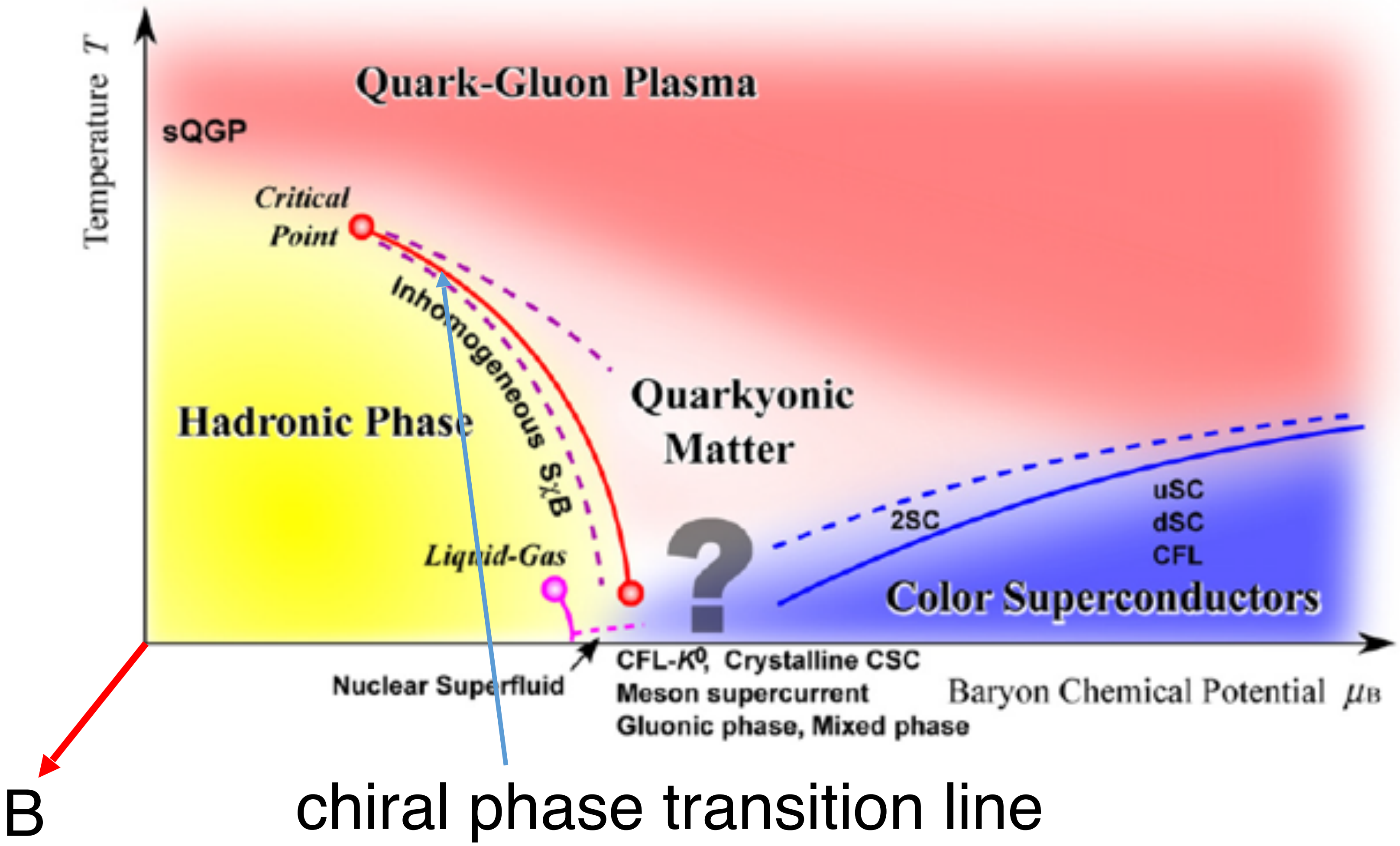

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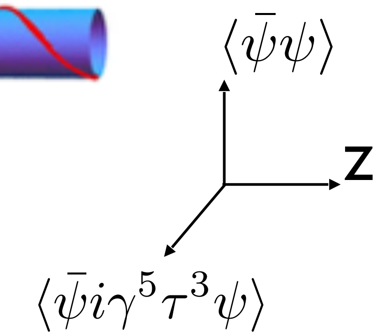
Inhomogeneous chiral phase

Inhomogeneous chiral condensate ※chiral limit

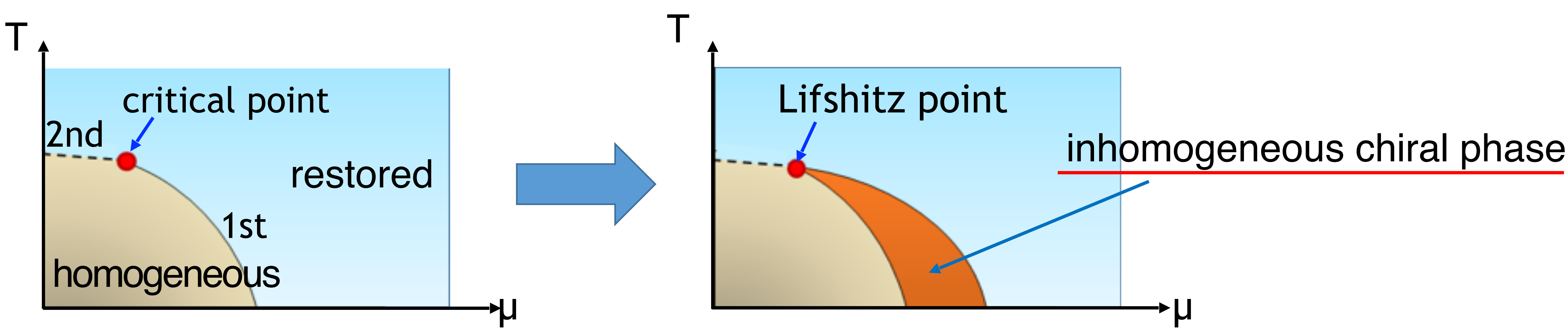
$$M(\mathbf{r}) \equiv -2G\left[\langle\bar{\psi}\psi\rangle + i\langle\bar{\psi}i\gamma^5\tau^3\psi\rangle\right] = m(\mathbf{r})e^{i\theta(\mathbf{r})}$$

Dual chiral density wave (DCDW) · · · $m(\mathbf{r}) = m,\ \theta(\mathbf{r}) = qz$



Real kink crystal (RKC) · · · $m(\mathbf{r}) \sim m\,\mathrm{sn}(qz),\ \theta(\mathbf{r}) = 0$

G. Basar, et al., PRD 79, 105012 (2009)



E. Nakano and T. Tatsumi, PRD 71, 114006 (2005)
D. Nickel, PRD 80, 074025 (2009)
D. Muller, M. Buballa and J. Wambach, PLB 727, 240 (2013)

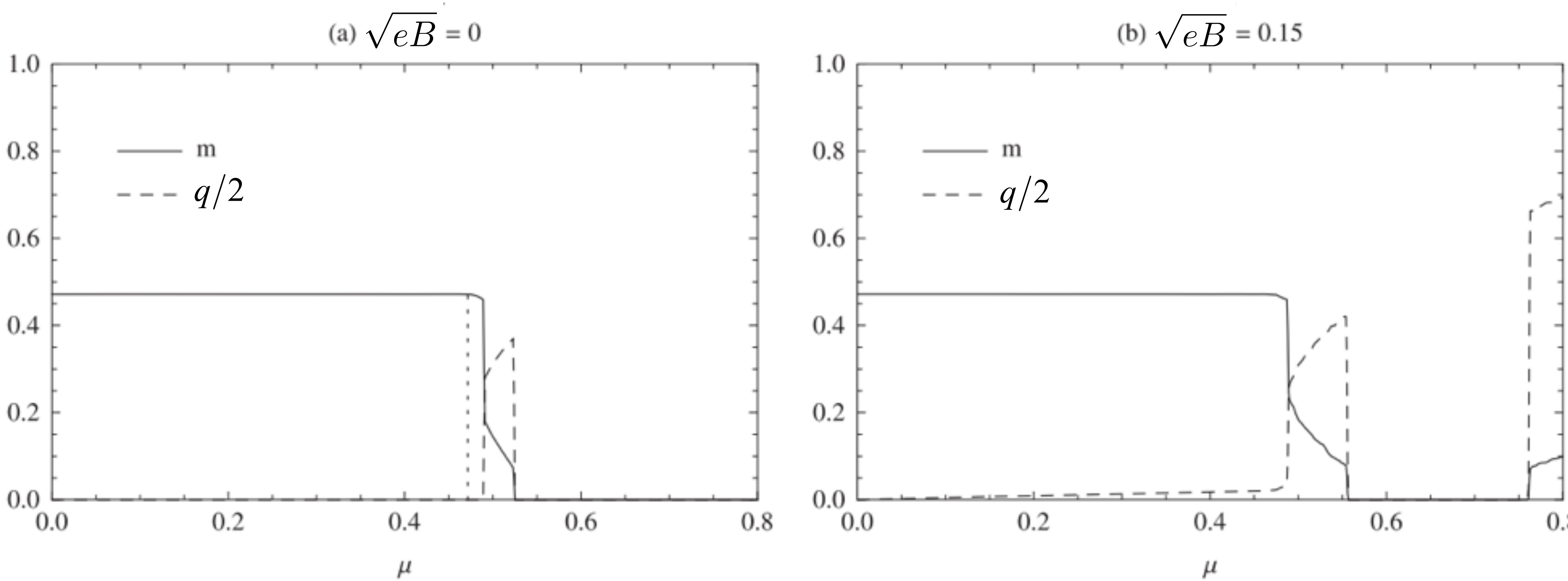
DCDW phase in the magnetic field

I. E. Frolov, et al., PRD 82, 076002 (2010)

2-flavor NJL model within the mean field approximation（T=0）

$$\mathcal{L}_{\rm MF} = \bar{\psi}\left[i\not{D} - m\left(\cos qz + i\gamma^5\tau^3 \sin qz\right)\right]\psi + \frac{m^2}{4G}$$

$\langle\bar{\psi}\psi\rangle$ $\langle\bar{\psi}i\gamma^5\tau^3\psi\rangle$



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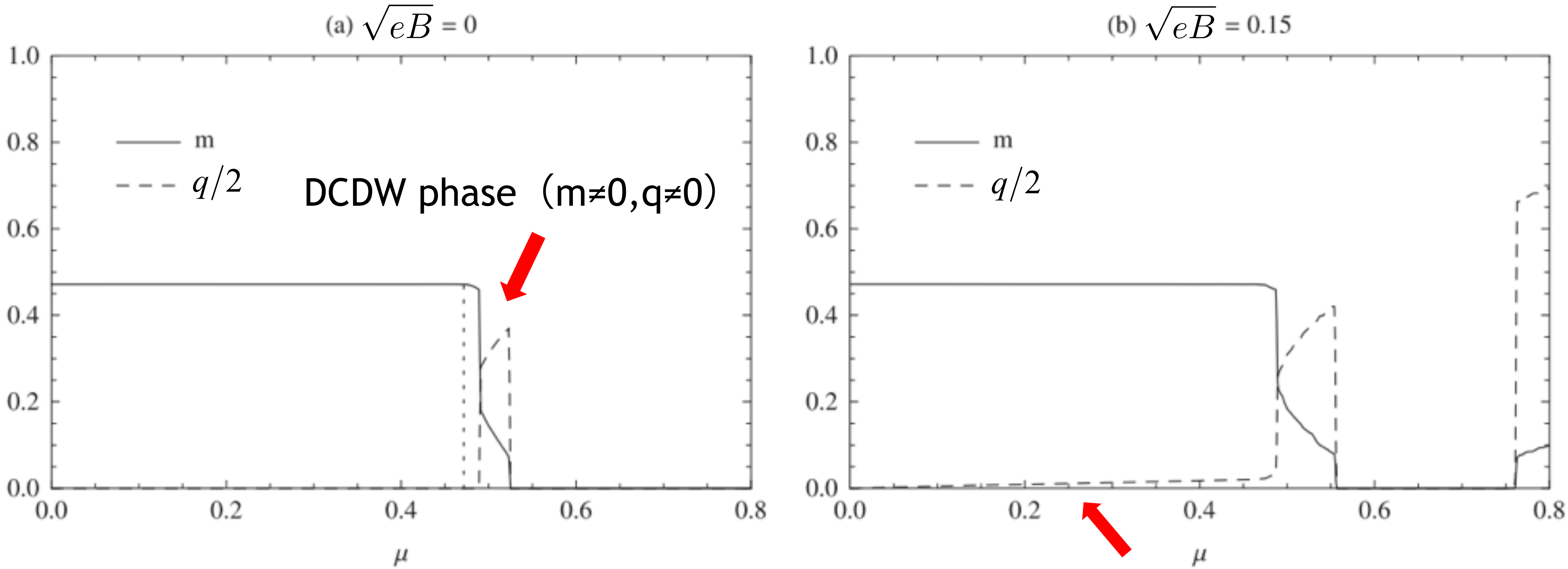

The weak DCDW phase appears at μ≠0.

B spreads the DCDW phase over the low μ region.

Analysis around the Lifshitz point

Generalized Ginzburg-Landau expansion around the Lifshitz point

$$\mathcal{L}_{\mathrm{MF}} = \bar{\psi}\left\{i\not{D} - \left[\mathrm{Re}M(\mathbf{r}) + i\gamma^5\tau^3\mathrm{Im}M(\mathbf{r})\right]\right\}\psi$$

D. Nickel, PRL 103, 072301 (2009)

$$M(\mathbf{r}) = m(\mathbf{r})e^{i\theta(\mathbf{r})}$$

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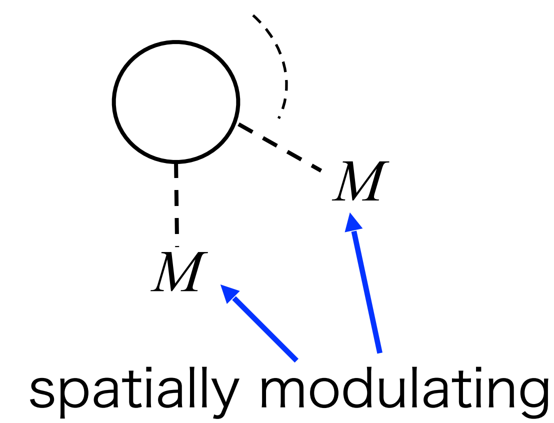
$$M(\mathbf{r}) = m(\mathbf{r})e^{i\theta(\mathbf{r})}$$

$$\Omega(\mu, T, B) = -\frac{T}{V}{\rm Tr\,Ln}\left[S_B^{-1} - ({\rm Re}M + i\gamma_5\tau_3{\rm Im}M)\right] + \frac{|M|^2}{4G}$$
$$= \Omega_0 - \frac{T}{V}\sum_{j\geq 1}{\rm Tr}\left[S_B\,({\rm Re}M + i\gamma_5\tau_3{\rm Im}M)\right]^j + \frac{|M|^2}{4G}$$

$$S_B = \frac{1}{i\gamma^\mu D_\mu + \mu\gamma^0}$$



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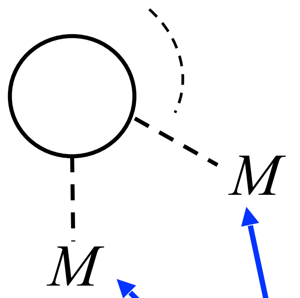

expand Ω about M and the derivative up to the 4th order

T. Tatsumi, K. Nishiyama and S. Karasawa, PLB 743, 66 (2015)

$$\Omega(\mu,T,B) = \Omega_0 + \int \frac{d^3\mathbf{x}}{V}\left(\frac{\alpha_2}{2}|M|^2 + \alpha_3{\rm Im}M^*M' + \frac{\alpha_{4a}}{4}|M|^4 + \frac{\alpha_{4b}}{4}|M'|^2 \right)$$

It is important M is complex. ➡ It is related to anomaly.

$$E(M) \leftrightarrow -E(M^*)$$

Lifshitz point in the magnetic field

In the case of DCDW T. Tatsumi, K. Nishiyama and S. Karasawa, PLB 743, 66 (2015)

$$\Omega(\mu, T, B) = \Omega_0 + \frac{\alpha_2}{2} m^2 + \alpha_3 m^2 q + \frac{\alpha_{4a}}{4} m^4 + \frac{\alpha_{4b}}{4} m^2 q^2$$

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$\alpha_3 \neq 0$ only when B≠0 and μ≠0

Ω always has a minimum point at q≠0

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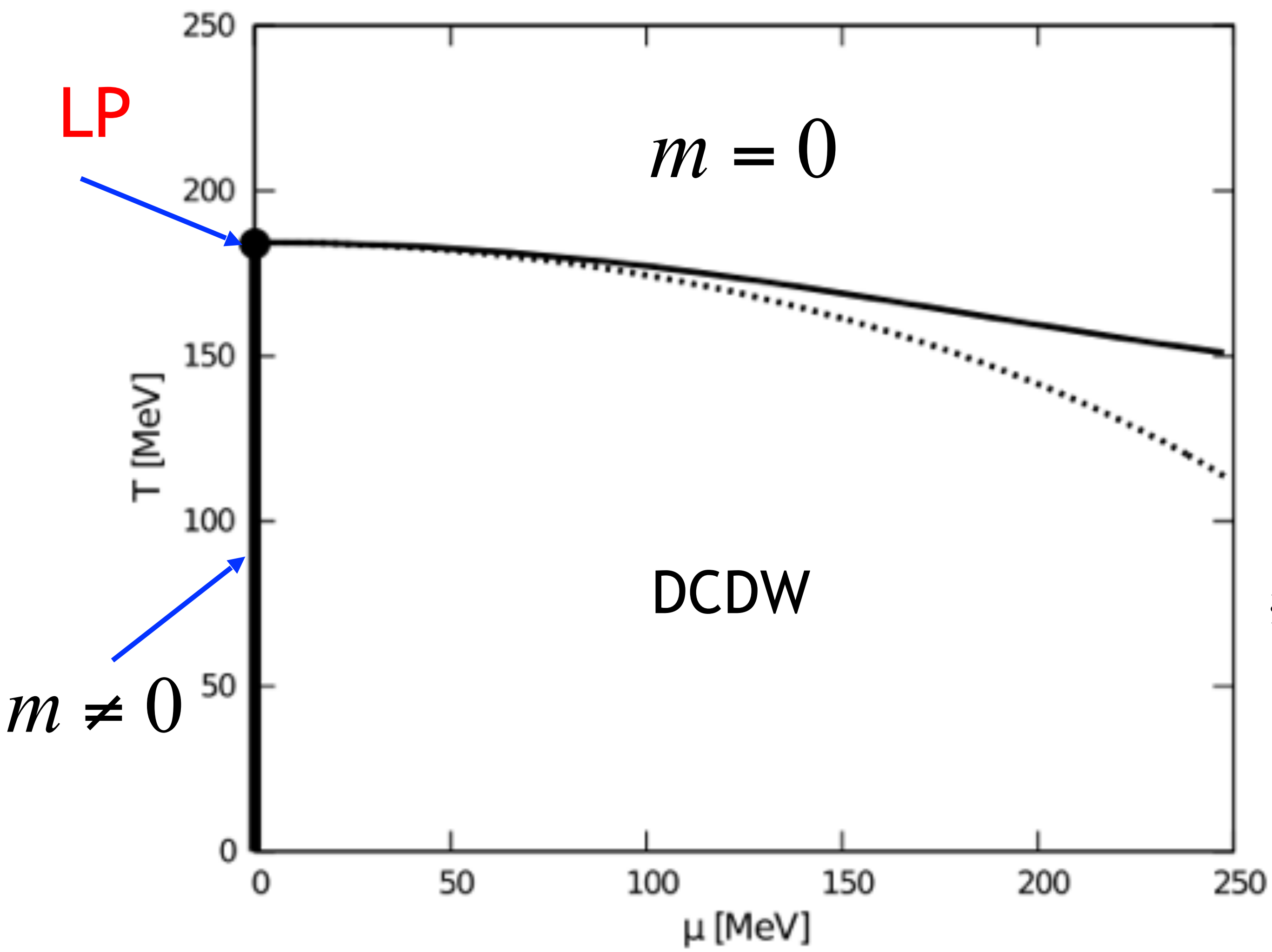
$\sqrt{eB} = 300\text{MeV}$



Lifshitz point appears on the T axis.
The DCDW phase is spread over μ≠0.

※The RKC phase is not spread dramatically.

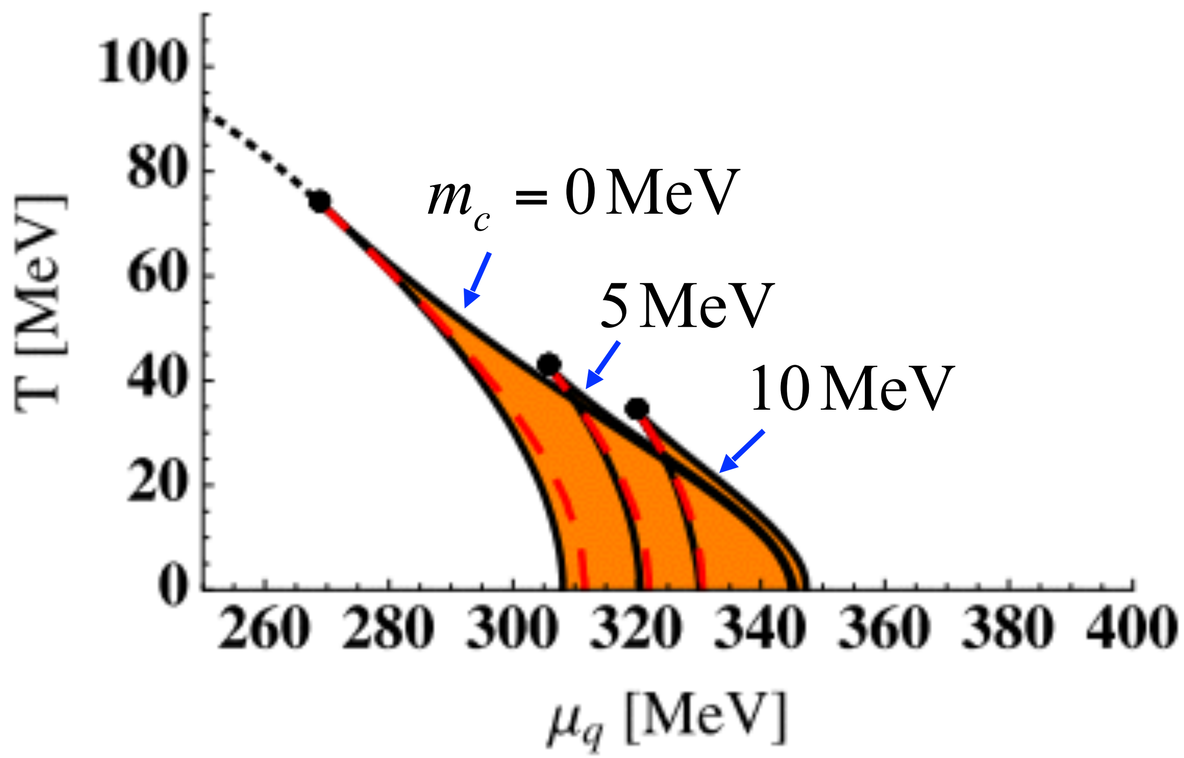
However,
DCDW is not a solution off the chiral limit.

Effect of the finite current quark mass (B=0)

RKC

RKC is a solution even if m_c is finite.

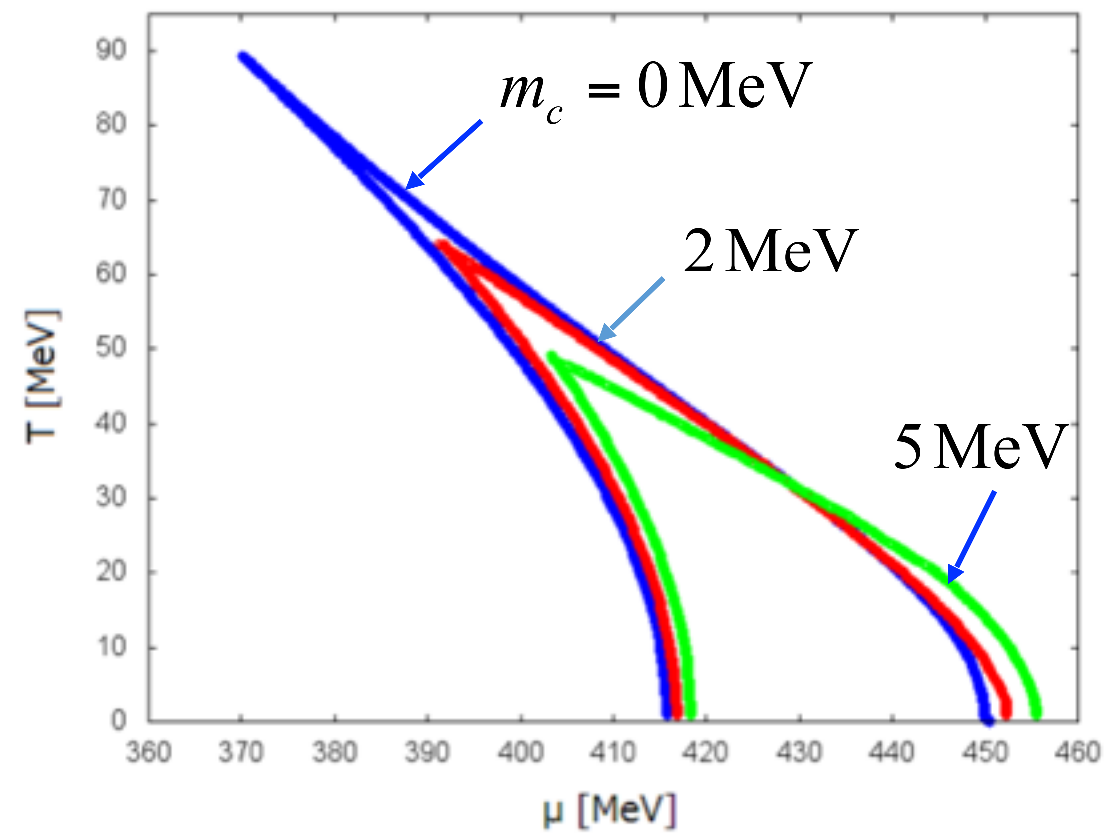
O. Schnetz, et al., Ann. Phys. 321, 2604 (2006)



D. Nickel, PRD 80, 074025 (2009)

DCDW

The deformation of DCDW is considered by the variational method.



S. Karasawa, T. Tatsumi, PRD 92, 116004 (2015)

The current quark mass reduces the region of the inhomogeneous chiral phase.

Goal

To consider the more realistic system,

We analyze the phase structure in the magnetic field including the finite m_c.

Analysis of the phase transition by the generalized Ginzburg-Landau expansion

m_c disfavors the inhomogeneous chiral phase.

↕

The magnetic field favors the DCDW phase.

Because it is expected the DCDW phase appears in the high T and low μ region,

We discuss the possibility of the observation in the lattice QCD.

K. Kashiwa, T.-G. Lee, K. Nishiyama and RY, arXiv:1507.08382 (2015)

The methods to explore the region at $\mu \neq 0$

- The Taylor expansion method
- The analytic continuation method from imaginary μ
- The reweighting method
- The canonical approach

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Generalized Gizburg-Landau expansion with m_c

Inhomogeneous chiral condensate： $M(\mathbf{r}) = m\, e^{i\theta(z)}$

consider only d.o.f. of the phase, which is important in B

NJL Lagrangian within the mean field approximation

$$\mathcal{L}_{\rm MF} = \bar{\psi}\left[i\gamma^\mu D_\mu \underbrace{- m_c - m\left(\cos\theta(z) + i\gamma^5\tau^3 \sin\theta(z)\right)}_{\widetilde{M}}\right]\psi - \frac{m^2}{4G}$$

We expand Ω about m and the derivative of θ up to the 4th order, and m_c up to the 1st order

$$\Omega(\mu, T, B) = \Omega_0 + \int \frac{d^3\mathbf{x}}{V}\left[\alpha_1 m\cos\theta + \frac{\alpha_2}{2}m^2 + \alpha_3 m^3\cos\theta + \tilde{\alpha}_3 m^2\theta' + \frac{\alpha_4}{4}\left(m^4 + m^2\theta'^2\right)\right]$$

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proportional to m_c (α_1, α_3)

$\tilde{\alpha}_3 \neq 0$ only when B≠0 and μ≠0

※The coefficients except for $\tilde{\alpha}_3$ have the UV divergence.

Determination of the configuration by the stationary condition

Stationary condition

$$m_*^2 \equiv -2\frac{\alpha_1 + m^2\alpha_3}{m\alpha_4}$$

$$\frac{\delta\Omega}{\delta\theta(z)} = 0 \Rightarrow \theta'' - m_*^2 \sin\theta = 0$$ (sine-Gordon equation)

$$\Rightarrow \theta(z) = 2\mathrm{am}\left(\frac{m_*}{k}z, k\right) + \pi$$

period $$l = \frac{2kK(k)}{m_*}$$

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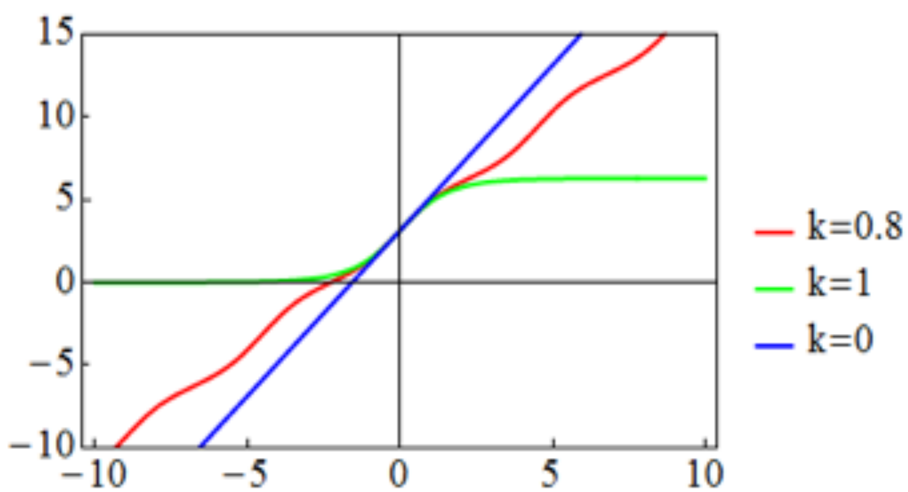
θ(z)



z

k=1 : equivalent to the homogeneous solution

0<k<1 : massive DCDW

※ $k, m_c \to 0$: equivalent to DCDW $\frac{2m_*}{k} \to q$

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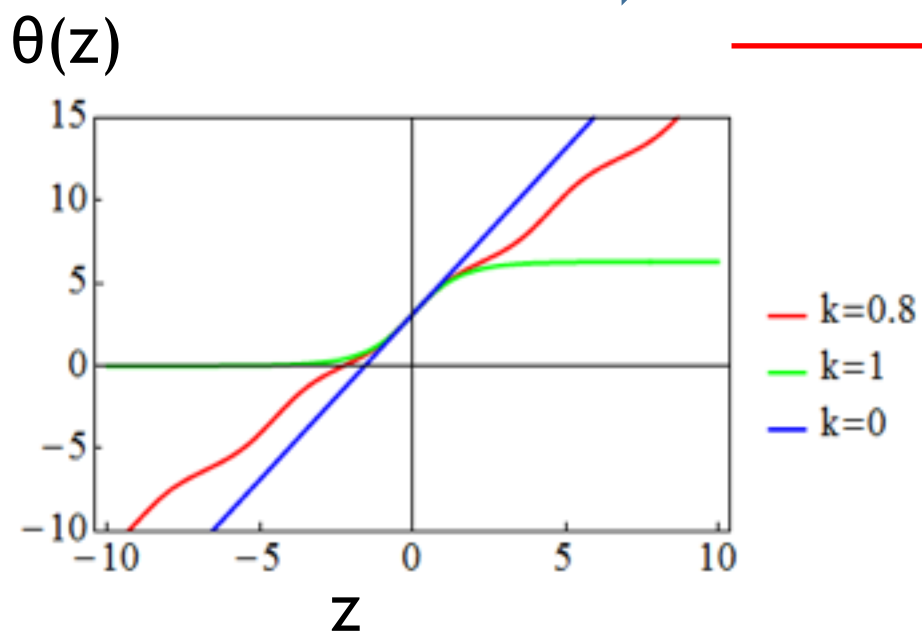


k=1 : equivalent to the homogeneous solution

0<k<1 : massive DCDW

※ $k, m_c \to 0$: equivalent to DCDW $\frac{2m_*}{k} \to q$

Thermodynamic potential

$$\Omega = \Omega_0 + \left(\alpha_1 m + \alpha_3 m^3\right) C_1(k) + \frac{\alpha_2}{2}m^2 + \tilde{\alpha}_3 m_* m^2 C_3(k) + \frac{\alpha_4}{4}m^4$$

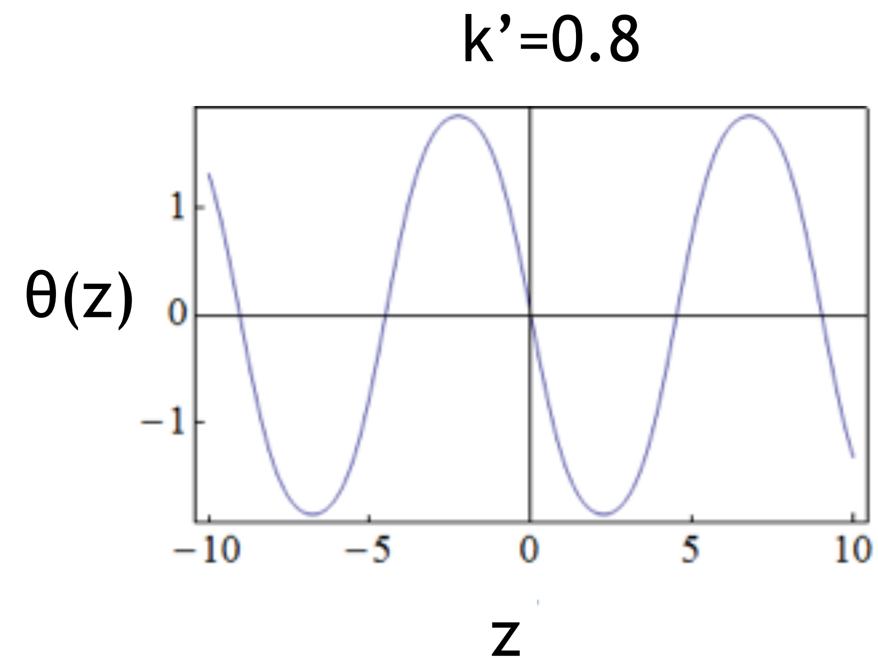
$$C_1(k) \equiv \frac{2}{k^2} - 1 - \frac{4E(k)}{k^2K(k)}, \quad C_3(k) \equiv \frac{\pi}{2kK(k)}$$

Order parameters : m, Q=2π/l

Other solution (oscillating solution)

$$\theta'' - m_*^2 \sin\theta = 0$$

$$\Rightarrow \quad \theta_{\mathrm{os}}(z) = 2\cos^{-1}\left[k' \,\mathrm{sn}\left(m_* z, k'\right)\right] - \pi$$



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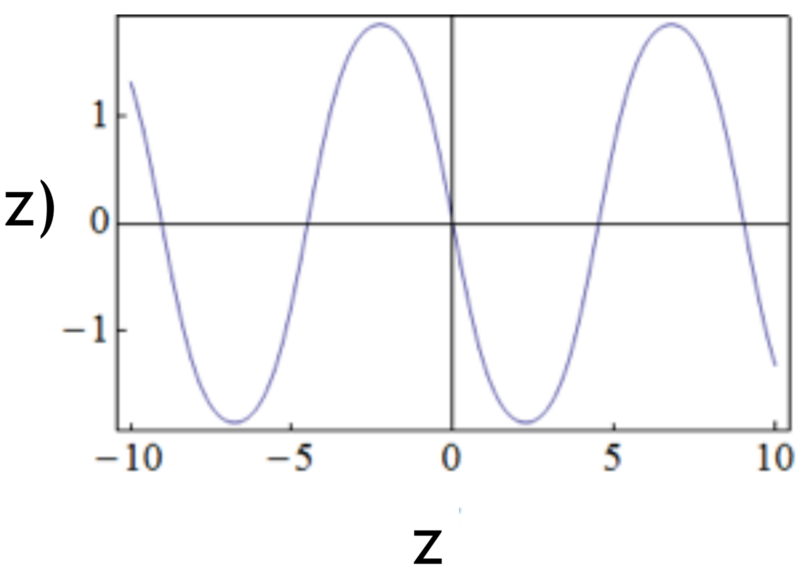
k’=0.8



Thermodynamic potential

$$\Omega = \Omega_0 + \left(\alpha_1 m + \alpha_3 m^3\right) C_1^{\rm os}(k') + \frac{\alpha_2}{2} m^2 + \frac{\alpha_4}{4} m^4$$

$\tilde{\alpha}_3$ term vanishes after the spatial integral.

$$C_1^{\rm os}(k') \equiv 3 - 2k'^2 - \frac{4E(k')}{K(k')}$$

→ Ω always takes minimum value at k’=1.

→ The homogeneous solution is always favored.

About anomaly

Anomalous particle number   A. J. Niemi, G. W. Semenoff, Phys. Rep. 135, 99 (1986)

$$N = -\frac{1}{2}\eta_H + \int dE \rho(E) \left[\frac{\theta(E)}{1 + e^{\beta(E-\mu)}} - \frac{\theta(-E)}{1 + e^{-\beta(E+\mu)}} \right]$$

anomalous   particle   anti-particle

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anomalous particle anti-particle

$$\eta_H = \lim_{s\to+0}\int dE\rho(E)\mathrm{sign}(E)|E|^{-s} \sim -\sum_f \frac{|e_f|}{2\pi^2}\int d^3\mathbf{x}\,\mathbf{B}\cdot\nabla\theta$$

Spectral asymmetry

T. Tatsumi, K. Nishiyama and S. Karasawa, PLB 743, 66 (2015)

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anomalous — particle — anti-particle

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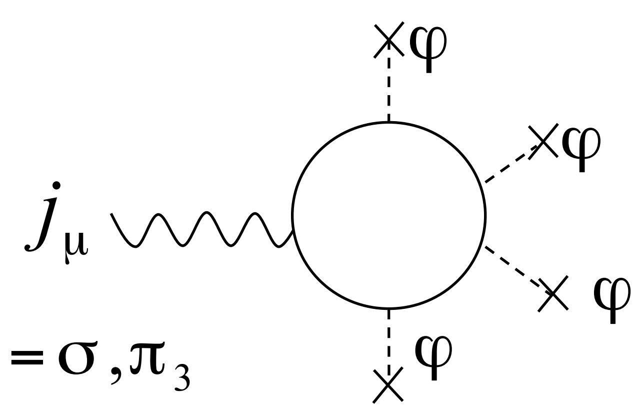
Chiral anomaly (WZW term)

D. T. Son, M. A. Stephanov, PRD 77, 014021 (2008)

$$S_{WZW} = -\int d^4x\left(A_\mu^B + \frac{e}{2}A_\mu\right)j_B^\mu = \frac{e}{4\pi^2}\int d^4x\,\mu\,\mathbf{B}\cdot\nabla\theta$$



$\varphi = \sigma, \pi_3$

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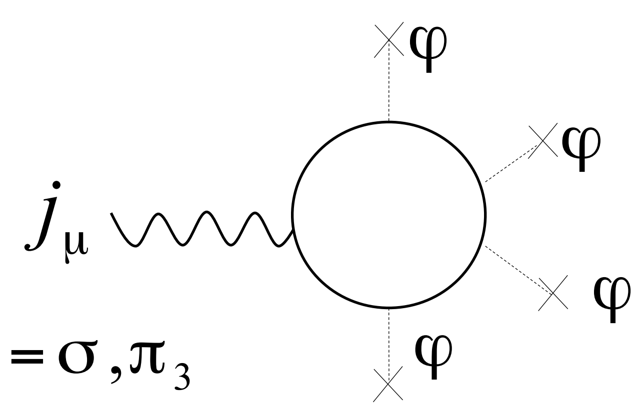

$\varphi = \sigma, \pi_3$

It seems the low order contribution of the GL expansion.

Anomaly and the generalized GL expansion

Particle number $$N = -V\frac{\partial\Omega}{\partial\mu}$$

$$N = -\int dE\rho(E)T\sum_k \frac{1}{E-\mu-i\omega_k}$$

Anomaly and the generalized GL expansion

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Local density of state : $\rho(\mathbf{x}, E) = \frac{1}{\pi}\mathrm{Im}\,\mathrm{tr}_{D,f,c}\left\langle \mathbf{x} \left| \frac{1}{H-E-i\epsilon} \right| \mathbf{x} \right\rangle$

Hamiltonian: $H = \vec{\alpha}\cdot\mathbf{P} + \gamma^0\left[m_c + m e^{i\gamma^5\tau_3\theta(\mathbf{r})}\right]$

Anomaly and the generalized GL expansion

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Hamiltonian: $H = \vec{\alpha}\cdot\mathbf{P} + \gamma^0\left[m_c + me^{i\gamma^5\tau_3\theta(\mathbf{r})}\right]$

the 1st order about $\nabla\theta$

$x = \sqrt{E^2 - m^2}$

$$N_{\nabla\theta} = \frac{N_c}{4\pi^2}\sum_f |e_f B|\partial_z\theta\left\{1 + \int_0^\infty dx\frac{\partial}{\partial\mu}\left[-\frac{1}{1+e^{\beta(\sqrt{x^2+m^2}-\mu)}} + \frac{1}{1+e^{\beta(\sqrt{x^2+m^2}+\mu)}}\right]\right\} + \mathcal{O}(m_c^2)$$

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Particle number $N = -V \frac{\partial \Omega}{\partial \mu}$

$$N = -\int dE \rho(E) T \sum_k \frac{1}{E - \mu - i\omega_k}$$

Local density of state : $\rho(\mathbf{x}, E) = \frac{1}{\pi} \mathrm{Im}\, \mathrm{tr}_{D,f,c} \left\langle \mathbf{x} \left| \frac{1}{H - E - i\epsilon} \right| \mathbf{x} \right\rangle$

Hamiltonian: $H = \vec{\alpha} \cdot \mathbf{P} + \gamma^0 \left[m_c + m e^{i\gamma^5 \tau_3 \theta(\mathbf{r})} \right]$

the 1st order about $\nabla\theta$

$x = \sqrt{E^2 - m^2}$

$$N_{\nabla\theta} = \frac{N_c}{4\pi^2} \sum_f |e_f B| \partial_z \theta \left\{ 1 + \int_0^\infty dx \frac{\partial}{\partial \mu} \left[-\frac{1}{1 + e^{\beta(\sqrt{x^2+m^2}-\mu)}} + \frac{1}{1 + e^{\beta(\sqrt{x^2+m^2}+\mu)}} \right] \right\} + \mathcal{O}(m_c^2)$$

In $m \to \infty$

(The theory where quarks are decoupled)

anomalous particle number
(WZW term)

Expand about m

m^0 term cancels out.

m^2 term coincides with $\tilde{\alpha}_3$ term.

Anomaly and the generalized GL expansion

Particle number $N = -V\frac{\partial \Omega}{\partial \mu}$

$$N = -\int dE \rho(E) T \sum_k \frac{1}{E - \mu - i\omega_k}$$

Local density of state : $\rho(\mathbf{x}, E) = \frac{1}{\pi} \mathrm{Im}\, \mathrm{tr}_{D,f,c} \left\langle \mathbf{x} \left| \frac{1}{H - E - i\epsilon} \right| \mathbf{x} \right\rangle$

Hamiltonian: $H = \vec{\alpha} \cdot \mathbf{P} + \gamma^0 \left[m_c + m e^{i\gamma^5 \tau_3 \theta(\mathbf{r})} \right]$

the 1st order about $\nabla\theta$

$x = \sqrt{E^2 - m^2}$

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In $m \to \infty$

(The theory where quarks are decoupled)

anomalous particle number
(WZW term)

Expand about m

m^0 term cancels out.

m^2 term coincides with $\tilde{\alpha}_3$ term.

Ω is reproduced by including anomaly.

- Introduction
- Construction of the thermodynamic potential and anomaly
- **Numerical results**
- Discussion and summary

The vacuum part is regularized by the Pauli-Villars regularization.

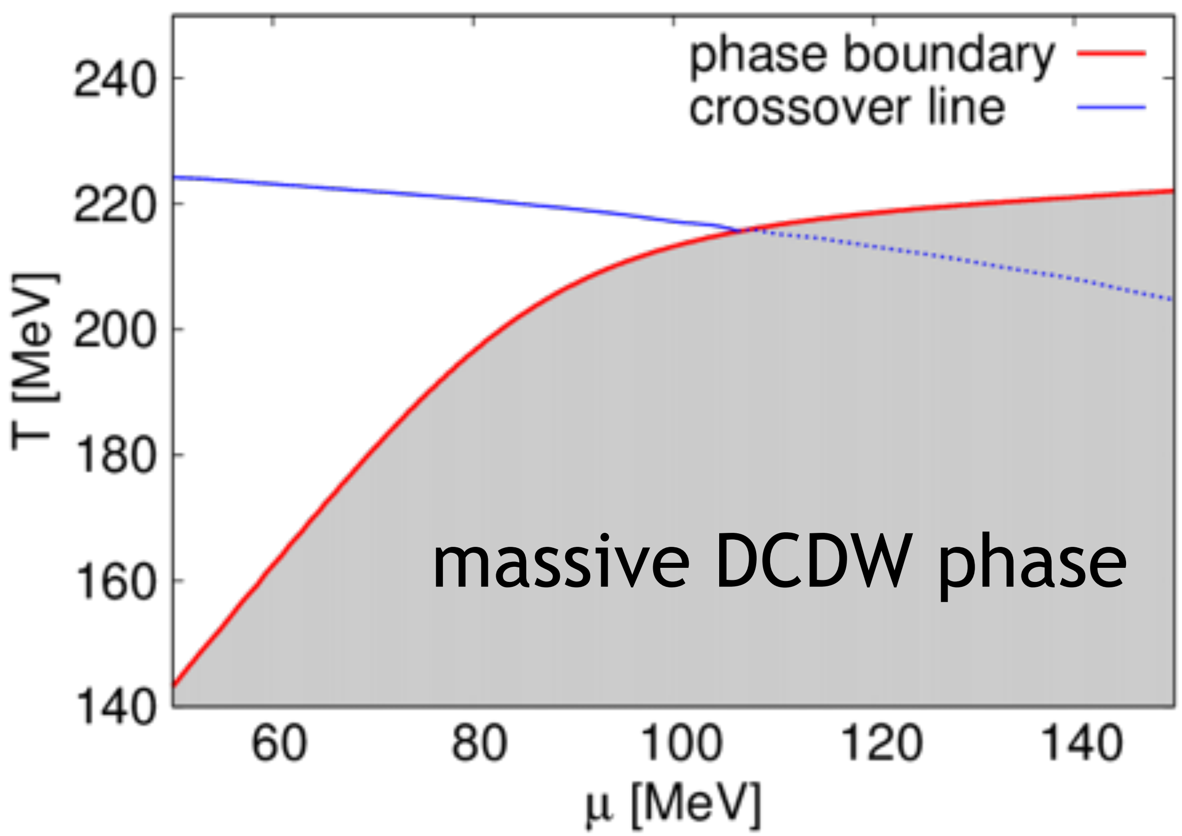
parameter set : $\Lambda = 851MeV,\ G\Lambda^2 = 2.87,\ m_c = 5.2MeV$

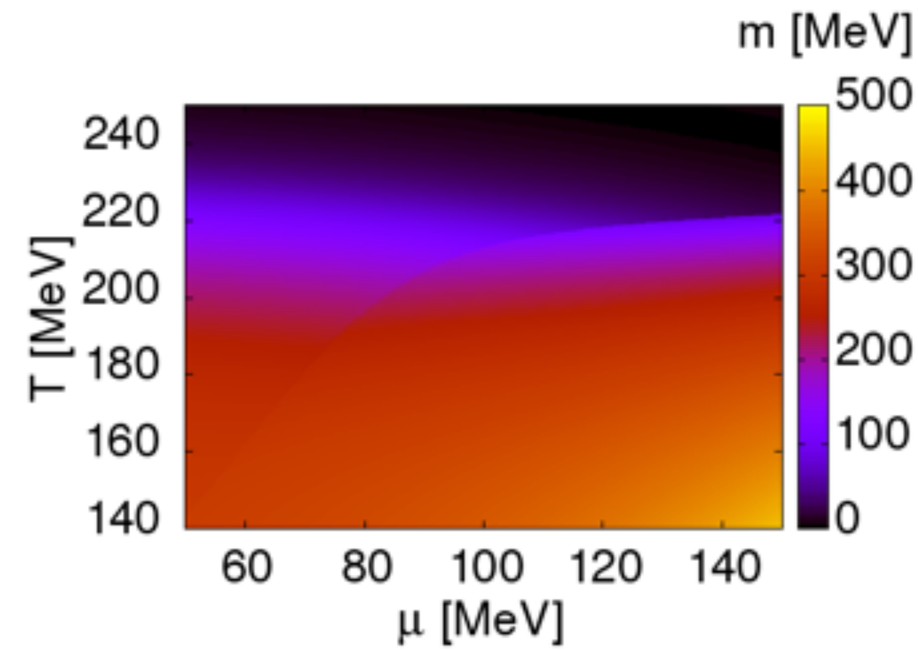
$$f_\pi = 93MeV,\ m_\pi = 135MeV,\ \langle\bar{\psi}\psi\rangle = (-250MeV)^3$$

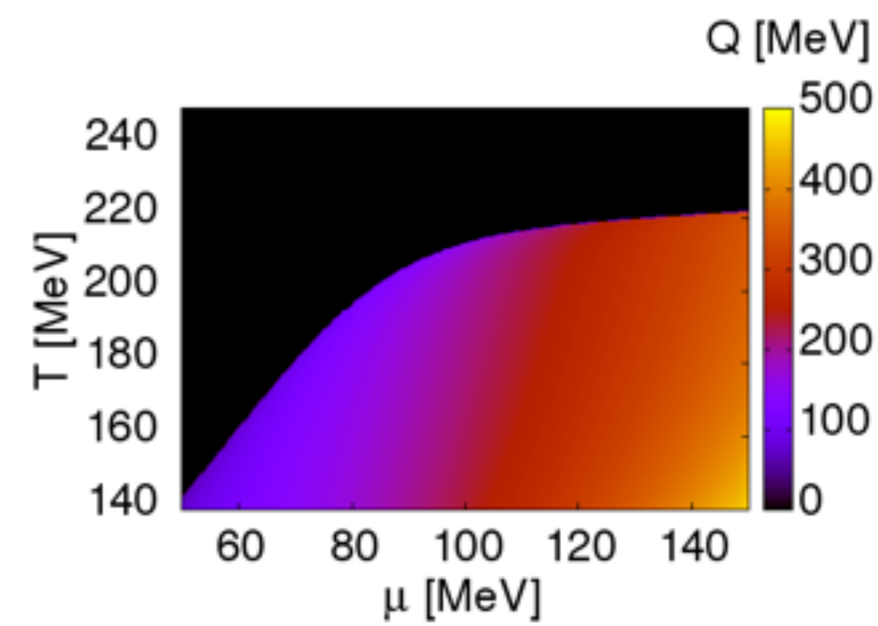
S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992)

Phase diagram

$m_c = 5\,\mathrm{MeV},\ \sqrt{eB} = 1\,\mathrm{GeV}$





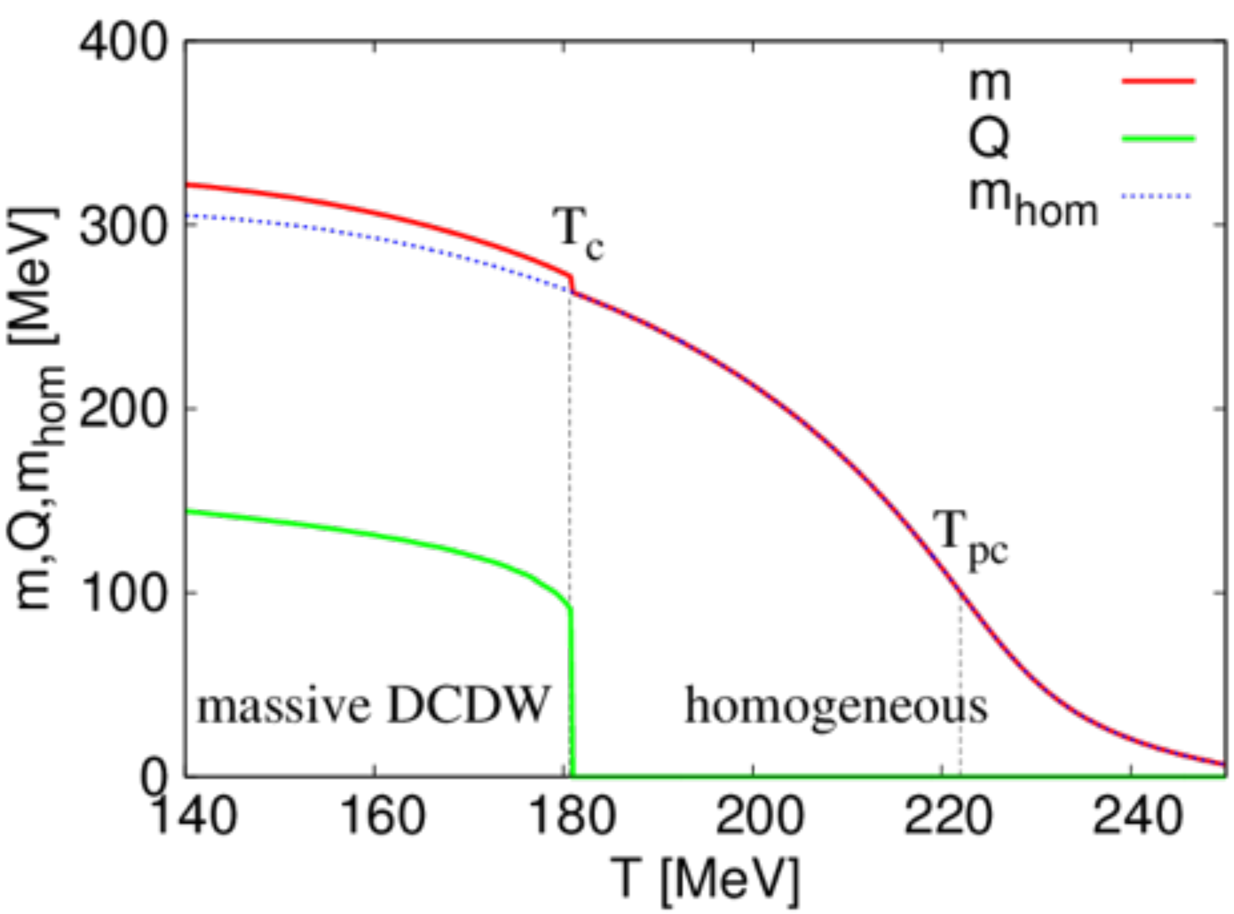


The massive DCDW phase appears in the high T and low μ region due to B.

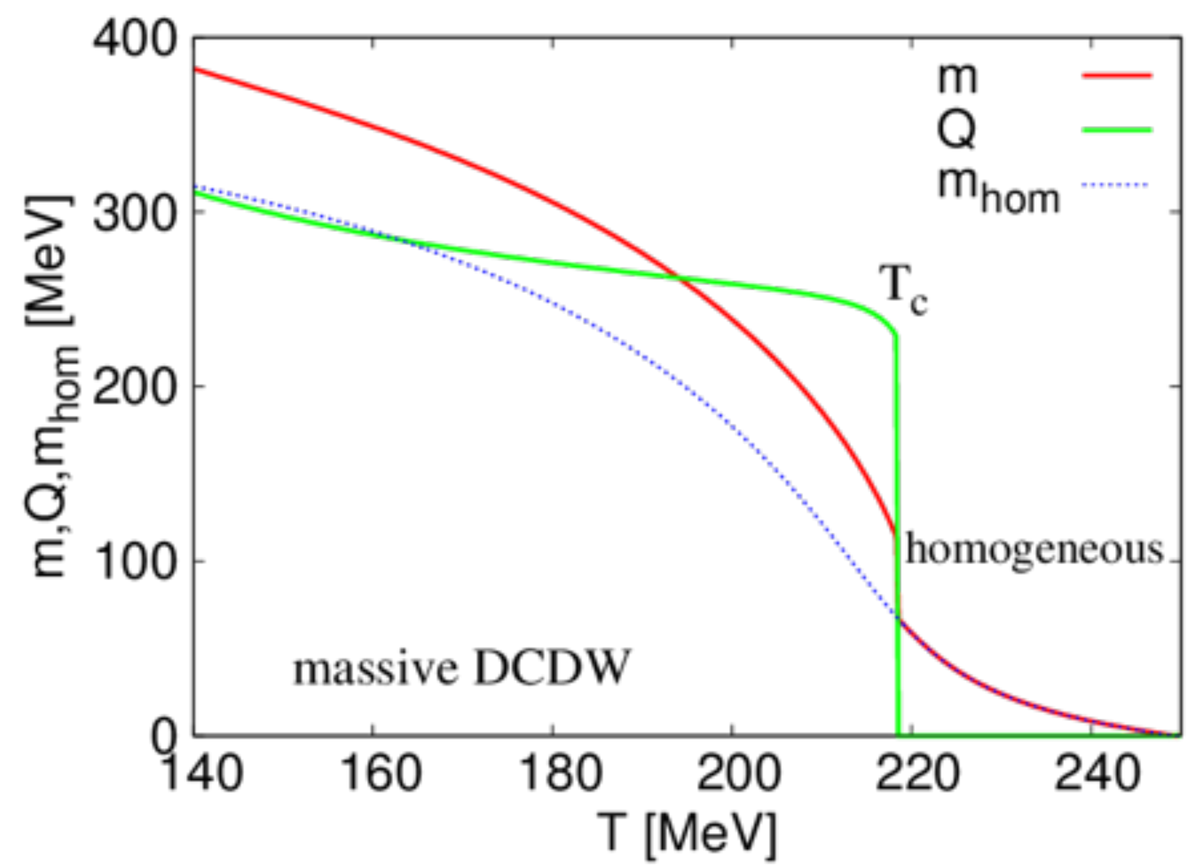
The region of the phase has been limited in $\mu \gtrsim 300\mathrm{MeV},\ \ T \lesssim 50\mathrm{MeV}$.

Change of the order parameters

$\mu = 70\,\mathrm{MeV}$



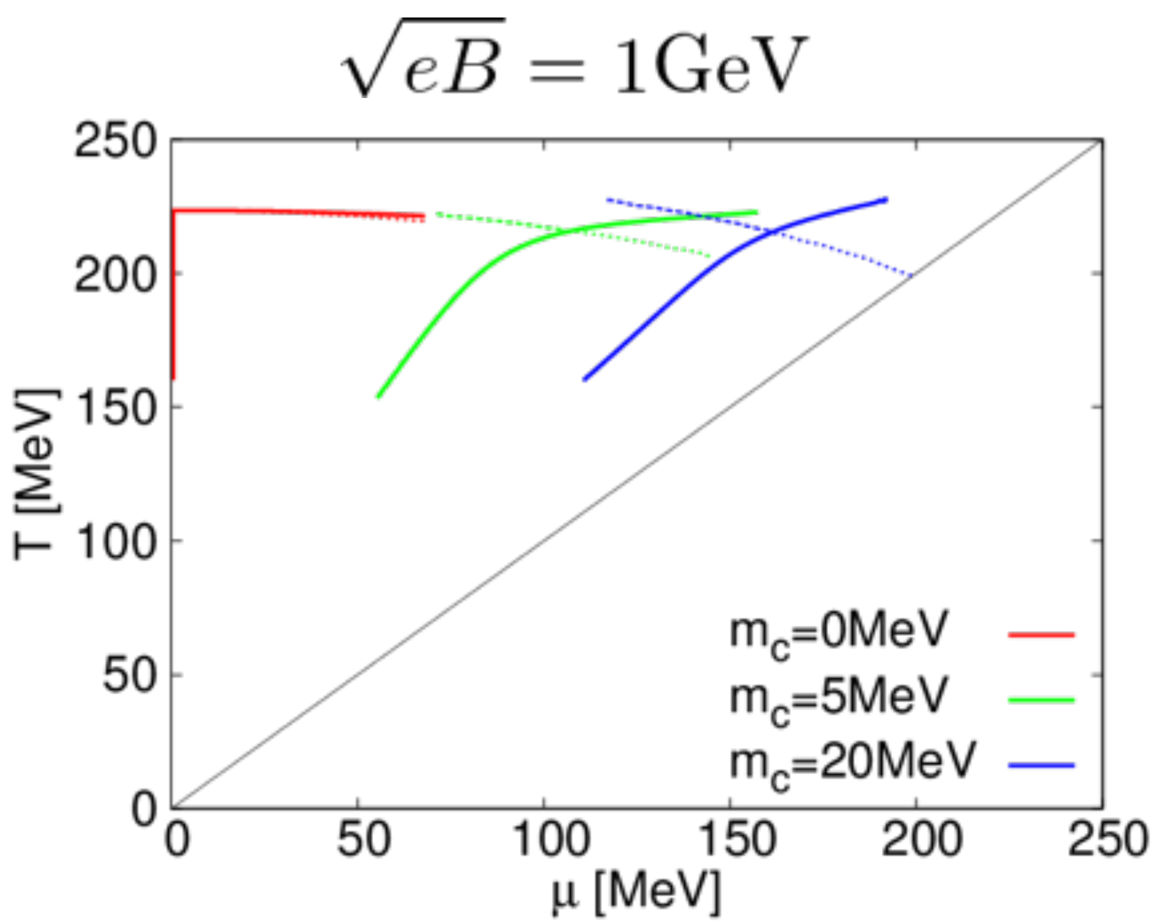
$\mu = 120\,\mathrm{MeV}$



- There is the 1st order phase transition and discontinuity of m and Q.
- The 1st order phase transition becomes weaker for lower μ.
- The dynamical mass in the massive DCDW phase is larger than the conventional one.

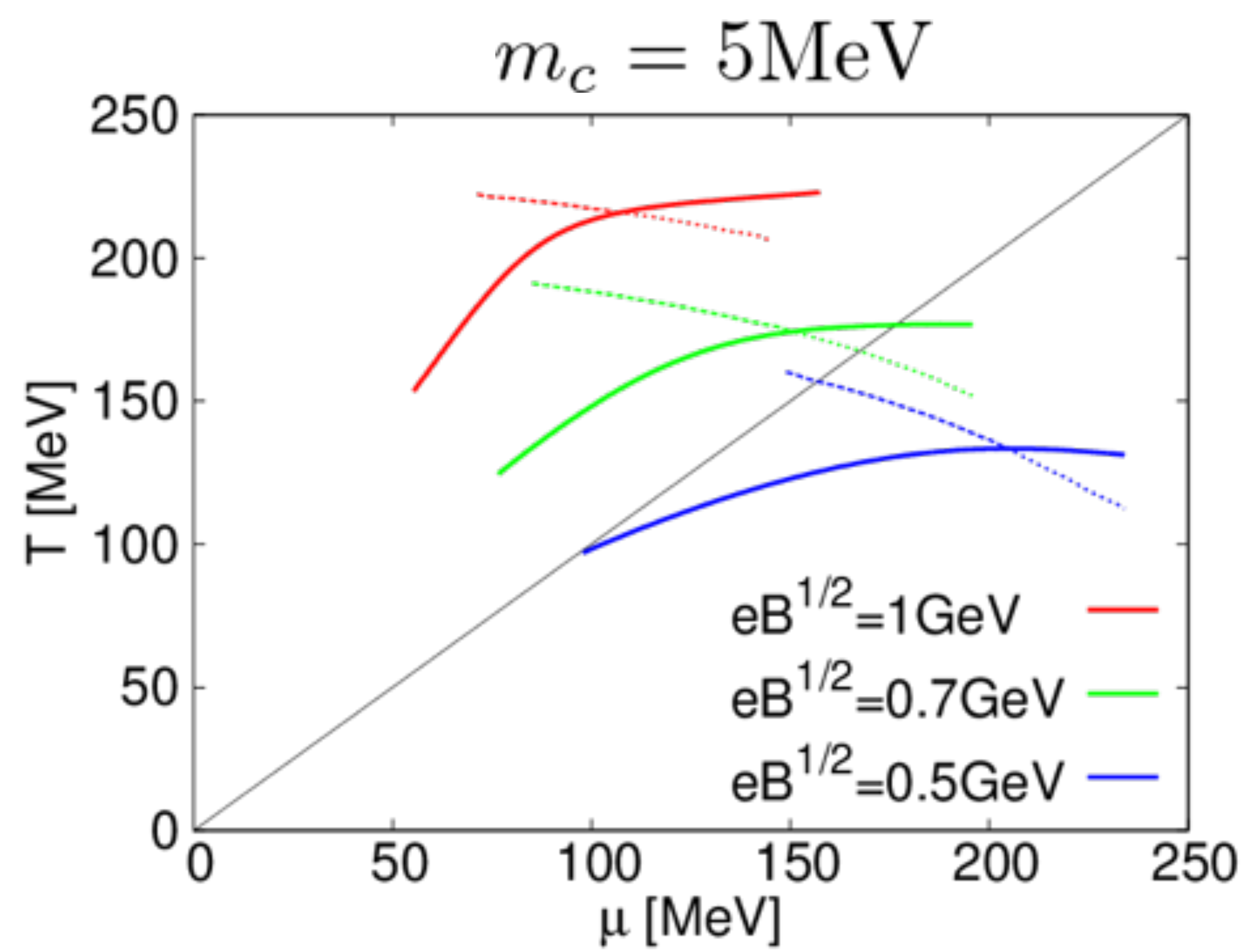
Change of the location of the phase boundary

m_c changes under the fixed B.



The massive DCDW phase is extended to the low μ region with the decrease of m_c.

B changes under the fixed m_c.



B raises the critical temperature.

※It is consistent with the magnetic catalysis.

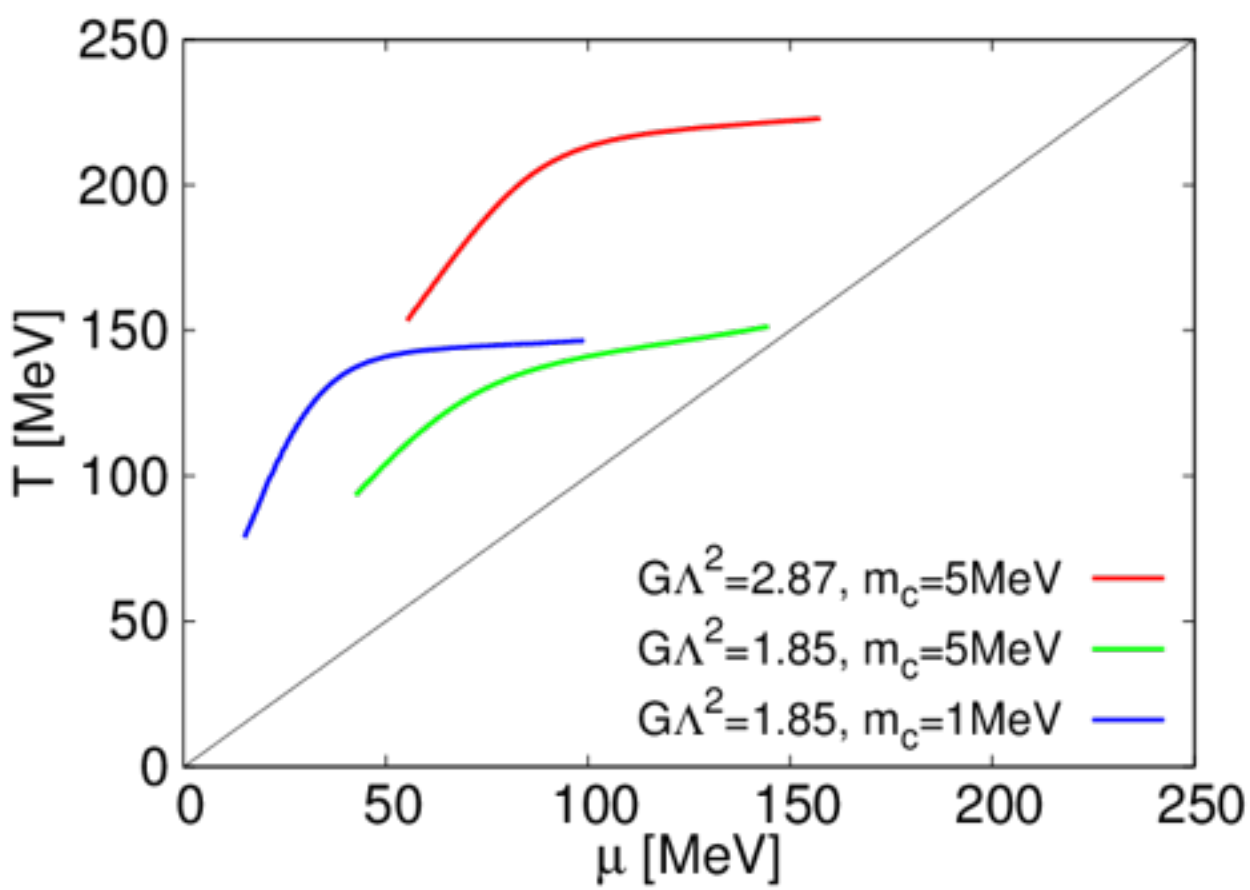
Inverse magnetic catalysis

It is assumed the coupling constant depends on B, where the result of the lattice simulation is reproduced at μ=0.

M. Ferreira et al., PRD 89, 116011 (2014)

$$\frac{T_{\rm pc}(\sqrt{eB}=1{\rm GeV})}{T_{\rm pc}(\sqrt{eB}=0)}=0.86 \quad \Rightarrow \quad G(\sqrt{eB}=1{\rm GeV})\Lambda^2=1.85$$

G. S. Bali and et al., PRD 86, 071502 (2012)



The critical temperature decreases due to the inverse magnetic catalysis.

The massive DCDW phase remains in the μ/T<1 region even if m_c is sufficiently small.

- Introduction
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Possibility of the observation in the lattice QCD

The Taylor expansion method
The analytic continuation method from imaginary μ

Because these method cannot describes singularity at μ≠0,
the massive DCDW phase with the 1st order phase transition cannot be grasped.

The reweighting method

Because the massive DCDW phase appears only in the region at μ≠0,
the overlap problem seems to be serious.

The canonical approach

The 1st order phase transition may be found,
but the phase transition cannot be identified as one to the massive DCDW phase.

Summary

- We investigate the structure of the inhomogeneous chiral phase in B and find the massive DCDW phase appears in the high T and low μ region even if m_c is finite.

- We discuss the possibility of the observation of the massive DCDW phase in the lattice QCD.

Perspective

Behavior in the low T and high μ region

➡ Extending the 3-flavor, Effect on compact stars

Observation of the lattice QCD

➡ Method of finding the inhomogeneity
Attempt in 2-color QCD

Appendix

Exploring the region at μ≠0 in the lattice QCD

The Taylor expansion method C. R. Allton, et al., PRD 71, 054508 (2005)

$$\langle\mathcal{O}(\mu)\rangle = \langle\mathcal{O}(\mu=0)\rangle + \frac{\partial\langle\mathcal{O}(\mu)\rangle}{\partial\mu}\Big|_{\mu=0}\mu + \frac{1}{2}\frac{\partial^2\langle\mathcal{O}(\mu)\rangle}{\partial\mu^2}\Big|_{\mu=0}\mu^2 + \cdots$$

Each coefficients are sign problem free.

The analytic continuation method from imaginary μ

P. de Forcrand, O. Philipen, Nucl. Phys. B 642, 290 (2002)

$$\langle\mathcal{O}(\mu = i\mu_I)\rangle$$

Observables are sign problem free at imaginary μ.

The reweighting method Z. Fodor, S. D. Katz, PLB 534, 87 (2002)

$$Z(\alpha) = \int \mathcal{D}\phi\, e^{-S_b(\alpha_0,\phi)}\det M(\alpha_0,\phi)\left[e^{-S_b(\alpha,\phi)+S_b(\alpha_0,\phi)}\frac{\det M(\alpha,\phi)}{\det M(\alpha_0,\phi)}\right]$$

The importance sampling is carried out for some parameter set, where there is no sign problem.

The canonical approach A. Alexandru, et al., PRD 72, 114513 (2005)

$$Z(V,T,\mu) = \sum_n Z_C(V,T,n)e^{n\mu/T}$$

The canonical partition function is considered

Coefficients

$$\alpha_2 = -2N_c \sum_f T \sum_k \frac{|e_f B|}{2\pi} \sum_{n\geq 0} \int \frac{dp}{2\pi} \frac{2-\delta_{n,0}}{(\omega_k + i\mu)^2 + p^2 + 2|e_f B| n} + \frac{1}{2G}$$

$$\alpha_4 = 2N_c \sum_f T \sum_k \frac{|e_f B|}{2\pi} \sum_{n\geq 0} \int \frac{dp}{2\pi} \frac{2-\delta_{n,0}}{\left[(\omega_k + i\mu)^2 + p^2 + 2|e_f B| n\right]^2}$$

$$\alpha_1 = m_c \left(\alpha_2 - \frac{1}{2G}\right)$$

$$\alpha_3 = m_c \alpha_4$$

$$\tilde{\alpha}_3 = N_c \sum_f \frac{|e_f B|}{16\pi^3 T} \mathrm{Im}\psi^{(1)} \left(\frac{1}{2} + i\frac{\mu}{2\pi T}\right)$$

regularization

α_2 and α_4 have the divergence.

$$\alpha_{2j} = (-1)^j N_c \sum_f \frac{|e_f B|}{2\pi} I_j(0)$$

$$I_j(\Lambda^2) \equiv 2T \sum_k \sum_{n\geq 0} \int \frac{dp}{2\pi} \frac{2-\delta_{n,0}}{[(\omega_k + i\mu)^2 + E_n^2(\Lambda^2)]^j} \qquad E_n(\Lambda^2) \equiv \sqrt{p^2 + \Lambda^2 + 2|e_f B| n}$$

Taking the Matsubara frequency,

$$I_1 = \sum_{n\geq 0} \int \frac{dp}{2\pi} \frac{2-\delta_{n,0}}{E_n} [1 - f_F(E_n+\mu) - f_F(E_n-\mu)]$$

$$I_2 = \frac{1}{2} \sum_{n\geq 0} (2-\delta_{n,0}) \int \frac{dp}{2\pi} \left\{ \frac{1}{E_n^3} [1 - f_F(E_n+\mu) - f_F(E_n-\mu)] + \frac{1}{E_n^2} [f_F'(E_n+\mu) + f_F'(E_n-\mu)] \right\}$$

$$I_{1,\mathrm{vac}} = \sum_{n\geq 0} \int \frac{dp}{2\pi} \frac{2-\delta_{n,0}}{E_n} \qquad I_{2,\mathrm{vac}} = \frac{1}{2} \sum_{n\geq 0} \int \frac{dp}{2\pi} \frac{2-\delta_{n,0}}{E_n^3}$$

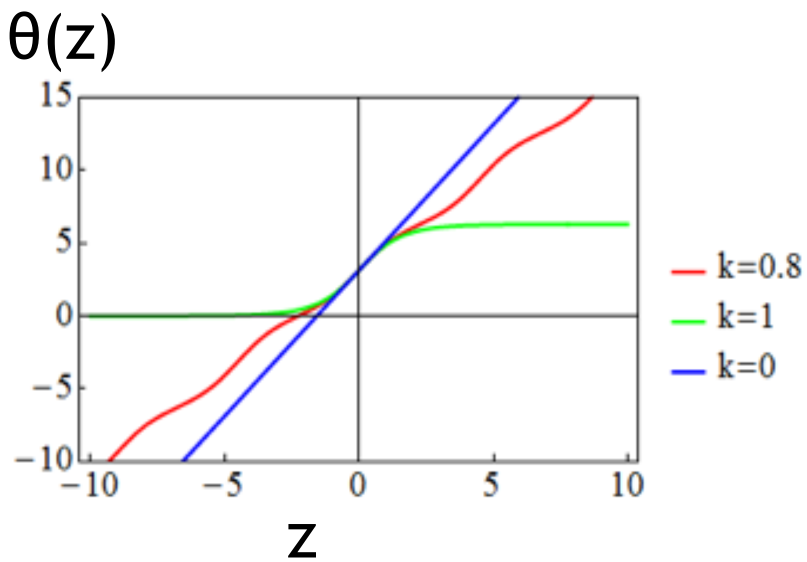
PVR ⇒

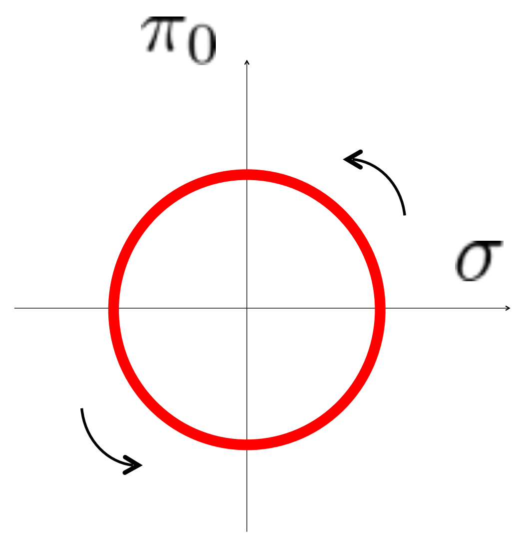
$$I_{1,\mathrm{vac}}(0) \to I_{1,\mathrm{vac}}(0) - 2I_{1,\mathrm{vac}}(\Lambda^2) + I_{1,\mathrm{vac}}(2\Lambda^2)$$

$$I_{2,\mathrm{vac}}(0) \to I_{2,\mathrm{vac}}(0) - I_{2,\mathrm{vac}}(\Lambda^2)$$

Topological configuration of solutions and anomaly

Massive DCDW



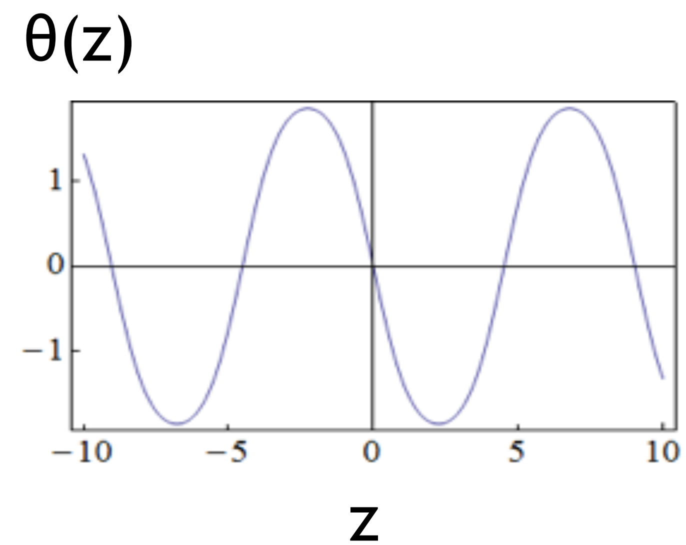


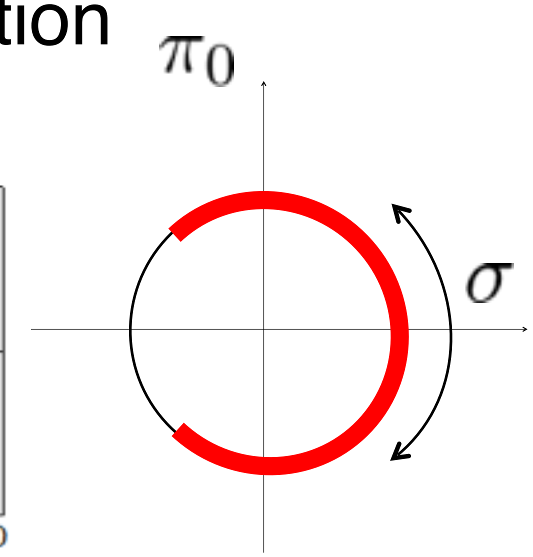
$$N_{\mathrm{anom}} = \sum_f \frac{e_f B}{4\pi^2} \int_{L^3} d^3\mathbf{x}\, \partial_z \theta = \sum_f \frac{e_f B}{2\pi} L_x L_y \frac{L_z}{l}$$

The winding number

With spectral asymmetry,
the massive DCDW is favored in B due to $\widetilde{\alpha}_3$.

Oscillating solution





$$N_{\mathrm{anom}} = \sum_f \frac{e_f B}{4\pi^2} \int_L d^3\mathbf{x}\, \partial_z \theta = 0$$

The winding number=0

Without spectral asymmetry,
the oscillating solution is not favored
because $\widetilde{\alpha}_3$ term vanishes.

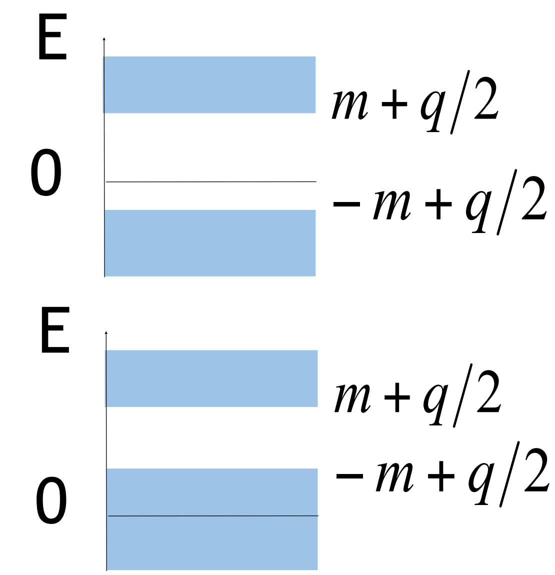
It is possibly favored in the high μ region.

In the case of DCDW in B

$$\eta_H = \begin{cases} -N_c|eB|q/2\pi^2 \\ -N_c|eB|q/2\pi^2 + \frac{N_c|eB|}{\pi^2}\sqrt{q^2/4 - m^2} \end{cases}$$

T. Tatsumi, K. Nishiyama and S. Karasawa, PLB 743, 66 (2015)

The spectrum of LLL



⇔ chiral anomaly (WZW term) $\Omega_{\mathrm{WZW}} = -\frac{N_c}{4\pi^2}\mu eBq$

Expanding about m,

$$\eta_H = -\frac{N_c|eB|}{\pi^2}\frac{m^2}{q} + \mathcal{O}(m^4)$$

$N_{\nabla\theta}$ の導出

Weinberg transformation $\psi \to \psi_W = e^{i\gamma^5\tau_3\theta(\mathbf{r})/2}\psi$

$$H \to \tilde{H} = \tilde{H}_0 + \delta\tilde{H}$$

$$\tilde{H}_0 \equiv \vec{\alpha}\cdot\mathbf{P} + \gamma_0 m$$

$$\delta\tilde{H} \equiv \gamma^0\left[m_c e^{-i\gamma^5\tau_3\theta(\mathbf{r})} - \frac{1}{2}\gamma^5\tau_3\vec{\gamma}\cdot\nabla\theta(\mathbf{r})\right]$$

Expanding the local density of state about $\delta\tilde{H}$,

$$\rho(\mathbf{x},E) = \frac{N_c}{\pi}\sum_f \mathrm{Im\,tr}\left\langle\mathbf{x}\left|\frac{1}{\tilde{H}_0 - E}\right|\mathbf{x}\right\rangle - \underline{\frac{N_c}{\pi}\sum_f \frac{\partial}{\partial E}\mathrm{Im\,tr}\left\langle\mathbf{x}\left|\frac{1}{\tilde{H}_0 - E - i\epsilon}\right|\mathbf{x}\right\rangle\delta\tilde{H}(\mathbf{x})} + \mathcal{O}\left(\partial(\delta\tilde{H}),(\delta\tilde{H})^2\right)$$

$$\Rightarrow \quad \rho_{\partial\theta}(\mathbf{x},E) = -\frac{N_c}{4\pi^2}\sum_f |e_f B|\partial_z\theta(\mathbf{x})\frac{\partial}{\partial E}\left[\frac{|E|}{\sqrt{E^2-m^2}}\theta\left(|E|-m\right)\right]$$

partial integral about E $\quad y = \sqrt{E^2 - m^2}$

$$N_{\partial\theta} = \frac{N_c}{4\pi^2}\sum_f |e_f B|\int d^3\mathbf{x}\partial_z\theta(\mathbf{x})\left\{1 + T\sum_k\int_0^\infty dy\left[\frac{1}{(\sqrt{y^2+m^2}-\mu-i\omega_k)^2} + \frac{1}{(\sqrt{y^2+m^2}+\mu+i\omega_k)^2}\right]\right\}$$