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# Composite particles in medium - effects of substructure

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## Abstract

The role of phase space occupation effects for the formation of two- and three-particle bound states in a dense medium is investigated within an algebraic approach suitable for systems with short-range interactions. While for two-fermion bound states due to Pauli blocking in a dense medium the binding energy is reduced and vanishes at a critical density (Mott effect), for three-fermion bound states it is shown that within a Faddeev approach the Bose enhancement of fermion pairs can partially compensate the Pauli blocking between the fermions. Therefore, three-fermion bound states in a medium can exist as Borromean states beyond the Mott density of the two-fermion bound state.


## The Lipkin Model for Composite Bosons

Introduction of composite boson operators (deuterons as nucleon pairs [1])

$$< R, r|D, 2K > = e^{2iKR}\,\phi(r) = \sum_q g_q\, e^{i(K+q)r_1}\, e^{i(K-q)r_2} \quad (1)$$

$$D^\dagger_{2K} = \sum_q g_q\, a^\dagger_{(K+q)\uparrow}\, a^\dagger_{(K-q)\downarrow},\quad D_{2K'} = \sum_q g_q\, a_{(K'-q)\downarrow}\, a_{(K'+q)\uparrow} \quad (2)$$

$$\left[D_{2K'}, D^\dagger_{2K}\right] = \delta_{KK'} - \Delta_{KK'} \quad (3)$$

$$\Delta_{KK'} = \sum_q g_q\, \{g_{(K'-K+q)}\, a^\dagger_{(2K-K'-q)\downarrow}\, a_{(K'-q)\downarrow} + g_{(K'-K-q)}\, a^\dagger_{(2K-K'+q)\uparrow}\, a_{(K'+q)\uparrow}\}$$

$$\Delta_{KK} = \sum_q g^2_{K-q}\, n_{q\downarrow} + g^2_{q-K}\, n_{q\uparrow} \quad (4)$$

## m Deuteron State

The interaction could be developed within the quasi-spin formalism. That allows the calculation of multi particle states. Now it is only a small step to calculate the result of $V = -\epsilon_D D_{2K} D^\dagger_{2K}$, acting on the $m$- deuteron state

$$V\left(D^\dagger_{2K}\right)^m|0\rangle = -(\epsilon_D/\Omega)\left(\frac{\Omega}{2}\left(\frac{\Omega}{2}+1\right) - \left(\frac{2m-\Omega}{2}\right)^2 + \frac{2m-\Omega}{2}\right)\left(D^\dagger_{2K}\right)^m|0\rangle \quad (5)$$

$$= -m\epsilon_D\left(1 - \frac{m-1}{\Omega}\right)\left(D^\dagger_{2K}\right)^m|0\rangle \quad (6)$$

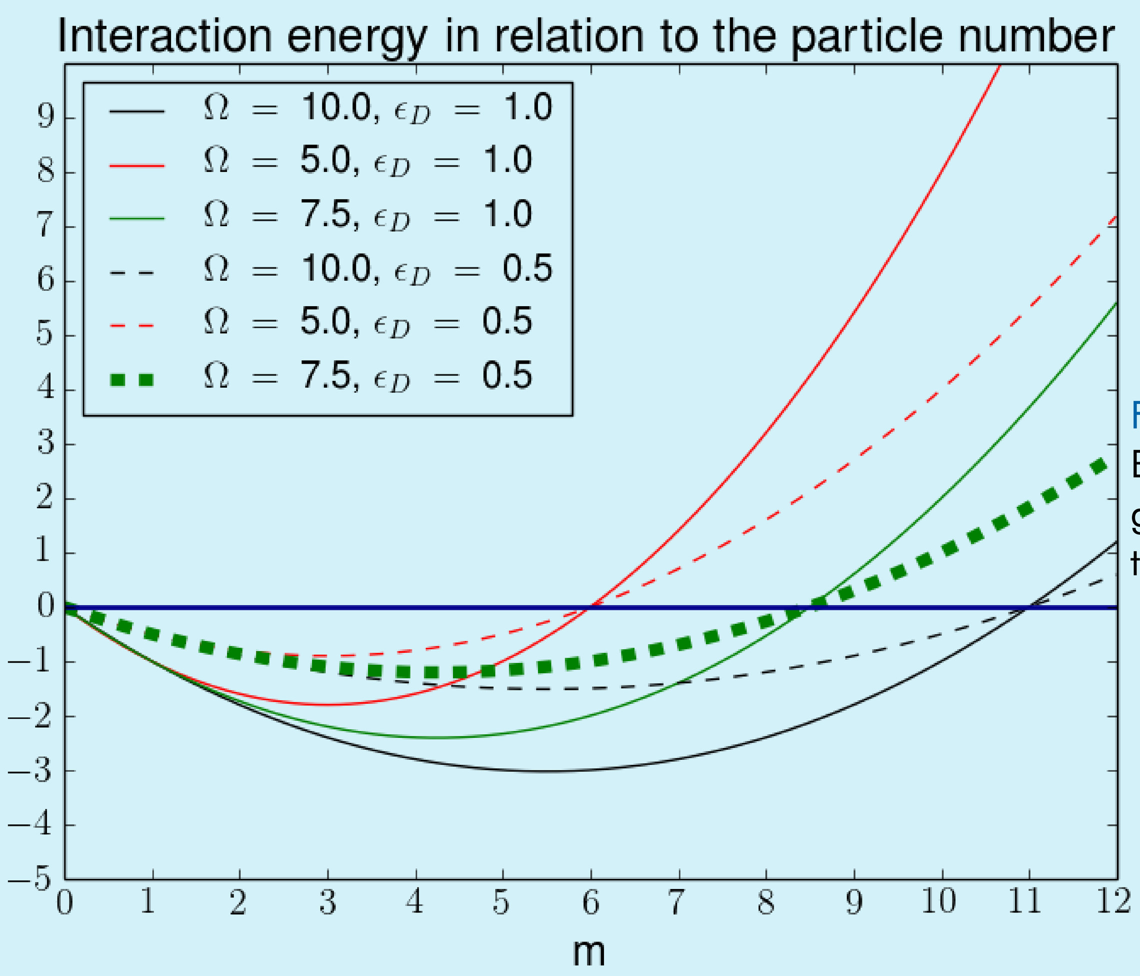


Fig. 1: The figure shows how Bose enhancement evolves with growing particle density and than the Pauli blocking takes over.

## Composite Baryon Model

Baryon composed by fermionic ($a^\dagger$) and bosonic ($b^\dagger$) creation operators for quarks and diquarks, resp.

$$N^\dagger_K = \sum_q g_q\, a^\dagger_{\frac{K}{2}+q} b^\dagger_{\frac{K}{2}-q},\quad N_K = \sum_q g_q\, b_{\frac{K}{2}-q} a_{\frac{K}{2}+q} \quad (7)$$

$$[N_{K'}, N^\dagger_K] = \delta_{K,K'} + \Delta_{K,K'} \quad (8)$$

$$\Delta_{K,K'} = \sum_q g_q\left(g_{\frac{K-K'}{2}-q}\, b^\dagger_{K-\frac{K'}{2}-q}\, b_{\frac{K'}{2}-q} - g_{\frac{K-K'}{2}+q}\, a^\dagger_{K-\frac{K'}{2}+q}\, a_{\frac{K'}{2}+q}\right) \quad (9)$$

The next step is to calculate multi particle states. It is possible to write down quasi spin operators like in the case of bosons, see Blaizot [2]. Phase space occupation factors as in PNJL model [3] show partial compensation of Pauli blocking and Bose enhancement –> baryon bound even when diquarks unbound (borromean state).


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## N-particle T-matrix in ladder approximation

$$T_N = V_N G^0_N T_N = \frac{V_N}{1 - V_N G_N} \quad (10)$$

two-particle propagator

$$G^0_2(\Omega, e_1, e_2) = \sum_{\omega_1} \frac{1}{\omega_1 - e_1} \frac{1}{\Omega - \omega_1 - e_2} = \frac{(1-f_1)(1-f_2) - f_1 f_2}{\Omega - e_1 - e_2} \quad (11)$$

three-particle propagator

$$G^0_3(\Omega, e_1, e_2, e_3) = \sum_{\omega_1\omega_2} \frac{1}{\omega_1 - e_1} \frac{1}{\omega_2 - e_2} \frac{1}{\Omega - \omega_1 - \omega_2 - e_3} = \frac{(1-f_1)(1-f_2)(1-f_3) + f_1 f_2 f_3}{\Omega - e_1 - e_2 - e_3} \quad (12)$$

Evaluation of the numerator in $G^0_2$ for $f = 1/2$ lead to zero. In case of $G^0_3$ one could discuss the influence of the pairing terms.

$$Q_2^{ladder} = 1 - f_1 - f_2 \quad (13)$$
$$Q_2^{brueckner} = (1-f_1)(1-f_2) = 1 - f_1 - f_2 + f_1 f_2 \quad (14)$$
$$Q_3^{linear\ approx} = 1 - f_1 - f_2 - f_3 \quad (15)$$
$$Q_3^{ladder} = 1 - f_1 - f_2 - f_3 + f_1 f_2 + f_2 f_3 + f_1 f_3 \quad (16)$$
$$Q_3^{fermi-bose} = \frac{1}{3}(1 - f_1 + g_{23}) + \frac{1}{3}(1 - f_2 + g_{13}) + \frac{1}{3}(1 - f_3 + g_{23}) \quad (17)$$

| $f_i$ | $g_{ij}$ | $Q_2^{ladder}$ | $Q_2^{brueckner}$ | $Q_3^{linear\ approx.}$ | $Q_3^{ladder}$ | $Q_3^{fermi-bose}$ | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | |
| 0.5 | 0 | 0 | 0.25 | -0.5 | 0.25 | 1.5 | borromean |
| 0.5 | 0.5 | 0 | 0.25 | -0.5 | 0.25 | 2.25 | borromean |

This is consistent with the introduction of 3 particle momenta and the numerical evaluation of the Pauli blocking operators

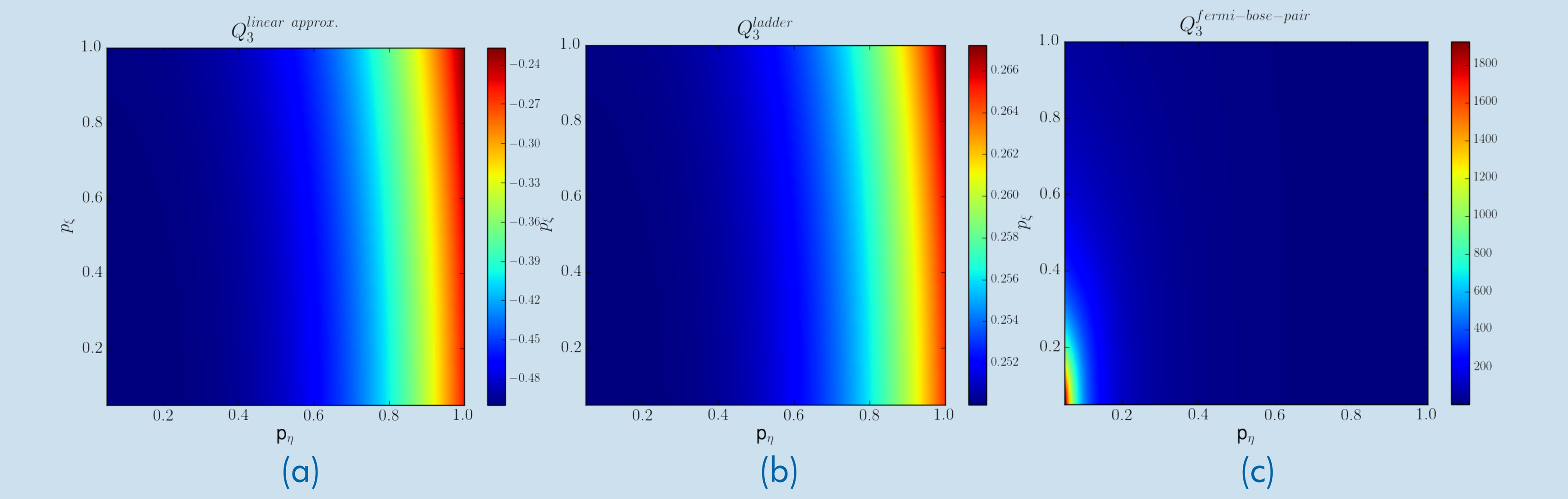


Fig. 2: Comparison of the different Pauli blocking operators $Q_3^{linear\ approx}$, $Q_3^{ladder}$ and $Q_3^{fermi-bose}$ were one gets a similar distribution for $Q_3^{linear\ approx.}$ and $Q_3^{ladder}$. Nevertheless there is a fundamental difference because values and especially the sign changes. In case of $Q_3^{fermi-bose-pair}$ one can see the region, were bosons form a condensate.

- Including ladder term changes sign
- Borromean three particle bound state possible while two particle state unbound.

## Summary

- two-fermion bound states: Pauli blocking leads to Mott dissosiation
- three-fermion (or fermion-boson) bound states can be stable while no two-fermion bound state possible under same conditions
- nucleon as in-medium Borromean state → quarkyonic phase


## References

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