

# vHLLE, a code for hydrodynamic modelling of relativistic heavy ion collisions



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## Abstract

vHLLE solves the equations of relativistic viscous hydrodynamics in 3+1 dimensions using Israel-Stewart framework. In addition to energy and momentum, charge densities are explicitly propagated and included in the equation of state, making the code suitable for simulations of matter expansion with finite baryon density. With the help of ideal-viscous splitting, we keep the ability to solve the equations of ideal hydrodynamics in the limit of zero viscosities using a Godunov-type algorithm. Milne coordinates are used to treat the predominant expansion in longitudinal (beam) direction effectively.

## Equations to solve

- Energy/momentum conservation:  $\partial_\nu T^{\mu\nu} = 0$

- Charge conservation:  $\partial_\nu N_i^\mu = 0$

Where

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$N_i^\mu = n_i u^\mu + V_i^\mu \text{ (Landau frame)}$$

- Evolution equations for shear stress/bulk:

$$\langle u^\gamma \partial_\gamma \pi^{\mu\nu} \rangle = -(\pi^{\mu\nu} - \pi_{NS}^{\mu\nu})/\tau_\pi - \frac{4}{3}\pi^{\mu\nu} \partial_\gamma u^\gamma$$

$$u^\gamma \partial_\gamma \Pi = -(\Pi - \Pi_{NS})/\tau_\Pi - \frac{4}{3}\Pi \partial_\gamma u^\gamma$$

$$\langle A^{\mu\nu} \rangle = \left(\frac{1}{2}\Delta_\alpha^\mu \Delta_\beta^\nu + \frac{1}{2}\Delta_\alpha^\nu \Delta_\beta^\mu - \frac{1}{3}\Delta^{\mu\nu} \Delta_{\alpha\beta}\right) A^{\alpha\beta}$$

$$\pi_{NS}^{\mu\nu} = \eta(\Delta^{\mu\lambda} \partial_\lambda u^\nu + \Delta^{\nu\lambda} \partial_\lambda u^\mu) - \frac{2}{3}\eta \Delta^{\mu\nu} \partial_\lambda u^\lambda$$

## Coordinate transformations

Milne coordinates are defined as follows:

$$0) \tau = \sqrt{t^2 - z^2}$$

$$1) x = x$$

$$2) y = y$$

$$3) \eta = \frac{1}{2} \ln \frac{t+z}{t-z}$$

$$g^{\mu\nu} = \text{diag}(1, -1, -1, -1/\tau^2)$$

Nonzero Christoffel symbols are:

$$\Gamma_{\tau\eta}^\eta = \Gamma_{\eta\tau}^\tau = 1/\tau, \quad \Gamma_{\eta\eta}^\tau = \tau$$

Which results in the explicit form of conservation equations solved numerically:

$$\partial_\nu(\tau T^{\tau\nu}) + \frac{1}{\tau}(\tau T^{\eta\nu}) = 0$$

$$\partial_\nu(\tau T^{\mu\nu}) = 0$$

Conservative variables are

$$\partial_\nu(\tau T^{\tau\nu}) = 0$$

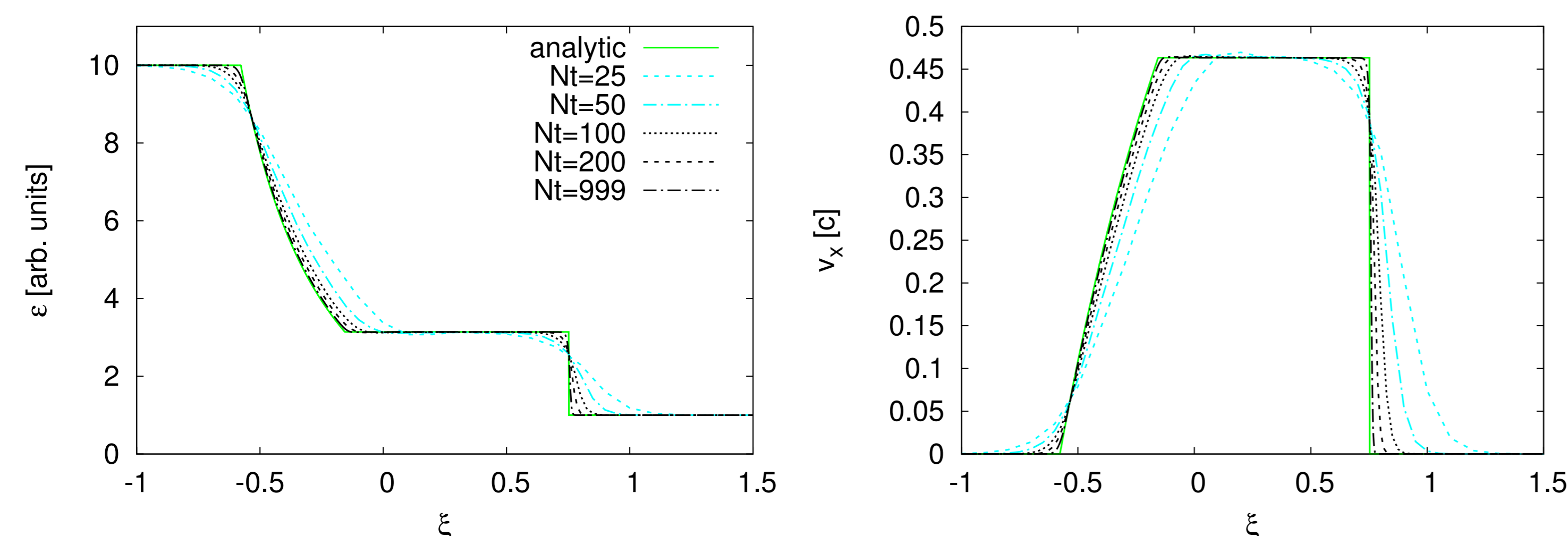
$$\partial_\nu(\tau T^{\eta\nu}) + \frac{1}{\tau} \tau T^{\eta\tau} = 0$$

$$Q^\mu = \tau \cdot T^{\tau\mu}$$

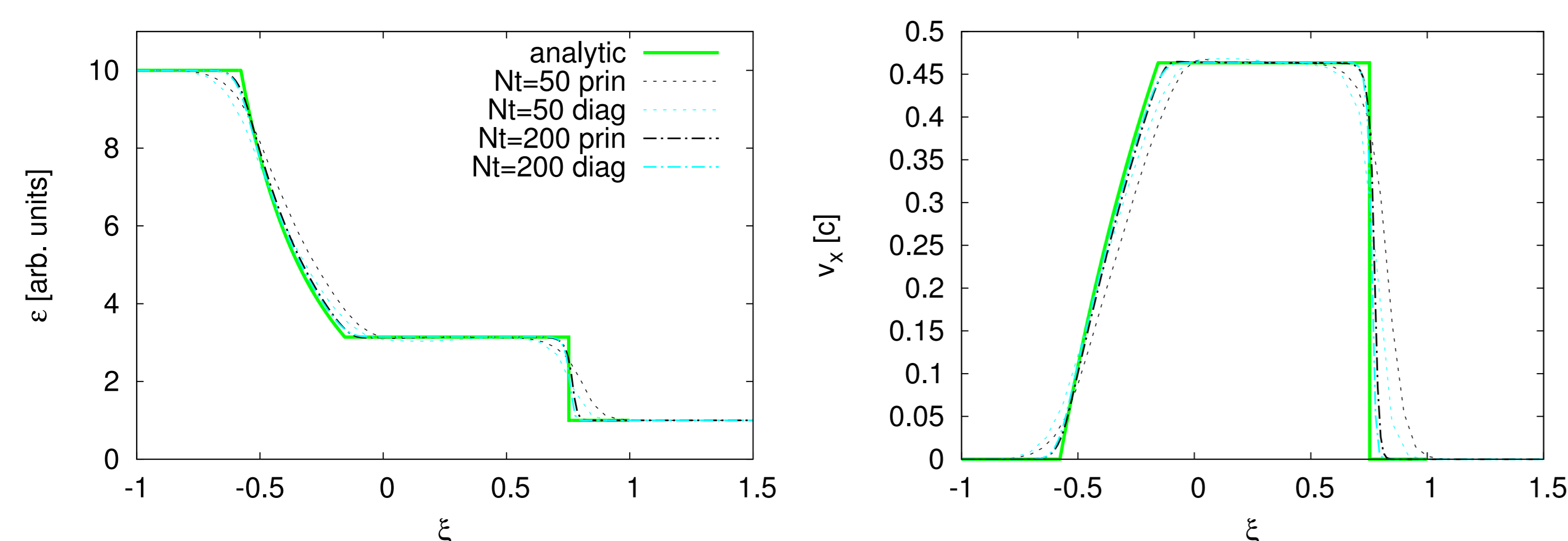
Plus corresponding source terms in the Israel-Stewart equations

## Shock tube test

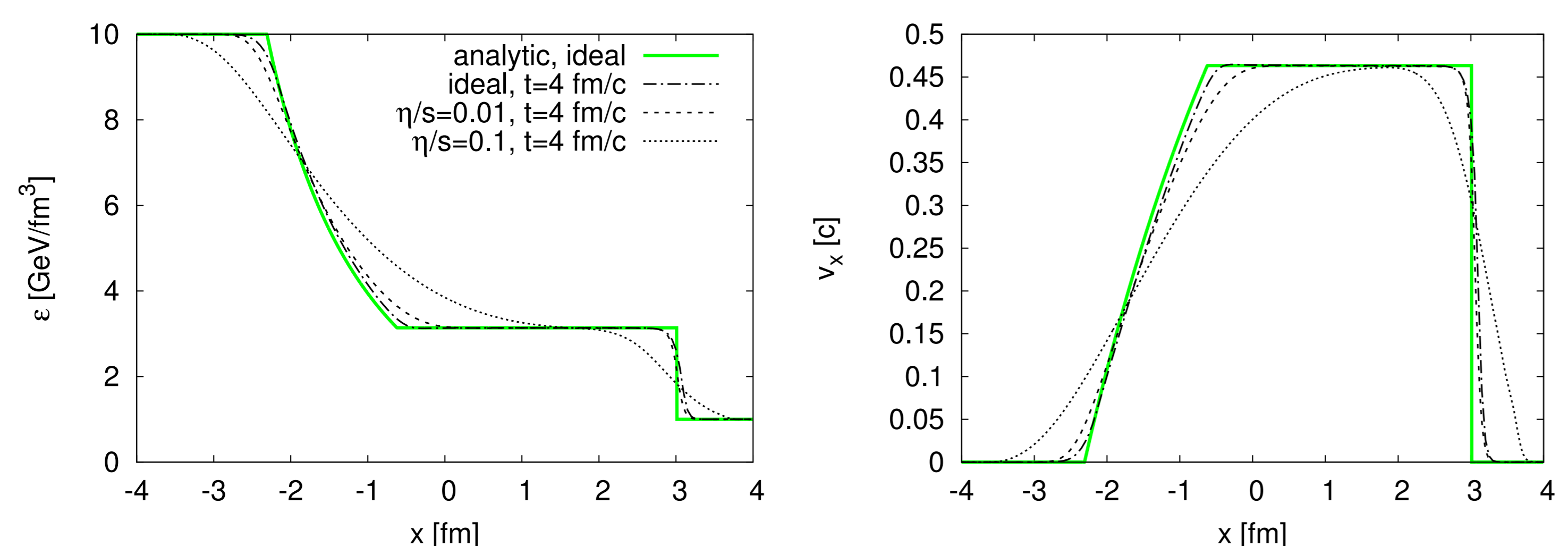
Approaching analytical solution with increasing  $N_t$



Shock tube test in principal (X) vs diagonal (XY) direction



Effects of nonzero shear viscosity



## References

- [1] Makoto Takamoto, Shu-ichiro Inutsuka, J.Comput.Phys. 230 (2011), 7002
- [2] K.Werner, B. Guiot, Iu.Karpenko, T.Pierog, Phys. Rev. C 89, 064903 (2014)
- [3] V.Yu. Naboka, Iu.A. Karpenko, Yu.M. Sinyukov, arXiv:1508.07204
- [4] Iu. A. Karpenko, P. Huovinen, H. Petersen, M. Bleicher, Phys. Rev. C 91, 064901 (2015)

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## Numerical implementation

$$\partial_\mu(T_{id}^{\mu\nu} + \delta T^{\mu\nu}) = S^\nu, \quad S = \text{geometrical source terms}$$

$$\partial_\tau \underbrace{(T_{id}^{\tau i} + \delta T^{\tau i})}_{Q_i} + \partial_j \underbrace{(T^{ji})}_{id.flux} + \partial_j \underbrace{(\delta T^{ji})}_{visc.flux} = \underbrace{S_{id}^\nu + \delta S^\nu}_{source terms}$$

Finite-volume realization:

$$\frac{1}{\Delta\tau} (Q_{id}^{n+1} + \delta Q^{n+1} - \underbrace{Q_{id}^{*n+1} + Q_{id}^{*n+1}}_{=0} - Q_{id}^n - \delta Q^n) + \frac{1}{\Delta x} (\Delta F_{id} + \Delta \delta F) = S_{id} + \delta S$$

Then, split the equation into two parts[1]:

$$\frac{1}{\Delta t} (Q_{id}^{*n+1} - Q_{id}^n) + \frac{1}{\Delta x} \Delta F_{id} = S_{id} \quad (\text{using finite volume, HLLC approx})$$

$$\frac{1}{\Delta t} (Q_{id}^{n+1} + \delta Q^{n+1} - Q_{id}^{*n+1} - \delta Q^n) + \frac{1}{\Delta x} \Delta \delta F = \delta S \quad (\text{upwind/Lax-Wendroff})$$

## Other techniques involved

- HLLC approximation to calculate fluxes  $F$  between cells
- linear reconstruction of  $Q^\mu$  at left/right boundaries of a cell for second order accuracy in space
- upwind/Lax-Wendroff method for the evolution equations for  $\pi^{\mu\nu}$  and  $\Pi$
- predictor-corrector (half-step) method for second order accuracy in time
- outflow (non-reflecting) boundary conditions via ghost cell method

## Viscous Gubser test

Semi-analytical I-S solution from H. Marrochio et al, Phys. Rev. C 91, 014903 (2015)

Parameters:  $\tau_0 = 1$  fm/c

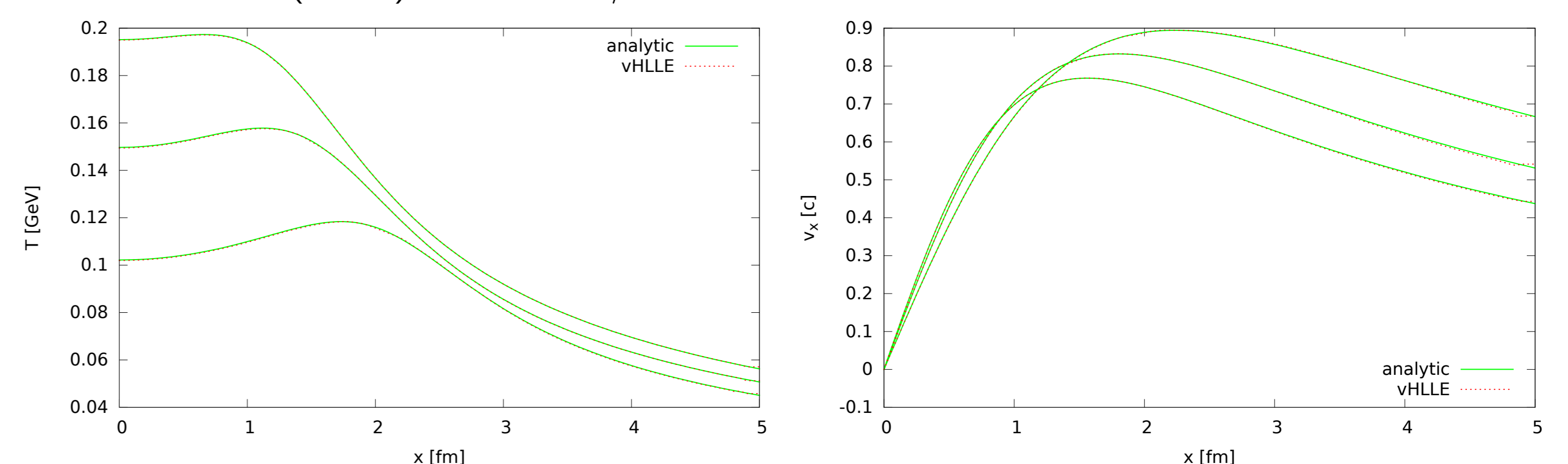
$\eta/s = 0.2$ ,  $\tau_\pi = 5\eta/(sT)$

Numerics:

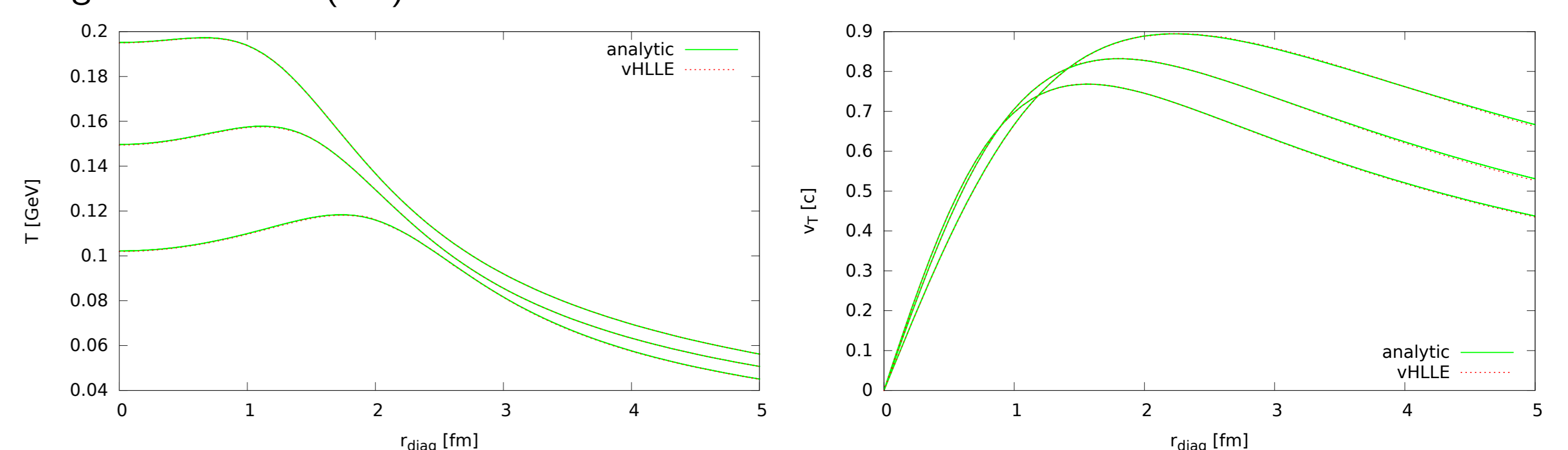
- $\Delta x = \Delta y = 0.05$  fm,  $\Delta\tau = 0.02$  fm/c
- van Leer monomod slope limiter with  $\theta = 1.8$

Temperature/transverse velocity profiles, principal direction (X)

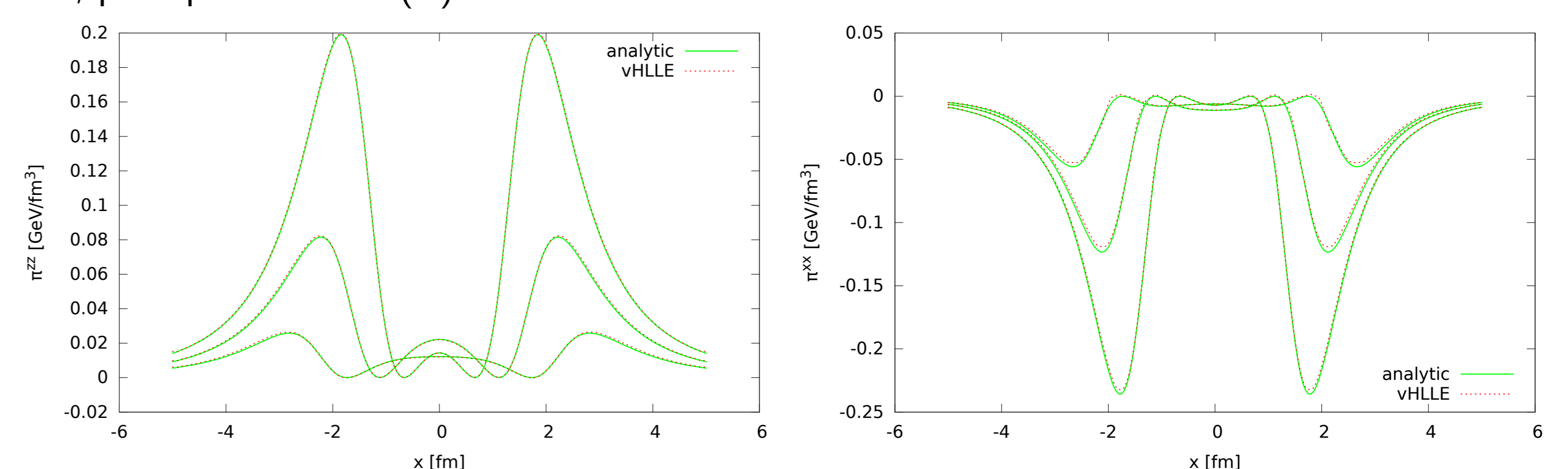
Curves:  $\tau = 1.2$  (initial), 1.5, 2.0 fm/c



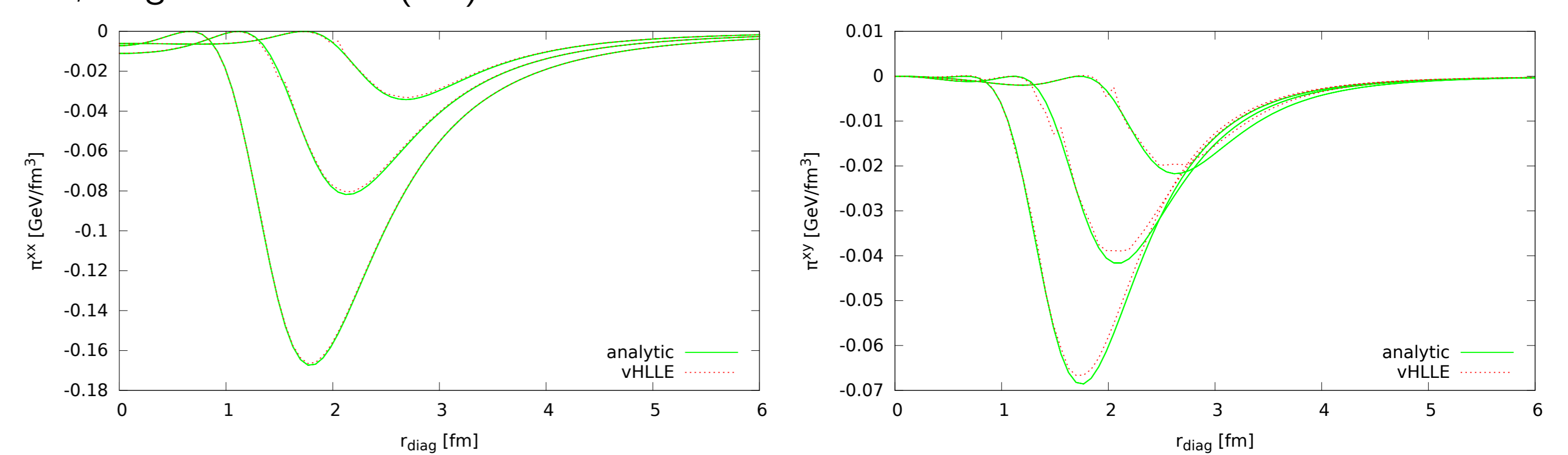
Diagonal direction (XY)



$\pi^{\mu\nu}$ , principal direction (X)



$\pi^{\mu\nu}$ , diagonal direction (XY)



## Applications

The code, as a part of EPOS3 [2], HKM [3] and vHLLC+UrQMD [4] hybrids, is applied for simulations of heavy ion and proton-lead collisions at LHC, RHIC and RHIC BES energies.

**The code is published and described at:**

Iu. Karpenko, P. Huovinen, M. Bleicher, Comput. Phys. Commun. **185** (2014), 3016

<https://github.com/yukarpenko/vhllc>