



# Quark-Nuclear Hybrid EoS with Excluded-Volume Effects

## Abstract

We outline a new 2-phase description of the quark-nuclear matter hybrid equation of state that takes into account effects of phase space occupation (excluded-volume) in both, the hadronic and the quark matter phases. For the nuclear matter phase, the reduction of the available volume at increasing density leads to a stiffening, while for the quark matter phase a reduction of the effective string tension in the confining density functional is obtained. The deconfinement phase transition in the resulting equation of state is sensitive to both excluded-volume effects. As an application, we consider matter under compact star constraints of electric neutrality and  $\beta$ -equilibrium. We obtain mass-radius relations for hybrid stars that fulfill the  $2M_{\odot}$  constraint and exhibit the high-mass twin phenomenon.

## Compact Stars

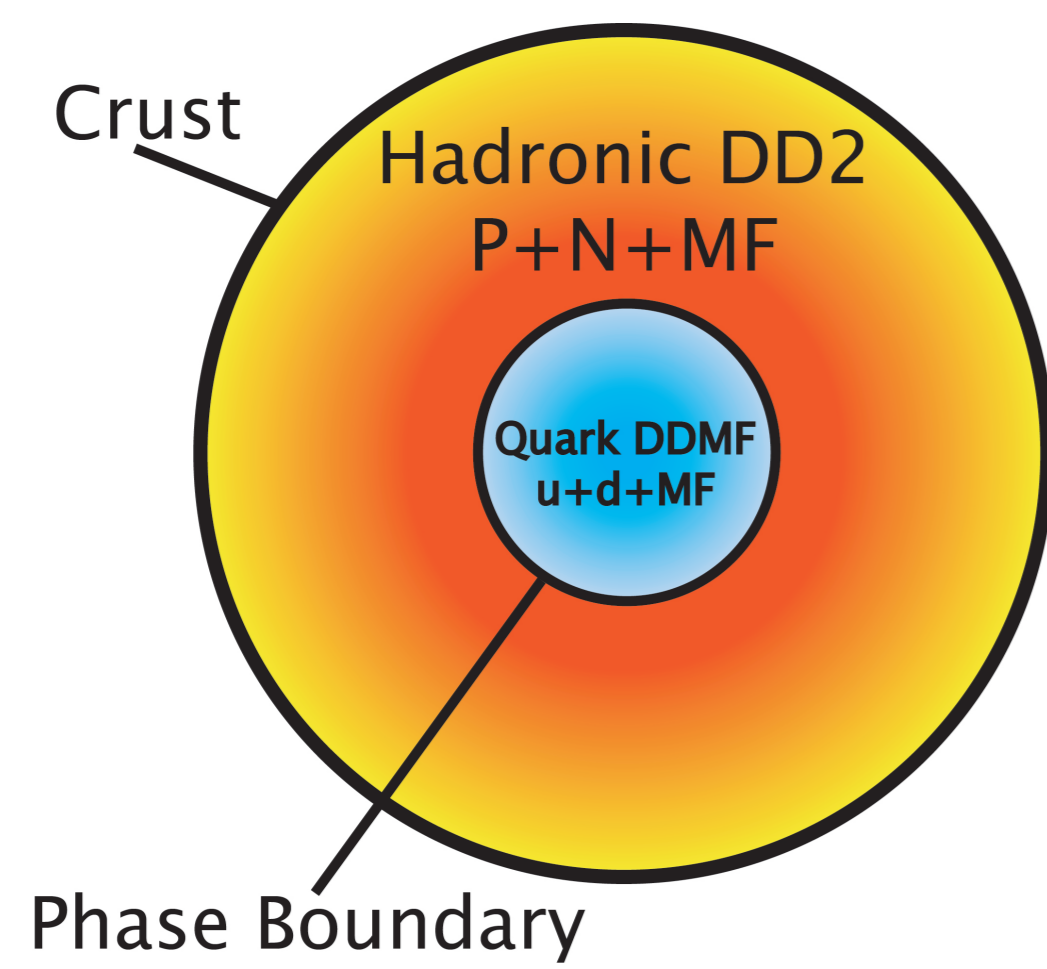


Figure 1: Two-phase model for compact stars in  $\beta$ -equilibrium at zero temperature. Hadronic model + Quark model.

### Two-Phase Description:

#### Hadronic phase:

- Relativistic Mean-Field DD2 model [3].
- Protons and neutrons, dressed with three meson fields.

#### Quark phase:

- Relativistic Mean-Field, two-flavor ( $u, d$ ) model [1].
- $u$  and  $d$  quarks, dressed with a vector potential and a scalar potential.
- Equation of State:

$$P = -\frac{\Omega}{V} = \frac{T}{V} \ln \mathcal{Z}$$

$$P(T \rightarrow 0) = \sum_i \frac{g_i}{6\pi^2} \int_0^{p_F} dp \frac{p^4}{E_i^*}$$

$p_F$  is the Fermi-momentum and  $g_i$  is the degeneracy for species  $i$ .  $E_i^*$  and  $\mu_i^*$  are the quasi-particle energies and chemical potentials.

$$E_i^* = \sqrt{p^2 + M_i^{*2}}; \quad M_i^* = m_i - U_S; \quad \mu_i^* = \mu_i - U_V.$$

$U_{S,V}$  are the scalar and vector mean-field potentials that dress the up and down quarks.

## Mean-Field Modifications

### Scalar MF: String-flip

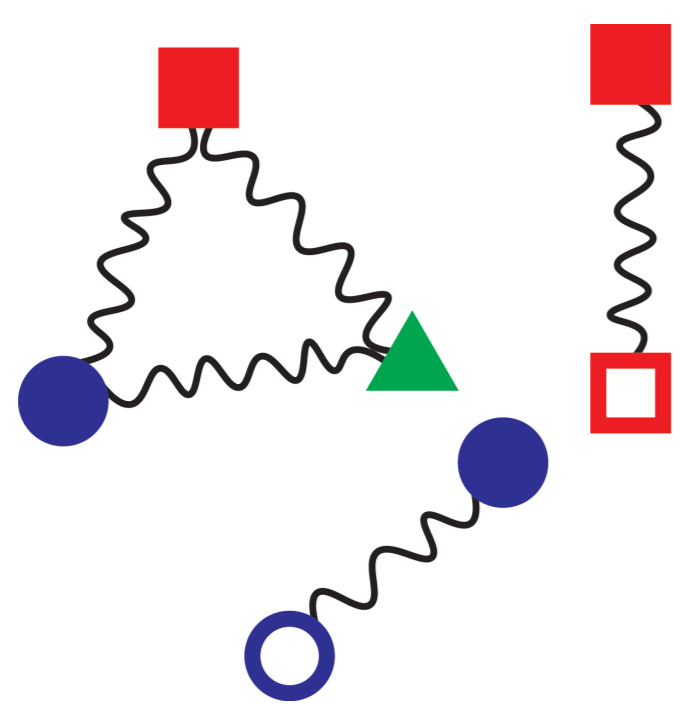


Figure 2: String configurations between quarks. Contributes to effective quark masses at deconfinement densities.

$$M_i^* = m_{i0} - U_{Si} = m_{i0} + \underbrace{D(n_S) n_S^{-\frac{1}{3}}}_{\text{confinement}} + \underbrace{C n_S^{\frac{1}{3}}}_{\text{Coulomb}}$$

The  $D(n_S)$  term is the effective in-medium string tension resulting from multiplying the vacuum string tension [5, 6],  $D_0$  between quarks with an available volume fraction,  $\Phi(n_S)$ :

$$D(n_S) = D_0 \Phi(n_S) = D_0 \exp\{-\alpha n_S^2\}, \quad (1)$$

where  $\alpha = V_{ex}^2$  and  $V_{ex}$  is the excluded-volume parameter.

### Vector MF: Multi-Quark collisions

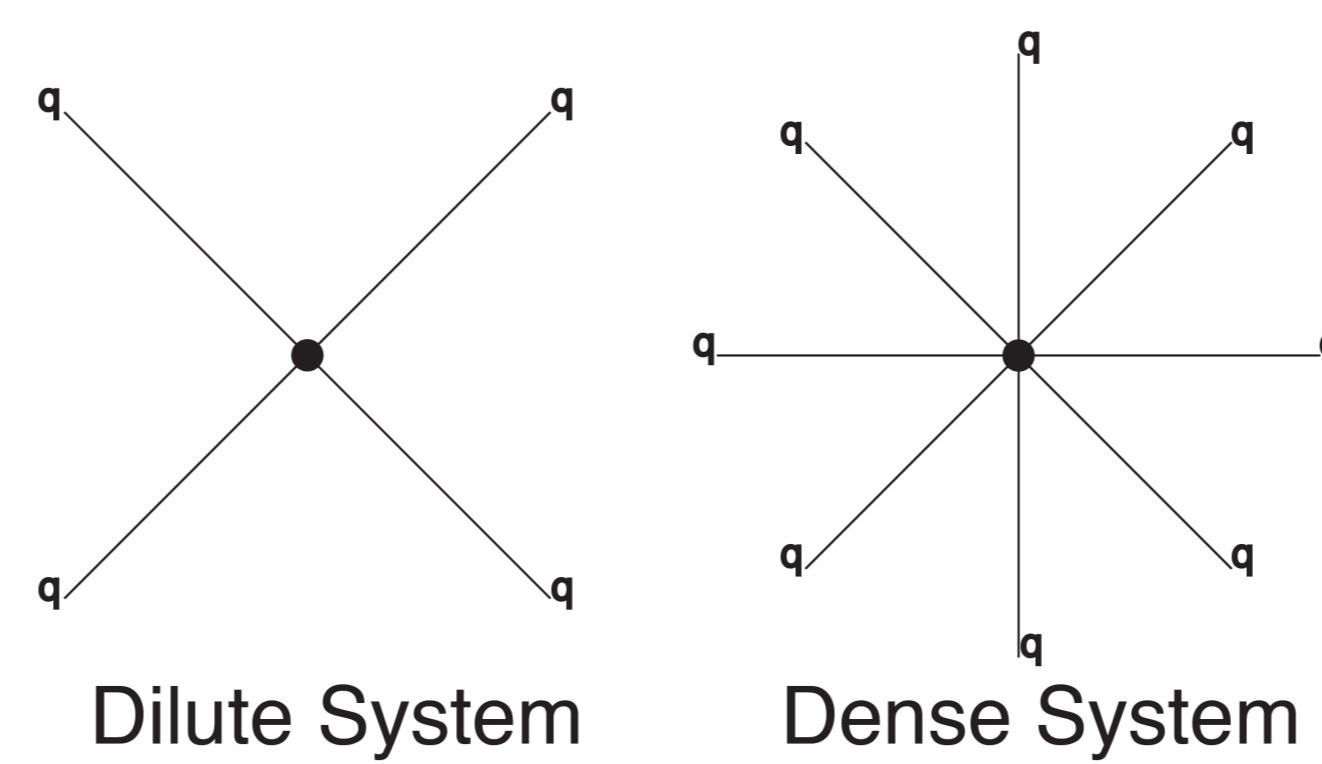


Figure 3: Multi-quark interactions (4-quark  $\sim n_V$  and 8-quark  $\sim n_V^2$ ) contribute strongly at densities larger than the onset of deconfinement, with the 4-quark contribution dominating first, followed by the domination of the 8-quark contribution at even larger densities.

The vector self-energy is taken from [2], giving us

$$\mu_i^* = \mu_{0,i} - U_{V,i} = \mu_{0,i} - a n_V - b n_V^2.$$

### Excluded-Volume

For the hadronic EoS, it is implemented as a repulsive potential between nucleons increasing with density, effectively stiffening the hadronic matter.

The excluded-volume in the quark phase is taken into account as an effective reduction of the string tension in dense matter by the available volume fraction,  $\Phi(n_S)$ , resulting in a softening of quark matter at deconfinement densities.

## High-Mass Twins

The large energy density jump  $\Delta\epsilon$  between phases from excluded-volume properties suggests there exists a hybrid star branch.

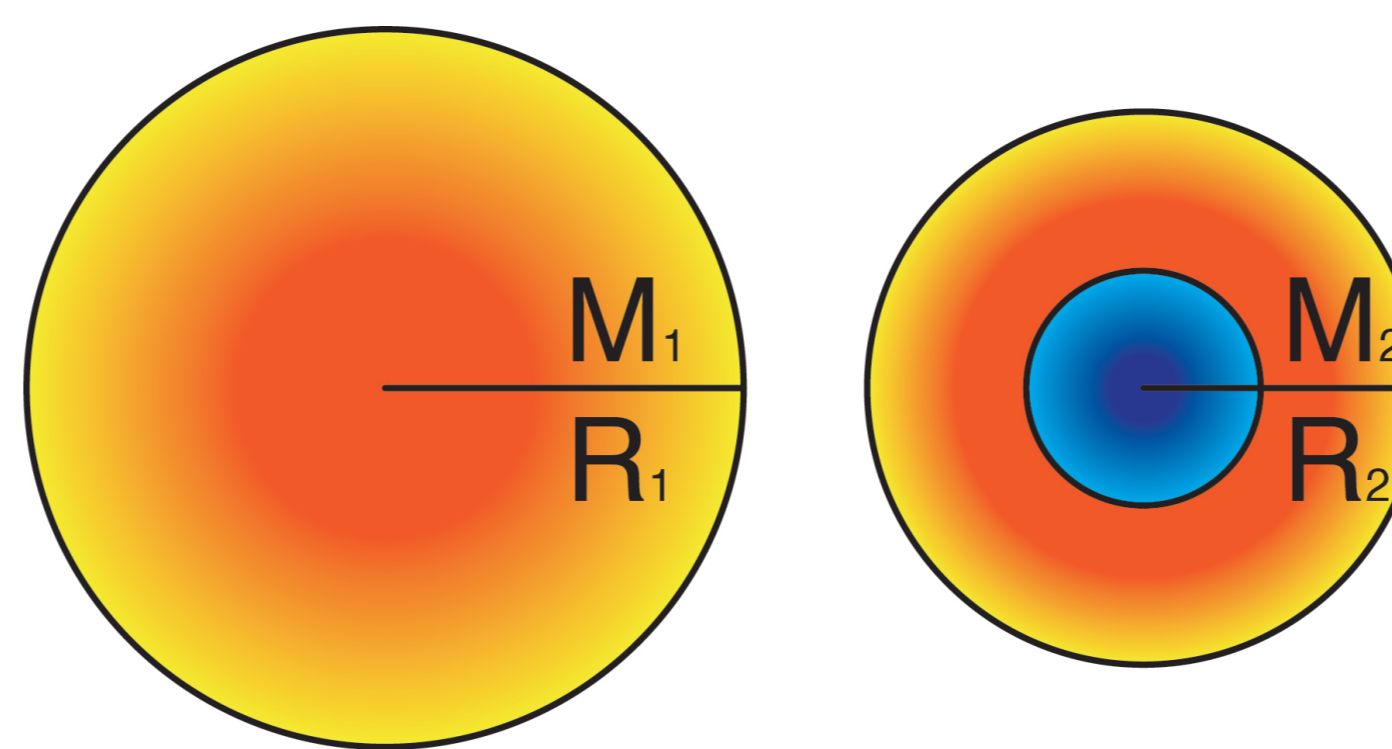


Figure 4: High-mass twin compact stars have identical masses,  $M_1 = M_2$ , but different radii,  $R_1 \neq R_2$ . If their radii could differ significantly, e.g., by 2 km, observable high-mass twin stars could be found, proving the existence of a strong first-order phase transition and thus the existence of a CEP.

### Critical Endpoint

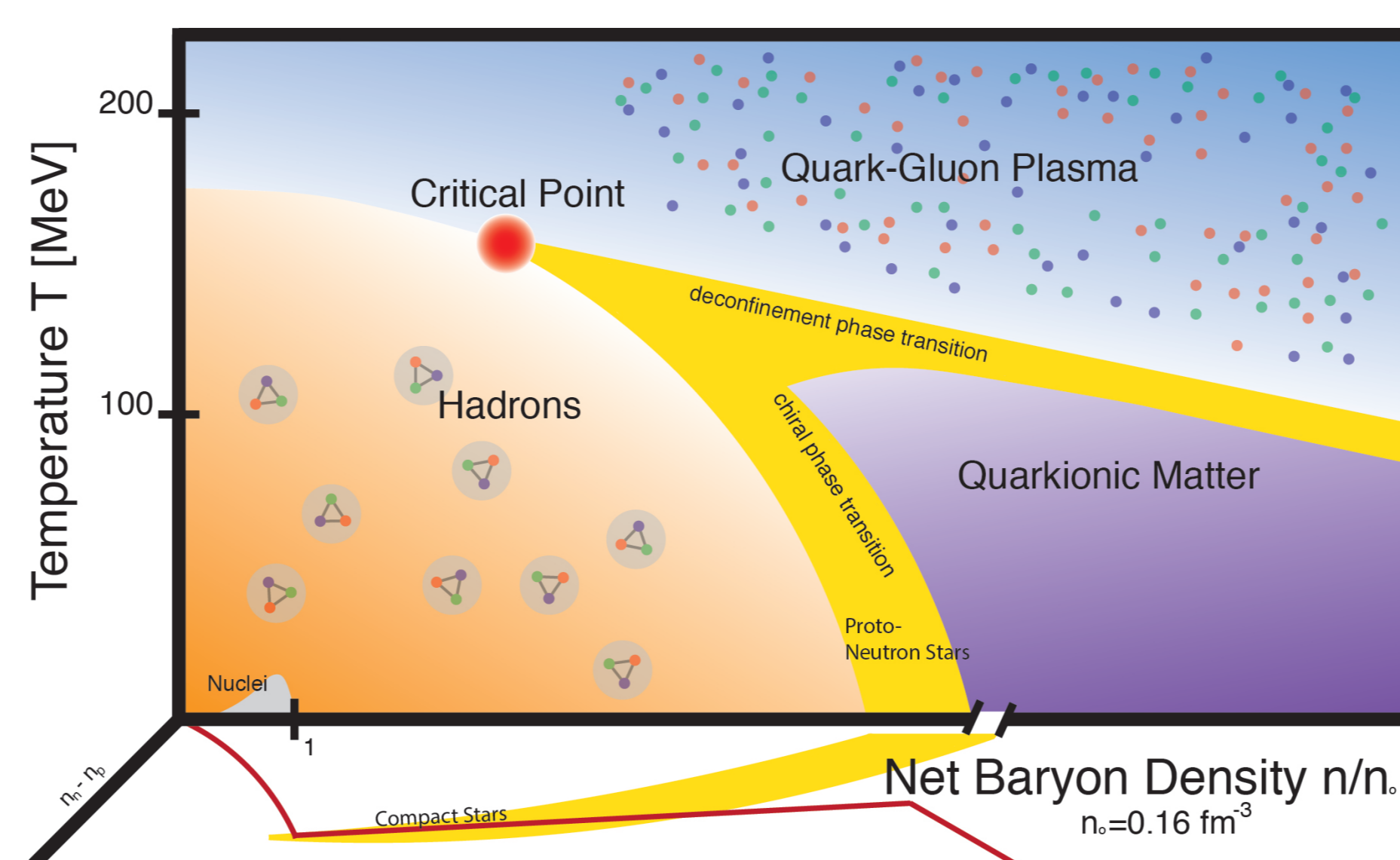


Figure 5: From lattice simulations, a cross-over phase transition exists at a high temperature. If a first order transition can be deduced from astrophysical observations of high-mass compact stars, the existence of the critical end-point will be confirmed. [4]

## Results

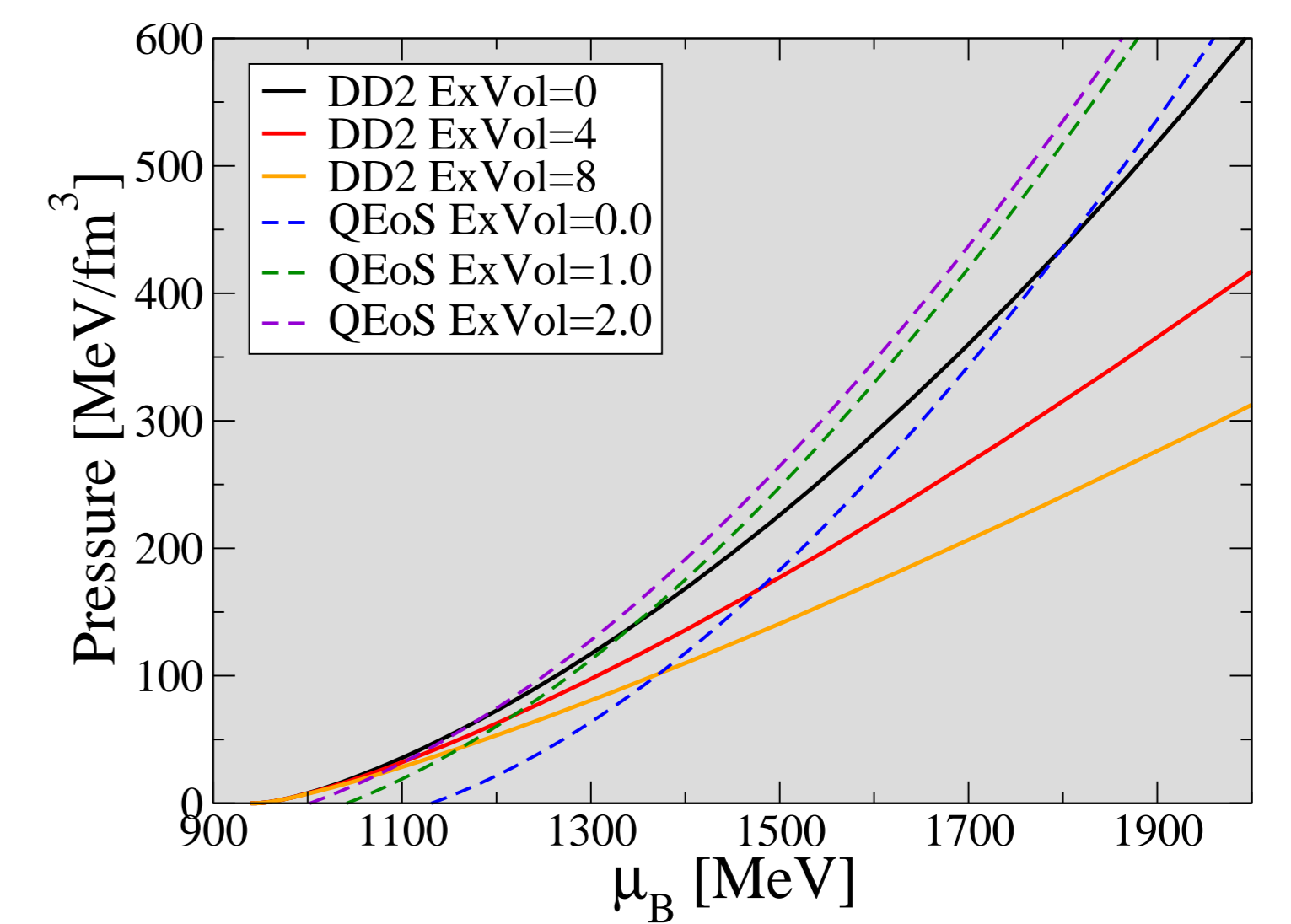


Figure 6: Hadronic phase with increasing excluded-volumes, pressure versus baryon chemical potential, crossed with quark-matter phase varying excluded-volume parameter. The crossing point represents the deconfinement phase transition from nuclear matter to quark matter.

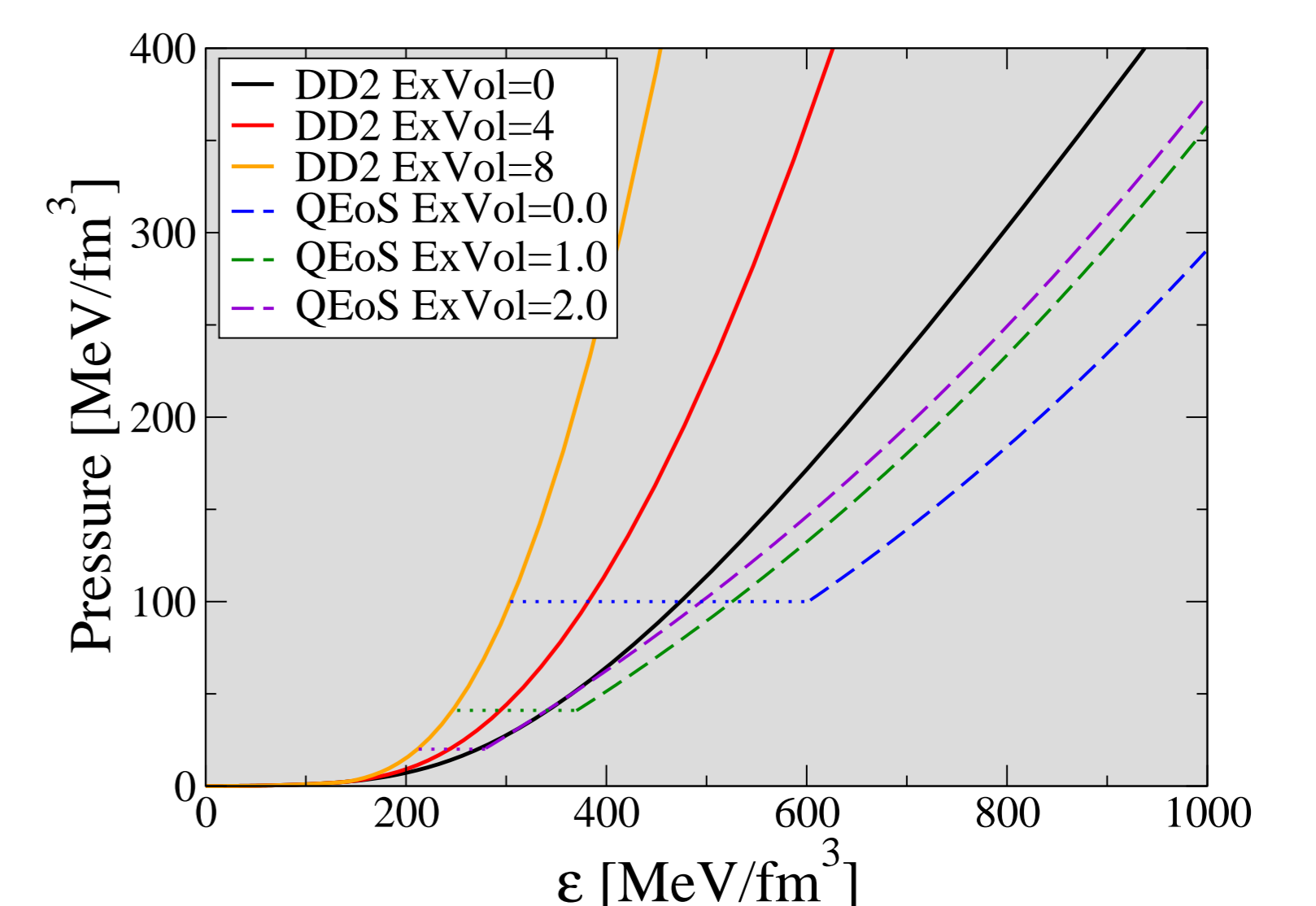


Figure 7: Hadronic phase with increasing excluded-volumes, pressure versus energy density, for the same parameters. The dashed lines connecting the DD2 ExVol=8 line with the quark lines represent the jump in energy density at the deconfinement phase transition from nuclear matter to quark matter.

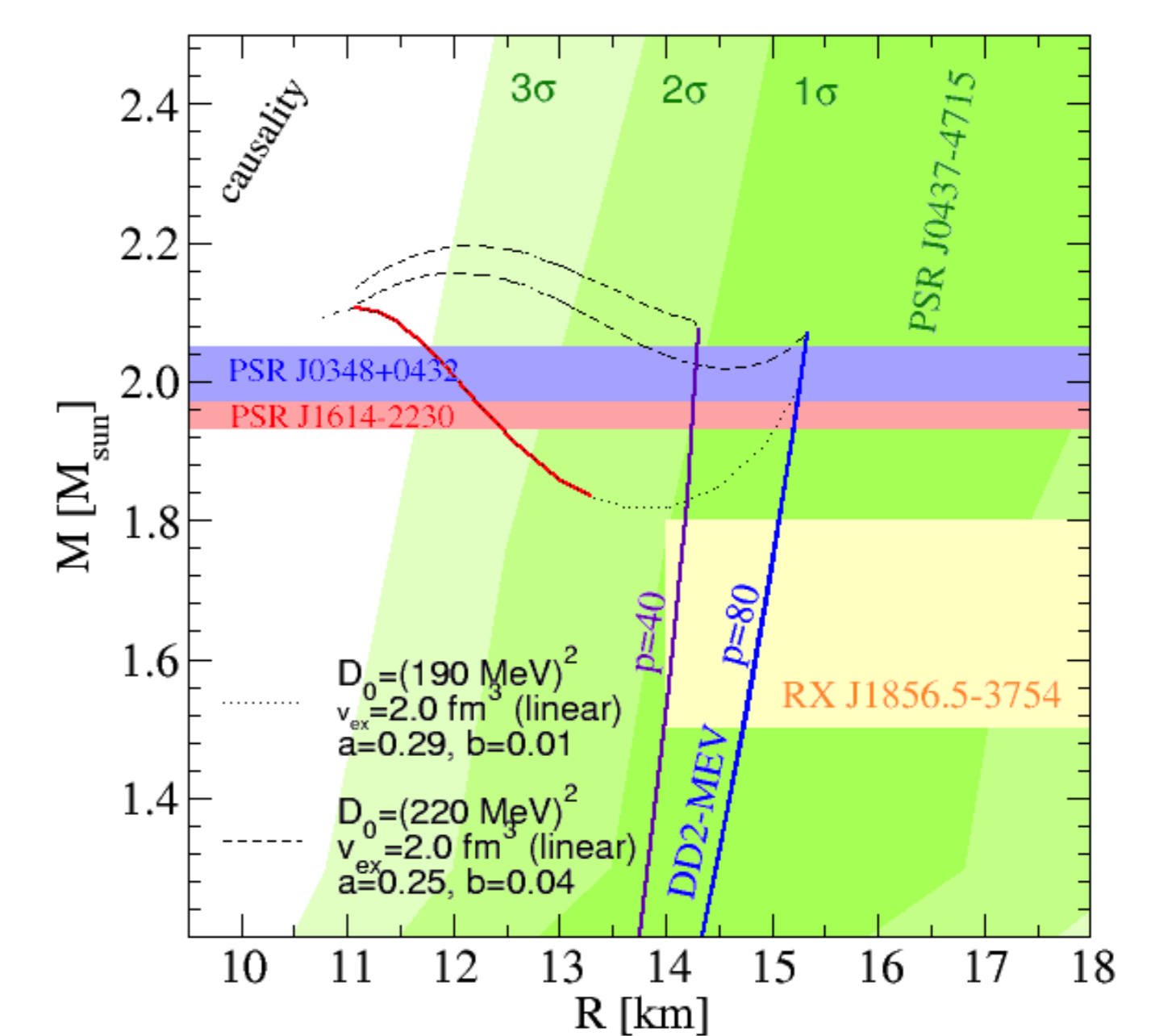


Figure 8: Mass-Radius plots for different parameterized equations of state, exhibiting a stable third family of compact stars with significant differences in radii in the high-mass region. If observable, the strong, first-order phase transition will prove the existence of the CEP.

## References

- [1] D. E. Alvarez-Castillo, M. A. R. Kaltenborn and D. Blaschke, J. Phys.: Conf. Ser. **668**, 012035 (2016).
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- [6] G. Röpke, D. Blaschke and H. Schulz, Phys. Rev. D **34**, 3499 (1986).