Mott-hadron resonance gas and lattice QCD thermodynamics

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Introduction

We present an effective model for low-energy QCD thermodynamics which provides a microscopic interpretation of the transition from a gas of hadron resonances to the quark-gluon plasma by Mott dissociation of hadrons and compare results with data from lattice QCD simulations. We consider the thermodynamics of the Polyakov-loop extended Nambu-Jona-Lasinio (PNJL) model within the self consistent approximation scheme of the Φ-derivable approach. This allows us to obtain the Generalized Beth-Uhlenbeck (GBU) equation of state. Our approach goes beyond the mean-field description of quark matter [1] by taking into account hadronic correlations (bound and scattering states) as well as their backreaction on the propagator of constituents. The next step in our work is to include more hadronic degrees of freedom than just the low-lying pseudoscalar mesons. For that purpose we discuss a model for the generic behavior of hadron masses and phase shifts at finite temperature which shares basic features with recent developments within the PNJL model for correlations in quark matter.

EoS for PNJL model

■ Thermodynamic potential in Gaussian approximation within the PNJL model has the form [2]

$$\Omega_{PNJL} = -P(T,\mu) = \Omega_{MF} + \sum_X \Omega_{X,FL} + \mathcal{U}(\Phi,\bar\Phi,T)\ ,$$

where mean field part and sum over fluctuation in different channels the Polyakov-loop potential $\mathcal{U}(\bar\Phi,\Phi,T)$ is chosen in logarithmic form [3].

■ MF-Thermodynamic potential

$$\Omega_{MF}(T,\mu) = \sum_{\alpha=u,d,s}\frac{\sigma_\alpha}{4G} + \Omega_Q(T,\mu)\ ,$$

with the condensate part and

■ Quark-Thermodynamic potential

$$\Omega_Q(T,\mu) = -2N_c\sum_{\alpha=u,d,s}\int\frac{d^3p}{(2\pi)^3}E_\alpha(p)$$
$$-2\sum_{\alpha=u,d,s}\int\frac{d^3p}{(2\pi)^3}\ln\left(\left[1+3\bar\Phi Y+3\Phi Y^2+Y^3\right]\left[1+3\Phi\bar Y+3\bar\Phi\bar Y^2+\bar Y^3\right]\right)\ ,$$

where $Y = e^{-\beta(E_\alpha-\mu_\alpha)}$ and $\bar Y = e^{-\beta(E_\alpha+\mu_\alpha)}$.
The fluctuation part in spectra function representation has the form

■ Beth-Uhlenbeck (BU) formula [1]

$$\Omega_{\rm X,BU}(T,\mu) = -d_{\rm X}\int\frac{{\rm d}^3q}{(2\pi)^3}\int\frac{{\rm d}\omega}{2\pi}\,[n_X^-(\omega)+n_X^+(\omega)]\delta_{\rm X}(\omega,\mathbf{q})\ ,$$

with Bose distribution function $n_X^\pm = 1/[exp[(\omega\pm\mu_X)/T]-1]$.

■ Phase shift

$\delta_X(\mathbf{q}) = -{\rm Im}\ln S_X^{-1}(\omega-\mu_X+i\epsilon,\mathbf{q})$, where hadronic propagator $S_X^{-1} = G^{-1}-\Pi_X(\omega-\mu_X+i\epsilon,\mathbf{q})$ with the polarization function [2]

$$\Pi^X_{ff'}(\omega+\mu_f-\mu_{f'}+i\eta) = 4\big\{I_1^f(T,\mu_f)+I_1^{f'}(T,\mu_{f'})$$
$$-\left[(\omega+\mu_f-\mu_{f'})^2-(m_f-m_{f'})^2\right]I_2^{ff'}(\omega,T,\mu_f-\mu_{f'})\big\}\ ,$$

where I_1 and I_2 standard one loop integrals.

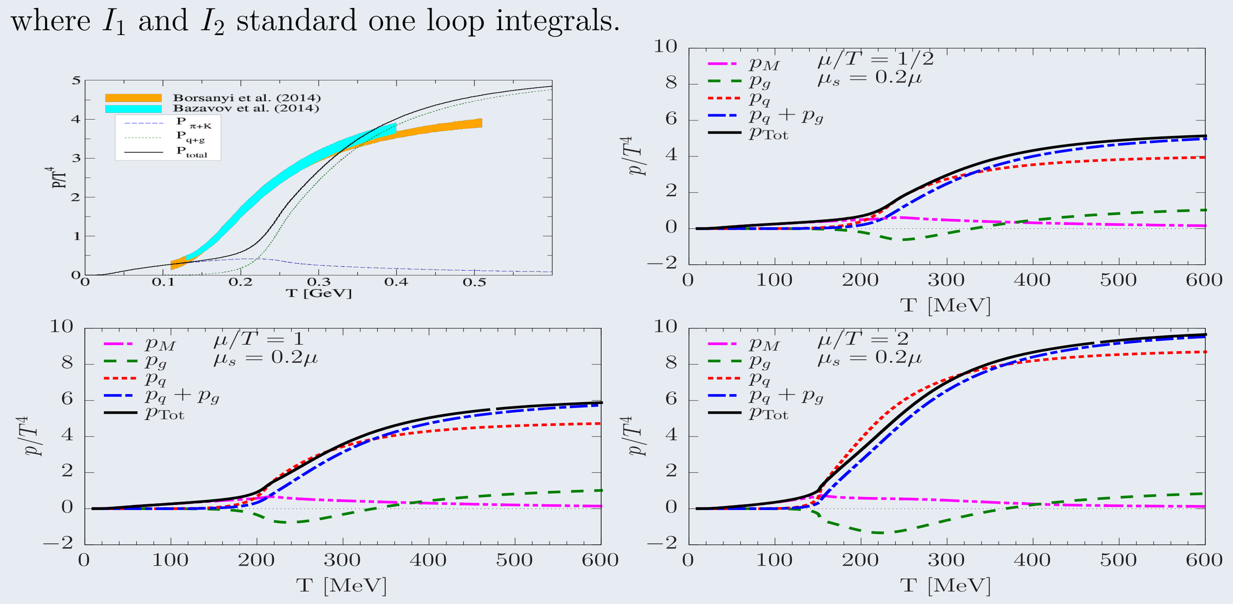

FIG. 1: Pressure of the 2 + 1 flavor PNJL model with scalar and pseudoscalar mesons as a function of temperature (black solid line) and its components for the cases $T/\mu = 0$, 0.5, 1.0.2.0. light and strange quarks (red dotted); gluon contribution from Polyakop loop potential $\mathcal{U}$ (green dashed); light and strange quarks plus gluons (blue dashed- dotted); mesonic contribution of pion, kaon, a0 and kappa (magenta dash-double-dotted). For the case $T/\mu = 0$ (first plot) the total pressure compared to the lattice QCD results of Ref. [?, ?].

Φ-derivable approach

■ The thermodynamic potential for hadron-quark-gluon matter within the approximate selfconsistent approach to QCD reads [4]

$$\Omega = \sum_{i=q,d,M,B}\frac{c_i}{2}\left[{\rm Tr}\ln(G_i^{-1}) - {\rm Tr}\,(\Sigma_i\,G_i)\right] + \Phi\left[G_i\right]\ ,$$

where $c_i = 1(-1)$ for bosons (fermions). Note that also the factor 1/2 here because we work in Nambu-Gorkov representation within 2×2 matrix propagators appropriate for entering fermion pair condensate phases at low temperatures and high densities.

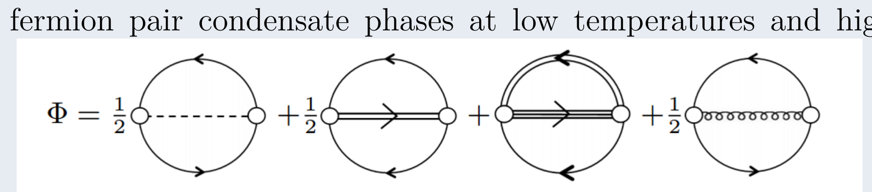

FIG. 2: Diagram choice for the Φ functional.

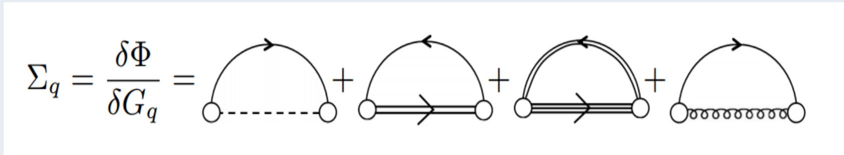

FIG. 3: Quark selfenergy therms for the the Φ functional of FIG. 2.

GBU EoS

■ GBU EoS for case mesons (first sunset diagram)

$$\Omega_{\rm M}(T,\mu) = -\frac{1}{2}\Big[{\rm Tr}\ln(G_{\rm M}^{-1}) - {\rm Tr}(\Sigma_{\rm M}G_{\rm M})\Big]\ ,$$

after calculation GBU EoS has the form

$$\Omega_{\rm M,GBU}(T,\mu) = -d_{\rm M}\int\frac{{\rm d}^3q}{(2\pi)^3}\int\frac{{\rm d}\omega}{2\pi}\,[n_{\rm M}^-(\omega)+n_{\rm M}^+(\omega)]\Big(\delta_{\rm M}(\omega,\mathbf{q}) - \cos\delta_{\rm M}(\omega,\mathbf{q})\sin\delta_{\rm M}(\omega,\mathbf{q})\Big)\ .$$

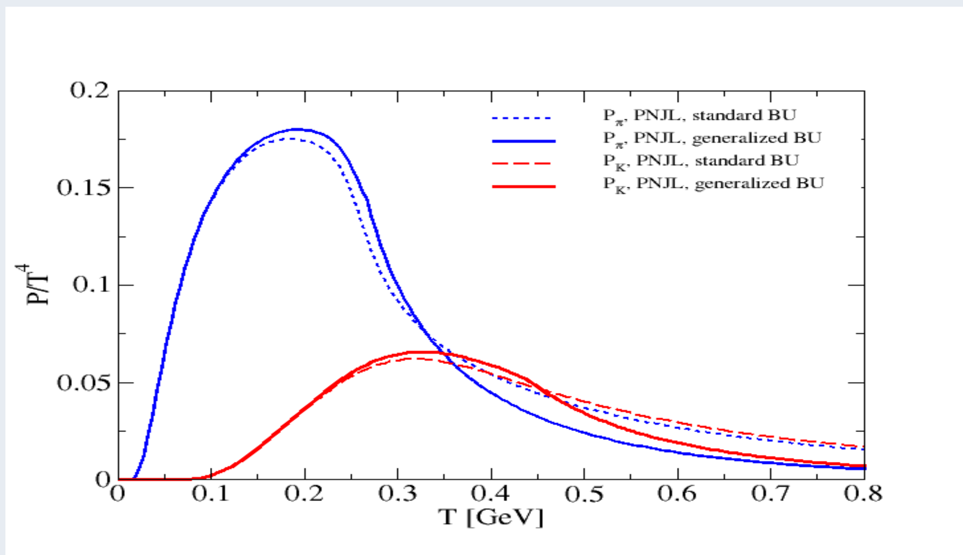

FIG. 3: Pressure of pions (blue color) and kaons (red color) as function of the temperature. Solid line correspond to the result for GBU equation and dot line correspond to the result for BU equation.

MHRG model

■ The total thermodynamic potential for MHRG model for vanishing chemical potential

$$\Omega(T) = \sum_{i=M,B}\Omega_{i,GBU}(T) + \Omega^\star_{PNJL}(T)\ ,$$

where the first therm - GBU formula describes a hadron resonance gas with Mott dissociation of the hadronic bound states and the underlying quark and gluon thermodynamics is described within the PNJL model in the form

$$\Omega^\star_{PNJL}(T) = \Omega^\star_{FG}(T) + \mathcal{U}(\Phi,T)\ ,$$

where the asterix denotes that we go beyond the standard meanfield level and introduce a quasiparticle picture

$$\Omega^\star_{FG}(T) = 4N_c\sum_{u,d,s}\int\frac{dp}{(2\pi)^3}\frac{d\omega}{\pi}f_\Phi\Big[\delta_q(\omega,\gamma) - \cos\delta_q(\omega,\gamma)\sin\delta_q(\omega,\gamma)\Big]\ ,$$

where the generalized Fermi distribution function of the PNJL model for the case of vanishing quark chemical potential considered here is defined as

$$f_\Phi = \frac{\Phi(1+2Y)Y+Y^3}{1+3\Phi(1+Y)Y+Y^3}\ .$$

The function

$$\delta_q(\omega,\gamma) = \frac{\pi}{2} + \arctan\Big[\frac{\omega-E_p}{\gamma}\Big]\ ,$$

plays the role of a quark phase shift due to the scattering off hadrons and the parameter γ stands for the collisional broadening.

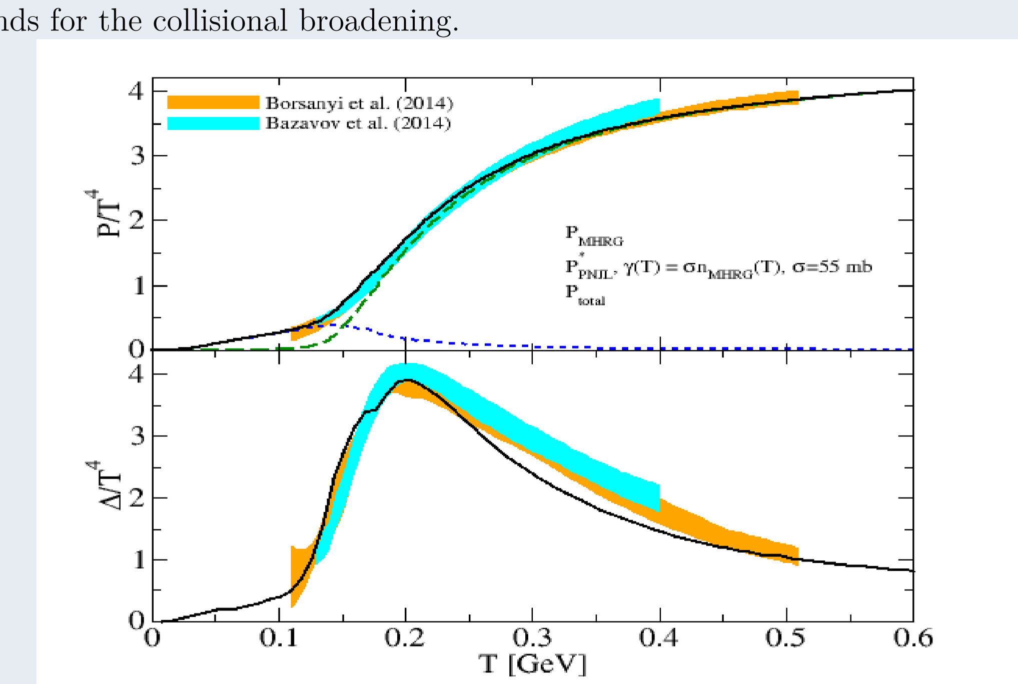

Fig. 4: Upper Panel: Temperature dependence of the total pressure of the present model (black solid line) compared to the lattice QCD results. For comparison, the hadron (blue dash-dotted line), quark-gluon (green dashed line) are shown.
Lowest Panel: Temperature dependence of the interaction measure $\triangle = (\epsilon - 3P)$.

■ Ansatz for quark masses

The temperature dependence of the quark masses are obtained using Lattice QCD data for the behaviour of the continuum extrapolated chiral condensate $\triangle_{l,s}(T)$,

$$m(T) = [m(0)-m_0]\,\triangle_{l,s}(T) + m_0\ ,$$
$$m_S(T) = m(T) + m_S - m_0\ ,$$

with $m_0 = 5.5$ MeV for light quark and for strange quark mass $m_s = 100$MeV.

■ We make the simplifying ansatz that the hadron masses are constant and temperature independent up to their Mott temperature, where they hit the threshold $m_{thr,M}(T)$. After that temperature, we assume that their mass rises with temperature in the same way as the resonance width $\Gamma_i(T)$

$$M_i(T) = M_i(0) + \Gamma_i(T)\ ,$$
$$\Gamma_i(T) = \sqrt{a\,(T-T_{\rm Mott,i}) + a^2\,(T-T_{\rm Mott,i})^2}\ ,$$

where $M_i(0)$ are the hadron masses according to the particle data group, for the parameters we choose a =2.5 GeV and b=6.25. The ansatz for the width is motivated by the Mott transition given in Ref. [5].
The Mott temperature determined from condition

$$M_i(T_{Mott,i}) = m_{thr.i}(T_{Mott,i})\ ,$$
$$m_{thr.i}(T_{Mott,i}) = (N_i - N_S)m(T) + N_S m_S(T)\ ,$$

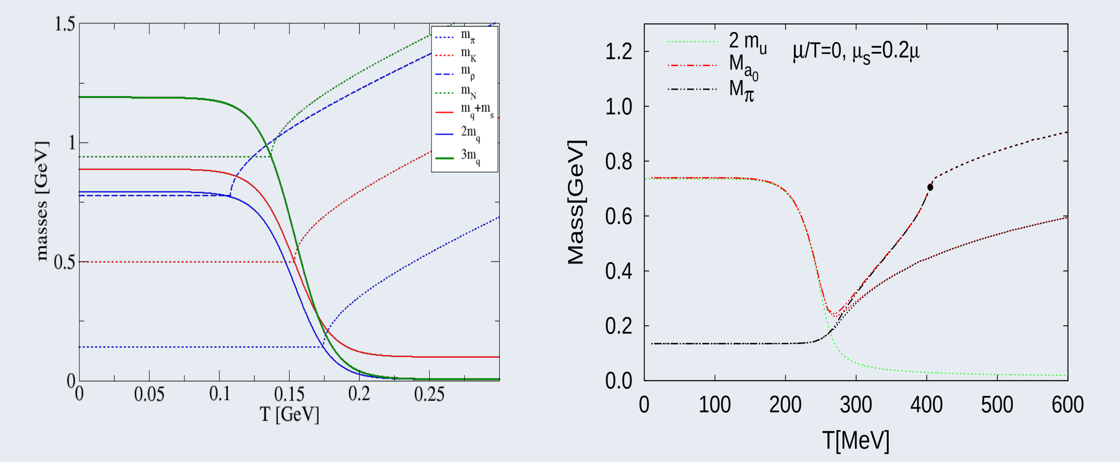

FIG. 5: Left panel: Temperature dependence of the light hadron masses and the corresponding 2-quark and 3-quark continuum thresholds (solid lines) within the MHRG model. Right panel: Temperature dependence of the pion and sigma meson masses and the corresponding 2-quark continuum thresholds within the PNJL model.

■ The generic behavior of the temperature dependence of hadronic phase shifts is realized by the following ansatz which holds for both, mesons and baryons

$$\delta_i(s;T) = F(s)\left[\frac{\pi}{2} + \arctan\left(\frac{s-M_i^2(T)}{M_i(T)\Gamma_i(T)}\right)\right]\Bigg\{\theta(m^2_{\rm thr,i}-s)$$
$$+\ \theta(s-m^2_{\rm thr,i})\theta(m^2_{\rm thr,i}+N_i^2\Lambda^2-s)\left[\frac{m^2_{\rm thr,i}+N_i^2\Lambda^2-s}{N_i^2\Lambda^2}\right]\Bigg\},$$

where the auxiliary function $F(x) = [sin(x)\Theta(\pi/2-x)+\Theta(x-\pi/2)]$ has been introduced in order to ensure that the phase shift at s = 0 shall always be zero, even at higher temperatures, where large values of the width parameter in the Breit-Wigner like ansatz would otherwise spoil this constraint.

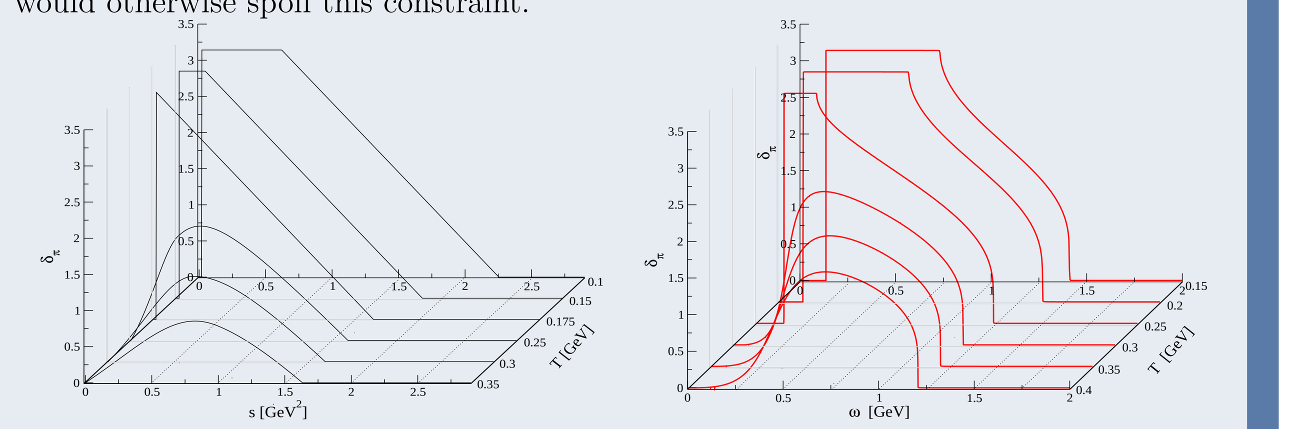

FIG. 6: Phase shift of pions within MHRG model (left panel) and within PNJL model (right panel) as function of the energy for different temperatures from T = 150 MeV to (350)400 MeV.

Conclusion

In this work we discribe the hadron-quark-gluon matter in framework of the (G)BU approach where main role play the phase shift. The (G)BU equation include an interaction between bound and scattering states in the medium, at the Mott transition where the bound state transforms to a resonance in the continuum. This transition is manifested in a vanishing of the binding energy as well as a jump by π of the phase shift at threshold in accordance with the Levinson theorem. Thus the hadronic pressure expressed in the (G)BU form is melting. The MHRG give results in quantitative agreement with recent ones from LQCD. To achieve this it was essential to realize a calculational scheme that is inspired by the Φ-derivable approach of Baym and Kadanoff. Due to the confining property of QCD it is of crucial importance to take into account the strong contributions of the hadron resonance gas components to the quark degrees of freedom which constitute them.

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