

Inclusive jet spectrum for small-radius jets

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based on [JHEP 1504 \(2015\) 039](#) and [JHEP 1606 \(2016\) 057](#)

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Outline

1. Precision & jets
2. Resummation & Matching
 - ▶ Small- R formalism and validity
 - ▶ Matching LL_R to fixed order
3. Non-perturbative effects & comparison to data
 - ▶ Hadronisation and underlying event
 - ▶ Comparison to ALICE and ATLAS data
4. Conclusion

PRECISION & JETS

Jets in the era of precision phenomenology

High precision will be a key element in the future of particle physics

- ▶ Higgs physics
- ▶ PDF extractions
- ▶ EW physics
- ▶ BSM searches

Many processes involve jets (used in about 2/3 of recent ATLAS and CMS papers)

- ▶ What are the limits on precision in such processes?
- ▶ How far can they be pushed?

Case study: the inclusive jet spectrum

Plays a **central role** in collider physics

- ▶ Important for PDFs, α_s extractions, new physics at high p_t , ...
- ▶ Challenging **experimentally** (JES errors) and **theoretically** (sensitive to perturbative & non-perturbative effects).
- ▶ Provides a **simple context** to study problems appearing also in **more complicated processes**.

Case study: the inclusive jet spectrum

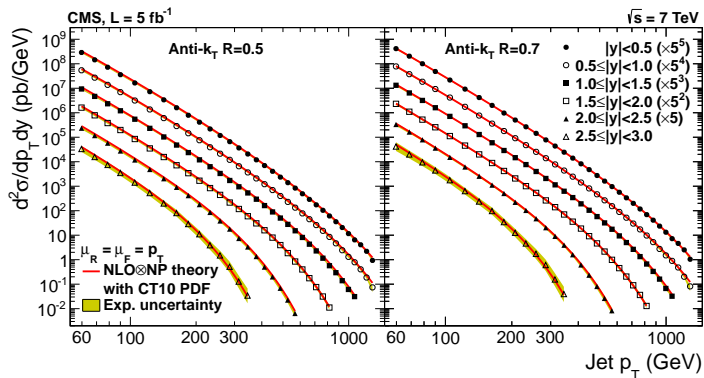
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- ▶ Provides a **simple context** to study problems appearing also in **more complicated processes**.

We aim to investigate the R -dependence of jet spectra, with particular focus on the small radius limit.

R dependence as a handle for validation

ATLAS, CMS, and ALICE have results for two separate R values.



Degree of consistency between experimental and theory comparisons at different R values provides powerful check of accuracy.

Fixed order calculations

- ▶ NLO from NLOJet++ dijet process.
- ▶ NNLO is still work in progress, but full NNLO R -dependent terms can be obtained from NLOJet++ 3-jet process.

Small- R resummation

- ▶ Generating functional approach to resum **Leading Logs of jet radius** in the small- R limit

$$\alpha_s^n \ln^n \frac{1}{R^2} .$$

- ▶ **Matched** to fixed order calculations using appropriate scheme.

Non-perturbative effects

- ▶ Start by examining analytical estimates.
- ▶ We will adopt **correction** factors **derived from Monte Carlo** generators.

RESUMMATION & MATCHING

Small- R resummation for the inclusive jet spectrum

Small- R inclusive “microjet” spectrum obtained from convolution of the **inclusive microjet fragmentation function** with the **LO inclusive spectrum**

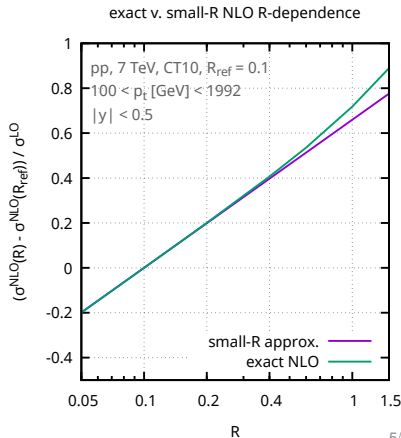
$$\sigma^{\text{LLR}}(p_t, R) \equiv \sum_k \int_{p_t} \frac{dp'_t}{p'_t} f_{\text{jet}/k}^{\text{incl}} \left(\frac{p_t}{p'_t}, t(R, R_0, \mu_R) \right) \frac{d\sigma^{(k)}}{dp'_t}$$

Validity of small- R approx. can be checked by looking at differences between R values.

Compare inclusive spectrum from NLOJet++ with small- R approximation

Overlap of the curves indicates that the small- R approximation is good.

[Nagy [Phys.Rev. D68 \(2003\) 094002](#)]



Matching NLO and LL_R

Precise resummed predictions require **matching to NLO**.

We adopt multiplicative matching

$$\sigma^{\text{NLO}+LL_R} = (\sigma_0 + \sigma_1(R_0)) \times \left[\frac{\sigma^{LL_R}(R)}{\sigma_0} \times \left(1 + \frac{\sigma_1(R) - \sigma_1(R_0) - \sigma_1^{LL_R}(R)}{\sigma_0} \right) \right]$$

large R_0 jet prod.

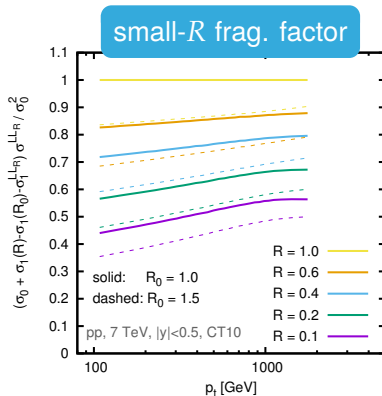
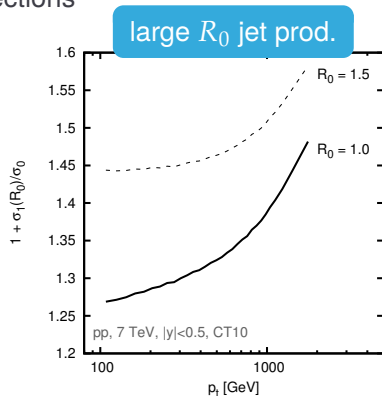
small R fragmentation factor

Physical interpretation of different terms suggests alternative expression for the NLO cross section

$$\sigma^{\text{NLO,mult.}} = (\sigma_0 + \sigma_1(R_0)) \times \left(1 + \frac{\sigma_1(R) - \sigma_1(R_0)}{\sigma_0} \right)$$

Unphysical cancellations in scale dependence

Different terms in matched predictions lead to K -factors going in opposite directions



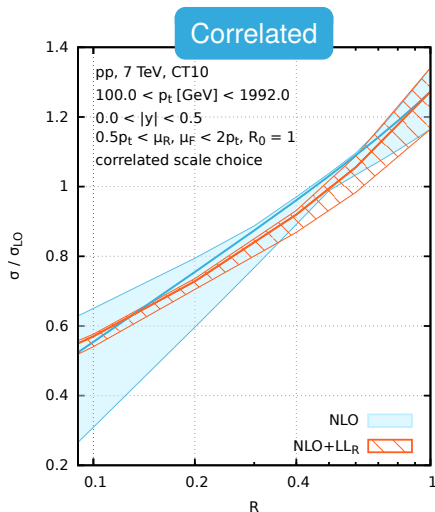
Partial cancellation in higher order effects can be dangerous when estimating scale uncertainties.

Evaluate independently and add in quadrature.

Correlated vs. uncorrelated scale variation

Uncorrelated scale variation gets rid of unphysical cancellations in uncertainty bands

Correlated scale variation:
Keep the same scale in both factors.

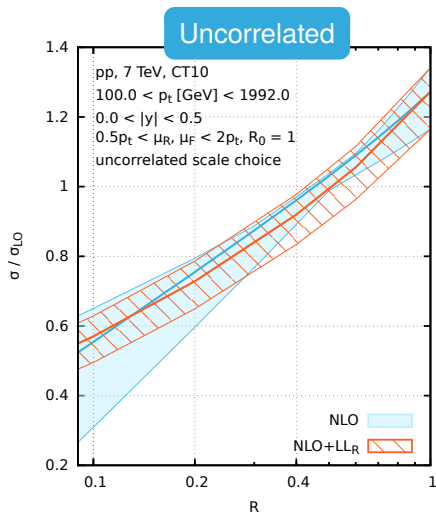


Correlated vs. uncorrelated scale variation

Uncorrelated scale variation gets rid of unphysical cancellations in uncertainty bands

Uncorrelated scale variation:

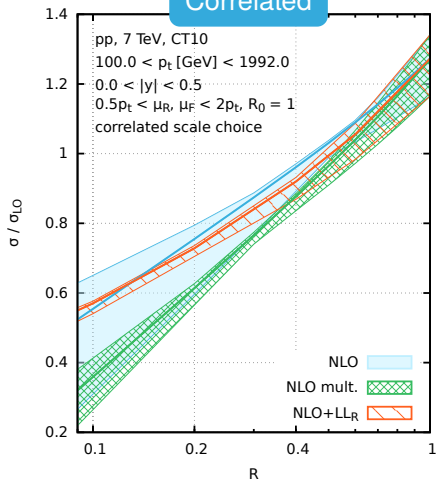
Vary scale independently in each factor, and add resulting uncertainties in quadrature.



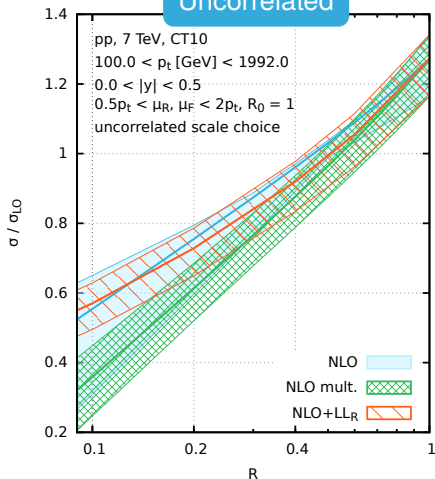
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Correlated



Uncorrelated



Small- R approximation beyond NLO

How important are subleading effects at higher orders?

Compute difference between R
values at NNLO

$$\begin{aligned}\sigma^{\text{NNLO}}(R) - \sigma^{\text{NNLO}}(R_{\text{ref}}) \\ = \sigma^{\text{NLO}_{3j}}(R) - \sigma^{\text{NLO}_{3j}}(R_{\text{ref}})\end{aligned}$$

Small- R approximation beyond NLO

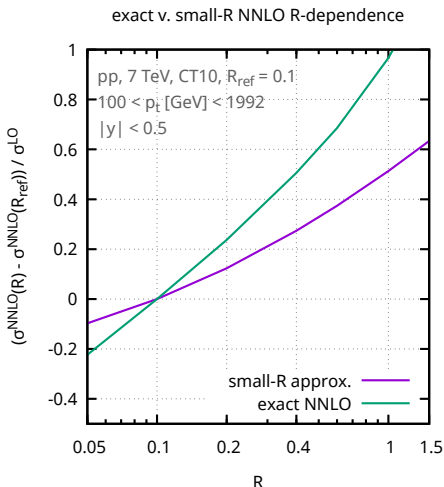
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Substantial subleading
 $\alpha_s^n \ln^{n-1} R$ contribution!

Ideally, one would like a full NLL_R resummation.



Including subleading terms

It is clear that formally **subleading $\alpha_s^n \ln^{n-1} R$ terms** can be sizeable.

A **full NLL_R resummation** is not possible at the moment, and would require substantial further work ...

...but we can at least include $\alpha_s^2 \ln R$ terms by matching to NNLO.

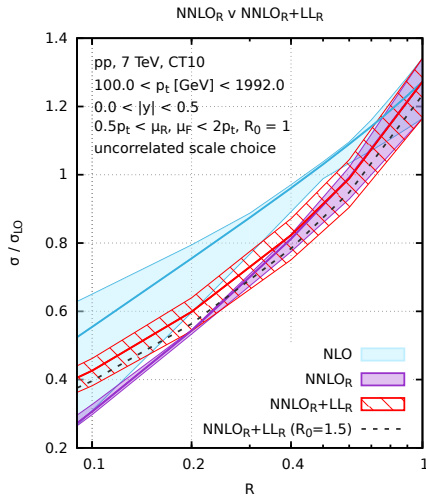
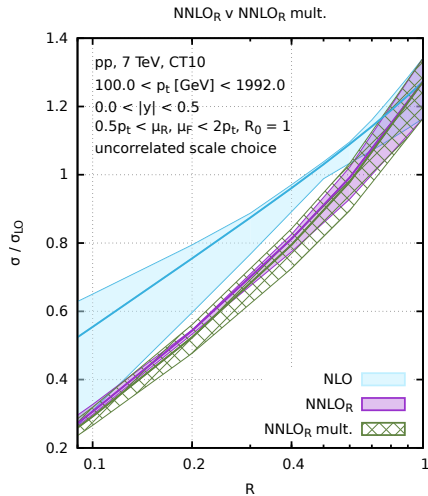
Since full calculation is not yet available, construct a stand-in for NNLO

$$\sigma^{\text{NNLO}_R}(R, R_m) \equiv \sigma_0 + \sigma_1(R) + [\underbrace{\sigma_2(R) - \sigma_2(R_m)}_{\text{from NLO 3-jet}}]$$

Which has NNLO accurate R -dependence. R_m is an arbitrary angular scale, taken to be $R_m = 1$.

Results at NNLO_R and NNLO_R+LL_R

NNLO_R brings large corrections at small radii, and steeper R dependence.



Impact of finite two-loop corrections

The NNLO_R predictions have all elements of full NNLO correction except those associated with **2-loop and squared 1-loop diagrams**.

To examine missing contributions, introduce **factor K** corresponding to NNLO/NLO ratio for a jet radius of R_m

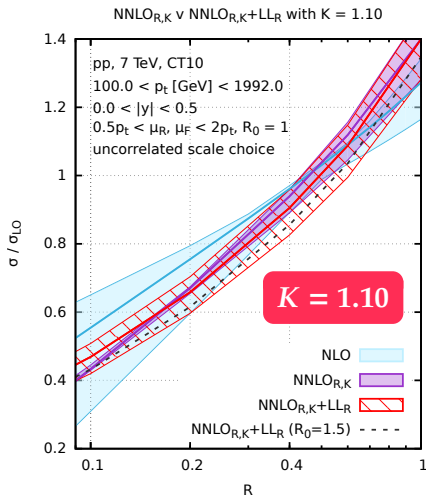
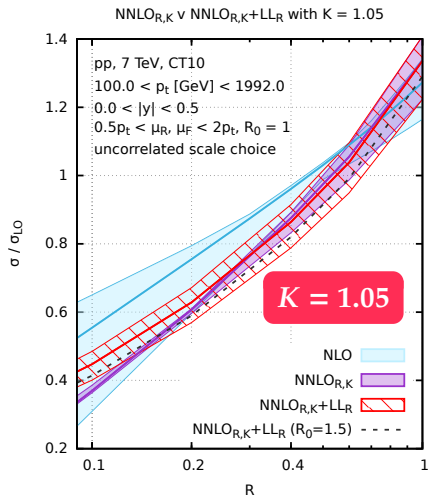
$$\sigma^{\text{NNLO}_{R,K}}(R_m) = K \times \sigma^{\text{NLO}}(R_m)$$

For other values of the jet radius, we have

$$\sigma^{\text{NNLO}_{R,K}}(R) = \sigma_0 \left[1 + \frac{\sigma_1(R)}{\sigma_0} + \Delta_2(R, R_m) + (K - 1) \times \left(1 + \frac{\sigma_1(R_m)}{\sigma_0} \right) \right]$$

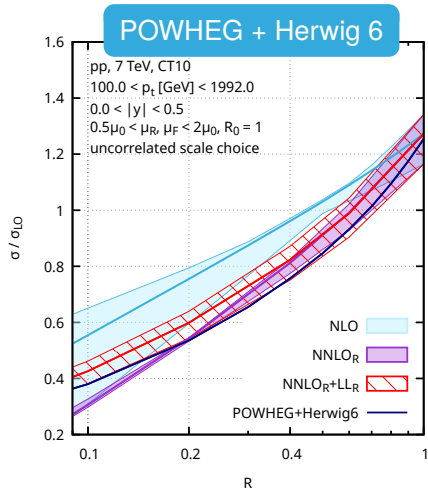
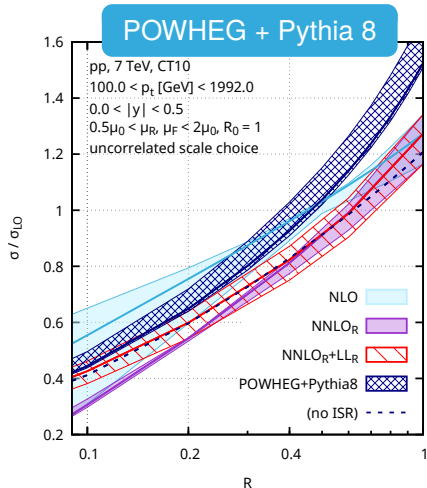
NNLO_{R,K} and NNLO_{R,K}+LL_R results with K -factor

Taking $K > 1$ increases overlap between NNLO_{R,K} and NNLO_{R,K}+LL_R.



Comparison to POWHEG

Compare with POWHEG's dijet process, showered with Pythia 8.186 and Herwig 6.521.



NON-PERTURBATIVE EFFECTS AND COMPARISON TO DATA

Non-perturbative effects

There are two main non-perturbative effects

- ▶ **Hadronisation** : the transition from parton-level to hadron-level
- ▶ **Underlying event** : multiple interactions between partons in the colliding protons

They are separate effects, and so it is important to examine them separately.

- ▶ **Hadronisation** shifts jet p_t by $\sim 1/R$, so it matters a lot at small R .
- ▶ **UE** shifts the jet p_t by $\sim R^2$, so it matters at large R .

Analytical hadronisation model

From event-shape measurements in DIS and e^+e^- collisions, it has been argued that the average hadronisation p_t shift should be roughly

$$\langle \Delta p_t \rangle \simeq -\frac{C}{C_F} \left(\frac{1}{R} + \mathcal{O}(1) \right) \times 0.5 \text{ GeV}$$

where C is the colour factor of the initiating parton, $C_F = \frac{4}{3}$ (quark) or $C_A = 3$ (gluon).

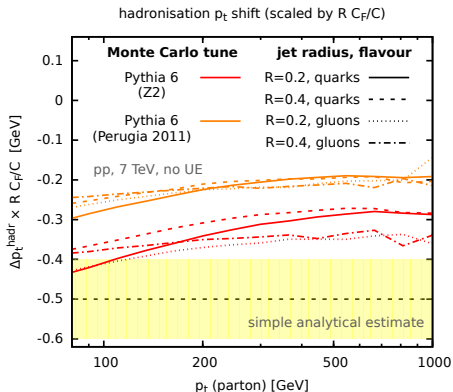
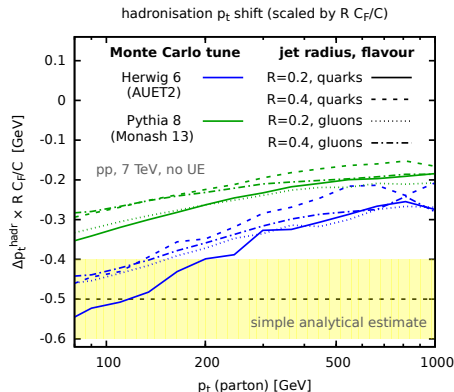
[Dasgupta, Magnea, Salam [JHEP 0802 \(2008\) 055](#)]

Predictions: the shift in jet p_t rescaled by RC_F/C

- ▶ should be identical for quarks and gluons and for different R values,
- ▶ should be p_t independent and cluster around 0.5 GeV.

Hadronisation for different Monte Carlo tunes

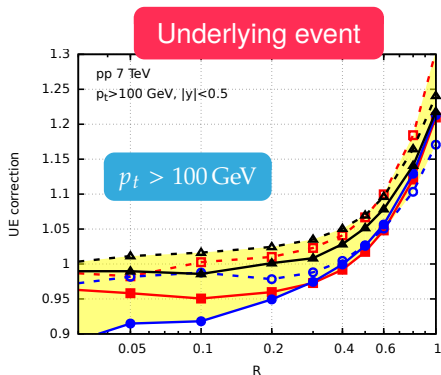
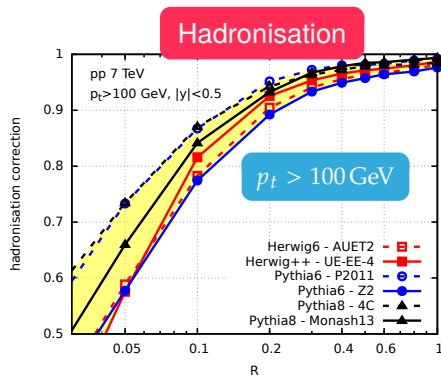
Shift in jet p_t induced by hadronisation, rescaled by a factor RC_F/C .



Reasonable agreement with analytical model for colour and R scaling, but **strong p_t dependence** and **large differences between tunes**.

Hadronisation and UE corrections

We will include non-perturbative effects by **rescaling spectra** with factors derived from Monte Carlo simulations.

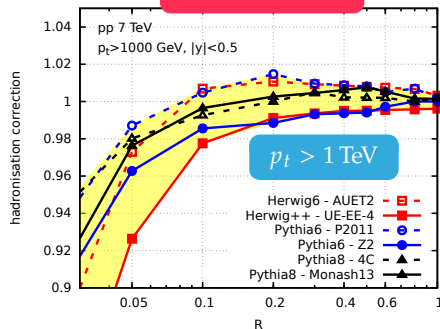


Surprising behaviour of UE corrections at small radii: some factors **smaller than one** (ie. removing energy), and not suppressed at high p_t .

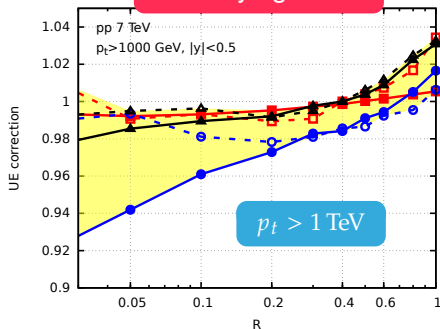
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Hadronisation



Underlying event



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What data? What settings?

Most of our new information is for smaller values of R

Therefore, we concentrate here on **data with smallest R** values

- ▶ **ALICE** data at $\sqrt{s} = 2.76$ TeV, for $R = 0.2$ and $R = 0.4$.
- ▶ **ATLAS** data at $\sqrt{s} = 7$ TeV, for $R = 0.4$ and $R = 0.6$.

Settings

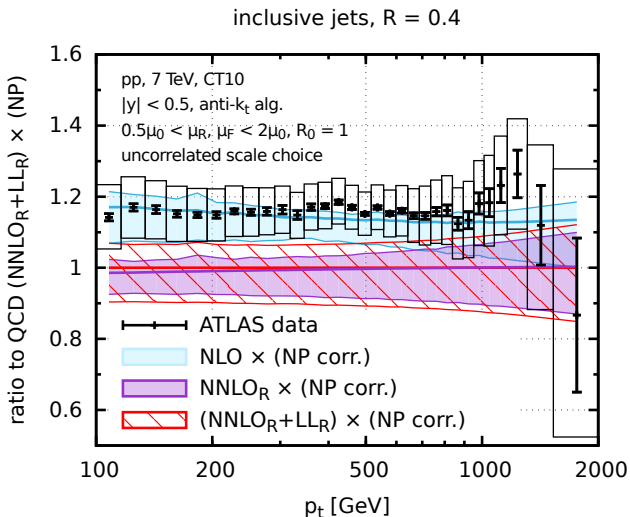
- ▶ Scale is set to hardest jet in the event, as clustered with $R = 1$

$$\mu_0 = p_{t,\max}^{R=1}, \quad \mu_0/2 < \mu_F, \mu_R < 2\mu_0 \text{ (uncorrelated variation)} .$$

- ▶ PDF used is **CT10 NLO** set.

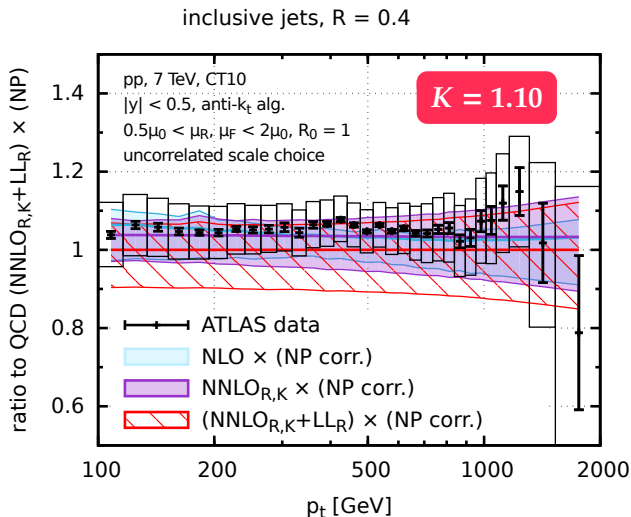
Comparison to data: ATLAS with $R = 0.4$

Partial NNLO_R results shift the predictions further away from data.



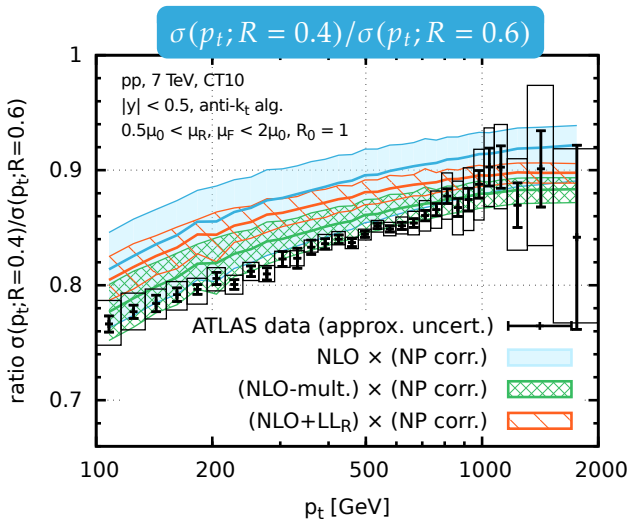
Comparison to data: ATLAS with $R = 0.4$

Better agreement with data when taking a constant K-factor modelling impact of two-loop corrections.



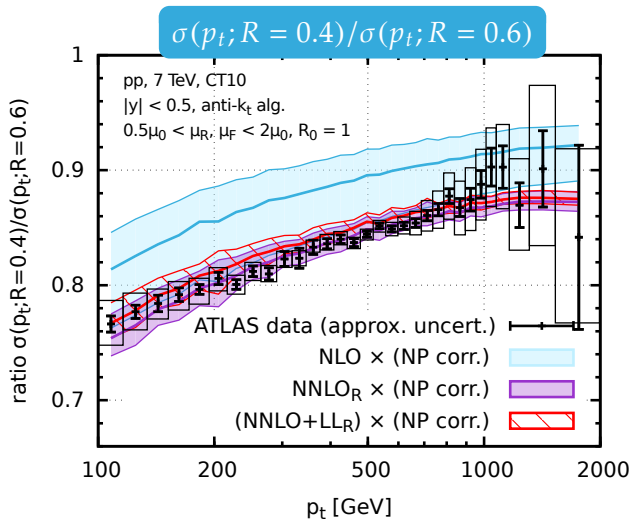
Comparison to ATLAS data: ratio of jet spectra

Take ratio of $R = 0.4$ and $R = 0.6$ spectra. Allows us to study directly the R -dependence



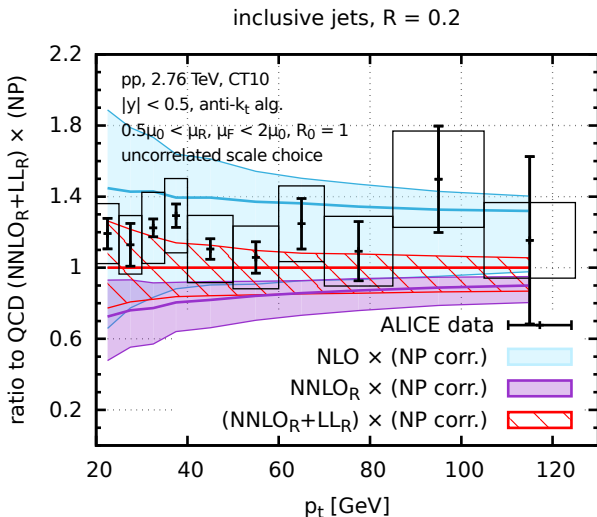
Comparison to ATLAS data: ratio of jet spectra

NLO not enough to get the ratio right! NNLO_R corrections are essential to have accurate predictions.



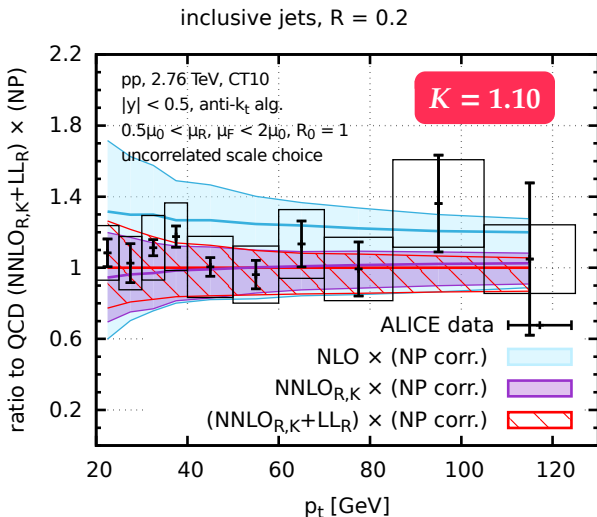
Comparison to data: ALICE with $R = 0.2$

NNLO_{R+LL_R} deviates from NNLO_R by up to 30% at low p_t , and provides best match for the data.



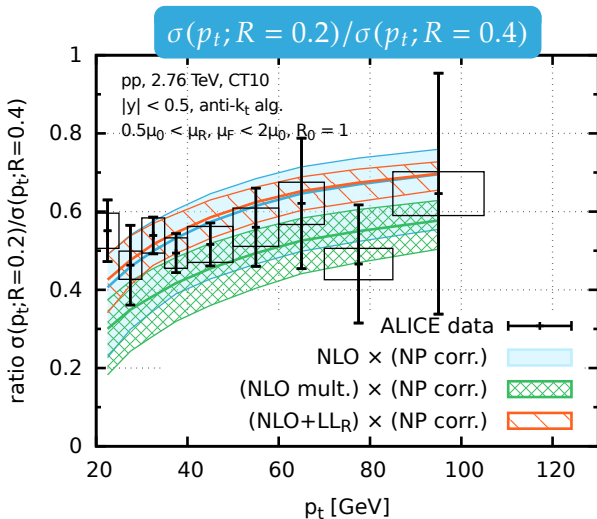
Comparison to data: ALICE with $R = 0.2$

Adding a K-factor $K = 1.10$ slightly improves agreement with data.



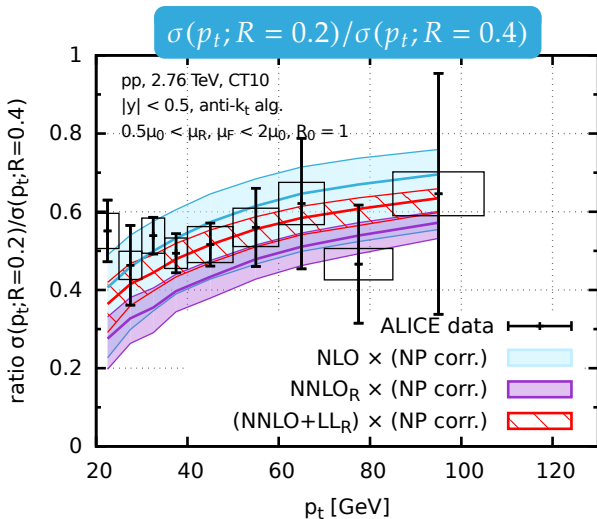
Comparison to ALICE data: ratio of jet spectra

Take ratio of $R = 0.2$ and $R = 0.4$ spectra. Allows us to study directly the R -dependence



Comparison to ALICE data: ratio of jet spectra

In this case, NNLO_{R+LL_R} provides best match for the data.



CONCLUSION

Conclusion

- ▶ Need perturbative control over full R range. We gain insight into what happens using NNLO_R and LL_R predictions.
 - ▶ R -dependence is strongly modified compared to NLO.
 - ▶ LL_R resummation can be **important for $R < 0.4$** .
- ▶ Comparison to data **ATLAS and ALICE data**: R -dependence works well, but an absolute comparison will require full NNLO calculation
- ▶ **NLO-mult** predictions and **uncorrelated scale variations** are important tools for reliably estimating missing higher-order uncertainties.

Code and plots available on microjets.hepforge.org.

BACKUP SLIDES

Microjet vetoes

Jet veto resummation for Higgs production has terms

$$\alpha_s^m \ln^{2m} \frac{M_H}{p_{t,\text{veto}}} + \text{subleading}$$

Among the subleading terms there are small-R enhanced terms

$$\alpha_s^{m+n} \ln^m \frac{M_H}{p_{t,\text{veto}}} \ln^n \frac{1}{R^2} + \dots$$

Suspected of having important impact, and calculated by several groups

NNLL jet vetoes	$n = 1$	[Banfi, Monni, Salam, Zanderighi PRL 109 (2012) 202001] + [Becher, Neubert, Rothen JHEP 1301 (2013) 125] + [Stewart, Tackmann, Walsh, Zuberi PRD 89 (2014) 054001]
Alioli & Walsh	$n = 2$ (numerically)	[JHEP 1403 (2014) 119 , corr. in arXiv-v3]
Our work	$n = 2$ (analytically)	[JHEP 1504 (2015) 039] + $n \rightarrow \infty$ (numerically)

Writing the probability of no gluon emissions above a scale p_t as

$$P(\text{no primary-parton veto}) = \exp \left[- \int_{p_t}^Q \frac{dk_t}{k_t} \bar{\alpha}_s(k_t) 2 \ln \frac{Q}{k_t} \right],$$

one can show that including small- R corrections and applying the veto on the hardest microjet, we have

$$\begin{aligned} \mathcal{U} &\equiv P(\text{no microjet veto})/P(\text{no primary-parton veto}) \\ &= \exp \left[- 2\bar{\alpha}_s(p_t) \ln \frac{Q}{p_t} \int_0^1 dz f^{\text{hardest}}(z, t(R, p_t)) \ln z \right]. \end{aligned}$$

The R -dependent correction generates a series of terms

$$\alpha_s^{m+n}(Q) \ln^m(Q/p_t) \ln^n R.$$

Definition

Reclustering of a jet on a smaller angular scale $R_{\text{filt}} < R_0$, discarding all but the n_{filt} hardest subjects.

Define $f^{k\text{-hardest}}(z)$ the probability that the k -th hardest subjet carries a momentum fraction z of the initial parton.

We can express the energy loss between the filtered jet and the initial parton as

$$\langle \Delta z \rangle^{\text{filt}, n} = \left[\sum_{k=1}^n \int dz z f^{k\text{-hardest}}(z) \right] - 1.$$

Definition

- ▶ Recluster all particles within a jet into subjects with $R_{\text{trim}} < R_0$.
- ▶ Resulting microjets with $p_t \geq f_{\text{cut}} p_t^{\text{parton}}$ are merged and form the trimmed jet, others are discarded.

Energy difference between the trimmed jet and the initial parton of flavour i can then be expressed as a function of f_{cut}

$$\langle \Delta z(f_{\text{cut}}) \rangle_i^{\text{trim}} = \left[\sum_j \int_{f_{\text{cut}}}^1 dz z f_{j/i}^{\text{incl}}(z, t) \right] - 1.$$

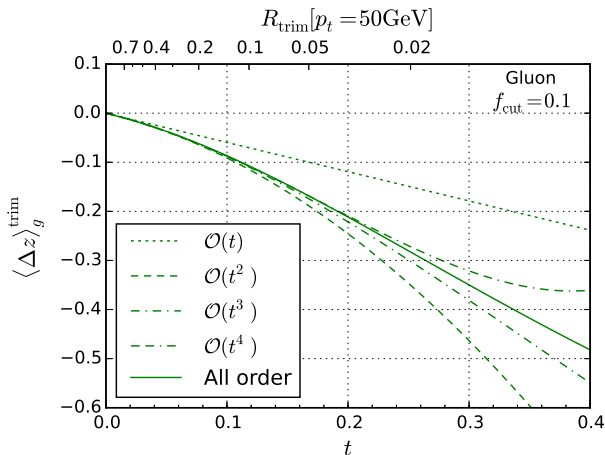
Caveat: this would need **double resummation** of $\ln R$ and $\ln f_{\text{cut}}$.

Trimming of gluon jets

As with filtering, energy loss from trimmed jets with a given R_{trim} is much reduced relative to that from a single microjet with that same radius.

Convergence of the power series is (maybe) better than for filtering.

Resummation of $\ln f_{\text{cut}}$ would be required as well.



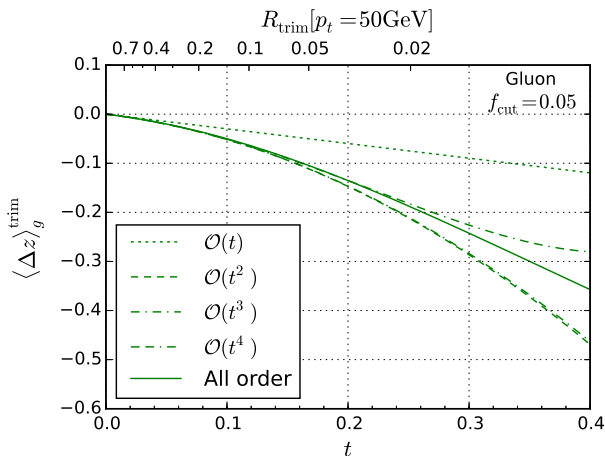
Average jet energy loss Δz after trimming with $f_{\text{cut}} = 0.1$.

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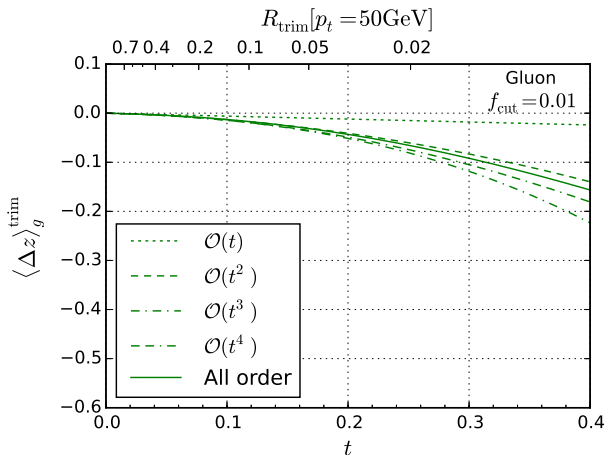
Average jet energy loss Δz after trimming with $f_{\text{cut}} = 0.05$.

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Convergence of the power series is (maybe) better than for filtering.

Resummation of $\ln f_{\text{cut}}$ would be required as well.



Average jet energy loss Δz after trimming with $f_{\text{cut}} = 0.01$.

Define quantity $\Delta_1(p_t, R, R_{\text{ref}})$, where

$$\Delta_i(p_t, R, R_{\text{ref}}) \equiv \frac{\sigma_i(p_t, R) - \sigma_i(p_t, R_{\text{ref}})}{\sigma_0(p_t)}$$

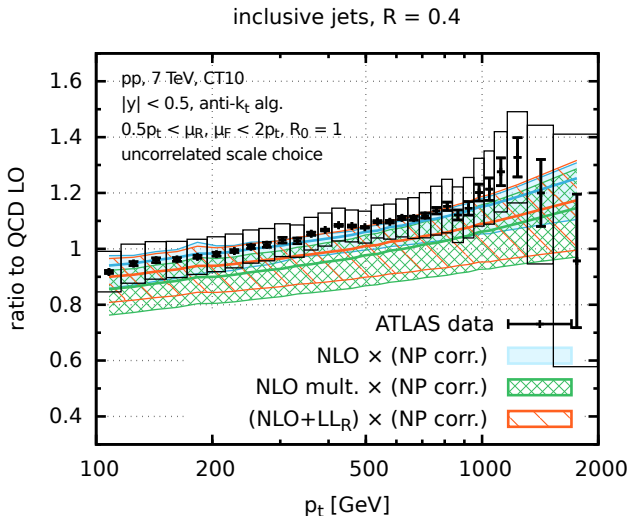
Here $\sigma_i(p_t)$ corresponds to the order α_s^{2+i} contribution to the inclusive jet cross section in a given bin of p_t .

At NNLO, we also define

$$\Delta_{1+2}(p_t, R, R_{\text{ref}}) \equiv \Delta_1(p_t, R, R_{\text{ref}}) + \Delta_2(p_t, R, R_{\text{ref}})$$

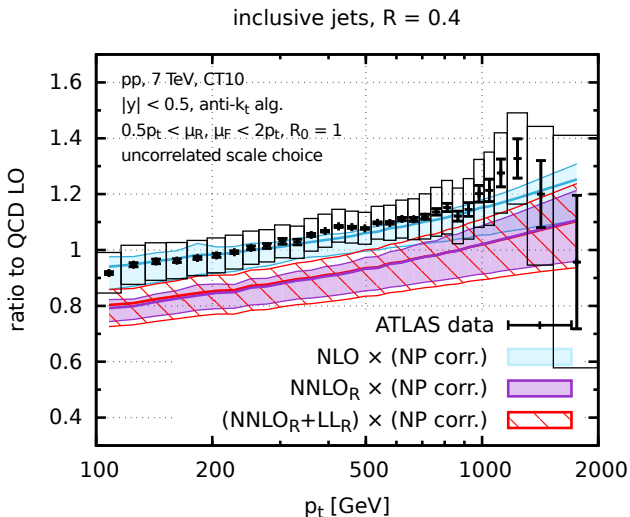
Comparison to data: ATLAS with $R = 0.4$

Small- R resummation shifts the spectrum by 5 – 10%, and increases the scale dependence of the NLO prediction.



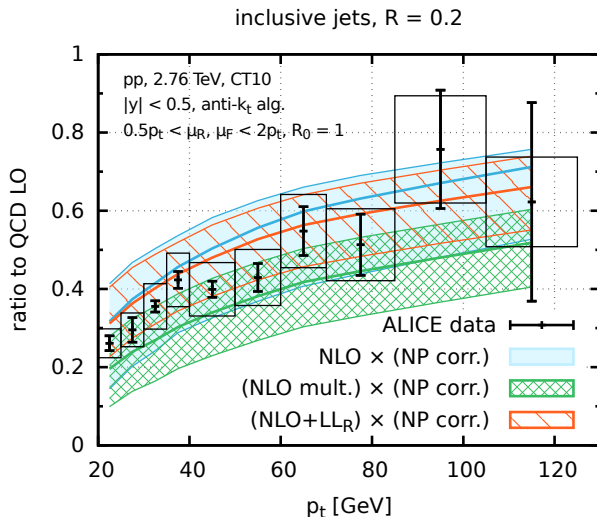
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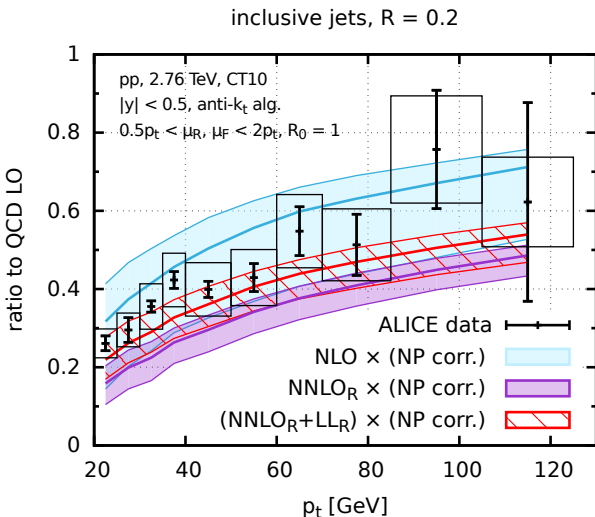
Comparison to data: ALICE with $R = 0.2$

Small- R resummation somewhat improves agreement with ALICE data, and reduces the scale dependence of the NLO prediction.



Comparison to data: ALICE with $R = 0.2$

NNLO_{R+LL_R} deviates from NNLO_R by up to 30% at low p_t , and provides best match for the data.



Example of jet algorithm: generalised k_t algorithms

Basic idea is to **invert QCD branching** process, clustering pairs which are closest in metric defined by the divergence structure of the theory.

Definition

1. For any pair of particles i, j find the minimum of

$$d_{ij} = \min\{k_{ti}^{2p}, k_{tj}^{2p}\} \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = k_{ti}^{2p}, \quad d_{jB} = k_{tj}^{2p}$$

where $\Delta R_{ij} = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$.

2. If the minimum distance is d_{iB} or d_{jB} , then the corresponding particle is removed from the list and defined as a jet, otherwise i and j are merged.
3. Repeat until no particles are left.

The index p defines the specific algorithm, with $p = \pm 1, 0$.

Jet radius value

Jet radius values for different experiments, excluding substructure R choices

	ATLAS	CMS	ALICE	LHCb
R	0.2*, 0.4 – 0.6	0.3*, 0.5, 0.7	0.2 – 0.4	0.5, 0.7

* for PbPb only

Matching NNLO and LL_R

Extend the multiplicative matching to NNLO

$$\begin{aligned} \sigma^{\text{NNLO}+\text{LL}_R} &= (\sigma_0 + \sigma_1(R_0) + \sigma_2(R_0)) \times \\ &\times \left[\frac{\sigma^{\text{LL}_R}(R)}{\sigma_0} \times \left(1 + \Delta_{1+2}(R, R_0) - \frac{\sigma_1^{\text{LL}_R}(R) + \sigma_2^{\text{LL}_R}(R)}{\sigma_0} \right. \right. \\ &\left. \left. - \frac{\sigma_1^{\text{LL}_R}(R) (\sigma_1(R) - \sigma_1^{\text{LL}_R}(R))}{\sigma_0^2} - \frac{\sigma_1(R_0) \left(\Delta_1(R, R_0) - \frac{\sigma_1^{\text{LL}_R}(R)}{\sigma_0} \right)}{\sigma_0} \right) \right] \end{aligned}$$

and define “NNLO mult.”, which factorises the production of large- R_0 jets from the fragmentation to small- R jets

$$\begin{aligned} \sigma^{\text{NNLO,mult.}} &= (\sigma_0 + \sigma_1(R_0) + \sigma_2(R_0)) \times \\ &\times \left(1 + \Delta_{1+2}(R, R_0) - \frac{\sigma_1(R_0)}{\sigma_0} \Delta_1(R, R_0) \right) \end{aligned}$$

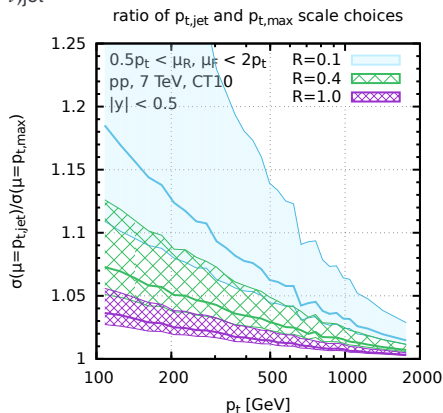
Choice of scale μ_0 beyond LO

Two prescriptions for central renormalisation and factorisation scale

- ▶ Single scale for whole event, set by p_t of hardest jet in the event, $\mu_0 = p_{t,\max}$.
- ▶ Different scale for each jet, $\mu_0 = p_{t,\text{jet}}$.

Prescriptions are identical at LO but can differ substantially starting from NLO.

Strong dependence on jet radius: For $R = 0.1$, $\mu_0 = p_{t,\text{jet}}$ scale increases σ by 20% at low p_t .



Choice of scale μ_0 beyond LO

We will use a single scale, taken to be the hardest jet in the event, as clustered with $R = 1$: $\mu_0 = p_{t,\max}^{R=1}$.

- ▶ At small R , NNLO correction suppress the cross section, so $\mu_0 = p_{t,\text{jet}}$ prescription goes in the **wrong direction**.
- ▶ Main difference between prescriptions comes from when **softest parton falls outside leading two jets**. One jet then has reduced p_t and the choice $\mu_0 = p_{t,\text{jet}}$ gives a smaller scale. This occurs with a probability that is **enhanced by $\ln 1/R$** .
- ▶ $\mu = p_{t,\text{jet}}$ scale choice introduces correction that goes in wrong direction because it leads to smaller scale (and larger α_s) for real part, but without corresponding modification of virtual part. Thus it **breaks the symmetry between real and virtual corrections**.

Comparison to POWHEG with K -factor

Compare with POWHEG's dijet process, showered with Pythia v8.186.

