The 2D-Oide effect

(http://arxiv.org/abs/1509.05747)

Submitted to PRST-AB as
Beam focusing limitation from synchrotron radiation in two dimensions

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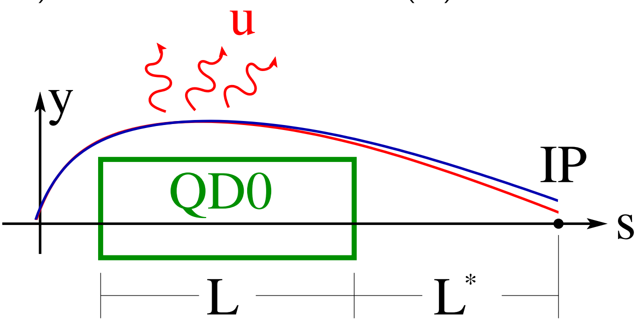
Oide effect

Radiation in a focusing magnet changes the energy of the particle and limits the focusing effect.
Important in strong focusing magnets like the last quad in a linear collider (QD0) before the Interaction Point (IP).



u is the energy of the photon radiated, k is the quadrupole gradient, L is the quad length, L^* is the distance to the IP, and y, s are the vertical and longitudinal coodinates

Oide limit

Radiation contributes to the beam size, σ. This sets a limit on the minimum achievable beam size which is independent of energy.

(K. Oide, PhysRevLett.61.1713, 1988)

$$\sigma^*_{y\ \min} = c_2 \left[F(\sqrt{K}L, \sqrt{K}l^*)\right]^{\frac{1}{7}} (\epsilon_{Ny})^{\frac{5}{7}}$$

We could remark that F is a function of the distance to the IP and the quad design parameters only.

Lattice	ϵ_N (nm)	σ_0 (nm)	σ_{oide} (nm)	σ (nm)	σ_{min} (nm)
CLIC 3 TeV	20	0.70	0.85	1.10	1.00
CLIC 500 GeV	25	2.3	0.08	2.3	1.17
ILC 500 GeV	40	5.7	0.04	5.7	1.85

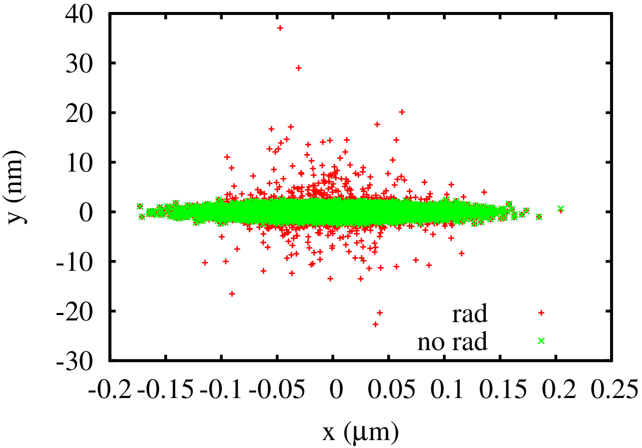
It is relevant for CLIC 3 TeV!

$\epsilon_N = \gamma\epsilon$ is the normalized emittance, σ_0 is the linear beam size (no rad), $\sigma_{oide}$is the contribution from radiation,

$\sigma = \sigma_0^2 + \sigma_{oide}^2$. c_2 is a constant

Oide in CLIC 3 TeV

Transverse beam profile at the IP from tracking from QD0 input.



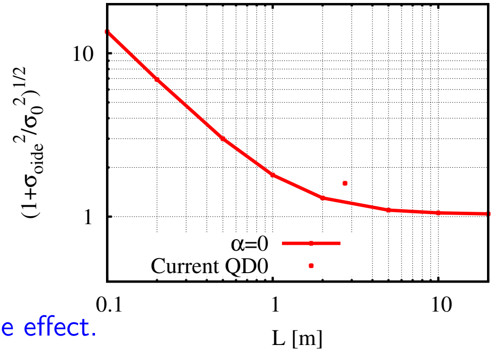
Typical mitigation method

To design the quad for the minimum reasonable value F.
This is normally achieved by **enlarging the quad** and reducing the quad strength while keeping the focal distance.

$$\sigma/\sigma_0 = \sqrt{1 + \sigma_{\text{oide}}^2/\sigma_0^2}$$

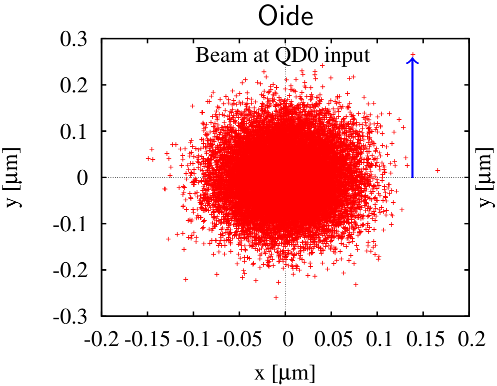
Red line is the beam size for the minimum focusing strength possible.

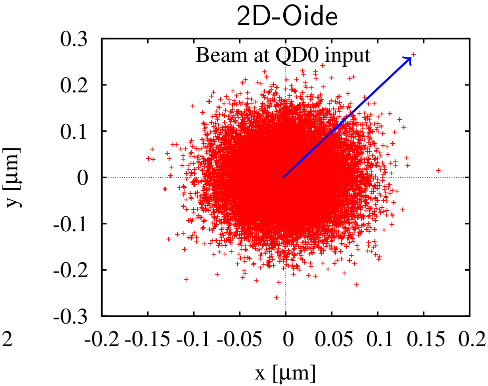
Magnets more than 10 m long do not improve the beam size.
The lack of other options, encourage us to review the Oide effect.



Oide and 2D-Oide

Transverse beam profile at the QD0 input.



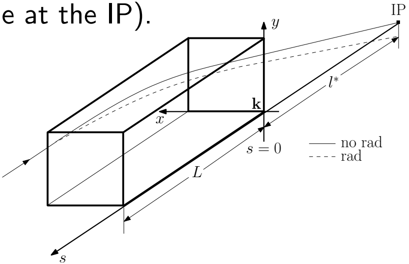


In the Oide calculation, only the vertical position is considered (blue arrow, right plot), and the horizontal component of the magnet field is neglected.
In 2D-Oide both components are considered (blue arrow, left plot).

$\overline{\Delta y} = y_{rad} - y_{no\ rad}$

We calculate theoretically the difference in vertical position due to the average radiation all along QD0, giving a cubic and a linear component. with respect to $\boldsymbol{y_0'}$ (the particle angle at the IP).

$$\overline{\Delta y} = a(\boldsymbol{y_0'^*})^3 + b(x_0'^*)^2\boldsymbol{y_0'^*}$$



And the second moment of the vertical displacement:

$$\overline{(\Delta y)^2} = c_3(y_0'^*)^2 \int_0^{\sqrt{k}L} \left(\left[\boldsymbol{y_0'^*} f_y(\phi)\right]^2 + \left[\boldsymbol{x_0'^*} f_x(\phi)\right]^2 \right)^{3/2} F_y^2(\phi) d\phi$$

Now the particle displacement is a function of the angles at the IP in x and y

a, b, c_3 are constants. f_x, f_y, F_y are part. propagation functions.

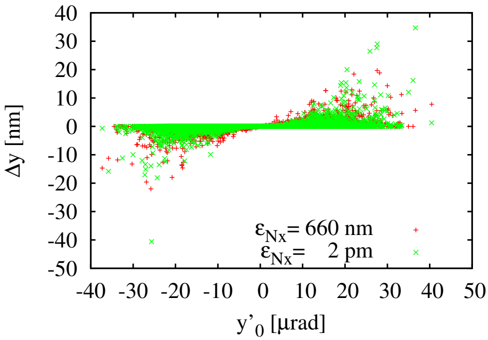
Tracking

And now we do tracking to compare Oide and 2D-Oide:

- Oide: beam with small horizontal emittance ($\epsilon_{Nx} = 2$ pm), where the horizontal particle position is negligible.
- 2D-Oide: beam with the CLIC 3 TeV horizontal emittance ($\epsilon_{Nx} = 660$ nm)

$$\overline{\Delta y} = a(\boldsymbol{y_0'^*})^3 + b(x_0'^*)^2 \boldsymbol{y_0'^*}$$

We assume gaussian distribution of the particles and calculate the expected values...



Compare Tracking, Oide and 2D-Oide (1 of 2)

Oide:($\epsilon_{Nx} = 2$ pm), 2D-Oide:($\epsilon_{Nx} = 660$ nm),

- a: There is a clear agreement in the order of magnitud when comparing Oide, 2D-Oide and tracking.
- $b\,\epsilon_{Nx}/(\gamma\beta_x)$: Similar agreement is clear between tracking and 2D-Oide when comparing.Extrangely tracking is much bigger than zero when the horizontal emittance is reduced

$\overline{\Delta y}$		a $[10^{-11}$ m]	$b\,\epsilon_{Nx}/(\gamma\beta_x)$ $[10^{-11}$ m]
$\epsilon_{Nx} = 2$ pm	Theory	9.0	0
	Tracking	9.5 ± 0.1	-1.3 ± 0.3
$\epsilon_{Nx} = 660$ nm	Theory	9.0	6.3
	Tracking	8.5 ± 0.1	5.4 ± 0.3

The differences have been attributed to limitations in the particle tracking and radiation simulations.

Compare Tracking, Oide and 2D-Oide (2 of 2)

Oide:($\epsilon_{Nx} = 2$ pm), 2D-Oide:($\epsilon_{Nx} = 660$ nm),
Now we compare the effect on the beam size

	$\langle\overline{(\Delta y)^2}\rangle^{1/2}$	[nm]
$\epsilon_{Nx} = 2$ pm	σ_{oide}	0.87
	$\sigma_{\text{2D-oide}}$	0.87 ± 0.03
	Tracking	0.92
$\epsilon_{Nx} = 660$ nm	$\sigma_{\text{2D-oide}}$	1.02 ± 0.03
	Tracking	1.00

17% bigger vertical beam size due to radiation.
Which corresponds to 11% larger vertical beam size.

Mitigation possibilities

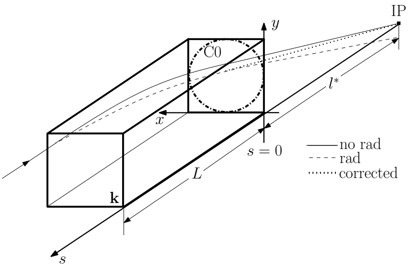
Most of the contribution to beam size comes from the cubic term

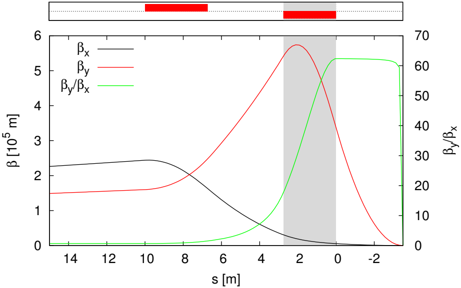
$$\overline{\Delta y} = a(\boldsymbol{y_0'^*})^3 + b(x_0'^*)^2 y_0'^*$$

In principle lattice could be tuned to correct this component (PhysRevSTAB.17.101002)

For a simple test an octupole (C0) is added to the line at the QD0 exit, giving a kick to the particles.

The β_y/β_y at the QD0 exit is maximum, therefore there will be minimal coupling effect.





Simplest mitigation (Octupole)

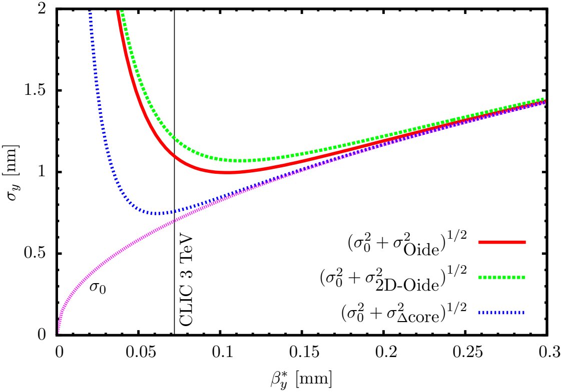
The octupole strength can be calculated to remove the component a.

	k_3 $[m^{-4}]$	σ_x [nm]	σ_y [nm]	L_{tot}	L_{peak} $[10^{34}cm^{-2} \cdot s^{-1}]$
NO RAD	0	47.45	0.69	7.7	2.9
RAD	0	47.45	1.18	7.5	2.7
RAD	3900	47.45	1.13	7.4	2.7

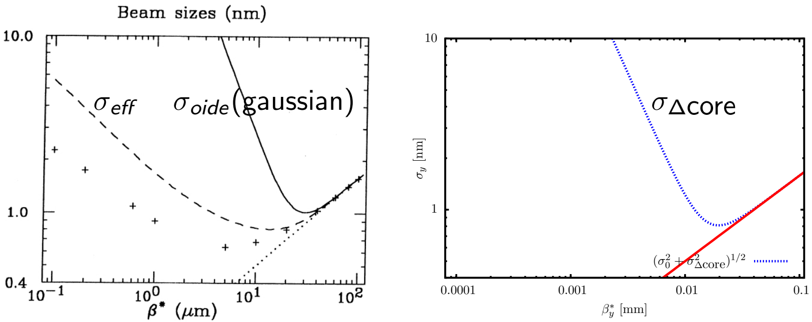
4% reduction of the vertical beam size at the IP
Further improvement could be possible if we tune upstream elements !
Negligible or negative effect on luminosity

Effect on luminosity

it is also possible to estimate the effect of radiation
using the expected absolute vertical displacement $\langle|\overline{\Delta y}|\rangle^2 = \sigma^2_{\Delta\text{core}}$



Effect on luminosity (cont.)



Oide (Phys.Lett.B224,437 1989) calculates an effective σ to measure the effect on luminosity using the Fourier transform of the beam distribution

Both have similar $\beta^*_{y,min}$, $\sigma_{y,min}$.

$\gamma = 10^6$, $\epsilon = 2.5 \times 10^{-14}$ m, $L = 0.4$ m, $l^* = 0.4$ m, $k = 8.5417$ m^{-2}

Conclusions

- Oide effect limits the minimum beam size due to radiation
  - Relevant for CLIC at 3 TeV
- The Oide effect considers only the focusing plane. When considering focusing and defocusing planes and additional 11% contribution to beam size appears.
- This is composed by a cubic and linear component in y'.
- One method to mitigate the effect is by adding an octupole. This reduces the vertical beam size at the IP in 4%.
- The effect of the mitigation in luminosity is negligible, however, it might improve if this correction is included in the tuning!
- $\sigma_{\Delta \mathrm{core}}$ has been created to simplify the calculation of beam size and minimum beta.