# In-medium quark mass-gap equation solutions

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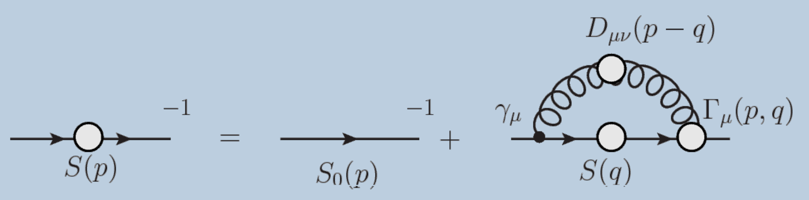
## Abstract

In this work we apply the Dyson–Schwinger formalism to investigate QCD's quark properties, taking advantage of the fact that within this formalism it is relatively easy to include finite chemical potential. We employ a rainbow truncation scheme and investigate two different ansätzes for the dressed gluon propagator. Both of them are simplifications of the general Maris–Tandy abelian gluon propagator [1] commonly used along the rainbow truncation [1][2], with either a dominant IR behavior (Munczek–Nemirovsky) or averaged interaction strength in momentum space with a hard UV cutoff (NJL). We illustrate the resulting in–medium mass gap solutions for both models.

## Dressed quark propagator eq.(4)




## Conclusions

An immediate observation is the lack of momentum dependence in the NJL model, coming from the explicit form of the solution coefficients

$$B(p^2,\tilde{p}_4) = m + \frac{16}{3m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{B(q^2,\tilde{q}_4)}{\vec{q}^2+\tilde{q}_4^2C^2(q^2,\tilde{q}_4)+B^2(q^2,\tilde{q}_4)} \qquad (1)$$

as opposed to those obtained within the MN model, where the momentum integral can be analytically evaluated

$$B(p^2,\tilde{p}_4) = m + \eta^2\frac{B(p^2,\tilde{p}_4)}{\tilde{p}^2A^2(p^2,\tilde{p}_4)+B^2(p^2,\tilde{p}_4)} \qquad (2)$$

The NJL model shows a clear 1st order phase transition. MN chiral quark in vacuum exhibits a phase transition from a massive Nambu–Goldstone phase to a chiraly symmetric Wigner–Weyl phase for high momenta. Particular for this model is a branch cut in the imaginary part of the effective mass, approximately at 4-momentum

$$\tilde{p}^2 = \frac{\eta^2}{4}, \qquad (3)$$

a feature which is not present in the standard quasiparticle picture. The dynamic mass has a rich momentum dependent structure without a strict 0 mass gap solution. We continue to study this model and calculate thermodynamic properties at finite temperature and chemical potential.

## References

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## Truncation

The expression relating the single flavor quark propagator to other Green's functions of QCD (dressed quark propagator) has the form:

$$S^{-1}(p^2,\tilde{p}_4) = i\vec{\gamma}\vec{p} + i\gamma_4\tilde{p}_4 + m + \int\frac{d^4q}{(2\pi)^4}g^2D^{\rho\sigma}(p-q)\frac{\lambda_\alpha}{2}S(q^2,\tilde{q}_4)\Gamma^\alpha_\rho(p,q) \qquad (4)$$

where $\vec{\gamma}\vec{p} = \sum_{i=1}^3\gamma_ip_i$, $\tilde{p}_4 = p_4 + i\mu$ and $\{\gamma_\mu,\gamma_\nu\} = 2\delta_{\mu\nu}$. The general solution to this equation has the form

$$S^{-1}(p^2,\tilde{p}_4) = i\vec{\gamma}\vec{p}A(p^2,\tilde{p}_4) + i\gamma_4\tilde{p}_4C(p^2,\tilde{p}_4) + B(p^2,\tilde{p}_4) \qquad (5)$$

For the dressed-quark-gluon vertex $\Gamma^\alpha_\rho(p,q)$ we have chosen the "rainbow" truncation (*i.e.* leading order approximation of the dressed vertex)

$$\Gamma^\alpha_\rho(p,q) = \frac{\lambda^\alpha}{2}\gamma_\sigma \qquad (6)$$

In the case of the dressed-gluon propagator, two schemes will be investigated.

**Munczek–Nemirovsky model** The original idea presented in [4], extended to finite chemical potential in [5]. The gluon propagator ansatz is

$$g^2D^{\rho\sigma}(k) = 3\pi^4\eta^2\delta^{\rho\sigma}\delta^{(4)}(k) \qquad (7)$$

**NJL-like model** Quark contact interaction achieved by modeling the gluon to be a delta function in configuration space with a hard 3-momentum cutoff to regularize divergent integrals [7]. What follows is a gluon propagator ansatz of the form

$$g^2D^{\rho\sigma}(p-q) = \frac{1}{m_G^2}\Theta(\Lambda^2-\vec{q}^2)\delta^{\rho\sigma} \qquad (8)$$

## Results

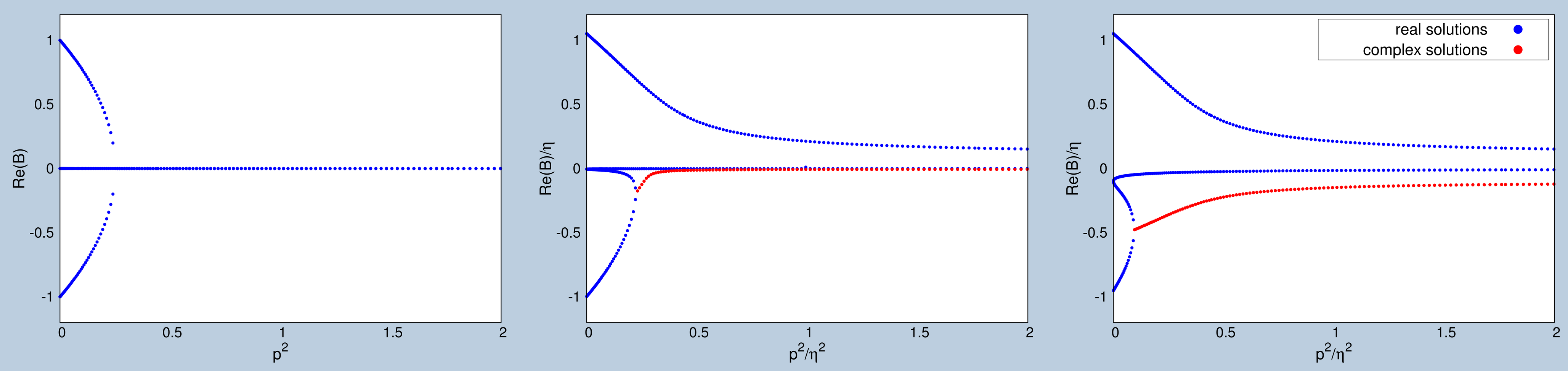


**Figure 1:** Mass and $p^2$ relation for the MN model, chiral limit (left) up and strange quark mass (center and right respectively)

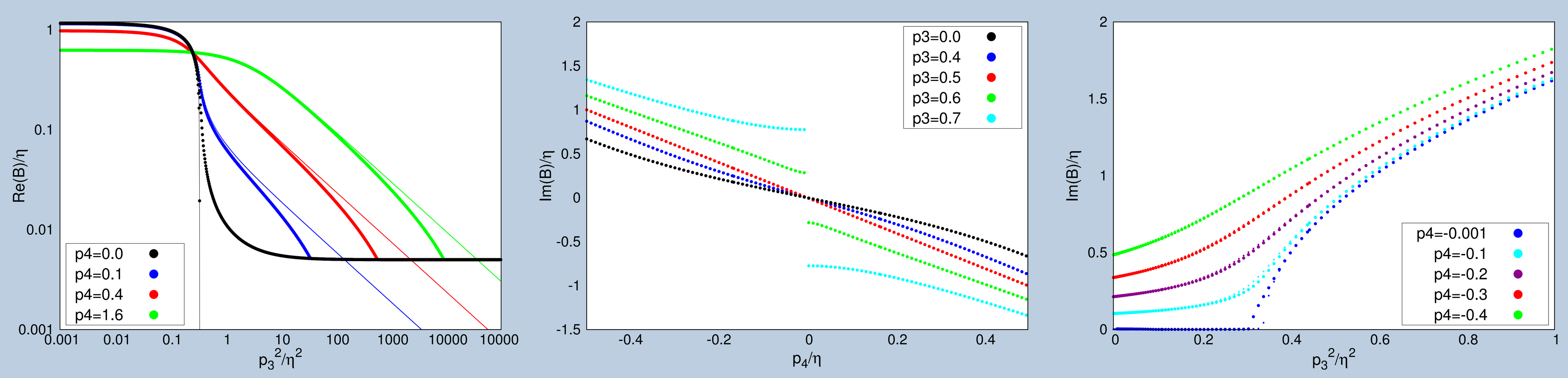


**Figure 2:** MN model, mass and $p_3$ relation (left, thin lines - chiral limit), imaginary mass branch cut (center) and the positive imaginary branch momentum behavior (right)

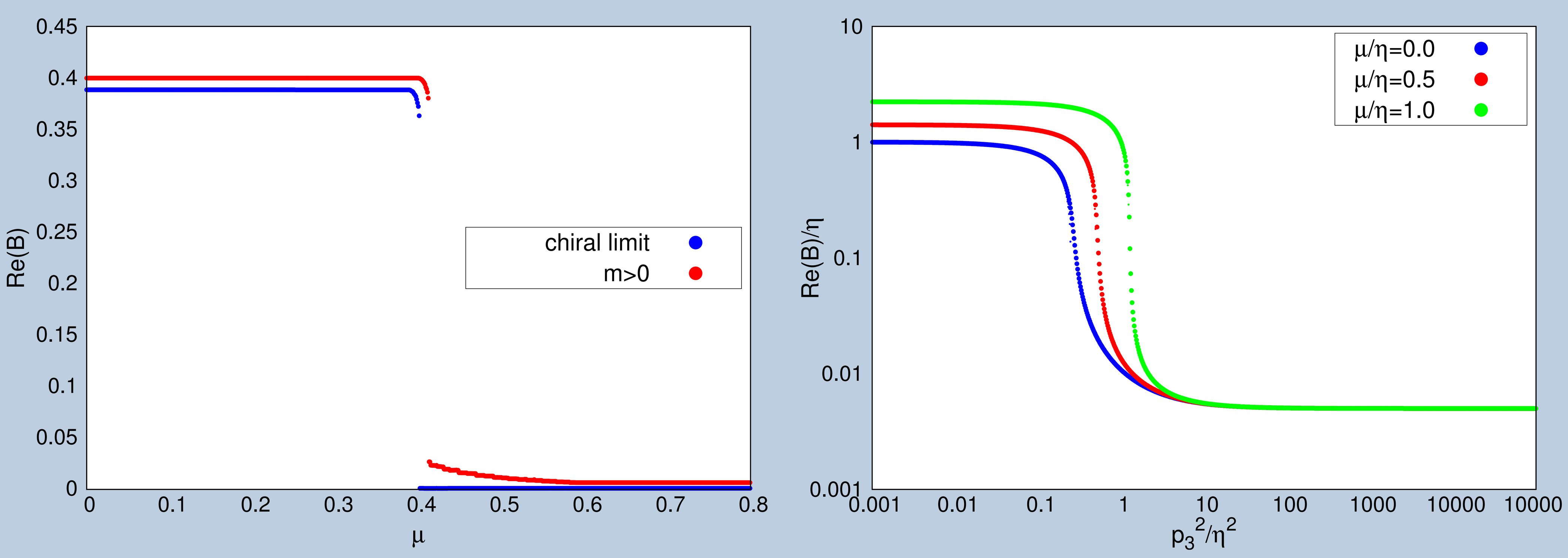


**Figure 3:** Mass and $\mu$ relation for the NJL model (left) and in–medium dynamic mass in the MN model (right)