$$P\,P \rightarrow H + X$$

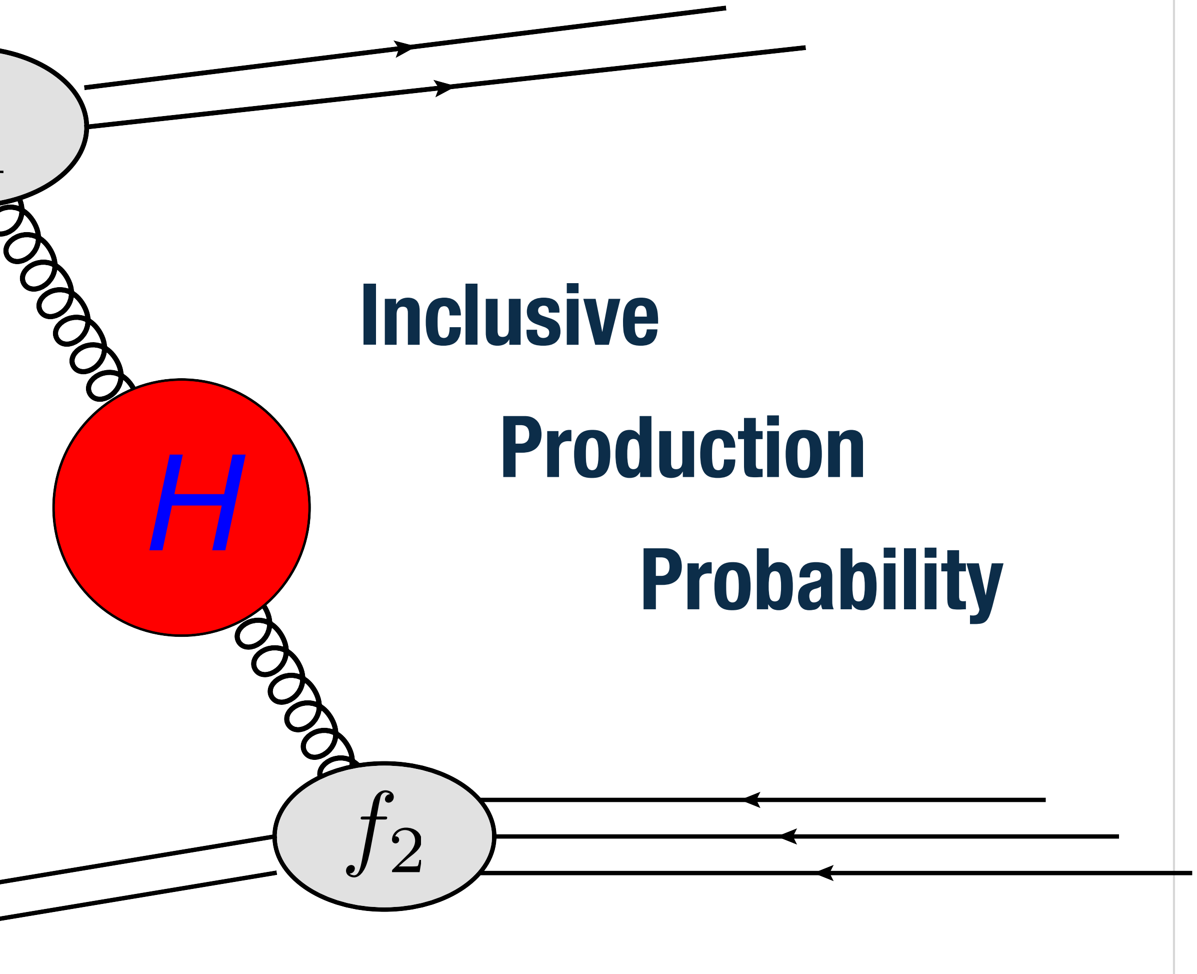
# Precision Predictions at N3LO for Inclusive Higgs Production

**Bernhard Mistlberger**

CERN

In collaboration with
Babis Anastasiou, Claude Duhr, Falko Dulat, Elisabetta Furlan,
Thomas Gehrmann, Franz Herzog, Achilleas Lazopoulos

# HIGGS PRODUCTION



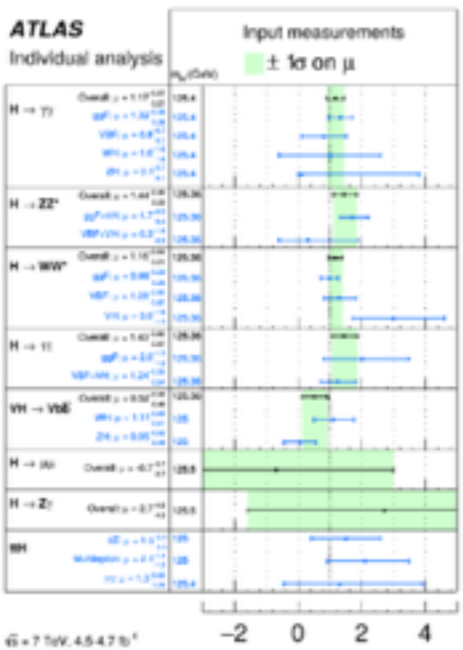

$$\sigma_{PP \to H+X}$$

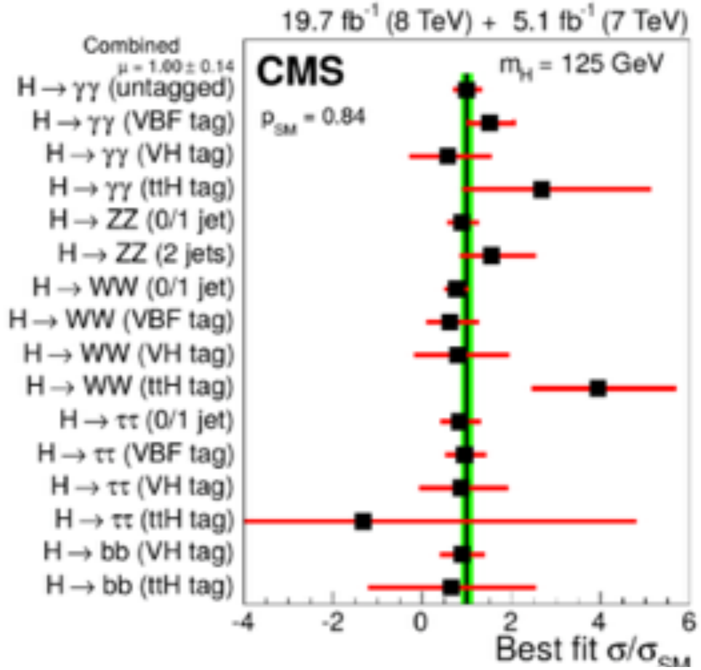
Inclusive

Production

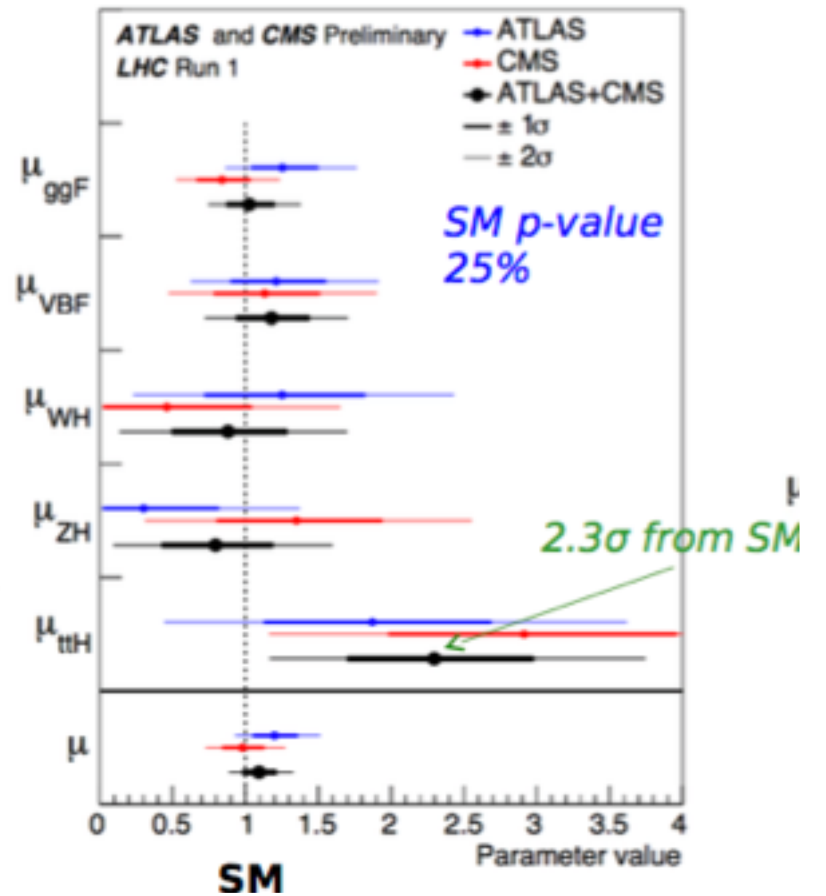
Probability

# HIGGS PRODUCTION



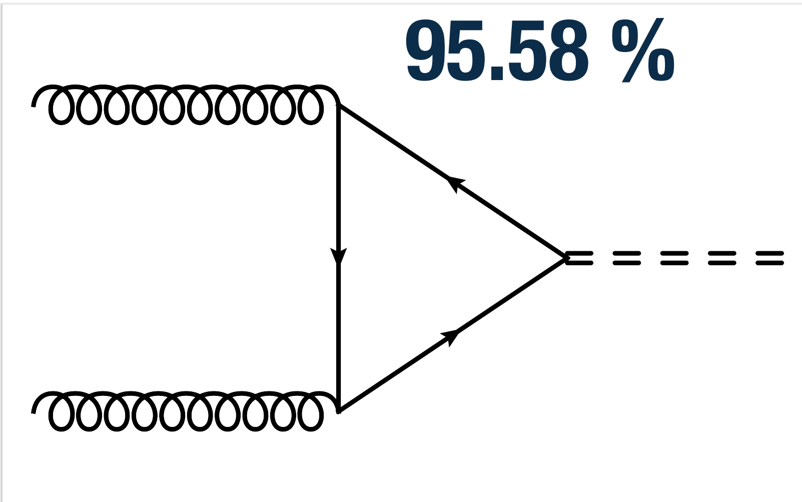

**Production signal strengths**
*(SM values of BRs assumed)*

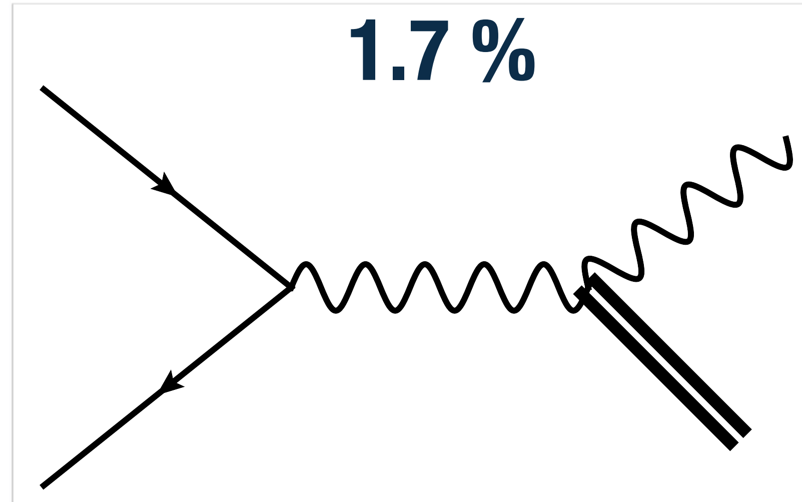


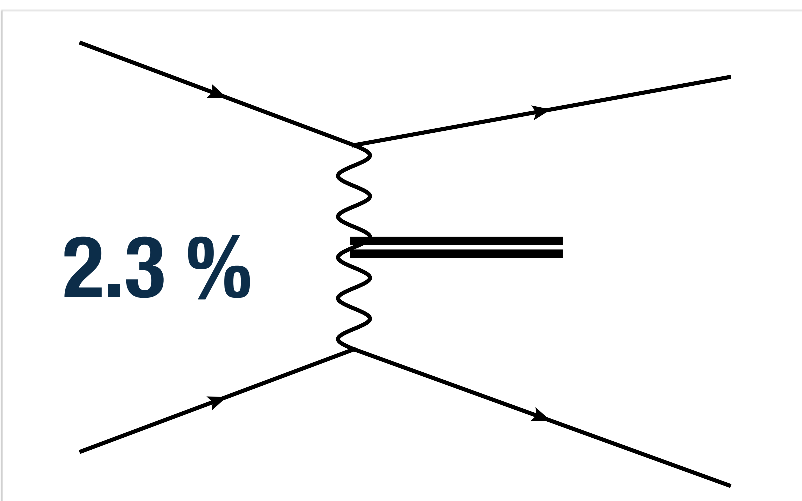

$$\mu = 1.09^{+0.11}_{-0.10}$$

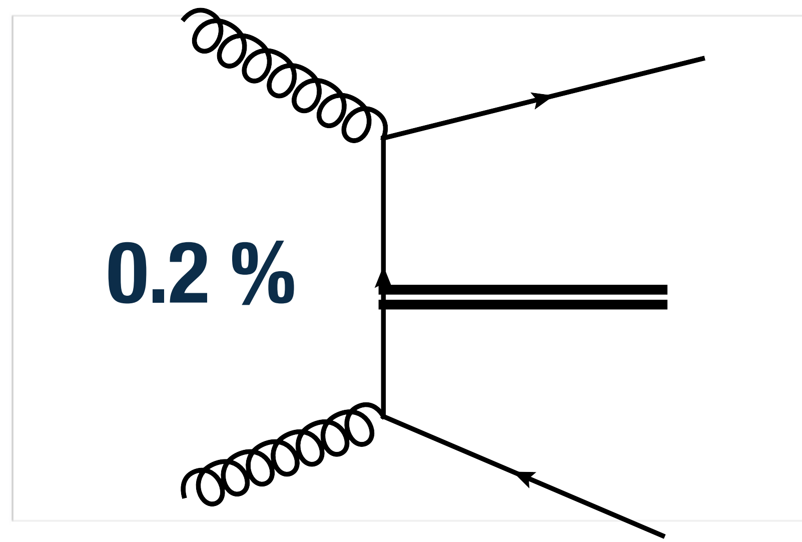
$$= 1.09^{+0.07}_{-0.07}\ (\text{stat})\ {}^{+0.04}_{-0.04}\ (\text{expt})\ {}^{+0.03}_{-0.03}\ (\text{thbgd})^{+0.07}_{-0.06}\ (\text{thsig})$$

# HIGGS PRODUCTION

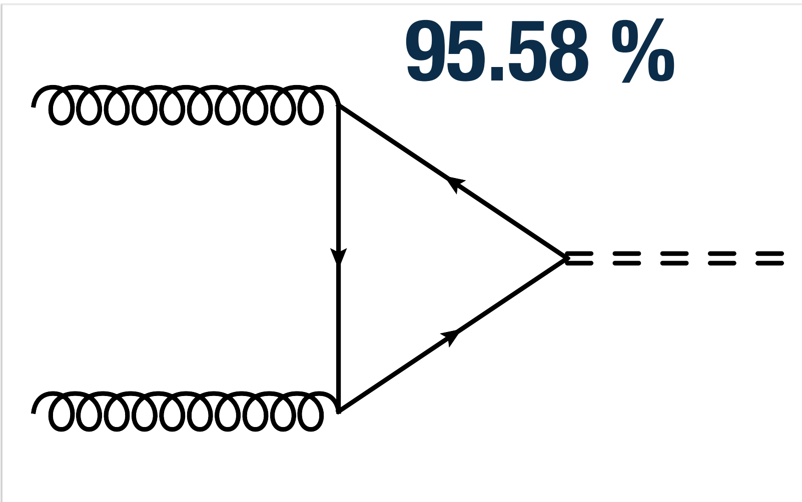


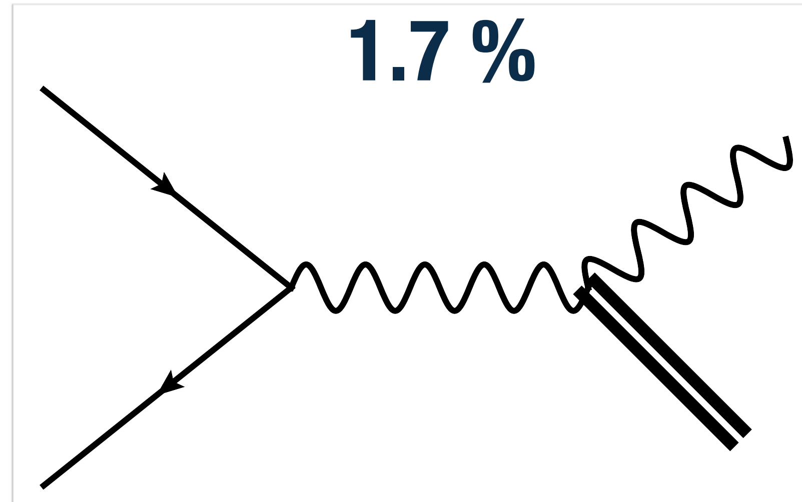



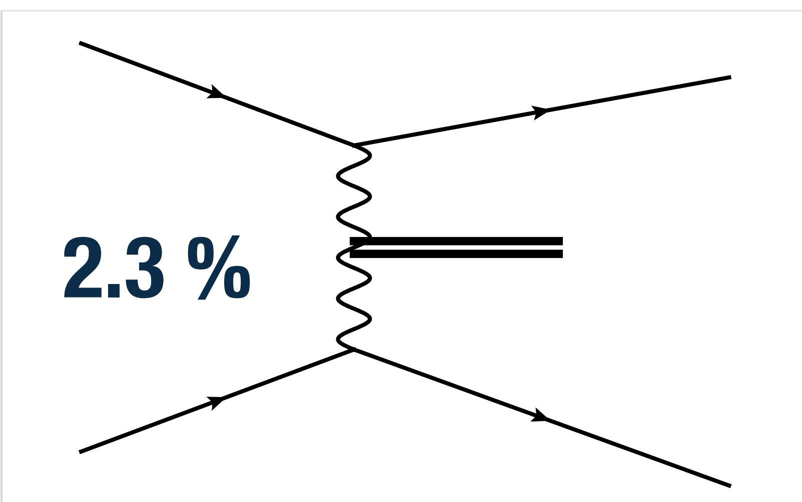



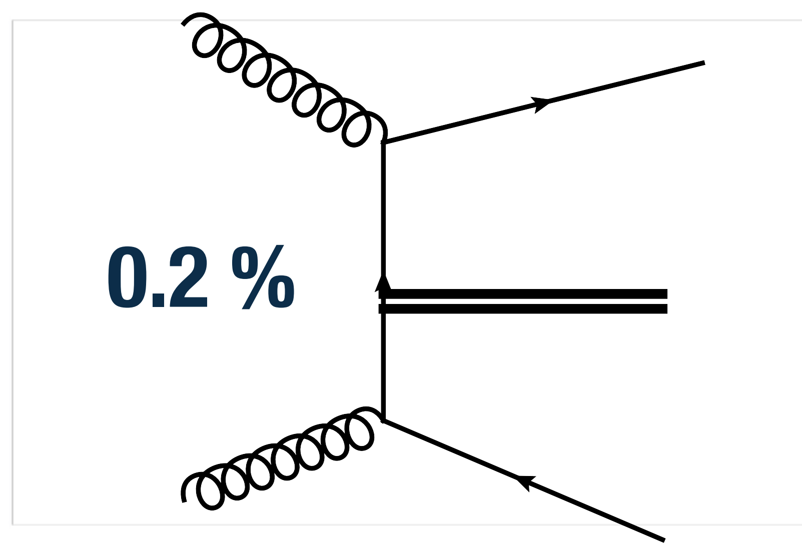



# HIGGS PRODUCTION



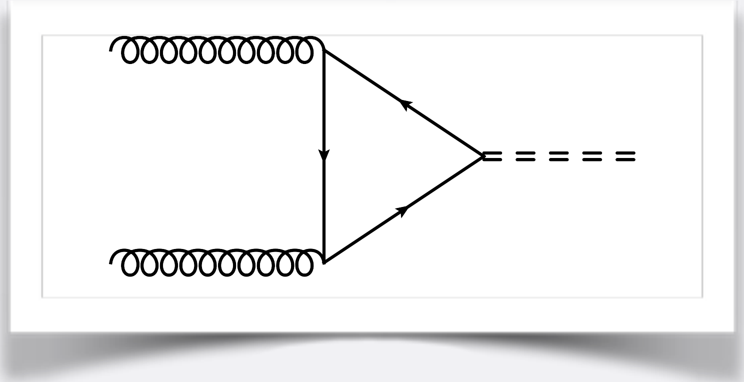



# HIGGS PRODUCTION

Dominant Mechanism: Gluon Fusion



Top-Quark coupling

Gateway to NP

Key ingredient for SM couplings

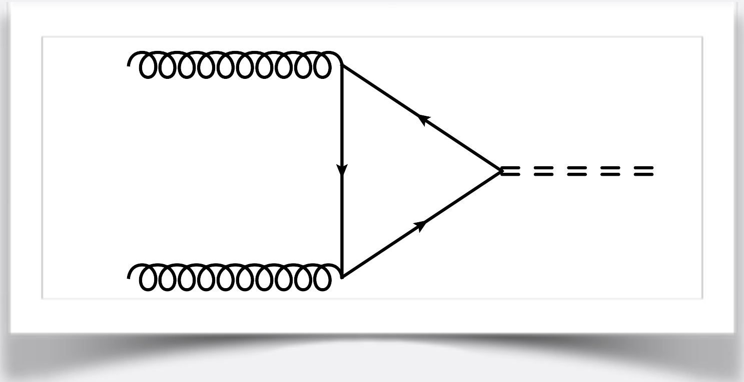
Ingredients:

- PDF
- QCD
- Electro-Weak

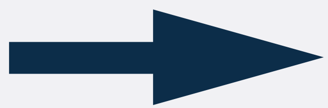
# EFFECTIVE THEORY

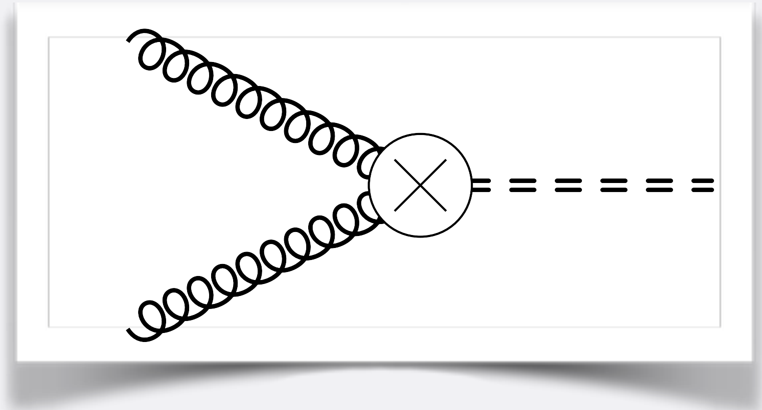
# EFFECTIVE THEORY

## QCD - Effective Theory

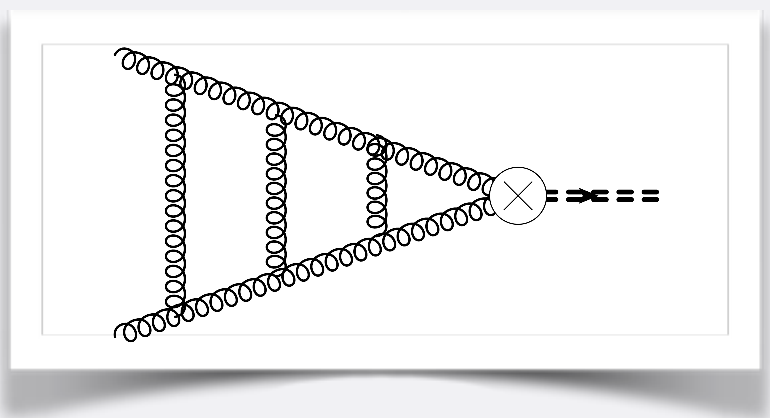


$m_t \to \infty$





## Current Status: N3LO



[Anastasiou,Duhr,Dulat,Furlan,
Gehrmann,Herzog,Lazopoulos,BM]

# HIGGS PRODUCTION

## Current Status: N3LO

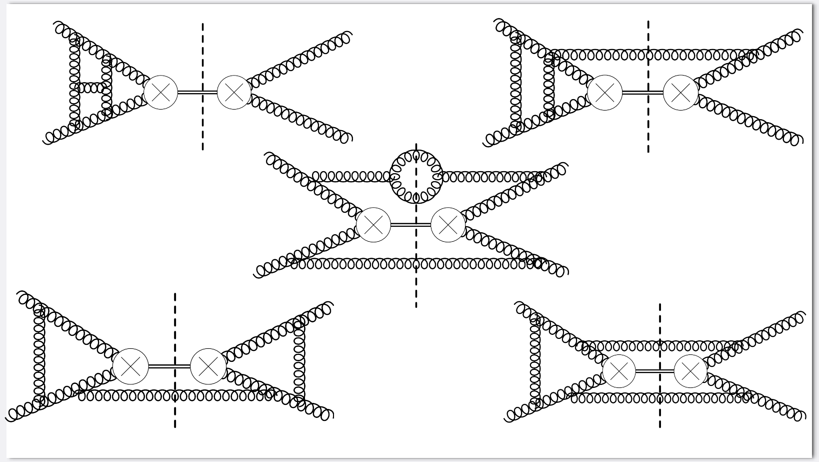
$$\hat{\sigma}(z) = \hat{\sigma}^{LO}(z) + \alpha_S \hat{\sigma}^{NLO}(z) + \alpha_S^2 \hat{\sigma}^{NNLO}(z) + \alpha_S^3 \hat{\sigma}^{N3LO}(z) + \mathcal{O}(\alpha_S^4)$$

✓ ✓ ✓ ✓

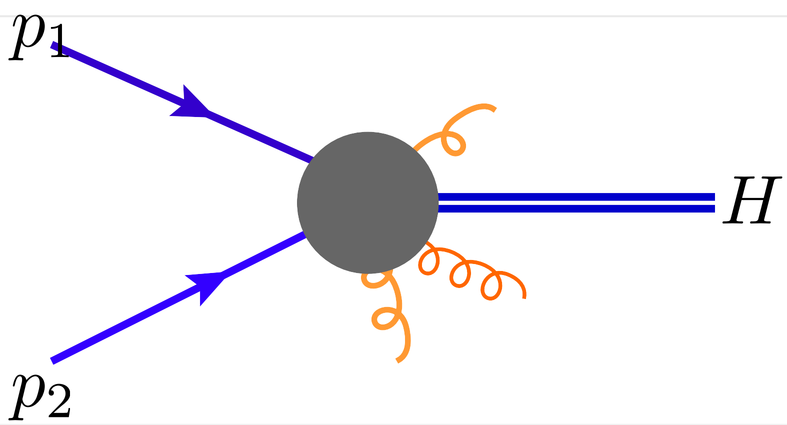
## Fully analytic formula

$$\sigma = \sum \int dx_1 dx_2 f(x_1) f(x_2) \hat{\sigma}(x_1 x_2)$$

## First Hadron Collider Observable at this order in QCD



# THRESHOLD EXPANSION



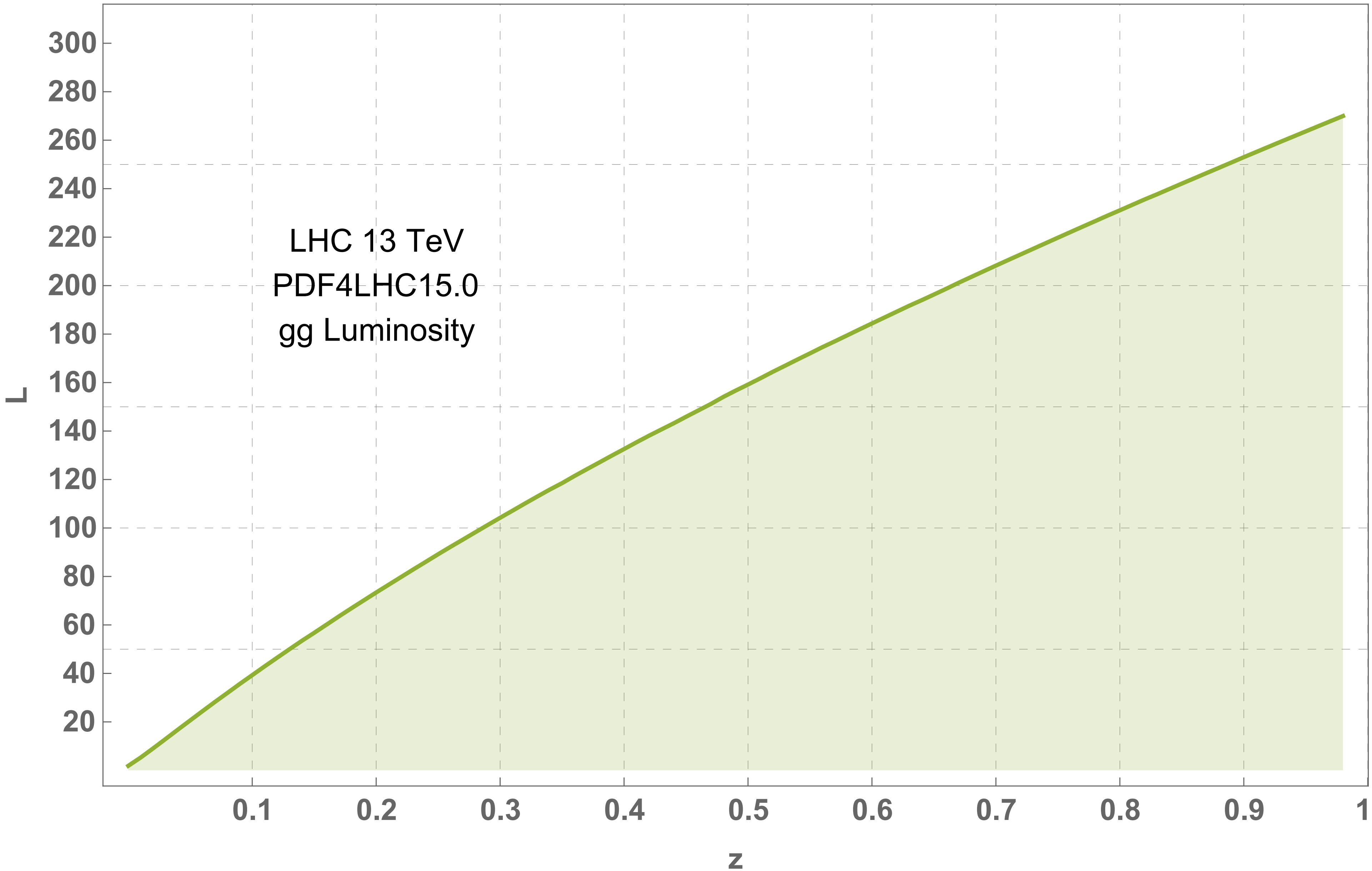

$$z = \frac{m_H^2}{\hat{s}} \sim 1$$

## Expansion around the production threshold

$$\bar{z} = 1 - z \quad \Rightarrow \quad \hat{\sigma}(\bar{z}) = \sigma^{SV} + \sigma^{(0)} + \bar{z}\sigma^{(1)} + \ldots$$

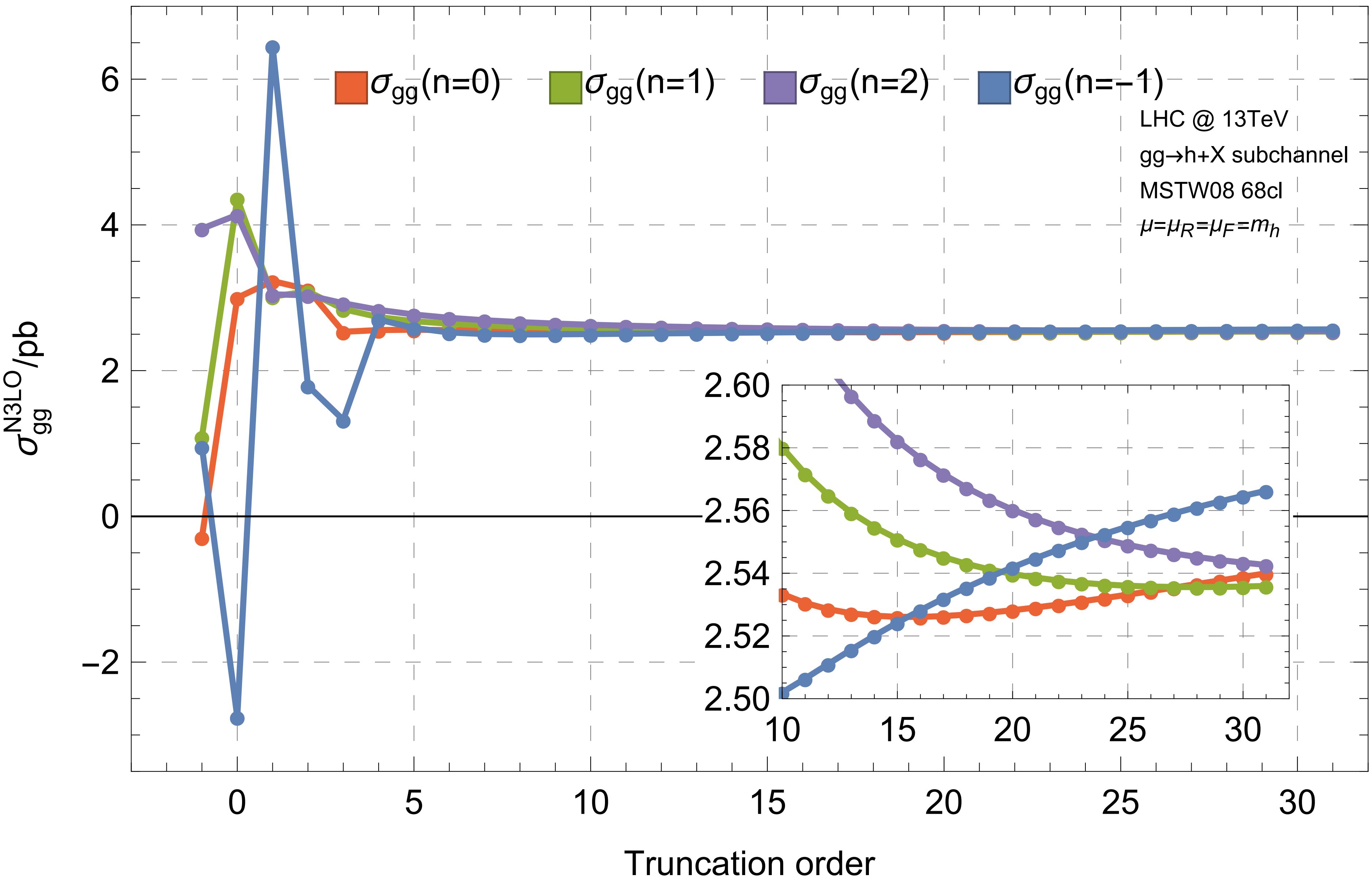
## Why is that a good idea?  Gluon Luminosity

# LUMINOSITY



LHC 13 TeV
PDF4LHC15.0
gg Luminosity
L
z
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.

# THRESHOLD EXPANSION



# ESTIMATING UNCERTAINTY

**Look at the progression of the series!**

**One way of estimating higher order uncertainties**

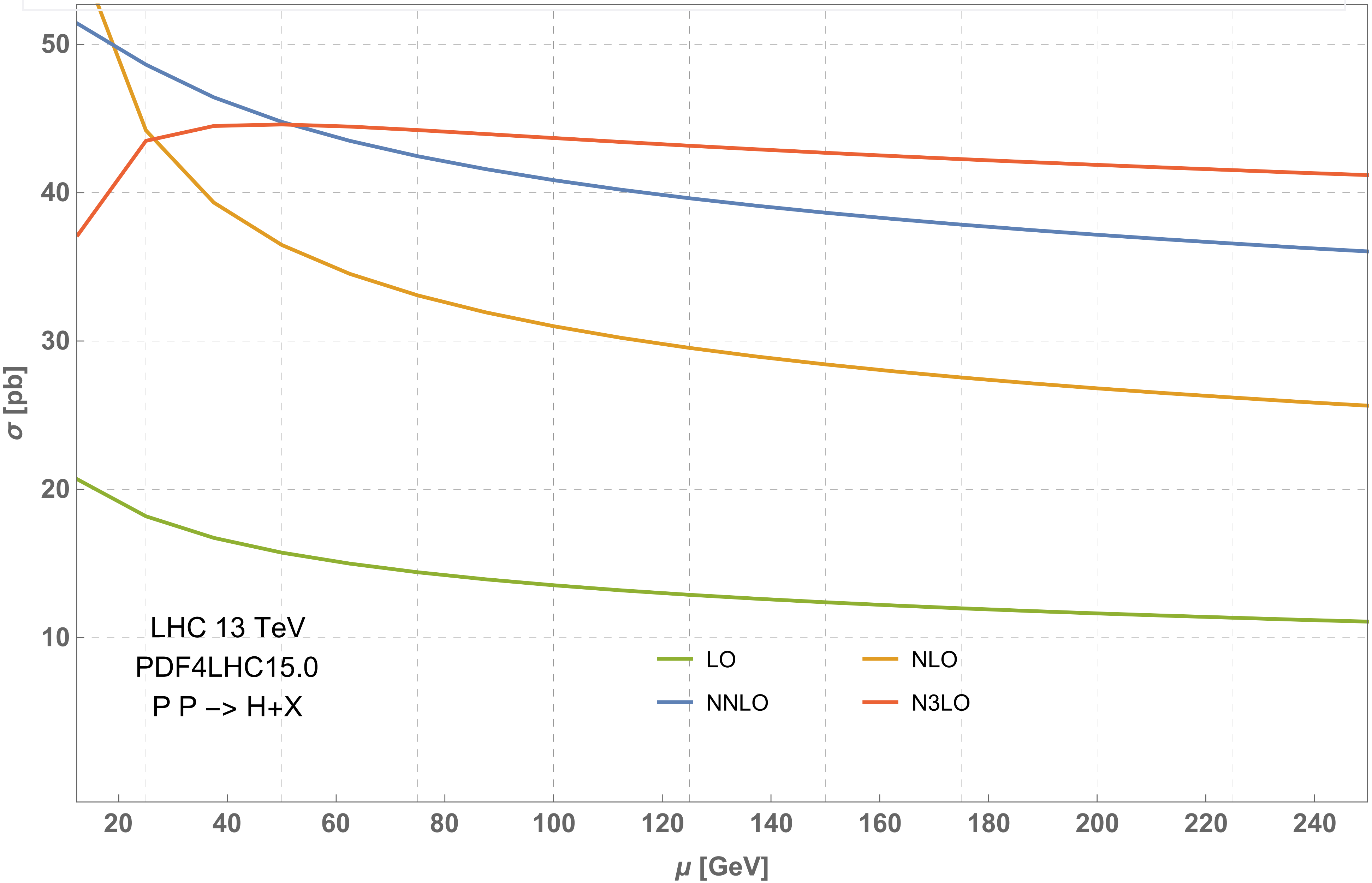
**Scale Variation**

$$\mu_{central} = \frac{m_h}{2}$$

**vary** $\mu \in \left[\frac{m_h}{4}, m_h\right]$

$\delta\mu$ **from maximum and minimum of variation**

# SCALE VARIATION



# SCALE VARIATION

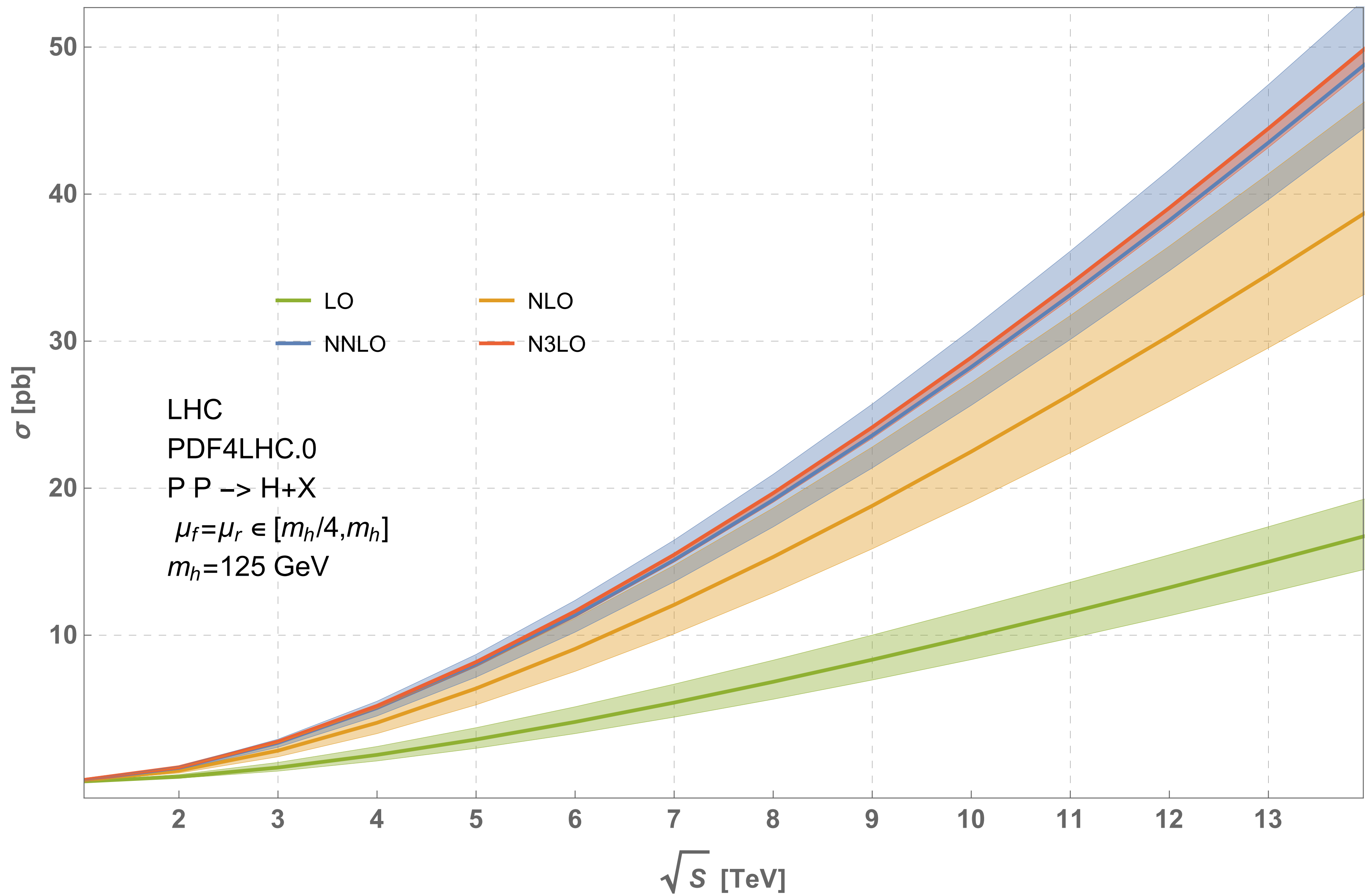
| | $\sigma^{eff}_{PP\to H+X}[pb]$ | $\delta_\mu[\%]$ |
|---|---|---|
| $LO$ | 15.0 | $^{15.9}_{-14.0}$ |
| $NLO$ | 34.5 | $^{19.8}_{-14.5}$ |
| $NNLO$ | 43.5 | $^{9.1}_{-8.9}$ |
| $N^3LO$ | 44.6 | $^{0.32}_{-2.91}$ |

Progression of series looks good

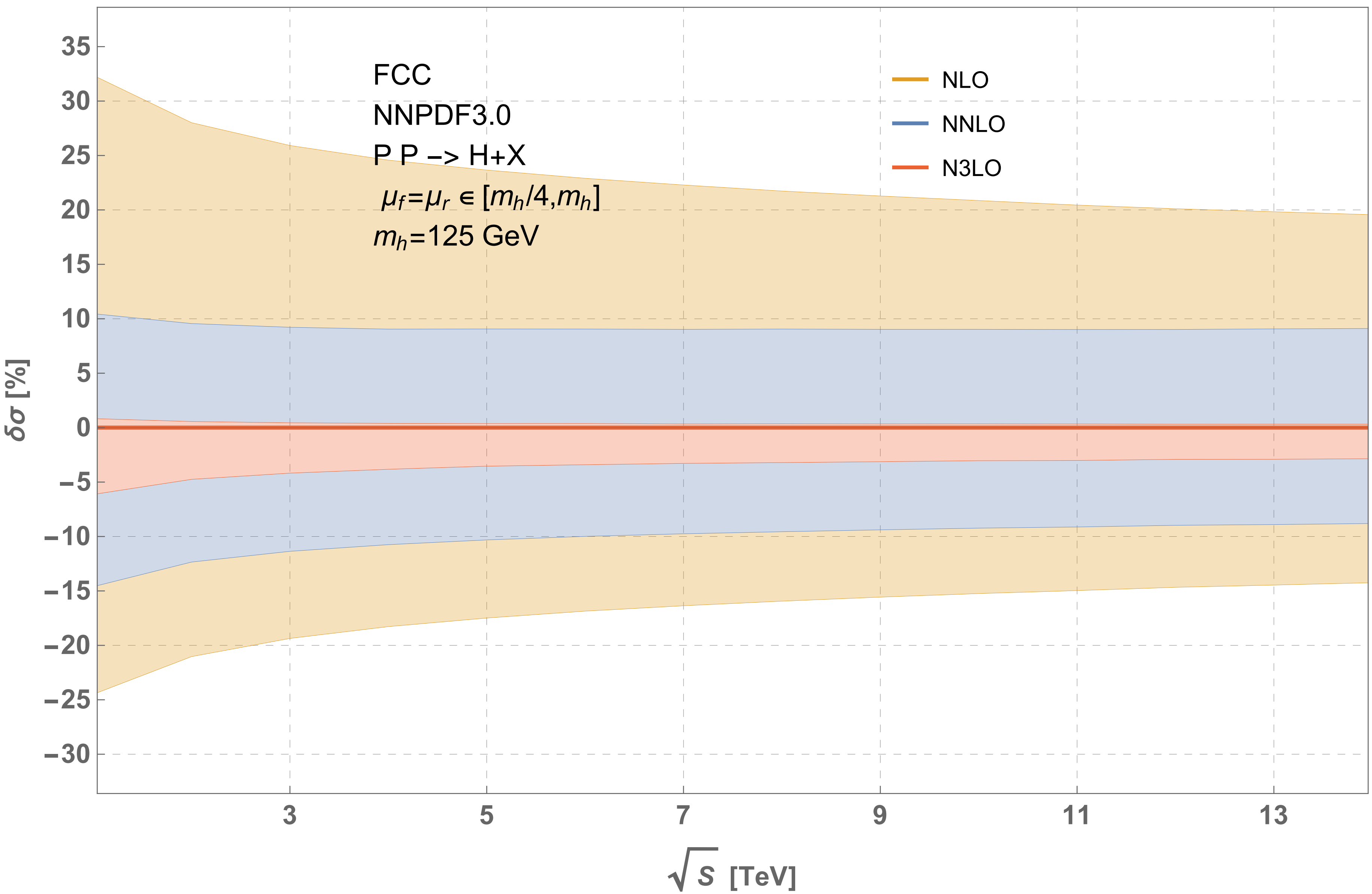
N3LO and NNLO central value similar!

Significantly smaller variation!

# ENERGY VARIATION



# ENERGY VARIATION



# SCALE VARIATION

## Separating scales

$$\sigma(\mu) = \alpha_S^2(\mu)\sigma^{LO}(\mu) + \alpha_S^3(\mu)\sigma^{NLO}(\mu) + \alpha_S^4(\mu)\sigma^{NNLO}(\mu) + \alpha_S^5(\mu)\sigma^{N^3LO}(\mu)$$

**Call** $\mu \to \mu_f$

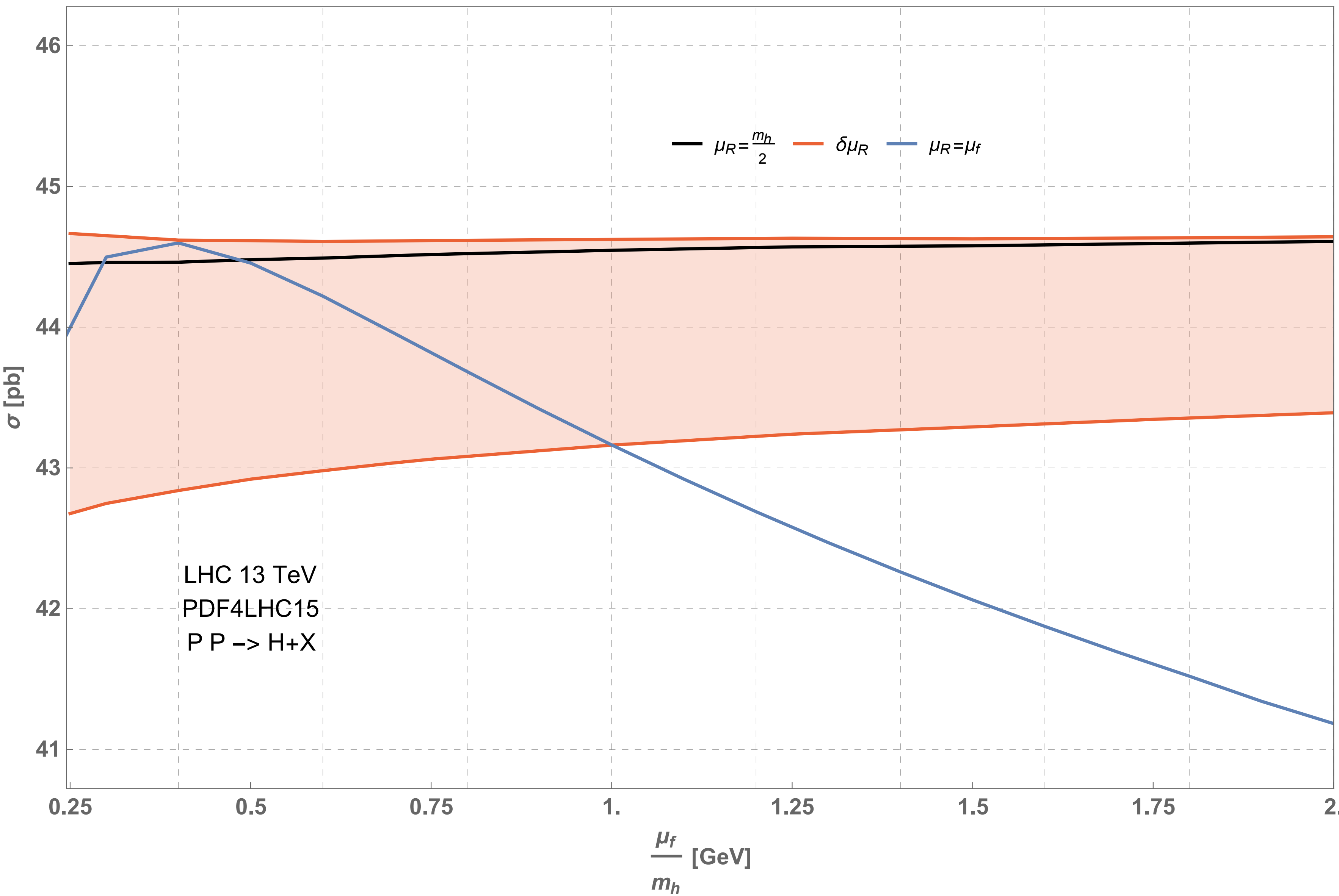
**Replace** $\alpha_S(\mu_f) = \alpha_S(\mu_R) - \frac{\alpha_S(\mu_R)^2}{\pi}\beta_0 \log\left(\frac{\mu_f^2}{\mu_R^2}\right) + \dots$

**Re-expand through** $\mathcal{O}(\alpha_S(\mu_R)^5)$

$$\mu_R^{central} = \frac{m_h}{2}$$

**Vary** $\mu_R \in \left[\frac{m_h}{4}, m_h\right]$

# SCALE VARIATION



# SCALE VARIATION

## Allowing for extreme variations

$$\sigma^{eff}_{PP\to H+X} = 44.6pb \pm^{0.42\%}_{-4.1\%}$$

## Extreme variations:

for example: $\mu_f = m_h \quad \mu_R = \frac{m_h}{4}$

## Commonly not used for estimates

# ALTERNATIVES?

Look for different representation
of an N3LO cross section

Threshold resummation

Deals with the first term in the threshold expansion

Predicts terms at higher order in $\alpha_S$

Most valid when threshold parameter is ~one

$$\tau = \frac{m_h^2}{S} = 0.0000924556$$

2(+) Schools:

SCET

"Conventional" Resummation

# CON. RESUMMATION

[Stermann;Catani,Trentadue;
Grazzini,de Florian,Nason;Forte&Co.,
Bonvini,Marzani;…]

**Factor the hadronic cross section by Mellin-Transform**

$$\sigma(N) = \int_0^1 d\tau \tau^{N-1} \sigma(\tau) = f_g^2 C_{gg} e^{G_H} + \mathcal{O}(\frac{1}{N})$$

**Threshold limit ~** $N \to \infty$

**Contains terms that are formally divergent and are resumed by exponential** $\log(N)$

**Numeric Mellin inversion**

**Logarithmic accuracy: N3LL**

# SCET

[Ahrens,Becher,Neuber,Li,…]

**Laplace transform to factor the cross section**

$$\sigma(\eta) = \int_0^\infty d\tau e^{\eta\tau}\sigma(\tau) = \sigma_0 H S C_t^2 f_g^2 \ +\text{''}\mathcal{O}((1-z)^0)\text{''}$$

**Solve evolution equation for factored functions**

$$\frac{\partial}{\partial\mu^2}\sigma(\eta) = 0$$

**Transform back to momentum space**

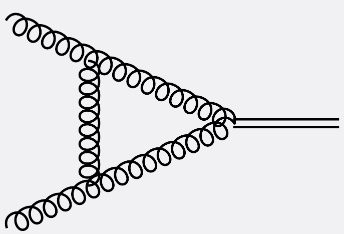
**Target: Reduce the scale dependence**

# SCET $\pi^2$

## Hard function

$$\frac{\partial}{\partial\mu} H(\mu) = C(\mu) H(\mu) \quad \rightarrow \quad H(\mu) = e^{\int_{\mu_0}^{\mu} d\mu' C(\mu')} H(\mu_0)$$

## H= "IR"-renormalised Form Factor



$$\sim \left(-\frac{m_h^2}{\mu^2}\right)^{-\#\ \text{loops}\ \times\epsilon} \qquad C$$

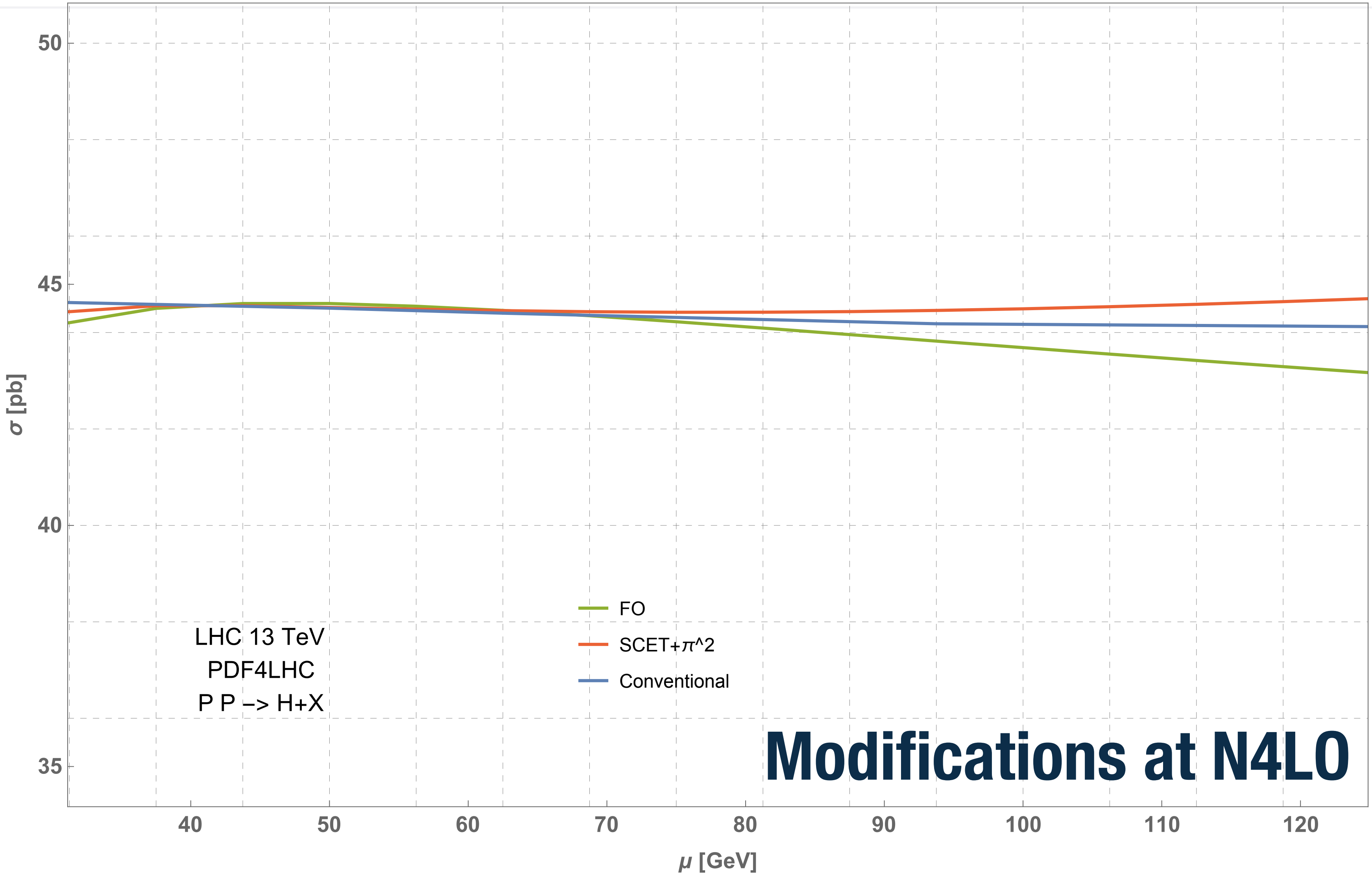
$$Re\left(\left(-1\right)^{-\epsilon}\right) = 1 - \frac{\pi^2\epsilon^2}{2} + O\left(\epsilon^4\right)$$

**Choose** $\mu_0^2 = -m_h^2$ **exponentiates some**

**NLO Virtual =** $\pi^2$ **Also responsible for large K-factor**

**Partly resummed by SCET**

# THRESHOLD RESUMMATION




**Modifications at N4LO**

PDFS

# PDFS

**PDF4LHC15**

**Combination of CT14, MMHT and NNPDF3**

**PDF uncertainty here: 100 MC**

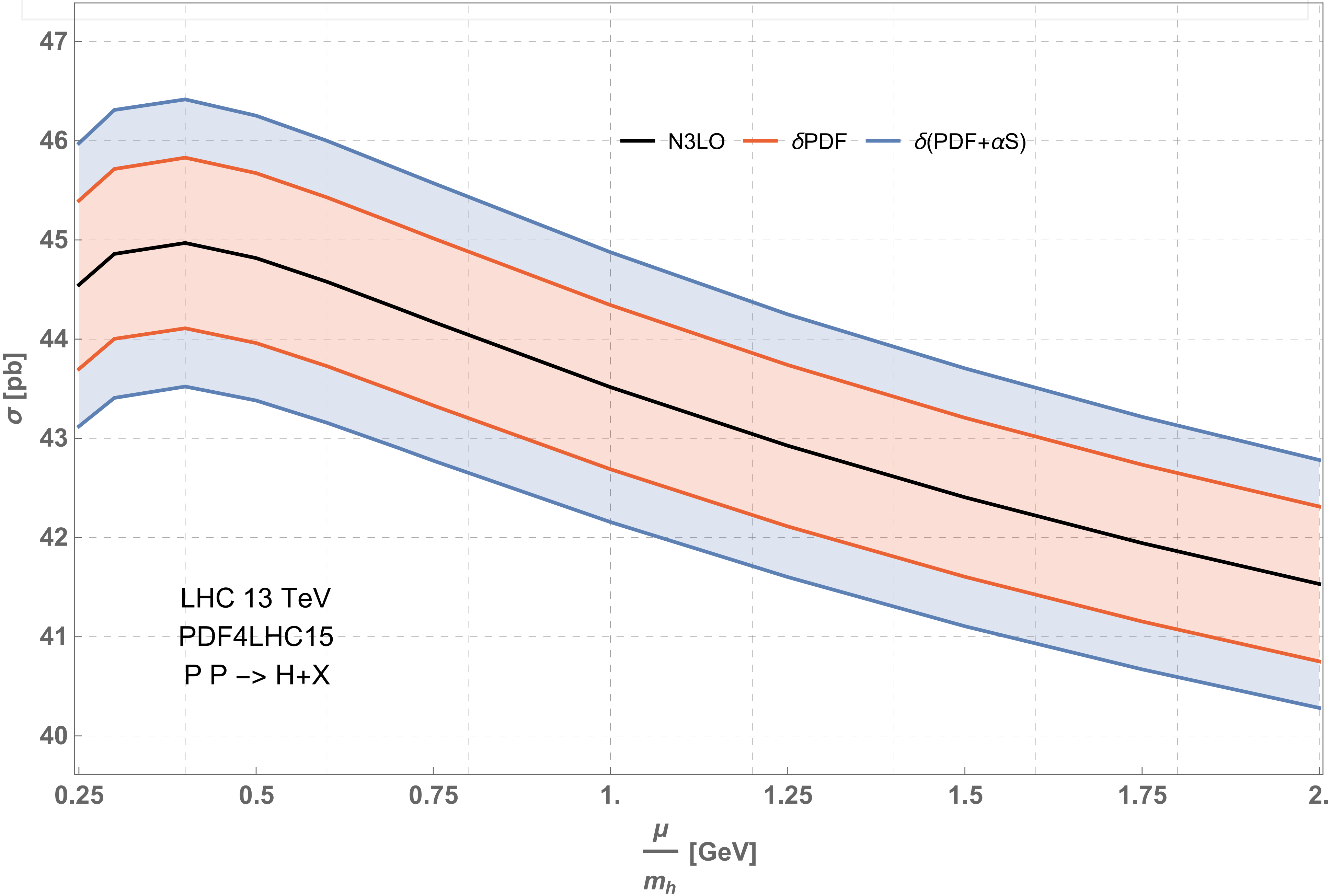
$\delta_{PDF} =$ **one standard deviation of all 100 MC results**

$$\alpha_S(m_Z) = 0.118 \pm 0.0015$$

$$\delta_{\alpha_S} = \frac{\sigma(\alpha_S = 0.1195) - \sigma(\alpha_S = 0.1165)}{2}$$ ⟵ **Error Sets!**

$$\delta_{\alpha_S + PDF} = \sqrt{\delta^2_{\alpha_S} + \delta^2_{PDF}}$$

# PDFS



N3LO
δPDF
δ(PDF+αS)
LHC 13 TeV
PDF4LHC15
P P -> H+X
σ [pb]
47
46
45
44
43
42
41
40
0.25
0.5
0.75
1.
1.25
1.5
1.75
2.
μ / m_h [GeV]

# PDFS

**Small PDF uncertainty**

$$\delta_{PDF} = 1.91\%$$

**Relatively large coupling uncertainty**

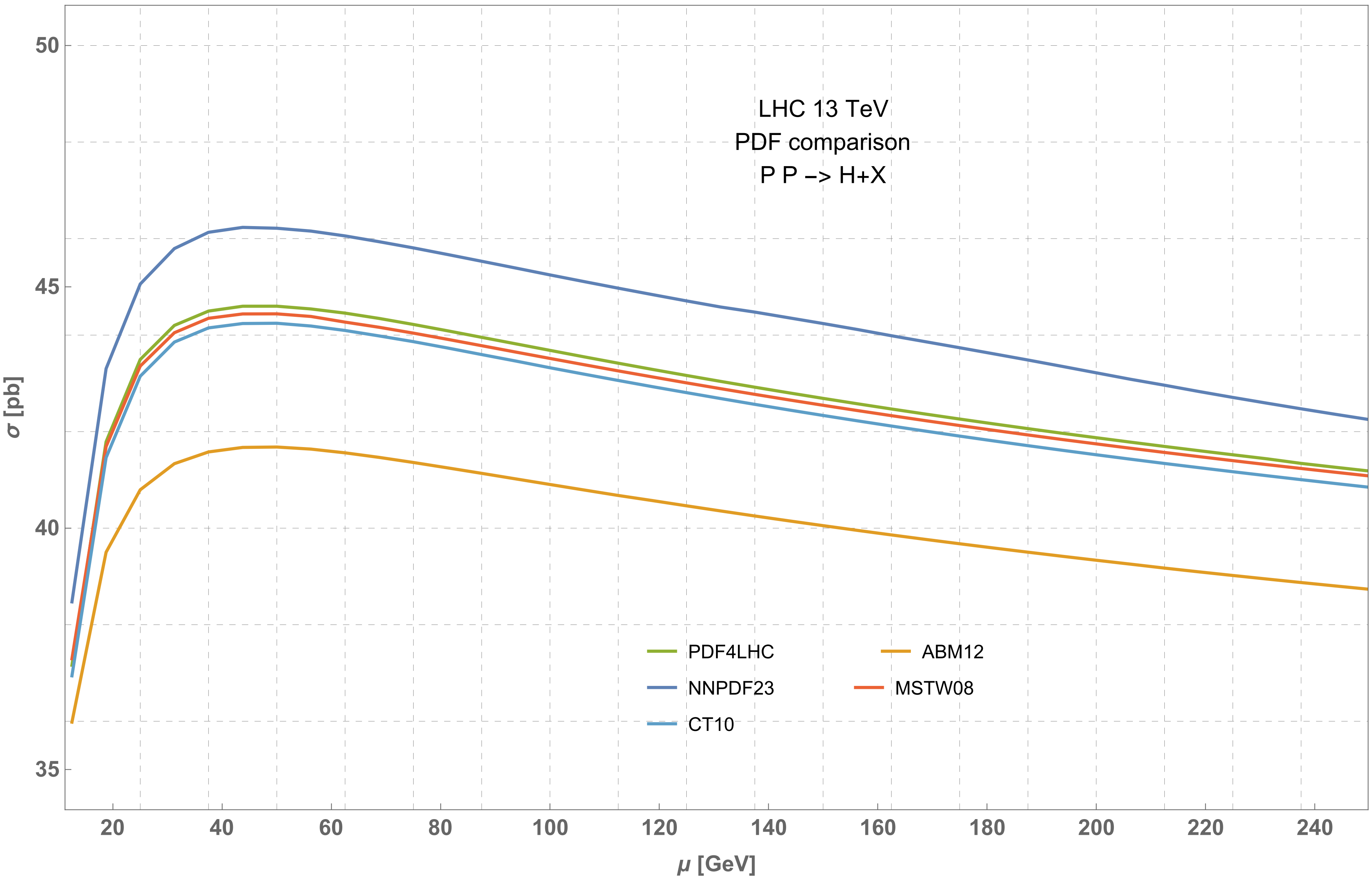
$$\delta_{\alpha_S} = 2.6\%$$

**Quadratic combination of uncertainties**

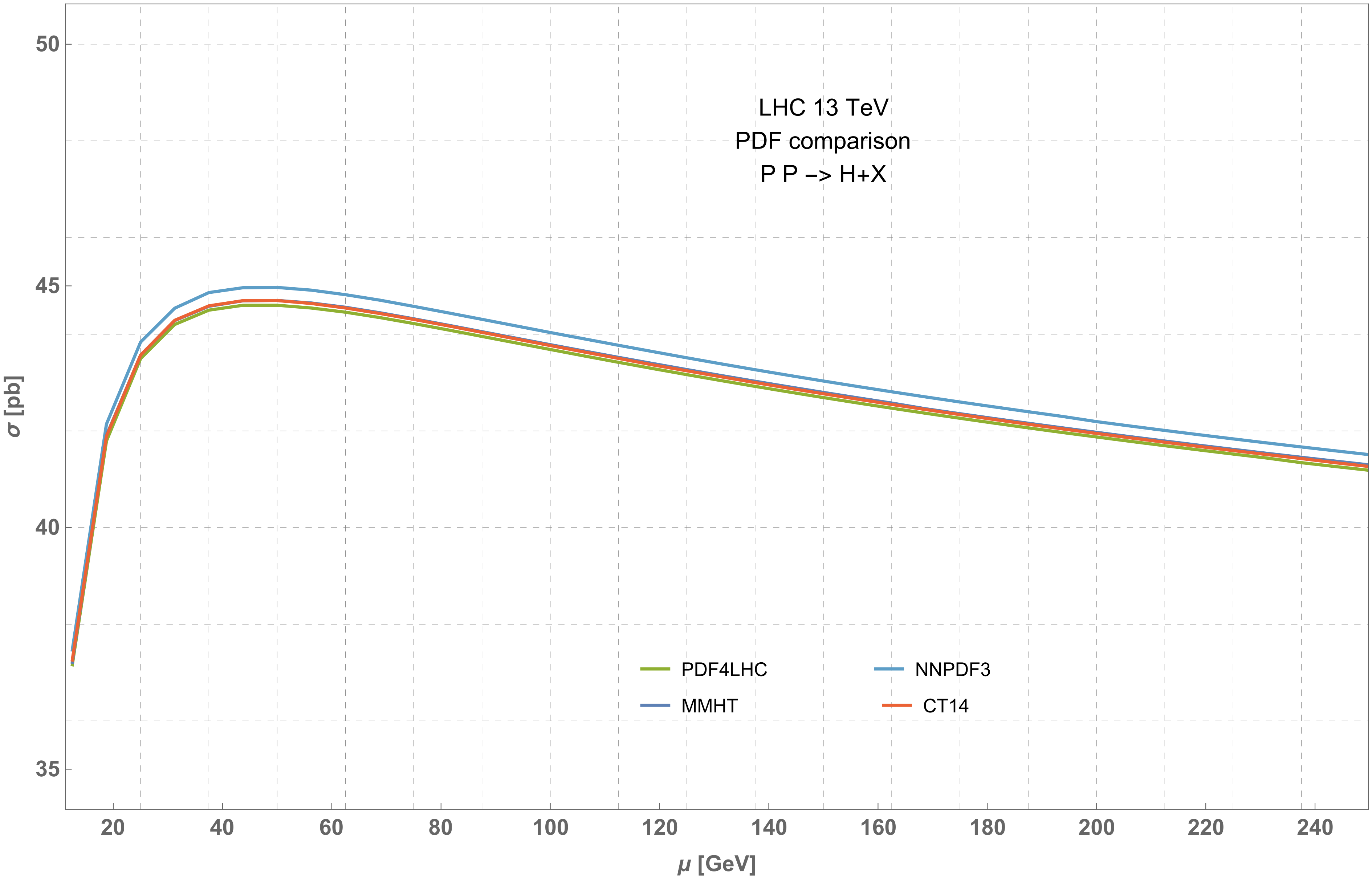
$$\delta_{PDF+\alpha_S} = 3.2\%$$

**Individual Sets? How much did they change?**

# PREVIOUS VERSIONS



# NEWER VERSIONS

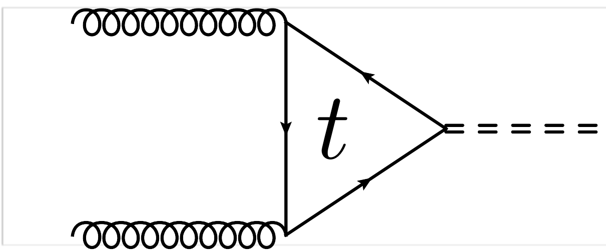


# QUARK MASSES


# MASS EFFECTS

We know:

- Full LO: Energy Independent Increase

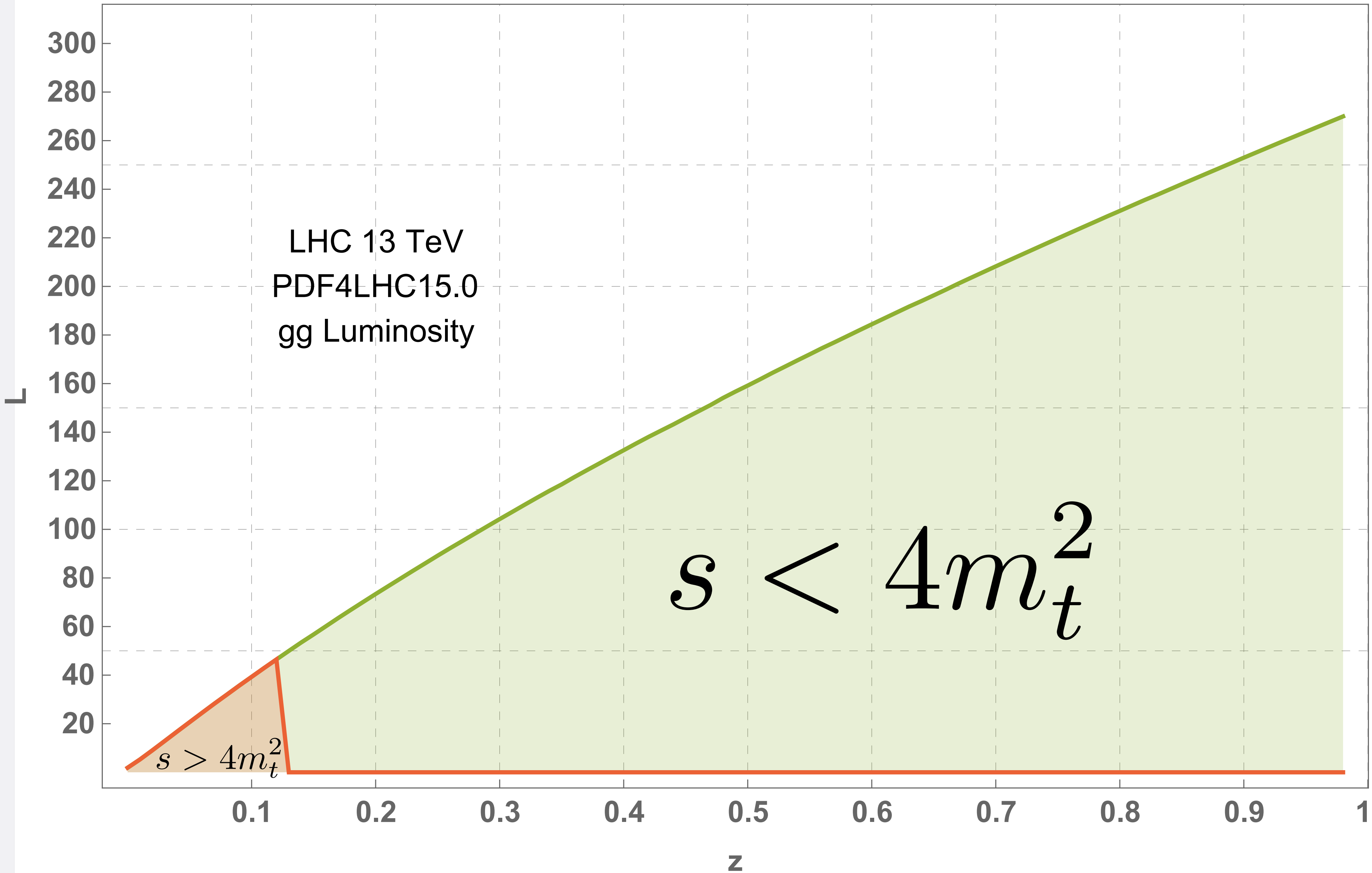



- Full NLO [Djouadi,Graudenz,Spira,Zerwas; Aglietti,Bonciani,Degrassi,Vicini;…]
- NNLO approximation [Harlander,Mantler,Marzani,Ozeren;Pak,Rogal,Steinhauser]

Approximation: Expansion in ~ $\frac{s}{4m_t^2}$

How well does the effective theory work?

# MASS EFFECTS



# MASS EFFECTS

**How to combine the effective theory with known results**

$$K_t = \frac{\sigma_{ex}^{LO}}{\sigma_{eft}^{LO}}$$

**only** $m_t \neq 0$

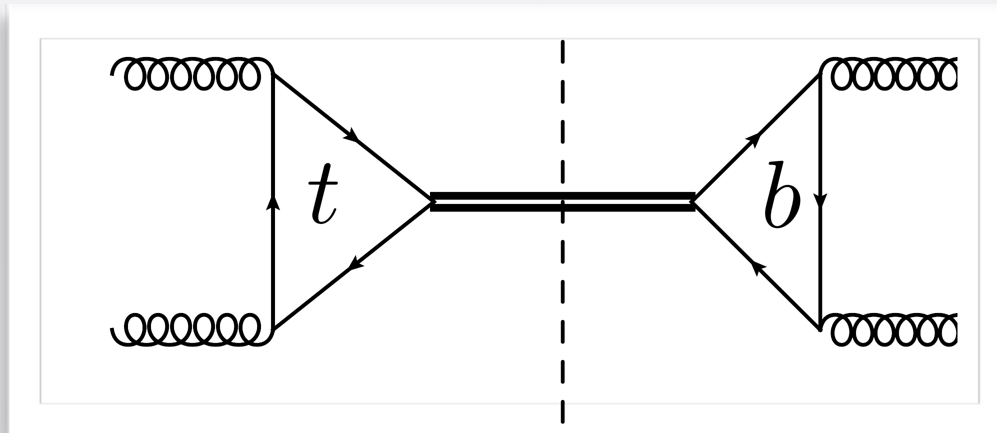
**Rescale effective theory**

$$\sigma_{eft,R} = K_t \left( \sigma_{eft}^{(0)} + \alpha_S \sigma_{eft}^{(1)} + \ldots \right)$$

**Combine higher order rescaled EFT contributions with lower order full mass contributions**

# MASS EFFECTS

## Bottom/Charm Quarks: Interference with top-quark loop




| | | |
|---|---|---|
| $\sigma^{LO}_{eft}$ | 15.13[7] | |
| $\sigma^{LO}_{eft;R}$ | 16.08[1] | |
| $\sigma^{LO}_{ex.;t}$ | 16.08[1] | $+6.2\%$ |
| $\sigma^{LO}_{ex.;t+b}$ | 15.02[1] | $-0.7\%$ |
| $\sigma^{LO}_{ex.;t+b+c}$ | 14.90[1] | $-1.5\%$ |

# MASS EFFECTS

| | | |
|---|---|---|
| $\sigma^{NLO}_{eft}$ | 34.81[1] | |
| $\sigma^{NLO}_{eft;R}$ | 37.00[2] | $+6.3\%$ |
| $\sigma^{NLO}_{ex;t}$ | 36.76[1] | $+5.6\%$ |
| $\sigma^{NLO}_{ex;t+b}$ | 35.09[1] | $+0.8\%$ |
| $\sigma^{NLO}_{ex;t+b+c}$ | 34.91[1] | $+0.3\%$ |

**Rescaled effective theory describes the full theory well**

**Interference contributions with light quarks are large!**

**NNLO: We only know an approximation**

**Correction: ~+0.4%      Uncertainty:~1 % ?**

# MASS EFFECTS

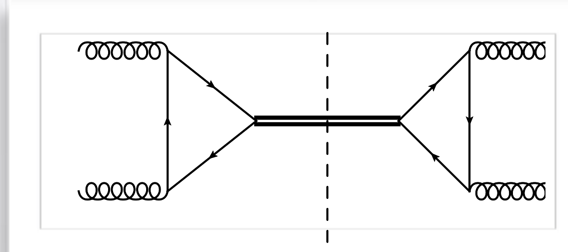
## Light quark masses sensitive to RG scheme!

| | $\overline{MS}$ | | OS |
|---|---|---|---|
| $\sigma^{LO}_{ex;t+b+c}$ | 14.90[1] | $\sigma^{LO}_{ex;t+b+c}$ | 16.12[1] |
| $\sigma^{NLO}_{ex;t}$ | 36.76[1] | $\sigma^{NLO}_{ex;t}$ | 36.80[1] |
| $\sigma^{NLO}_{ex;t+b}$ | 35.09[1] | $\sigma^{NLO}_{ex;t+b}$ | 34.63[1] |
| $\sigma^{NLO}_{ex;t+b+c}$ | 34.91[1] | $\sigma^{NLO}_{ex;t+b+c}$ | 34.15 [1] |

**2.1%**

## Top quark scheme has hardly any impact

## Interference of b ant t only NLO



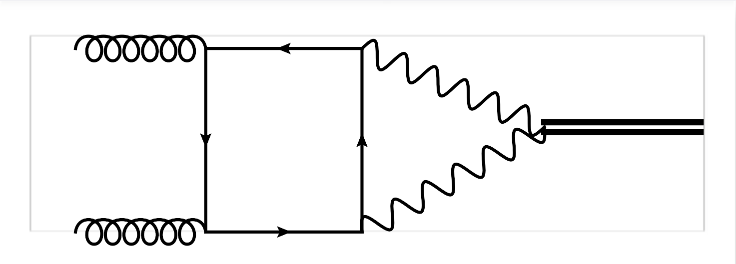
## Full NNLO could remove 2-3% uncertainty!

# ELECTRO-WEAK

# ELECTRO-WEAK

Energy independent EWK corrections

Only virtual corrections to Born



[Actis,Passarino,Sturm,Uccirati;Degrassi,Maltoni;…]

The problem: How to combine?

$$\sigma = \sigma_{QCD} \times (1 + \delta_{EWK})$$ +5.2%

$$\sigma = \sigma_{QCD} + \sigma^{LO} \delta_{EWK}$$ +2.5%

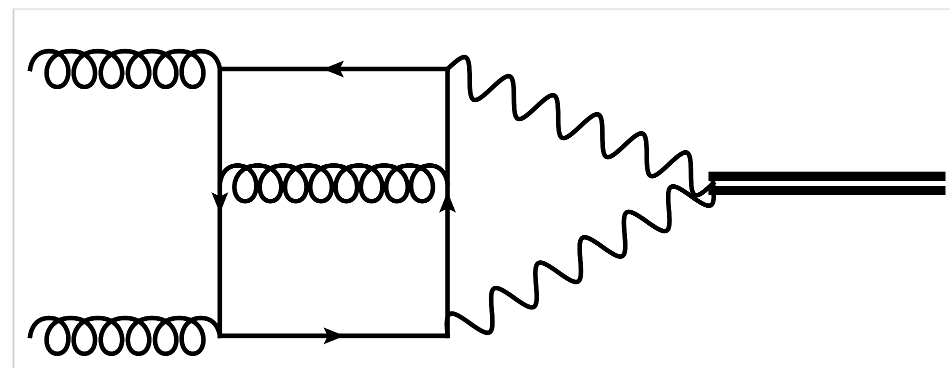
QCD K-Factor! Does it factor?

# ELECTRO-WEAK

$$\sigma = \sigma_{QCD} \times (1 + \delta_{EWK})$$ **+5.2%**

$$\sigma = \sigma_{QCD} + \sigma^{LO}\delta_{EWK}$$ **~+2.5%**

**We are missing** $\mathcal{O}(\alpha\alpha_S)$



**Approximation:**

[Anastasiou,Boughezal,Petriello]

**EFT approach to light quark contributions: Wilson Coefficient**

$$C_{QCD} \to C_{QCD} + \lambda_{EWK}\left(1 + C_{w1}\frac{\alpha_S}{\pi} + C_{w2}\frac{\alpha_S^2}{\pi} + \ldots\right)$$

**Result:** $C_{w1} = \frac{7}{6}$ **Almost complete factorization** **+5.1%**

**Requires** $m_W > m_h$

# CONCLUSIONS

**QCD Effective: N3LO**

**QCD Full: NLO+**

**EWK: $\mathcal{O}(\alpha)$ +**

**We know a lot**

$$\sigma^{ggf}_{PP\to H+X} = K_t \sigma^{N3LO}_{eft} + \delta\sigma_{EWK} + \delta\sigma_{m_t,m_b,m_c}$$

$$= 1.063 \times 44.62 + 2.37 - 2.12pb = 47.67pb$$

**preliminary**

**Many effects contributing!**

# CONCLUSIONS

**preliminary**

$$\sigma^{ggf}_{PP\to H+X} = 47.67pb\,^{0.32\%}_{-2.91\%} \pm 1.91\% \pm 2.6\% \pm 2\% \pm 2.5\%$$

$$\underbrace{\quad}_{\delta_\mu} \quad \underbrace{\quad}_{\delta_{PDF}} \quad \underbrace{\quad}_{\delta_{\alpha_S}} \quad \underbrace{\quad}_{\delta_{t+b+c}} \quad \underbrace{\quad}_{\delta_{EWK}}$$

Higher luminosity and energy soon: Statistical uncertainty gone

Many fronts to improve on current precision

## Outlook:

**N3LO Precision for LHC!**