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# Status of the chromaticity correction in the arcs

Bastian Haerer (CERN, Geneva; KIT, Karlsruhe) for the FCC-ee lattice design team  
Acknowledgement: Thanks to Luis Medina for performing the DA study.



# Outline

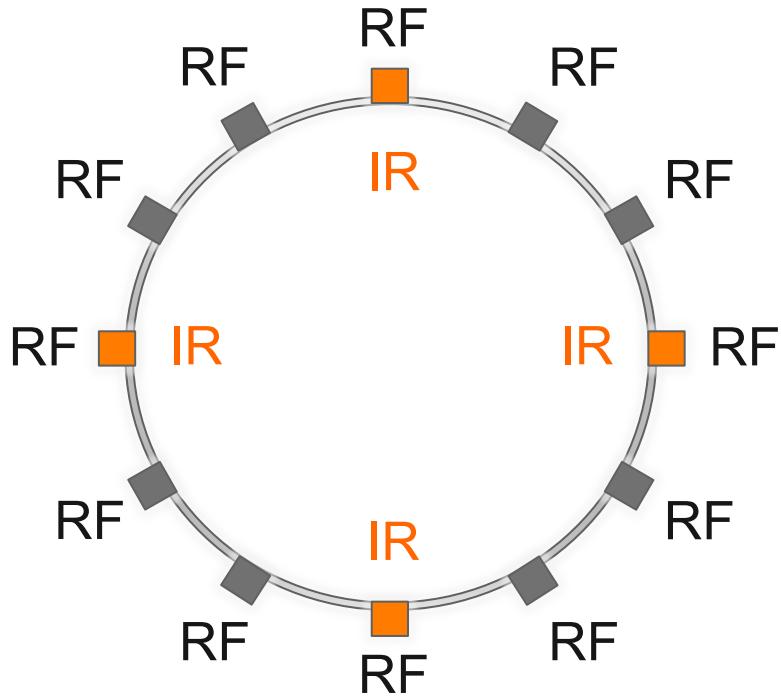
- First DA Results (Luis Medina)
- $\beta^*_y = 1 \text{ mm}$ : tuning the momentum acceptance
- $\beta^*_y = 2 \text{ mm}$ : momentum acceptance and chromaticities
- Summary and schedule until the Review



# 1) First DA study (Luis Medina)

## Lattices for first DA test:

- 1) 12-fold layout, 4 IPs
- 2) 12-fold layout, 2 IPs
- 3) 12-fold layout, 4 IPs,  
0.249/0.166 phase advance
- 4) Baseline layout (2 IPs)



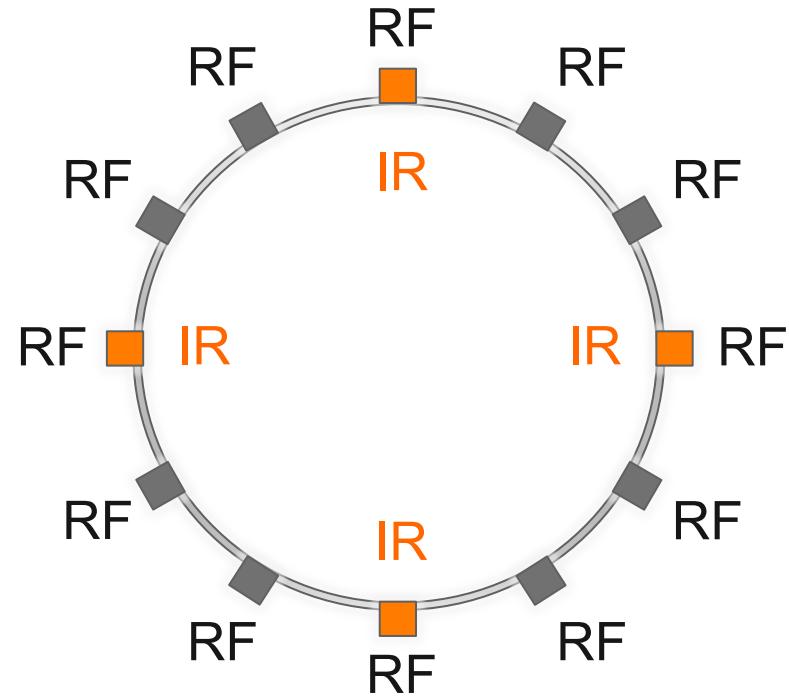
Chromaticity corrected with Montague formalism

# 1) First DA study (Luis Medina)

## 12-fold layout:

Circumference: 100 km  
Arc length: 6.8 km  
Straight section length: 1.5 km  
Phase advance per cell:  $90^\circ/60^\circ$

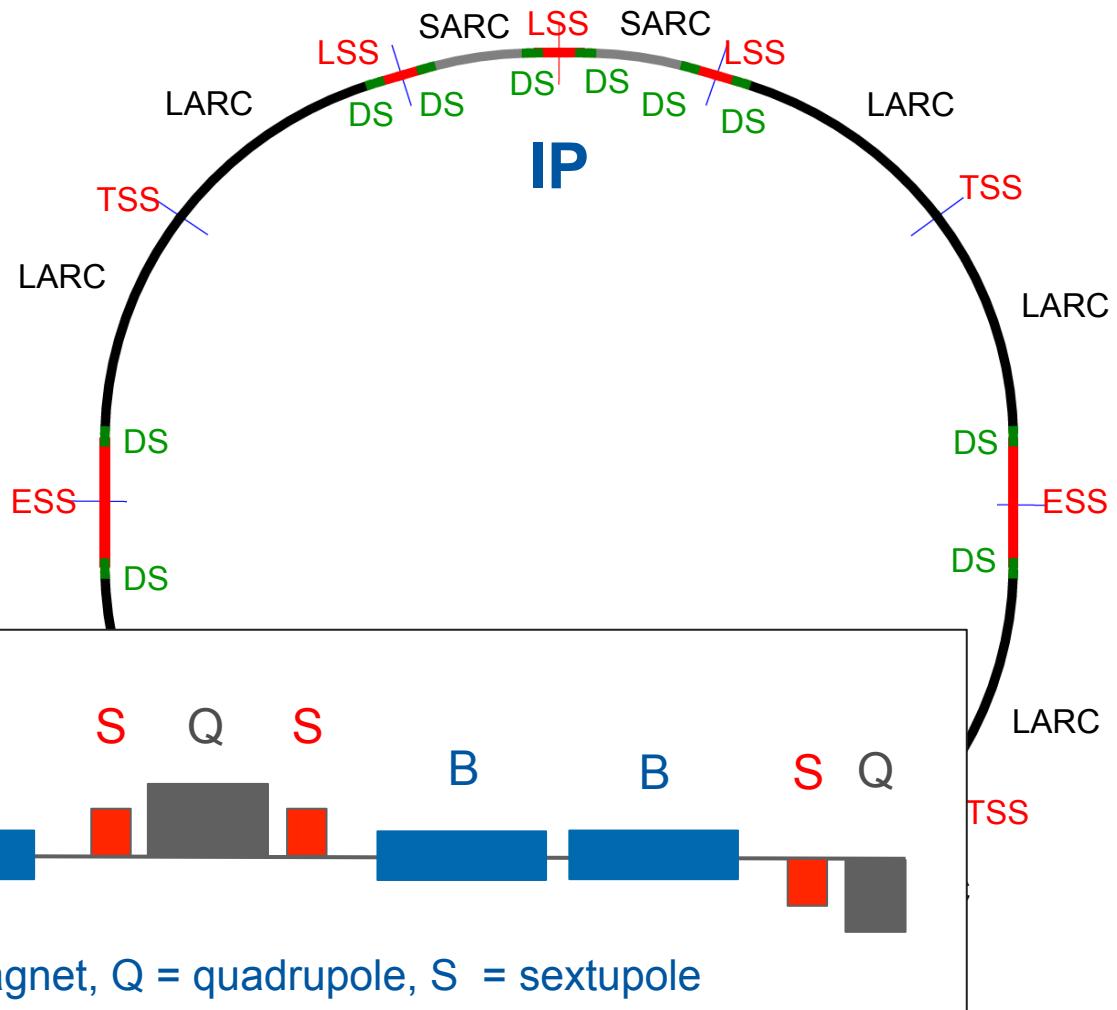
4 interaction regions (IR)  
with mini-beta insertions



NO LOCAL CHROMATICITY CORRECTION

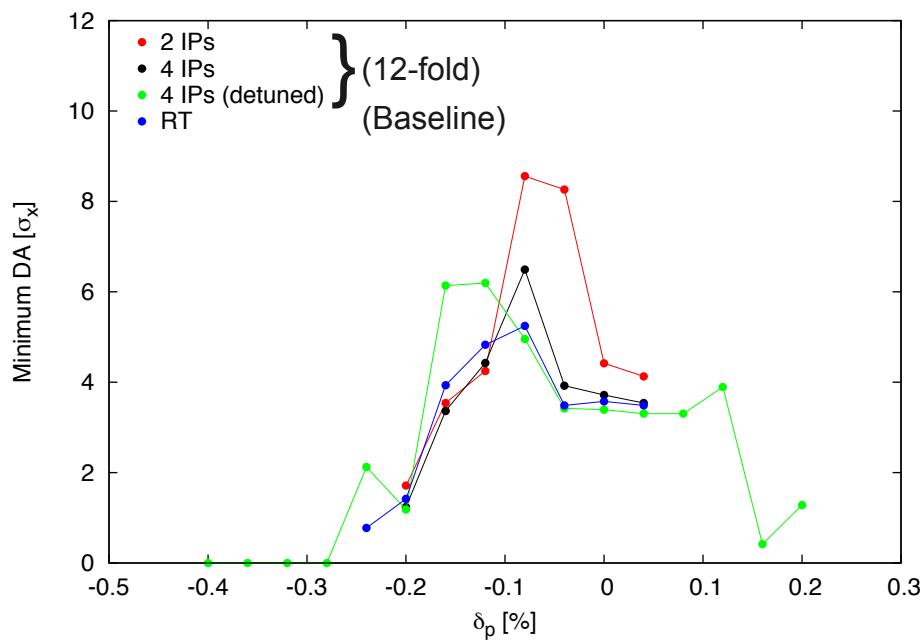
# Baseline Layout (V17)

- 100 km circumference
- 2 IPs
- No local CCS

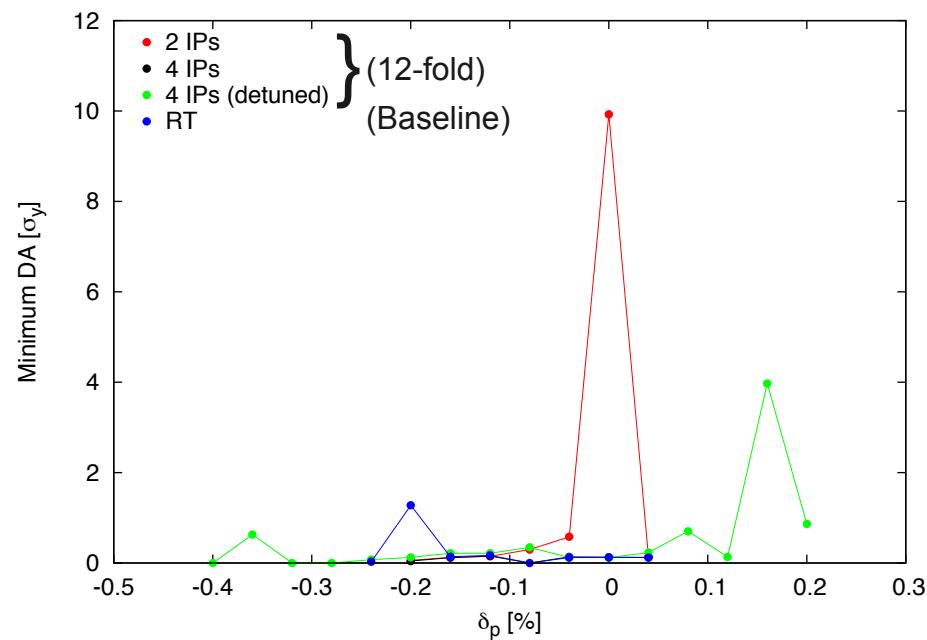


# First DA results (Luis Medina)

Horizontal plane



Vertical plane



- Reduction No. of IP does not increase momentum acceptance but DA
- Detuning of the FODO cells increases momentum acceptance slightly

More DA studies of the Baseline Layout in progress ...

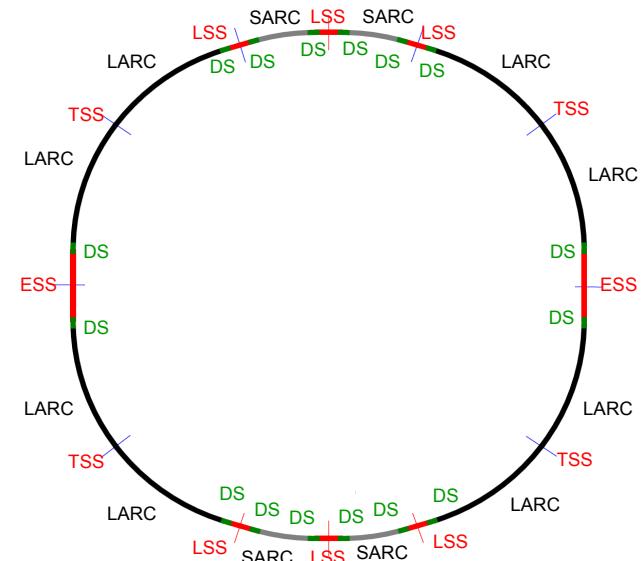
DA calculations and plots by Luis Medina



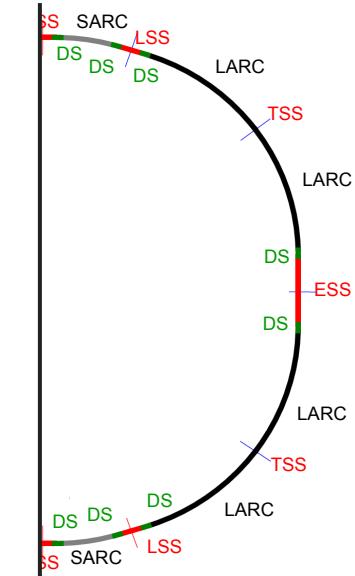
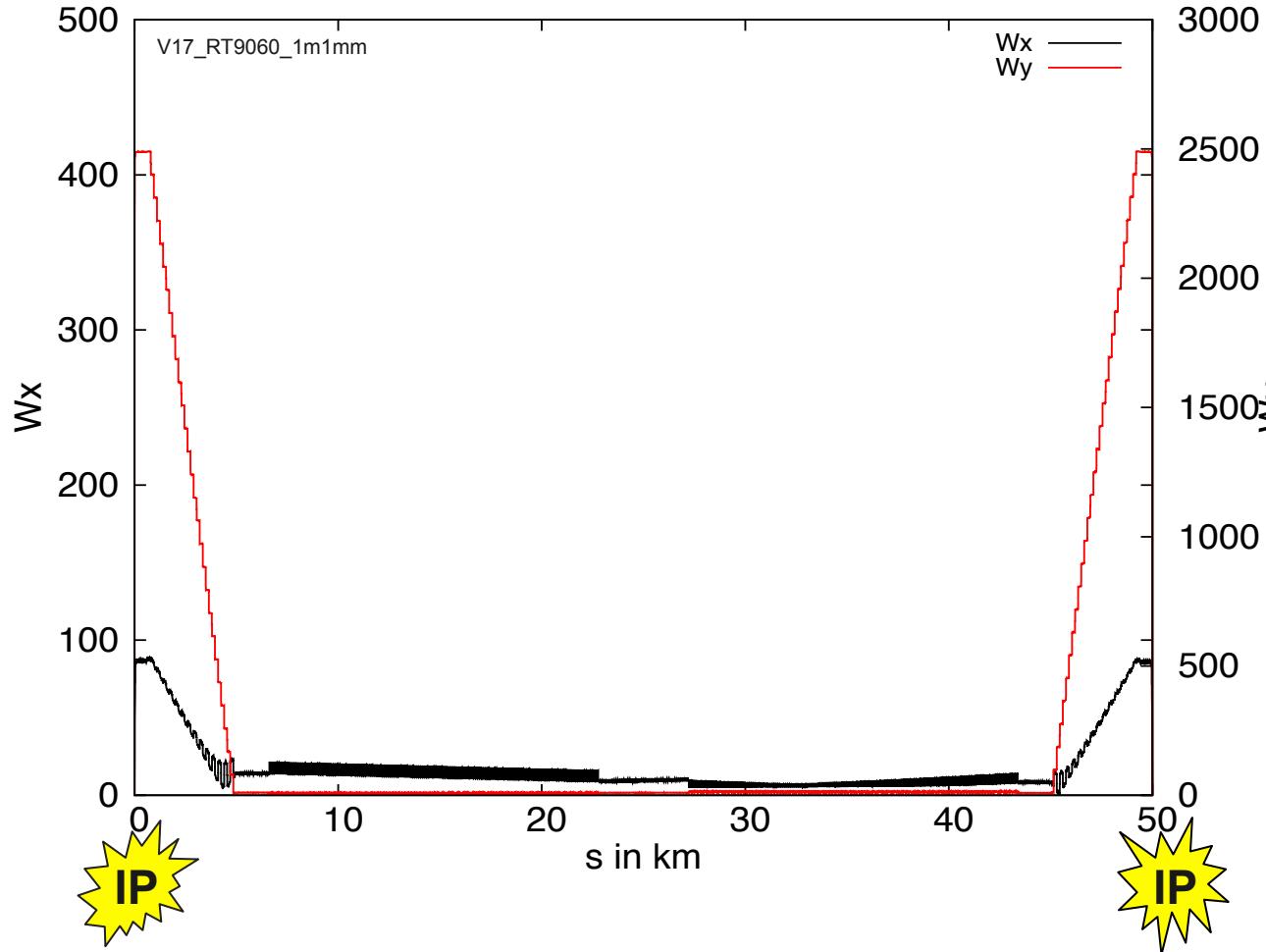
## 2) Momentum acceptance for

$$\beta^*_y = 1 \text{ mm}$$

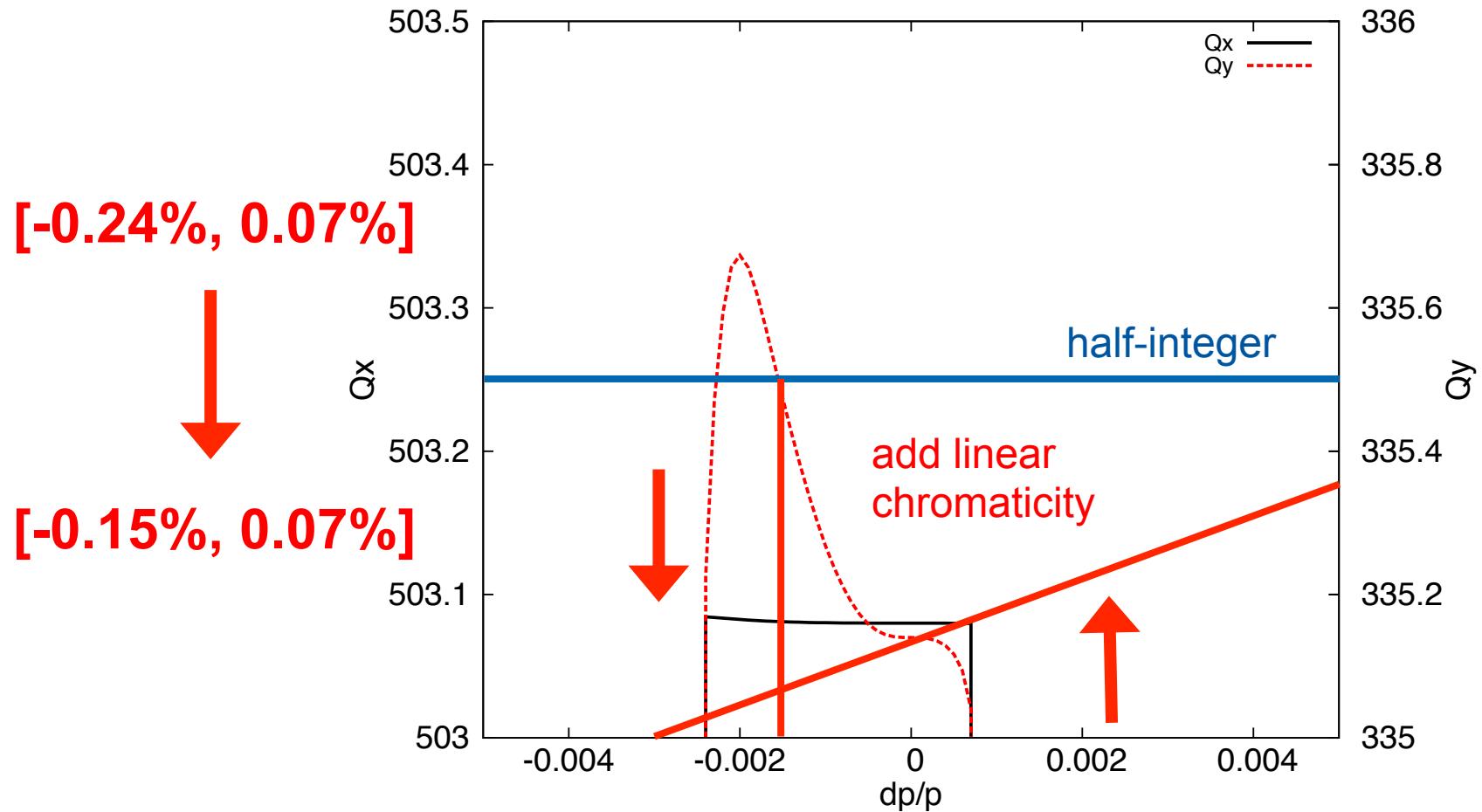
- Baseline layout
- Phase advance per cell:  $90^\circ/60^\circ$
- 2 IRs with  $\beta^*_x = 1 \text{ m}$ ,  $\beta^*_y = 1 \text{ mm}$
- Chromaticity corrected with Montague formalism



# W functions in the half-ring



# Initial momentum acceptance

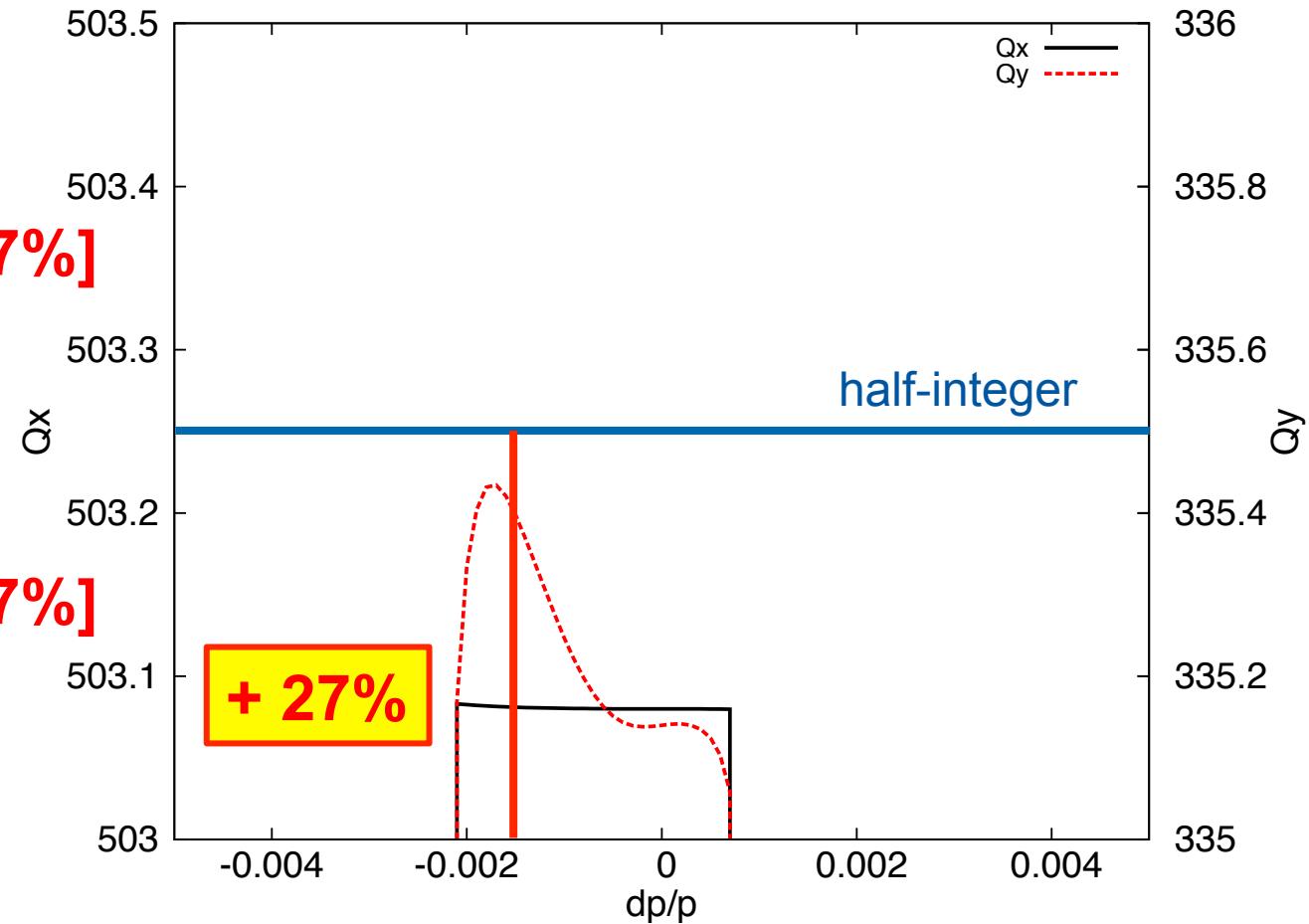


# Tuned momentum acceptance

$Q'_y = 15:$   
[-0.21%, 0.07%]



[-0.15%, 0.07%]



# 3) Momentum acceptance for $\beta^*_y = 2 \text{ mm}$

- Baseline layout
- Phase advance per cell:  $90^\circ/60^\circ$
- Parameters for the Review
  - 2 IRs with  $\beta^*_x = 1 \text{ m}$ ,  $\beta^*_y = 2 \text{ mm}$
- Chromaticity corrected with Montague formalism



# Natural chromaticity compared

	$\beta^*_y = 1 \text{ mm}$	$\beta^*_y = 2 \text{ mm}$	
$Q_x$	503.08	505.08	
$Q_x'$	-584.26	-587.67	
$Q_x''$	-3818.40	-3847.84	
$Q_x^{(3)}$	$-1.43 \times 10^8$	$-1.52 \times 10^8$	
$Q_x^{(4)}$	$1.45 \times 10^{13}$	$-1.41 \times 10^{13}$	
$Q_y$	335.14	337.14	
$Q_y'$	-2059.23	-860.42	(- 58 %)
$Q_y''$	$-4.18 \times 10^6$	$-1.04 \times 10^6$	(- 75 %)
$Q_y^{(3)}$	$-1.19 \times 10^{11}$	$-0.21 \times 10^{11}$	(- 82 %)
$Q_y^{(4)}$	$-4.53 \times 10^{15}$	$-0.53 \times 10^{15}$	(- 88 %)

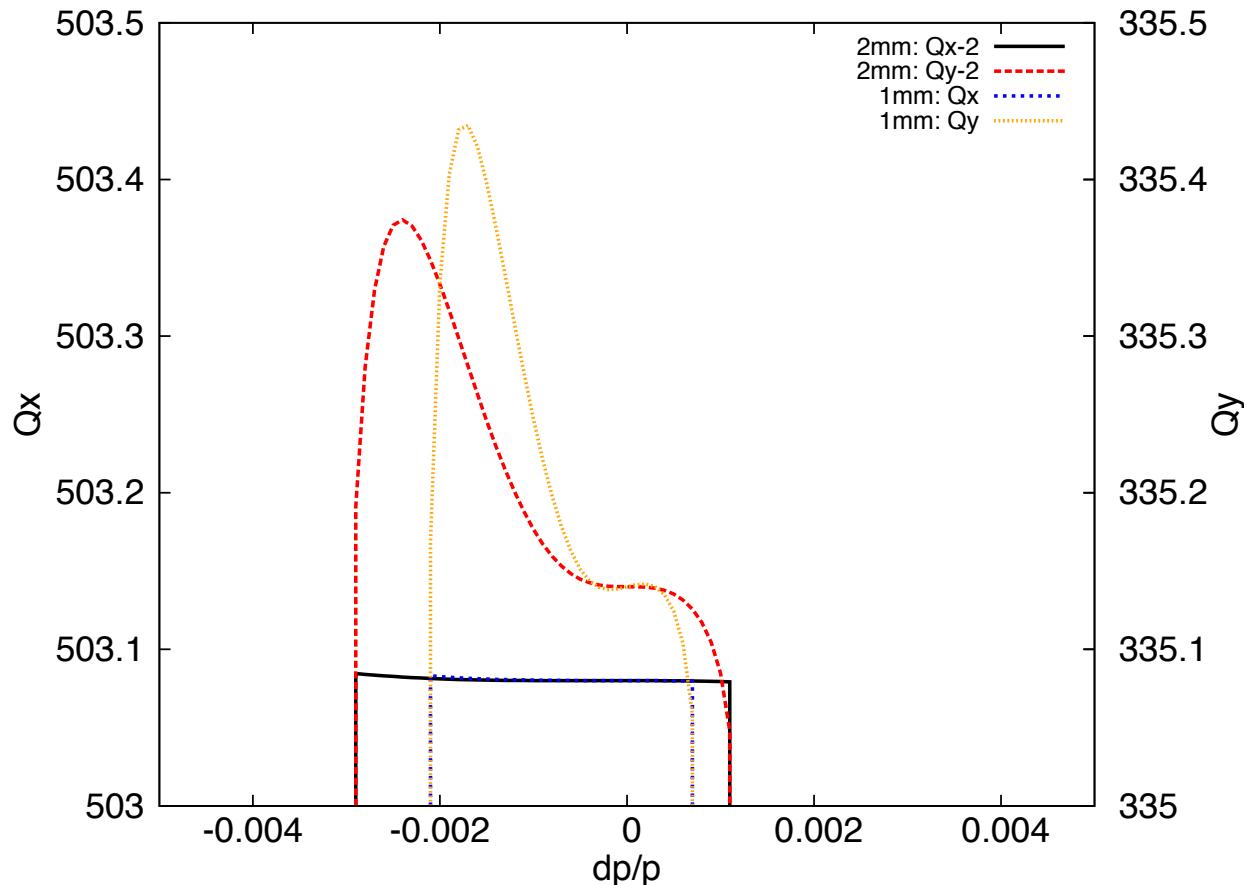


# Corrected chromaticity compared

	$\beta^*_y = 1 \text{ mm}$	$\beta^*_y = 2 \text{ mm}$
$Q_x$	503.08	505.08
$Q_x'$	$-1.53 \times 10^{-6}$	$-4.28 \times 10^{-5}$
$Q_x''$	$4.30 \times 10^2$	$-2.57 \times 10^2$
$Q_x^{(3)}$	$-5.51 \times 10^6$	$-5.97 \times 10^6$
$Q_x^{(4)}$	$3.64 \times 10^{12}$	$-1.36 \times 10^{13}$
$Q_y$	335.14	337.14
$Q_y'$	15.00	$-4.66 \times 10^{-5}$
$Q_y''$	$-2.34 \times 10^3$	$1.34 \times 10^3$
$Q_y^{(3)}$	$-9.20 \times 10^8$	$-2.16 \times 10^8$
$Q_y^{(4)}$	$2.28 \times 10^{13}$	$1.02 \times 10^{13}$



# Mom. acceptance compared

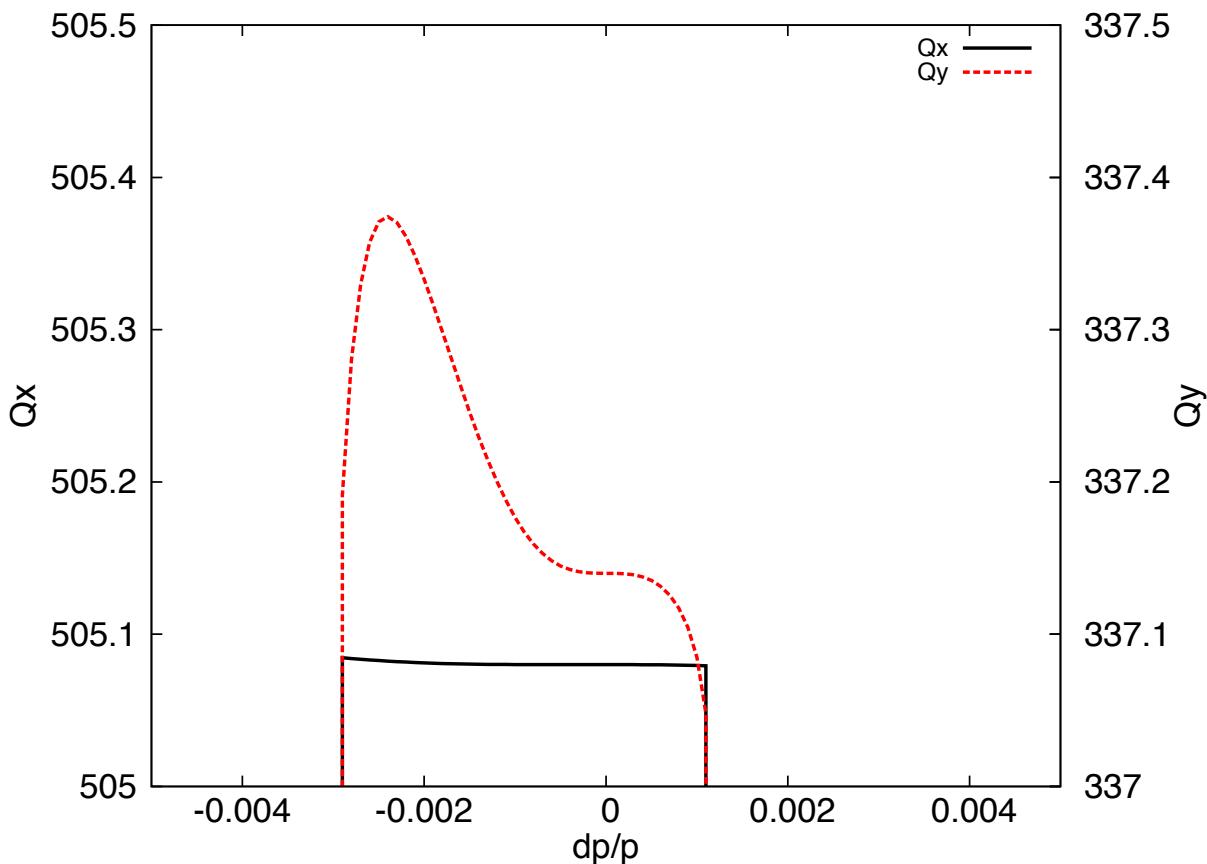


$\beta^*_y = 1 \text{ mm:}$   
[-0.21%, 0.07%]

$\beta^*_y = 2 \text{ mm:}$   
[-0.29%, 0.11%]

+43 %

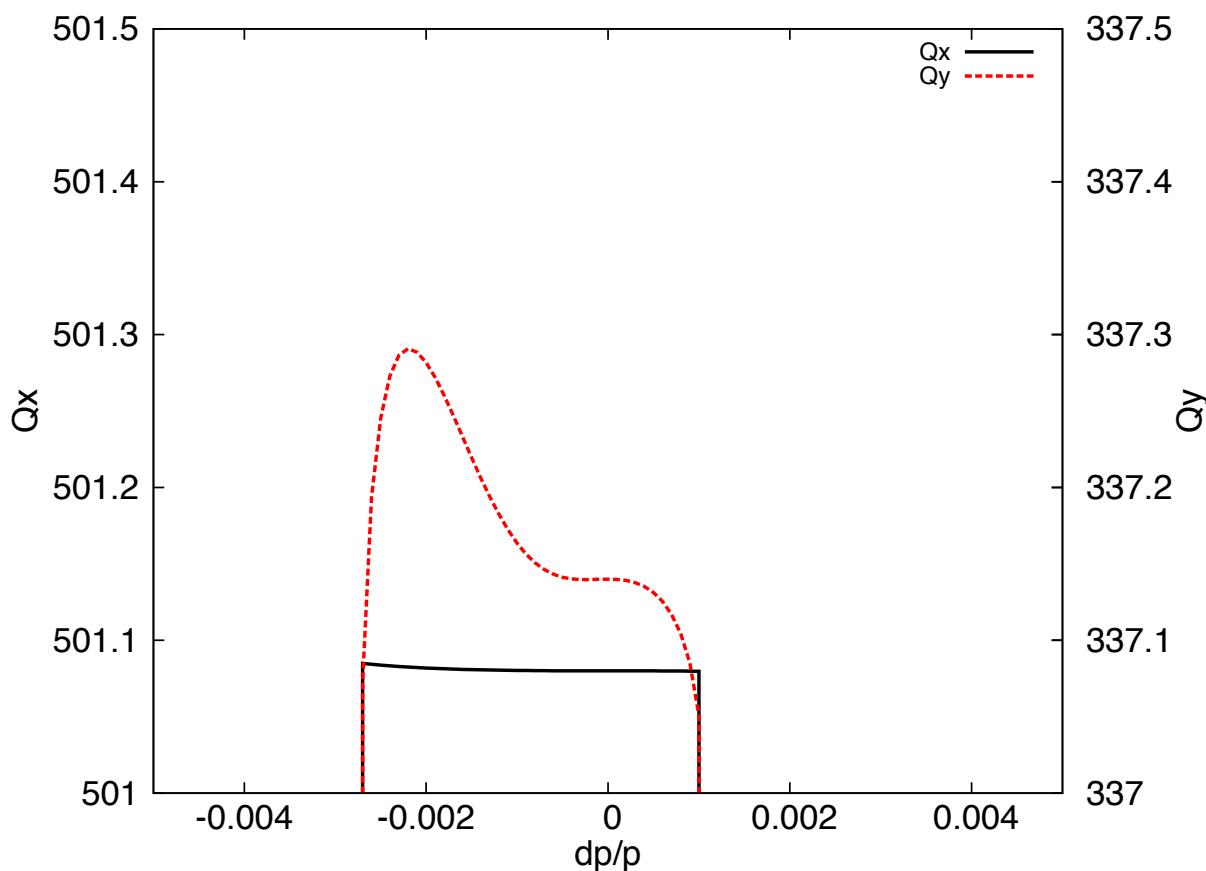
# Baseline layout, $\beta^*_y = 2$ mm



$\beta^*_y = 2$ mm	
$Q_x$	505.08
$Q_x'$	$-4.28 \times 10^{-5}$
$Q_x''$	$-2.57 \times 10^2$
$Q_x^{(3)}$	$-5.97 \times 10^6$
$Q_x^{(4)}$	$-1.36 \times 10^{13}$
$Q_y$	337.14
$Q_y'$	$-4.66 \times 10^{-5}$
$Q_y''$	$1.34 \times 10^3$
$Q_y^{(3)}$	$-2.16 \times 10^8$
$Q_y^{(4)}$	$1.02 \times 10^{13}$

**[-0.29%, 0.11%]**

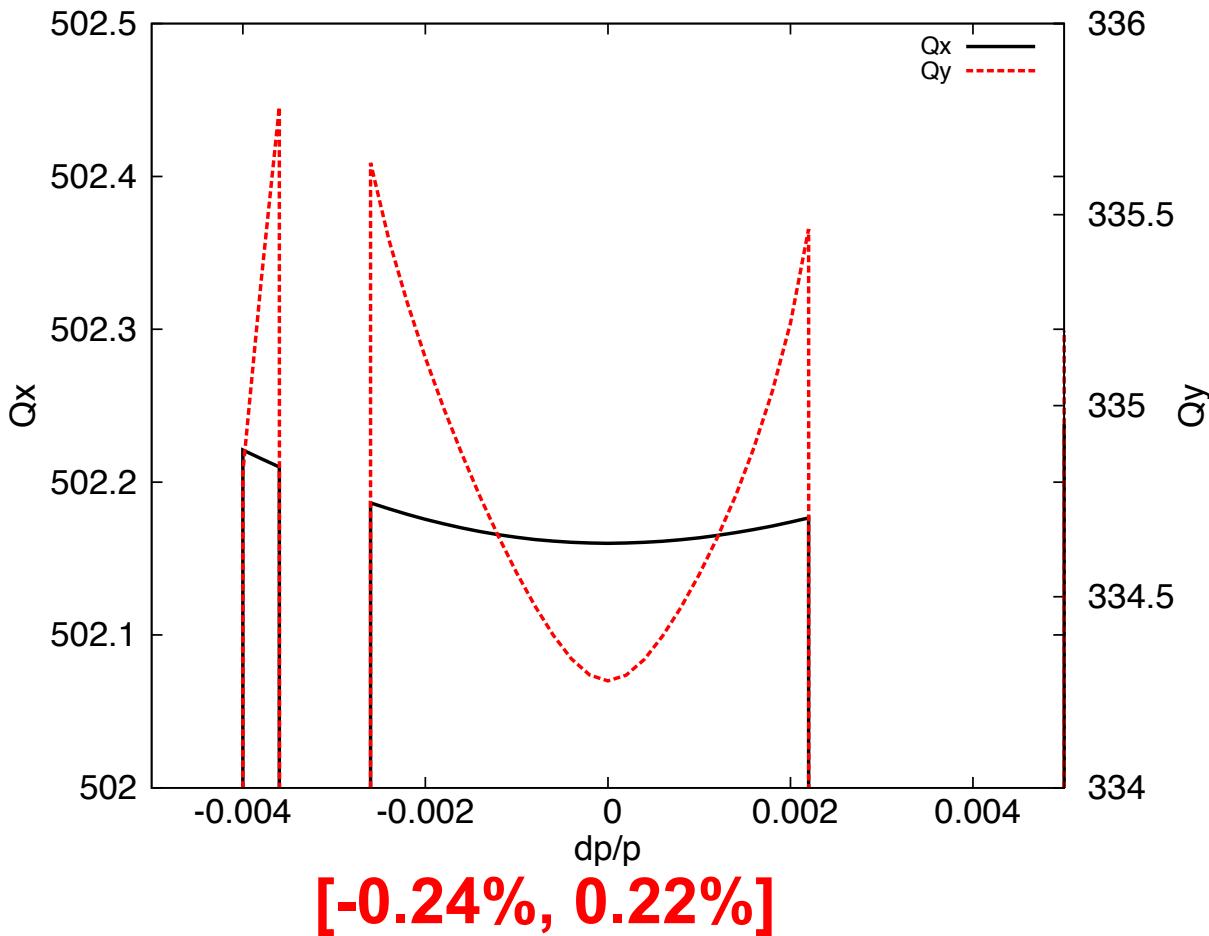
$$\beta_y^* = 2 \text{ mm}, \mu_{\text{cell}} = 0.249/0.167$$



**[-0.27%, 0.10%]**

$\beta_y^* = 2 \text{ mm}$	
$Q_x$	501.08
$Q'_x$	$-3.51 \times 10^{-5}$
$Q''_x$	$2.27 \times 10^2$
$Q^{(3)}_x$	$5.68 \times 10^6$
$Q^{(4)}_x$	$3.37 \times 10^{13}$
$Q_y$	337.14
$Q'_y$	$-1.76 \times 10^{-6}$
$Q''_y$	$-2.81 \times 10^4$
$Q^{(3)}_y$	$-2.22 \times 10^8$
$Q^{(4)}_y$	$8.64 \times 10^{12}$

# 12-fold layout, 4 IPs, $\beta^*_y = 1$ mm, $\mu_{\text{cell}} = 0.249/0.167$



$\beta^*_y = 1$ mm	
$Q_x$	502.16
$Q_x'$	$2.27 \times 10^{-7}$
$Q_x''$	$7.51 \times 10^3$
$Q_x^{(3)}$	$-2.90 \times 10^6$
$Q_x^{(4)}$	$1.82 \times 10^{12}$
$Q_y$	334.28
$Q_y'$	$4.60 \times 10^{-6}$
$Q_y''$	$7.76 \times 10^5$
$Q_y^{(3)}$	$-5.23 \times 10^7$
$Q_y^{(4)}$	$-2.36 \times 10^{13}$

# Derivatives of the $\beta$ function

The derivatives indicate places for an effective higher order chromaticity correction

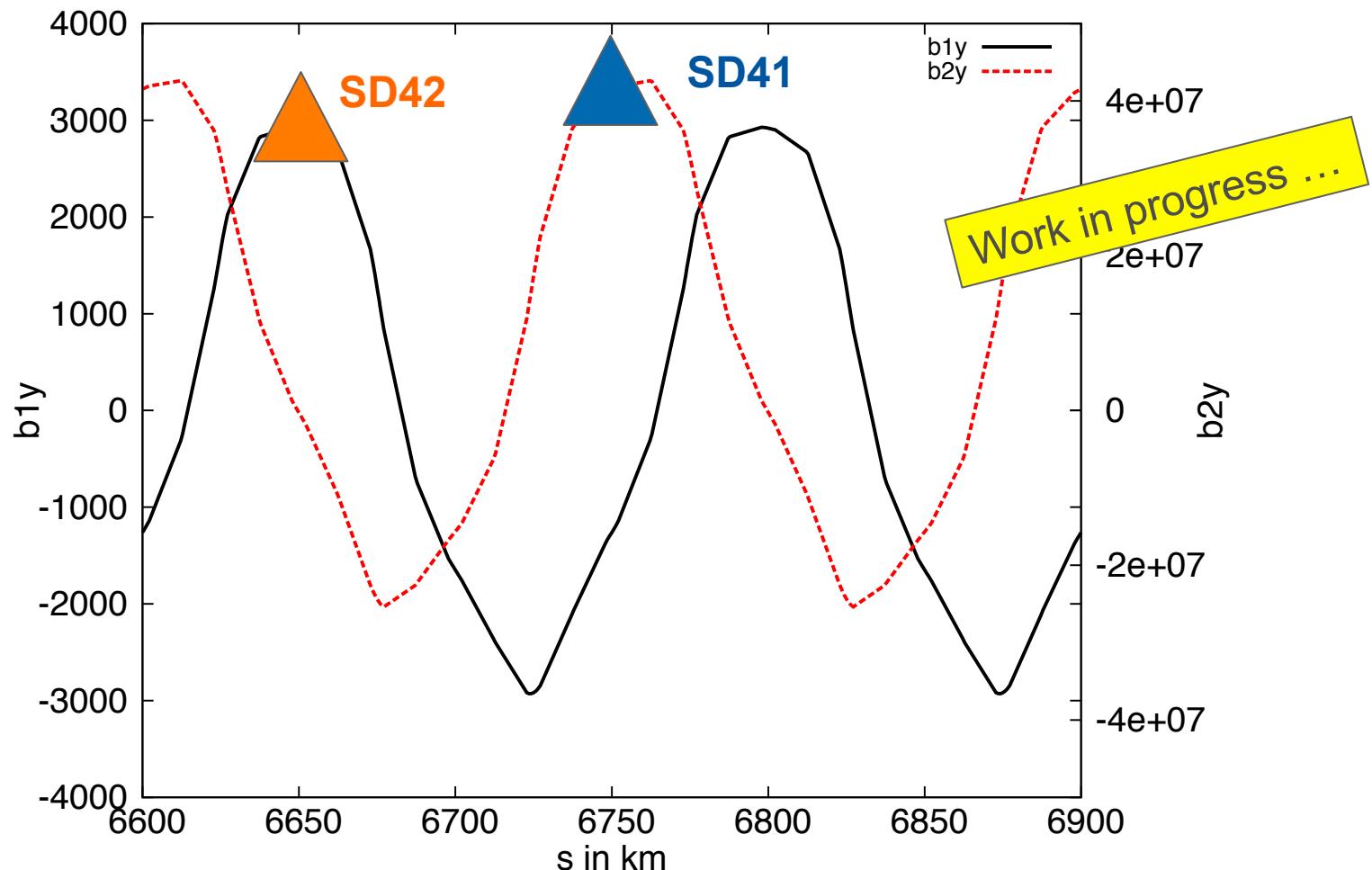
$$b_1 = \frac{\partial \beta}{\partial \delta} \quad b_2 = \frac{1}{\beta} \frac{\partial^2 \beta}{\partial \delta^2}$$

$$\begin{aligned}\frac{\partial^2 \varphi_y}{\partial \delta^2} &= -2 \frac{\partial \varphi_y}{\partial \delta} - \int_0^\Pi \beta_y K_2 \eta_1 ds + \frac{1}{2} \int_0^\Pi \beta_y b_{y,1} (K_1 - K_2 \eta_0) ds, \\ \frac{\partial^3 \varphi_y}{\partial \delta^3} &= 6 \frac{\partial \varphi_y}{\partial \delta} - \int_0^\Pi \beta_y (K_1 - K_2 \eta_0) (a_{y,1}^2 + b_{y,1}^2) ds + \\ &+ 3 \int_0^\Pi \beta_y (K_2 \eta_1 - K_2 \eta_2) ds + \frac{3}{2} \int_0^\Pi \beta_y b_{y,2} (K_1 - K_2 \eta_0) ds.\end{aligned}$$

(A. Bogomyagkov: “Crab waist interaction region for FCC-ee and the arc second attempt”, presentation in the FCC-ee meeting no. 13, 09 February 2015)



# Derivatives of the $\beta$ function



# Summary

- First DA studies performed:
  - DA very tight in vertical plane
  - Reducing No. of IPs did not increase momentum acceptance but DA
  - Detuning of the FODO cells increases momentum acceptance slightly
- Artificial positive linear chromaticity can help to avoid crossing resonances



# Summary II

- Increasing  $\beta^*_y$  to 2 mm decreases some chromaticities about more than 80 %
- Using the same methods the momentum acceptance could be increased by 43 %
- Further investigation necessary to understand behavior for 0.249/0.167 phase advance



# Schedule until the Review

1. Analysis of the derivatives of the  $\beta$  function  
(in progress)
2. DA studies of the Baseline Layout Lattice  
(in progress)
3. Lattice with split quadrupoles around one sextupole (Frank's proposal)
4. Lattice with individually powered interleaved sextupole pairs (complementary to Katsunobu's non-interleaved pairs)





Thank you for your attention!