QCD resummation in the framework of supersymmetry

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Parton showers, event generators and resummation 2016

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Outline

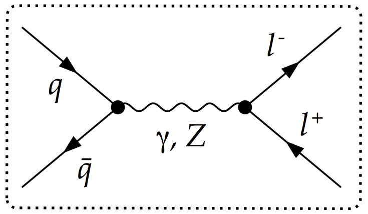
1. Motivation for precision calculations
2. Resummation calculations in QCD
3. Selection of results for supersymmetric particle pair production at the LHC
4. Summary - conclusions

Why precision: Drell-Yan production

✦ Leading-order (LO): $d\sigma \approx d\sigma^{(0)}$

✤ Naive approach, easy calculations

✤ Rough estimate, sometimes far too naive



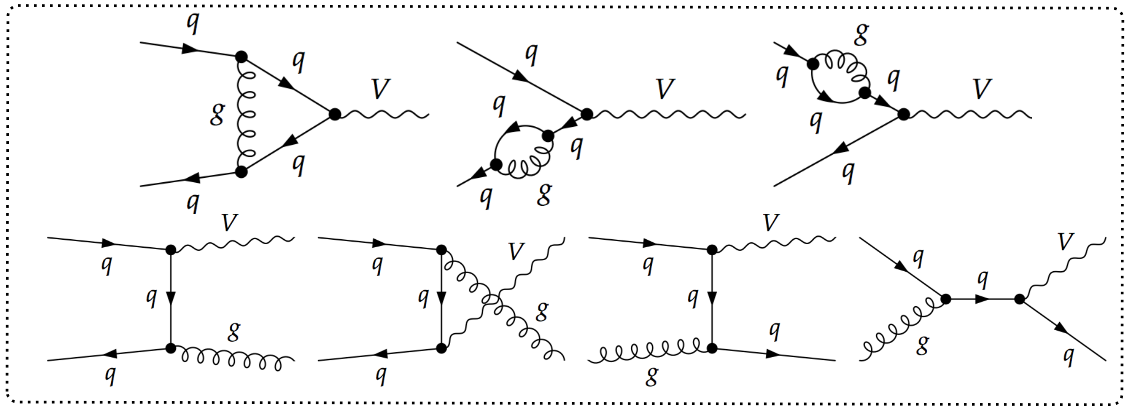
✦ Next-to-leading-order (NLO): $d\sigma \approx d\sigma^{(0)} + \alpha_s\, d\sigma^{(1)}$

✤ Virtual and real contributions

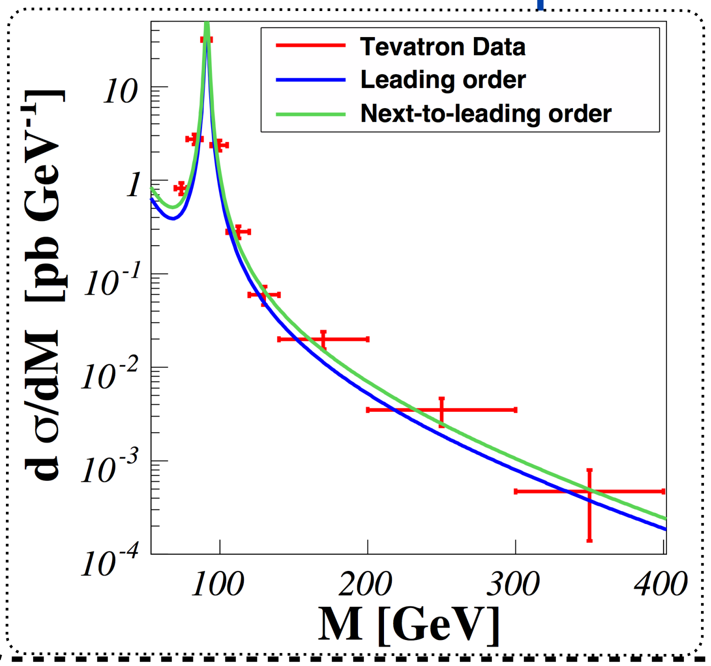
✤ First order where loops can compensate the scale dependence

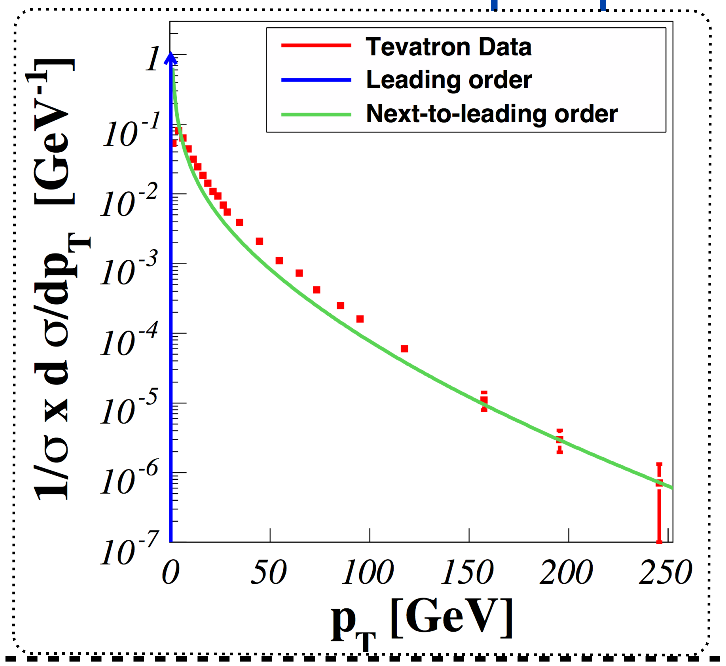
✤ A better agreement can be achieved

✤ Still sometimes too naive



✦ Illustration: dilepton invariant-mass and p_T spectra @ the Tevatron





✤ LO: disagreement between theory and experiment

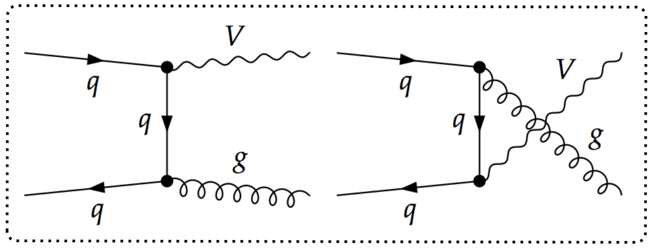
✤ Invariant-mass spectrum
- ★ Good agreement @ NLO

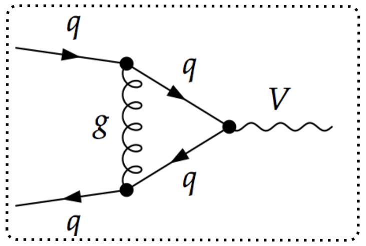
✤ Transverse-momentum spectrum
- ★ Good agreement at large p_T
- ★ Underestimation at medium p_T
- ★ Divergences at small p_T

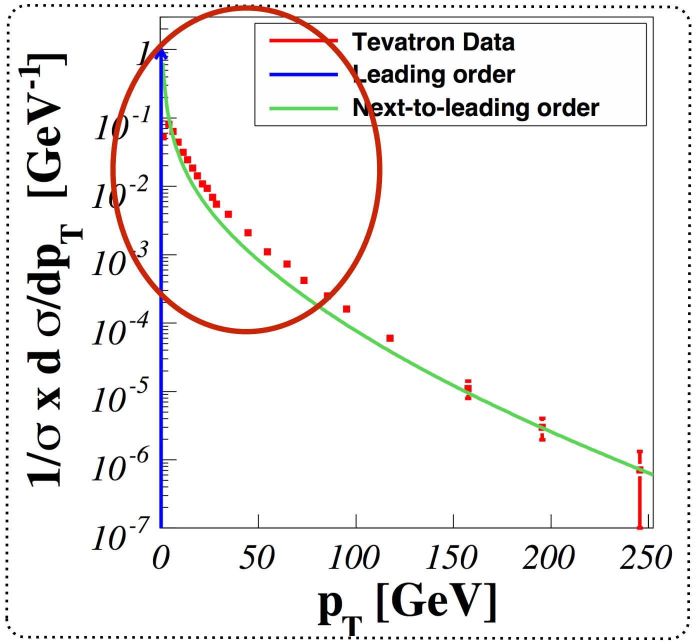
Investigating the NLO contributions

✦ Clear issue in the soft limit: starting point

- The soft limit: the behavior for small p_T values
- Computation of the NLO matrix element in this limit
  - ★ Real emission
  - ★ Virtual corrections







✦ Computation of $d\sigma^{(1)}$ in the soft limit

- Real emission

$$iM \approx g_s T^a \left[\frac{\epsilon^* \cdot k_2}{k_2^0 \mathbf{k_g^0}(\mathbf{1}+\cos\theta)} - \frac{k_1 \cdot \epsilon^*}{k_1^0 \mathbf{k_g^0}(\mathbf{1}-\cos\theta)}\right] \mathbf{iM}^{\rm Born}$$

Soft and collinear radiation diverges

- Virtual corrections

$$iM \approx (i\, g_s^2) \int \mathrm{d}k_g \frac{k_1 \cdot k_2}{k_g^2\, (k_1^0 \mathbf{k_g^0}(\mathbf{1}-\cos\theta))(k_2^0 \mathbf{k_g^0}(\mathbf{1}+\cos\theta))} \mathbf{iM}^{\rm Born}$$

Virtual contributions diverge in the soft and collinear limit

Soft and collinear radiation

✦ After summing real and virtual contributions

- ✤ The poles cancel
- ✤ An infrared behavior remains: logarithmic terms
  - ★ Invariant mass distribution ($z=M^2/s$)

    $$\alpha_s \left(\frac{\ln(1-z)}{1-z}\right)_+$$

  - ★ p_T distribution

    $$\frac{\alpha_s}{p_T^2} \ln \frac{M^2}{p_T^2}$$

- ✤ The fixed order theory is unreliable in critical regions (where the logs are large)
  - ★ Small p_T
  - ★ Large z (close to 1): but effects reduced by the PDFs

✦ Improving the NLO results

- ✤ QCD resummation
  - ★ Correct treatment of the soft and collinear radiation, accounted for to all orders
  - ★ Perturbative method
  - ★ Parton-level calculation
- ✤ Matching with a parton-shower algorithm
  - ★ Approximate treatment of the soft and collinear radiation
  - ★ Suitable for a proper description of the collision (hadronisation, detector simulation, *etc.*)
  - ★ Beyond the parton-level

Factorization, setup and conjugate spaces

✦ Doubly-differential cross section (*cf.* factorization theorem)

$$M^2 \frac{\mathrm{d}^2\sigma_{AB}}{\mathrm{d}M^2\mathrm{d}p_T^2}\left(\frac{M^2}{S_h}\right) = \sum_{ab}\int \mathrm{d}x_a\,\mathrm{d}x_b\,\mathrm{d}z \left[x_a\ f_{a/A}(x_a;\mu^2)\right]\left[x_b\ f_{b/B}(x_b;\mu^2)\right]\left[z\ \hat\sigma_{ab}\Big(z, M^2, \frac{M^2}{p_T^2}, \frac{M^2}{\mu^2}\Big)\ \delta\Big(\frac{M^2}{S_h} - x_a x_b z\Big)\right]$$

- ✤ Variables of interest: the p_T and the invariant mass M of the final state system
- ✤ $\hat\sigma$ is the hard-scattering function

✦ In Mellin space: two key quantities

$$F(N) = \int_0^1 \mathrm{d}X\ X^{N-1} F(X)$$

- ✤ Hadronic quantity
  - ★ The convolution is now a product

$$M^2 \frac{\mathrm{d}^2\sigma_{AB}}{\mathrm{d}M^2\mathrm{d}p_T^2}(N-1) = \sum_{ab}\left[f_{a/A}(N;\mu^2)\right]\left[f_{b/B}(N;\mu^2)\right]\left[\hat\sigma_{ab}\Big(N, M^2, \frac{M^2}{p_T^2}, \frac{M^2}{\mu^2}\Big)\right]$$

- ✤ The hard-scattering function can be linked to a partonic cross section

$$M^2 \frac{\mathrm{d}^2\sigma_{ab}}{\mathrm{d}M^2\mathrm{d}p_T^2}(N-1) = \sum_{cd}\left[\phi_{c/a}(N;\mu^2)\right]\left[\phi_{d/b}(N;\mu^2)\right]\left[\hat\sigma_{cd}\Big(N, M^2, \frac{M^2}{p_T^2}, \frac{M^2}{\mu^2}\Big)\right]$$

  - ★ ϕ are parton-in-parton distributions (evolution governed by DGLAP equations)
  - ★ ϕ absorb all collinear singularities of the cross section ($\overline{\text{MS}}$: ϕ are pure divergences)
  - ★ $\hat\sigma$ is singular at phase space boundaries ($z=1$, $p_T=0$)
  - ★ $\hat\sigma$ has an infrared logarithmic structure than can be resummed to all orders

Resummation: the philosophy

✦ We consider an infrared sensitive quantity R (like the cross section $d^2\sigma$):

- ✤ Dependence on a hard scale M
- ✤ Dependence on a scale m measuring the distance from the critical region
- ✤ Dependence on the ratio of the scales (that can be large)

✦ Resummation to all orders ≡ separation of the two scales

$$R(M^2, m^2) = H(M^2/\mu^2)\ S(m^2/\mu^2)$$

- ✤ R has been refactorized
  - ★ Refactorization holds in general in conjugate spaces ➢ Mellin transforms
- ✤ The functions H and S obey to

$$\frac{\partial H}{\partial \ln \mu^2} = -\frac{\partial S}{\partial \ln \mu^2} = \gamma_S(\mu^2)$$

- ✤ Special case: we choose $\mu = M$: introduction of the Sudakov form factor

$$\mathbf{R(M^2, m^2) = H(1) S(1) \exp\left[-\int_{m^2}^{M^2} \frac{dq^2}{q^2} \gamma_S(q^2) \right]}$$

  - ★ Exponentiation
  - ★ No logarithm of large ratio of scales remains

Example: threshold resummation (1/2)

✦ We reorganize terms of the form $\alpha_s^n(\ln^m(1-z)/1-z)_+$ in the large-N limit

✤ In Mellin space, the partonic cross section reads

$$M^2\frac{\mathrm{d}^2\sigma_{ab}}{\mathrm{d}M^2\mathrm{d}p_T^2}(N-1)=\sum_{cd}\Big[\phi_{c/a}(N;\mu^2)\Big]\Big[\phi_{d/b}(N;\mu^2)\Big]\Big[\hat\sigma_{cd}\Big(N,M^2,\frac{M^2}{p_T^2},\frac{M^2}{\mu^2}\Big)\Big]$$

★ The large logarithms are of the form $\ln^{m+l} N$
★ The critical region is the large N region
★ The off-diagonal terms (c ≠ a and b ≠ d) are subdominant in the large-N limit ➢ neglected

✦ The cross section can be refactorized (the N-dependence is factorized)

✤ After integrating upon p_T:

$$M^2\frac{\mathrm{d}\sigma_{ab}}{\mathrm{d}M^2}(N-1)=\Big[\psi_{a/a}(N;M^2)\Big]\Big[\psi_{b/b}(N;M^2)\Big]\,S_{ab}\Big(N,\frac{M^2}{\mu^2}\Big)\Big[H_{ab}\Big(M^2,\frac{M^2}{\mu^2}\Big)\Big]+\mathcal{O}\Big(\frac{1}{N}\Big)$$

★ The hard function H organizes the infrared-safe components (independent of N)
- perturbatively computable

★ The ψ functions are parton-in-parton distributions
- whose evolution depends on the N-independent parts of the DGLAP splitting functions
- perturbatively computable

★ The soft function S organizes large angle soft-gluon emission
- computable in the eikonal approximation

[Sterman (NPB'87)]

Example: threshold resummation (2/2)

✦ We can extract the hard scattering function in the large-N limit

$$M^2 \frac{d^2\sigma_{ab}}{dM^2 dp_T^2}(N-1) = \sum_{cd} \left[\phi_{c/a}(N;\mu^2)\right]\left[\phi_{d/b}(N;\mu^2)\right]\left[\hat\sigma_{cd}\left(N, M^2, \frac{M^2}{p_T^2}, \frac{M^2}{\mu^2}\right)\right]$$

$$M^2 \frac{d\sigma_{ab}}{dM^2}(N-1) = \left[\psi_{a/a}(N;M^2)\right]\left[\psi_{b/b}(N;M^2)\right] S_{ab}\left(N, \frac{M^2}{\mu^2}\right)\left[H_{ab}\left(M^2, \frac{M^2}{\mu^2}\right)\right] + \mathcal{O}\left(\frac{1}{N}\right)$$

$$\hat\sigma_{ab}\left(N, M^2, \frac{M^2}{\mu^2}\right) = H_{ab}\left(M^2, \frac{M^2}{\mu^2}\right) \frac{\psi_{a/a}(N;M^2)\psi_{b/b}(N;M^2)}{\phi_{a/a}(N;\mu^2)\phi_{b/b}(N;\mu^2)} S_{ab}\left(N, \frac{M^2}{\mu^2}\right) + \mathcal{O}\left(\frac{1}{N}\right)$$

✦ Exponentiation of the hard-scattering function

- The evolution of the parton densities can be solved in the large-N limit
- The soft function exponentiates [Gatheral (PLB'83)]

$$\hat\sigma_{ab}\left(N, M^2, \frac{M^2}{\mu^2}\right) = H_{ab}\left(M^2, \frac{M^2}{\mu^2}\right) \exp\left[G_{ab}\left(N, M^2, \frac{M^2}{\mu^2}\right)\right] + \mathcal{O}\left(\frac{1}{N}\right)$$

- The G function can be perturbatively computed: $G_{ab} = \ln\Delta_a + \ln\Delta_b + \ln\Delta_{ab}$

$$\ln\Delta_i\left(N, M^2, \frac{M^2}{\mu^2}\right) = \int_0^1 dz \frac{z^{N-1}-1}{1-z}\int_{\mu^2}^{(1-z)^2M^2}\frac{dq^2}{q^2}A_i(\alpha_s(q^2))$$

Soft/collinear radiation

$$\ln\Delta_{ab}\left(N, M^2, \frac{M^2}{\mu^2}\right) = \int_0^1 dz \frac{z^{N-1}-1}{1-z} D_{ab}(\alpha_s((1-z)^2M^2))$$

Large-angle soft emission

Resummation regimes

✦ Resummation is based on properties holding in conjugate spaces

$$d\sigma^{(\mathrm{res})}(N,b) = \sum_{a,b} f_{a/h_1}(N+1)\, f_{b/h_2}(N+1)\, \mathcal{H}_{ab}(N) \exp\Big\{\mathcal{G}(N,b)\Big\}$$

[b is the impact parameter, Fourier-conjugate to p_T]

- The process-dependent function $\mathcal{H}$: real and virtual collinear radiation + hard pieces
- The universal Sudakov form-factor $\mathcal{G}$: soft-collinear radiation (the logs)
- Both function are perturbatively computable

✦ Other resummation regimes: resummation formulas derived analogously

Matching to the fixed order

- Fixed-order calculations in QCD
  - Reliable far from the critical regions where the logarithms are large
  - Spoiled in the critical regions
- Resummation calculations in QCD
  - Reliable in the critical regions where the logarithms are large
  - Not justified far from the critical regions

Both calculations are relevant in the intermediate regions

- Information from both resummation and fixed order is required: matching
  - Addition of both results
  - Subtraction of the resummed result expanded at given α_s order
  - Prevention from double-counting the logarithms

$$d\sigma = d\sigma^{(\mathrm{F.O.})} + d\sigma^{(\mathrm{res})} - d\sigma^{(\mathrm{exp})}$$

- Inverse transforms are finally performed to get back to the physical space
  - Prescriptions needed to avoid poles

Precision on the total production rates

✦ Total production rate for electroweakino/slepton pair production at the LHC

✤ Benchmark #31 of the LPCC points

$$m_{\tilde\chi_1^0} = 250 \text{ GeV} , \quad m_{\tilde\chi_2^0} = m_{\tilde\chi_1^\pm} = 480 \text{ GeV}$$
$$m_{\tilde e_L} = m_{\tilde\mu_L} = 565 \text{ GeV} , \quad m_{\tilde e_R} = m_{\tilde\mu_R} = 460 \text{ GeV}$$
$$m_{\tilde\tau_1} = 295 \text{ GeV} , \quad m_{\tilde\tau_2} = 535 \text{ GeV} .$$

[Squarks and gluino above 1-1.5 TeV]

[AbdusSalam *et al.* (EPJC'11)]

✦ Comparing LO, NLO and NLO+NLL (8 TeV)

✤ LO *vs.* NLO
- ★ Important *K* factor (20-30%)
- ★ Important scale uncertainty reduction

✤ NLO *vs.* NLO+NLL
- ★ Mild modification of the *K* factor
- ★ Further reduction of the scale uncertainties

Scale uncertainties are under control

Final state	LO [fb]	NLO [fb]	NLO+NLL [fb]
$\tilde\chi_1^0\tilde\chi_1^0$	$0.1245^{+8.6\%}_{-7.5\%}$	$0.1605^{+3.6\%}_{-3.6\%}$	$0.1554^{+0.2\%}_{-0.0\%}$
$\tilde\chi_2^0\tilde\chi_2^0$	$0.0875^{+12\%}_{-10\%}$	$0.1065^{+4.5\%}_{-3.7\%}$	$0.1043^{+0.3\%}_{-0.0\%}$
$\tilde\chi_1^+\tilde\chi_2^0$	$4.3674^{+9.9\%}_{-8.5\%}$	$4.8750^{+2.0\%}_{-2.4\%}$	$4.8248^{+0.3\%}_{-0.5\%}$
$\tilde\chi_1^-\tilde\chi_2^0$	$1.4986^{+10\%}_{-8.6\%}$	$1.7333^{+2.1\%}_{-2.4\%}$	$1.7111^{+0.6\%}_{-1.1\%}$
$\tilde\chi_1^+\tilde\chi_1^-$	$2.8874^{+9.9\%}_{-8.5\%}$	$3.3463^{+3.3\%}_{-3.3\%}$	$3.3086^{+0.7\%}_{-0.3\%}$
$\tilde\ell_R^+\tilde\ell_R^-$	$0.0749^{+11\%}_{-9.1\%}$	$0.0868^{+2.7\%}_{-3.0\%}$	$0.0854^{+0.2\%}_{-0.4\%}$
$\tilde\ell_L^+\tilde\ell_L^-$	$0.0477^{+12\%}_{-10\%}$	$0.0543^{+2.8\%}_{-3.4\%}$	$0.0534^{+0.5\%}_{-0.3\%}$
$\tilde\tau_1^+\tilde\tau_1^-$	$0.5878^{+7.6\%}_{-5.3\%}$	$0.7093^{+2.5\%}_{-2.5\%}$	$0.6985^{+0.0\%}_{-0.2\%}$

[Results produced with RESUMMINO]
[BF, Klasen, Lamprea & Rothering (EPJC'13)]

Differential distributions (1)

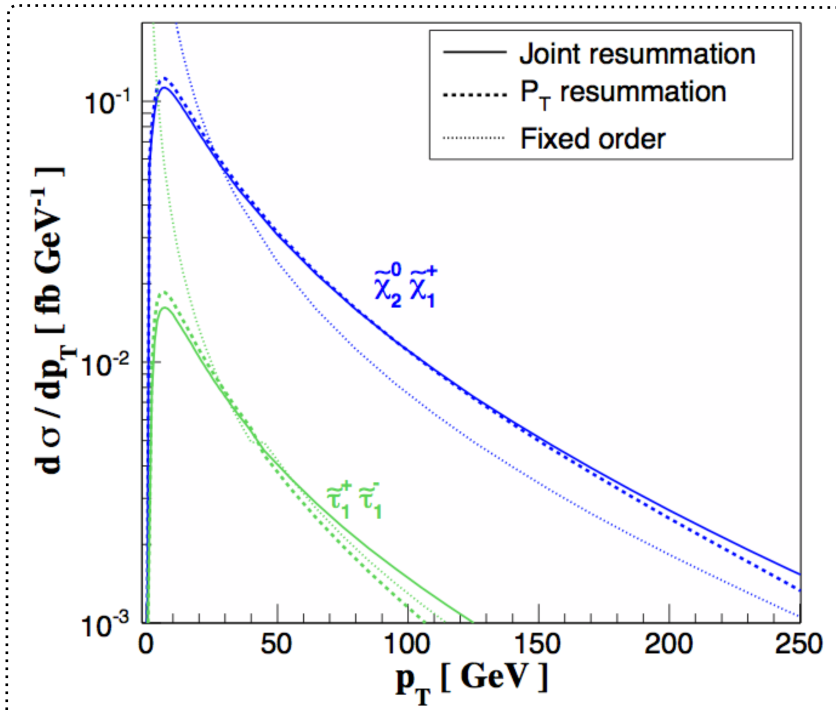
✦ p_T-spectrum of the ewkino pair (@ 8 TeV)

✤ LPCC benchmark #31: (M_χ,M_τ)≈(500,300) GeV

✤ The critical region (at small p_T)
- ★ The NLO results diverge
- ★ Finite behavior restored by resummation (peak at ~ 10 GeV)

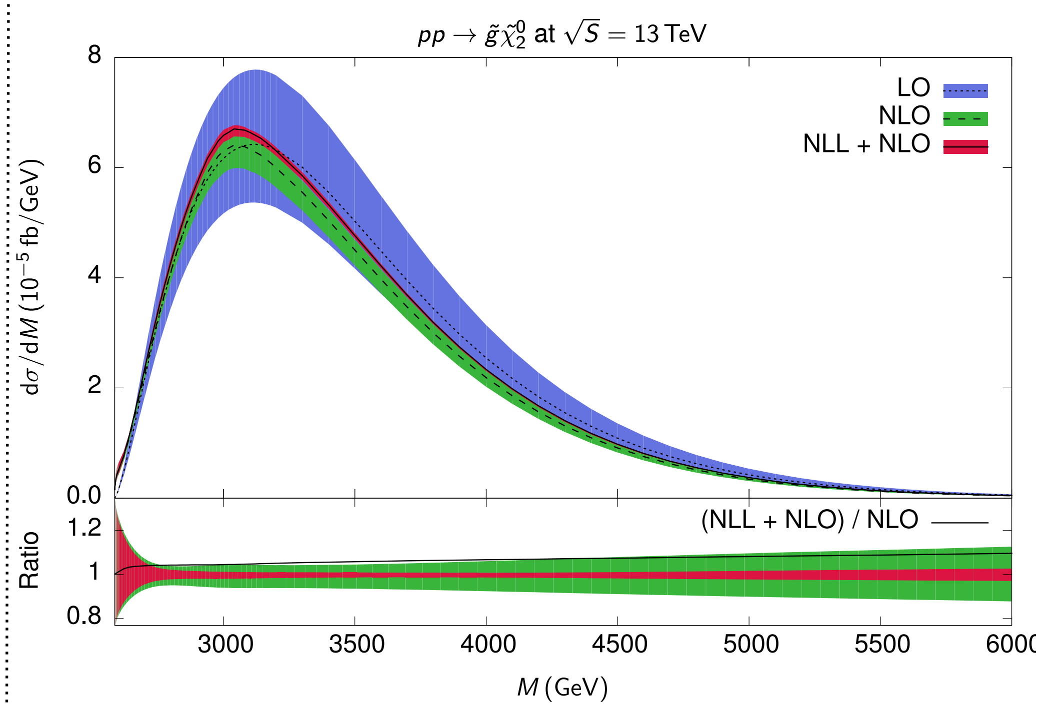
✤ Joint *vs.* p_T resummation: mild differences (a few %)

✤ Large resummation effects even far from critical regions



[BF, Klasen, Lamprea & Rothering (JHEP'12&'14)]

[BF, Klasen & Rothering (JHEP'16)]



✦ Gluino-neutralino invariant-mass (13 TeV)

✤ Benchmark: $M_{\tilde{g}}$≈2 TeV; $M_{\tilde{\chi}}$ ≈ 600 GeV

✤ CT14 (LO/NLO)

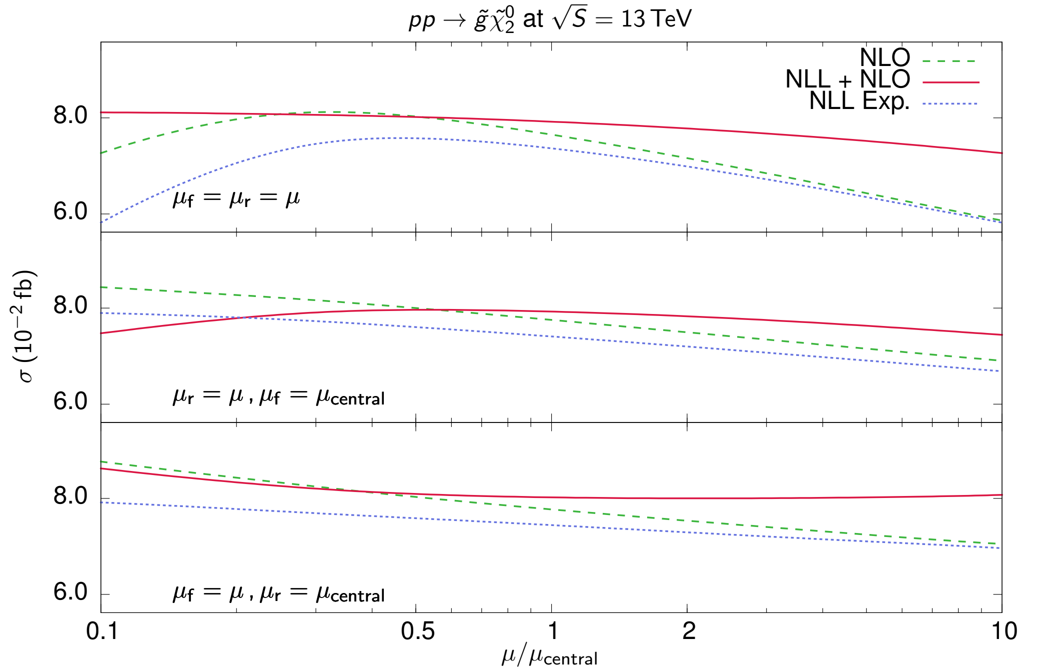
✤ LO/NLO: important spectrum distortions

✤ NLO/NLO+NLL
- ★ Mild increase of the *K*-factor
- ★ Resummation effects reduced by the PDFs

✤ Important reduction of the uncertainties

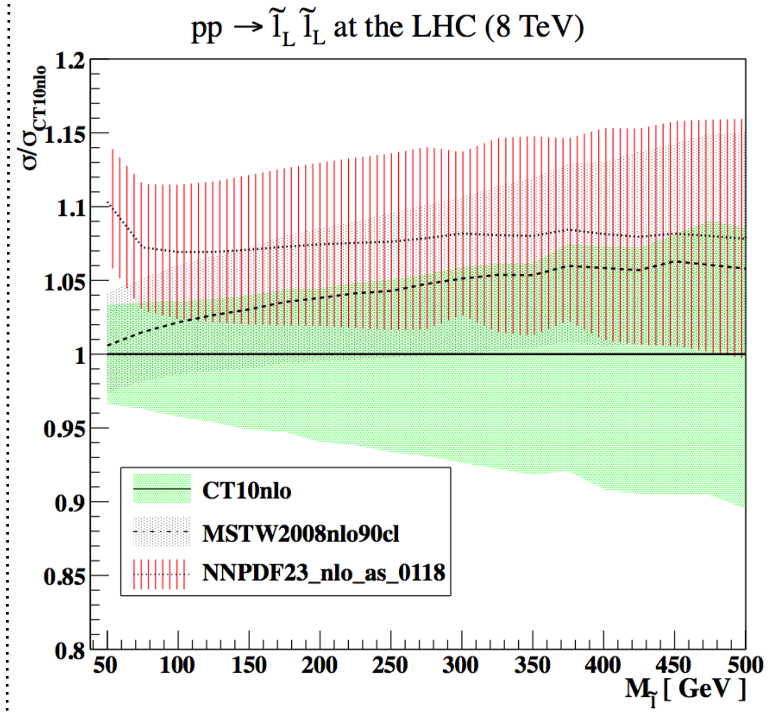
Differential distributions (2)

[BF, Klasen & Rothering (JHEP'16)]



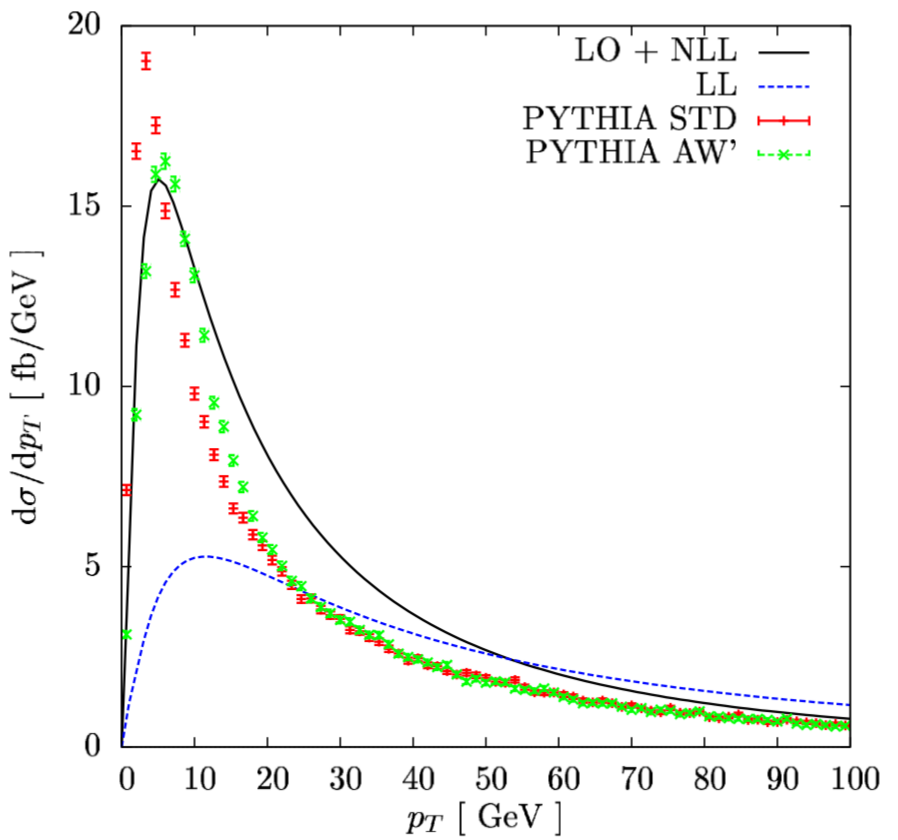
- ✦ Total rates for gluino-neutralino production
  - ✤ Scale dependence further stabilized
  - ✤ NLO *vs.* NLO+NLL
    - ★ Mild modification of the total rate
    - ★ Important reduction of the scale uncertainties

- ✦ Total cross section for slepton pair production
  - ✤ Rates shown as a function of the slepton mass
  - ✤ Normalized to the CT10 parton density choice
  - ✤ All parton density choices in agreement
    - ★ After accounting for the uncertainties



[BF, Klasen, Lamprea & Rothering (JHEP'14)]

Comparison with MC event generators (1/2)



[Debove, BF & Klasen (PLB'10)]

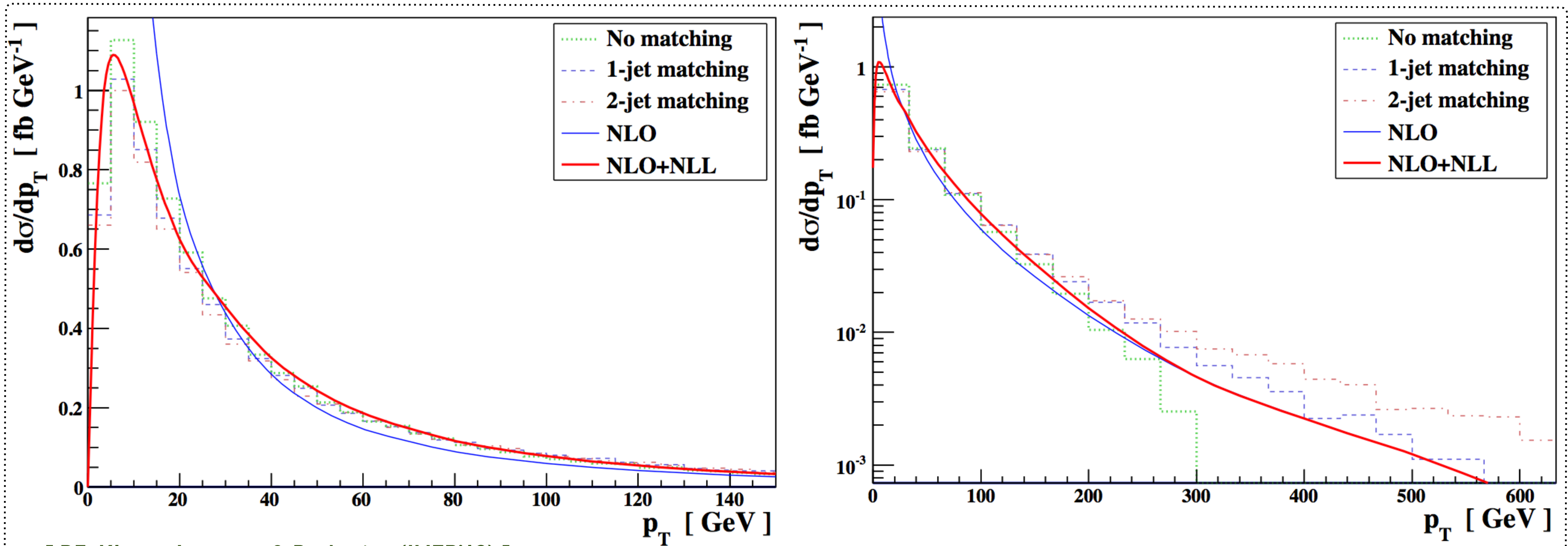
✦ PYTHIA *vs.* resummation (gaugino pair prod.)

✤ Benchmark: $M_{\tilde{\chi}} \approx 180$ GeV

✤ PYTHIA predictions are still widely used
- ★ Leading log approximation
- ★ Different Monte Carlo tunes:
  - The CDF AW tune (red)
  - Our AW' tune (green)

✤ PYTHIA *vs.* LO+LL (blue) and NLO+NLL (black)
- ★ PYTHIA improves the LL picture
- ★ MC tuning could help to reproduce the peak
- ★ Clear underestimation for the intermediate p_T

Comparison with MC event generators (2/2)



[BF, Klasen, Lamprea & Rothering (JHEP'12)]

✦ MG5_AMC + PYTHIA; merging up to 2 extra jets *vs.* resummation (chargino pair production)

- Benchmark: $M_{\tilde\chi} \approx 300$ GeV; Normalization of the MG5_AMC results to NLO+NLL
- Very good agreement between MG5_AMC and NLO+NLL
  - ★ Too soft spectrum in the no-merging case (green curve)
  - ★ MG5_AMC predictions harder when merging matrix elements with up to two extra jets

Summary - conclusions

- ✦ Soft and collinear radiation
  - ✤ Large logarithmic corrections in transverse-momentum and invariant-mass spectra
  - ✤ Need for resummation of the soft-collinear radiation

- ✦ Threshold, p_T and joint resummations have been implemented in RESUMMINO
  - ✤ Reliable perturbative results
  - ✤ Correct treatment of the soft-collinear radiation
  - ✤ Important effects, even far from the critical regions
  - ✤ Theoretical uncertainties under good control

- ✦ Comparison with Monte Carlo event generators
  - ✤ Merging of matrix elements containing 0, 1, 2, … N extra partons *à la* MLM
  - ✤ Same precision as resummation for the shapes of the distributions
  - ✤ NLO+NLL rates now employed in all ATLAS and CMS supersymmetry analyses