Towards NNLL Joint Threshold and q_T Resummation for Drell-Yan Processes

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Introduction

- Differential cross sections relevant at LHC
- Additional scales $\rightarrow$ Different types of logarithms
- Threshold resummation improved precision and small q_T logarithms

Theoretical status

- Original formalism for joint q_T and threshold *[Laenen, Sterman, Vogelsang, '00]*
- Applied at NLL to prompt photon *[Laenen, Sterman, Vogelsang, '00]*, Higgs and DY *[Kulesza, Sterman, Vogelsang, '02,'03]*, top *[Banfi, Laenen, '05]*, EW SUSY *[Fuks et al., '13]*
- Recent resurgence *[Li, Neill, Zhu, '16] [Lustermans, Waalewijn, Zuene, '16] [Ferrera, Marzani, VT, in progress]*
- Parton shower with threshold resummation *[Nagy, Soper, '16]*
- This talk → theoretical overview of extension to NNLL

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NLO Eikonal integral

$$E_{\mathrm{DY}}^{\mathrm{eik,\ NLO}}(\mathbf{b},N) = \alpha_s 4\pi C_F \int \frac{d^4k}{(2\pi)^3}\theta(k_0)\,\delta(k^2)\exp\left[-\sqrt{2}\frac{k_+ + k_-}{2Q}N - i\mathbf{b}\cdot\mathbf{k_T}\right]\frac{2}{k_T^2}$$
$$= \alpha_s 8\pi C_F \int \frac{d^2k_T}{(2\pi)^3} e^{-i\mathbf{b}\cdot\mathbf{k_T}}\frac{2}{k_T^2}K_0\left(\frac{2Nk_T}{Q}\right)$$

Including virtual and collinear counter term:

$$E_{\mathrm{DY}}^{\mathrm{eik,\ NLO}}(b,N) = 2\frac{\alpha_s}{\pi}C_F\int_0^{Q^2}\frac{dk_T^2}{k_T^2}\left[J_0(bk_T)\,K_0\left(\frac{2Nk_T}{Q}\right) + \log\left(\frac{\bar{N}k_T}{Q}\right)\right]$$

$\bar{N} = Ne^{\gamma_E}$

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NLL exponent

Generalizes to:

$$E_{ab}\left(b, N, Q, \mu_F\right) = \int_0^{Q^2} \frac{dk_T^2}{k_T^2} \left\{ \sum_{i=a,b} A_i(\alpha_s(k_T)) \left[J_0\left(bk_T\right) K_0\left(\frac{2Nk_T}{Q}\right) + \log\left(\frac{\bar{N}k_T}{Q}\right) \right] \right\}$$

$$- \log \bar{N} \int_{\mu_F^2}^{Q^2} \frac{dk_T^2}{k_T^2} \sum_{i=a,b} A_i(\alpha_s(k_T))$$

Approximates to:

$$E_{a\bar{a}}\left(\chi, N, Q, \mu_F\right) = 2 \int_{Q^2/\chi^2}^{Q^2} \frac{dk_T^2}{k_T^2} A_a(\alpha_s(k_T)) \log\left(\frac{\bar{N}k_T}{Q}\right)$$

$$- 2 \log \bar{N} \int_{\mu_F^2}^{Q^2} \frac{dk_T^2}{k_T^2} A_a(\alpha_s(k_T))$$

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Joint function χ

- χ reproduces threshold limit: $\lim_{N\to\infty} \chi(\bar{N}, \bar{b}) = \bar{N}$
- χ reproduces q_T limit: $\lim_{b\to\infty} \chi(\bar{N}, \bar{b}) = \bar{b} = Qbe^{\gamma_E}/2$
- Function determines power suppressed terms
- Contour needs to avoid poles and branch-cuts

Examples:

- $\chi = \bar{b} + \bar{N}$, simple pole and branch-cut structure, $\bar{N}/\bar{b}$ and $\bar{b}/\bar{N}$ power suppressed terms
- $\chi = \bar{b} + \frac{\bar{N}}{1+\bar{b}\eta/\bar{N}}$, $\eta > 0$, $(\bar{N}/\bar{b})^2$ and $\bar{b}/\bar{N}$ power suppressed terms, more complicated pole and branch-cut structure, angular restrictions for $\eta \neq 1/4$

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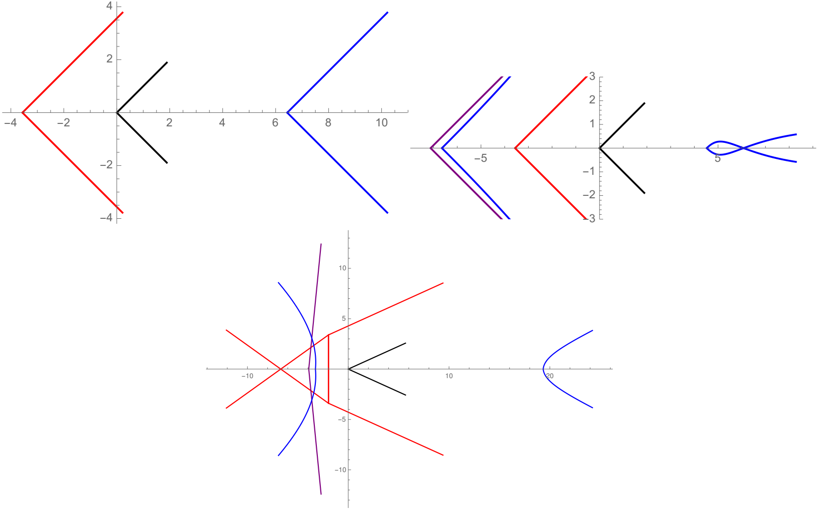
Contour



$\chi = 0$, $\chi = \exp[1/(2\alpha_s b_0)]$, $\chi = \infty$, $\bar{b}$

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Comparison to threshold and q_T

$$E_{a\bar{a}}\left(\chi, N, Q, \mu_F\right) = 2\int_{Q^2/\chi^2}^{Q^2} \frac{dk_T^2}{k_T^2} A_a(\alpha_s(k_T)) \log\left(\frac{\bar{N} k_T}{Q}\right) - 2\log\bar{N}\int_{\mu_F^2}^{Q^2} \frac{dk_T^2}{k_T^2} A_a(\alpha_s(k_T))$$

Becomes threshold exponential for $\chi \to \bar{N}$

$$E_{a\bar{a}}\left(\chi, N, Q, \mu_F\right) = -\int_{Q^2/\chi^2}^{Q^2} \frac{dk_T^2}{k_T^2} \left[A_a(\alpha_s(k_T)) \log\left(\frac{Q^2}{k_T^2}\right) + B(\alpha_s(k_T))\right] + \int_{\mu_F^2}^{Q^2/\chi^2} \frac{dk_T^2}{k_T^2} \left[-2\log\bar{N} A_a(\alpha_s(k_T)) - B(\alpha_s(k_T))\right]$$

Becomes q_T exponential for $\chi \to \bar{b}$

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Overview at NLL

Written similar to *[Bozzi, Catani, de Florian, Grazzini, '06]*

$$\frac{d\sigma_F^{(\mathrm{res})}}{dQ_T^2} = \int_0^\infty db \frac{b}{2} J_0(bQ_T) \int_{C_T} \frac{dN}{2\pi i} \left(\frac{Q^2}{s}\right)^{-N+1} \tilde{W}^F(b,N,Q)$$

$$\tilde{W}^F(N,b,Q) = \sum_{c,d} \sum_{\{I\}} \mathcal{H}_{cd}^{\{I\},\,F}(N,Q,\alpha_s(\mu_R),Q^2/\mu_R^2,Q^2/\mu_F^2,Q^2/\mu_Q^2)$$

$$\times\ \tilde{f}_{c/h_1}(N,\mu_F^2)\, \tilde{f}_{d/h_2}(N,\mu_F^2)$$

$$\times\ \exp\left\{E_{\{I\}}(\alpha_s(\mu_R),b,N,Q^2/\mu_R^2,Q^2/\mu_Q^2)\right\}$$

$$\mathcal{H}^{\,F}(N,Q,\alpha_s(\mu_R),Q^2/\mu_R^2,Q^2/\mu_F^2,Q^2/\mu_Q^2) = \sigma^{(0)}_{a\bar{a}\to F}(\alpha_s(Q)) H_a^F(\alpha_s(Q)) \left(\tilde{C}(N,\alpha_s(Q))\right)^2$$

$$E_{\{I\}}(\alpha_s(\mu_R),b,N,Q^2/\mu_R^2,Q^2/\mu_Q^2) = -\int_{Q^2/\chi^2}^{Q^2} \frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{Q^2}{q^2} + \tilde{B}_N(\alpha_s(q))\right]$$

$$+ \int_{\mu_F^2}^{Q^2} \frac{dq^2}{q^2} 2\gamma(N,\alpha_s(q))$$

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Overview at NLL

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$$\frac{d\sigma_F^{(\mathrm{res})}}{dQ_T^2} = \int_0^\infty db \frac{b}{2} J_0(bQ_T) \int_{C_T} \frac{dN}{2\pi i} \left(\frac{Q^2}{s}\right)^{-N+1} \tilde{W}^F(b, N, Q)$$

$$\tilde{W}^F(N, b, Q) = \sum_{c,d} \sum_{\{I\}} \mathcal{H}_{cd}^{\{I\},\ F}(N, Q, \alpha_s(\mu_R), Q^2/\mu_R^2, Q^2/\mu_F^2, Q^2/\mu_Q^2)$$

$$\times\ \tilde{f}_{c/h_1}(N, \mu_F^2)\ \tilde{f}_{d/h_2}(N, \mu_F^2)$$

$$\times\ \exp\left\{E_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_Q^2)\right\}$$

$$\mathcal{H}^{\ F}(N, Q, \alpha_s(\mu_R), Q^2/\mu_R^2, Q^2/\mu_F^2, Q^2/\mu_Q^2) = \sigma_{a\bar{a}\to F}^{(0)}(\alpha_s(Q)) H_a^F(\alpha_s(Q)) \left(\tilde{C}(N, \alpha_s(Q))\right)^2$$

$$E_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_Q^2) = -\int_{Q^2/\chi^2}^{Q^2} \frac{dq^2}{q^2}\left[A(\alpha_s(q)) \log\frac{Q^2}{q^2} + \tilde{B}_N(\alpha_s(q))\right]$$

$$+ \int_{\mu_F^2}^{Q^2} \frac{dq^2}{q^2} 2\gamma(N, \alpha_s(q))$$

Hard contribution

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Overview at NLL

Written similar to *[Bozzi, Catani, de Florian, Grazzini, '06]*

$$\frac{d\sigma_F^{(\mathrm{res})}}{dQ_T^2} = \int_0^\infty db\, \frac{b}{2}\, J_0(bQ_T) \int_{C_T} \frac{dN}{2\pi i} \left(\frac{Q^2}{s}\right)^{-N+1} \tilde{W}^F(b, N, Q)$$

$$\tilde{W}^F(N, b, Q) = \sum_{c,d} \sum_{\{I\}} \mathcal{H}_{cd}^{\{I\},\,F}(N, Q, \alpha_s(\mu_R), Q^2/\mu_R^2, Q^2/\mu_F^2, Q^2/\mu_Q^2)$$

$$\times\ \tilde{f}_{c/h_1}(N, \mu_F^2)\ \tilde{f}_{d/h_2}(N, \mu_F^2)$$

$$\times\ \exp\left\{E_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_Q^2)\right\}$$

$$\mathcal{H}^{\,F}(N, Q, \alpha_s(\mu_R), Q^2/\mu_R^2, Q^2/\mu_F^2, Q^2/\mu_Q^2) = \sigma^{(0)}_{a\bar{a}\to F}(\alpha_s(Q)) H_a^F(\alpha_s(Q)) \left(\tilde{C}(N, \alpha_s(Q))\right)^2$$

$$E_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_Q^2) = -\int_{Q^2/\chi^2}^{Q^2} \frac{dq^2}{q^2}\left[A(\alpha_s(q)) \log\frac{Q^2}{q^2} + \tilde{B}_N(\alpha_s(q))\right] + \int_{\mu_F^2}^{Q^2} \frac{dq^2}{q^2} 2\gamma(N, \alpha_s(q))$$

Sudakov factor

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Interlude: $\tilde{B}_N$

Usually $\tilde{C}(N, \alpha_s(Q/\chi))$

All hard contributions computed at the same scale (Q), can be described as:

$$\tilde{B}_N(\alpha_s) = B(\alpha_s) + 2\beta(\alpha_s)\frac{d\log C_N(\alpha_s)}{d\log\alpha_s} + 2\gamma(N, \alpha_s)$$

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DGLAP

Include $\log \bar{N}$ contribution of NNLO DGLAP can contribute at NNLL threshold, but N^{3}LL q_T

$$E_{\rm thr}^{\rm DGLAP} = -2 \int_{\mu_F^2}^{Q^2/\chi^2} \frac{dk_T^2}{k_T^2} \log \bar{N} A_a(\alpha_{\rm s}(k_T))$$

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Soft wide-angle

Treated similar to $t\bar{t}$ production *[Banfi, Laenen, '05]*

$$E_{\rm thr}^{\rm wide-angle,\, NNLL} = -\frac{1}{2}\int_{Q^2/\bar{N}^2}^{Q^2} \frac{dq^2}{q^2}\tilde{D}_a(\alpha_{\rm s}(q))$$

Logarithmic contributions are in $\tilde{C}_{qq}$

$$\mathcal{H}^{(2)} \to \mathcal{H}^{(2)} + \tilde{D}^{(2)} \log \bar{N}$$

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Collinear-anomaly

Take into account difference in threshold and q_T values of $A^{(3)}$

[Becher, Neubert, '10]

$$E_{\rm thr}^{\rm col-anom} = -\int_{Q^2(\bar{b}^2+1)/\chi^2}^{Q^2} \frac{dq^2}{q^2} \left(A_{Q_T}^{(3)} - A_{\rm thr}^{(3)}\right) \left(\frac{\alpha_{\rm s}(q)}{\pi}\right)^3 \log \frac{Q^2}{q^2}$$

Hard contribution

Difference hard contribution threshold and q_T.

Can be computed based on eikonal integral

$$\Delta\mathcal{H}^{(1)} = A^{(1)}\left[2\log^2\bar{N} + \mathrm{Li}_2\left(-\frac{\bar{b}^2}{\bar{N}^2}\right) + \zeta_2 + 2\log^2\chi - 4\log\chi\log\bar{N}\right]$$

$$= A^{(1)}\left[\zeta_2 + \mathrm{Li}_2\left(-\frac{\bar{b}^2}{\bar{N}^2}\right) + 2\log^2\left(\chi/\bar{N}\right)\right] \simeq A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]$$

Changes C_{qq} affects $\tilde{B}$

$$\tilde{B}^{(2)} \to \tilde{B}^{(2)} - 2\beta_0\Delta\mathcal{H}^{(1)}$$

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Overview NNLL

$$\mathcal{H}^{F,\,\mathrm{NNLL}} = \frac{\alpha_s}{\pi}\left\{\mathcal{H}^{F,(1)} + A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\right\} + \left(\frac{\alpha_s}{\pi}\right)^2\left\{\mathcal{H}^{F,(2)} + \tilde{D}^{(2)}\log\bar{N}\right\}$$

$$\begin{aligned}
E^{\mathrm{NNLL}}_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_Q^2) = &-\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{Q^2}{q^2} + \tilde{B}_N(\alpha_s(q))\right] \\
&+\int_{\mu_F^2}^{Q^2}\frac{dq^2}{q^2}2\gamma(N,\alpha_s(q)) - 2\log\bar{N}\int_{\mu_F^2}^{Q^2/\chi^2}\frac{dk_T^2}{k_T^2}A_a(\alpha_s(k_T)) \\
&-\frac{1}{2}\int_{Q^2/\bar{N}^2}^{Q^2}\frac{dq^2}{q^2}\tilde{D}_a(\alpha_s(q)) + 2\beta_0 A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(\frac{\alpha_s(q)}{\pi}\right)^2 \\
&-\int_{Q^2(\bar{b}^2+1)/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(A^{(3)}_{Q_T} - A^{(3)}_{\mathrm{thr}}\right)\left(\frac{\alpha_s(q)}{\pi}\right)^3\log\frac{Q^2}{q^2}
\end{aligned}$$

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Overview NNLL

$$\mathcal{H}^{F,\,\mathrm{NNLL}} = \frac{\alpha_s}{\pi}\left\{\mathcal{H}^{F,(1)} + A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\right\} + \left(\frac{\alpha_s}{\pi}\right)^2\left\{\mathcal{H}^{F,(2)} + \tilde{D}^{(2)}\log\bar{N}\right\}$$

$$\begin{aligned}
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&+\int_{\mu_F^2}^{Q^2}\frac{dq^2}{q^2}2\gamma(N,\alpha_s(q)) - 2\log\bar{N}\int_{\mu_F^2}^{Q^2/\chi^2}\frac{dk_T^2}{k_T^2}A_a(\alpha_s(k_T)) \\
&-\frac{1}{2}\int_{Q^2/\bar{N}^2}^{Q^2}\frac{dq^2}{q^2}\tilde{D}_a(\alpha_s(q)) + 2\beta_0 A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(\frac{\alpha_s(q)}{\pi}\right)^2 \\
&-\int_{Q^2(\bar{b}^2+1)/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(A^{(3)}_{Q_T} - A^{(3)}_{\mathrm{thr}}\right)\left(\frac{\alpha_s(q)}{\pi}\right)^3\log\frac{Q^2}{q^2}
\end{aligned}$$

Change Hard contribution

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Overview NNLL

$$\mathcal{H}^{F,\,\mathrm{NNLL}} = \frac{\alpha_s}{\pi}\left\{\mathcal{H}^{F,(1)} + A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\right\} + \left(\frac{\alpha_s}{\pi}\right)^2\left\{\mathcal{H}^{F,(2)} + \tilde{D}^{(2)}\log\bar{N}\right\}$$

$$\begin{aligned}
E^{\mathrm{NNLL}}_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_Q^2) = & -\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{Q^2}{q^2} + \tilde{B}_N(\alpha_s(q))\right] \\
& + \int_{\mu_F^2}^{Q^2}\frac{dq^2}{q^2}2\gamma(N,\alpha_s(q)) - 2\log\bar{N}\int_{\mu_F^2}^{Q^2/\chi^2}\frac{dk_T^2}{k_T^2}A_a(\alpha_s(k_T)) \\
& - \frac{1}{2}\int_{Q^2/\bar{N}^2}^{Q^2}\frac{dq^2}{q^2}\tilde{D}_a(\alpha_s(q)) + 2\beta_0 A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(\frac{\alpha_s(q)}{\pi}\right)^2 \\
& - \int_{Q^2(\bar{b}^2+1)/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(A^{(3)}_{Q_T} - A^{(3)}_{\mathrm{thr}}\right)\left(\frac{\alpha_s(q)}{\pi}\right)^3\log\frac{Q^2}{q^2}
\end{aligned}$$

Exponentiation soft wide-angle

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Overview NNLL

$$\mathcal{H}^{F,\,\mathrm{NNLL}} = \frac{\alpha_s}{\pi}\left\{\mathcal{H}^{F,(1)} + A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\right\} + \left(\frac{\alpha_s}{\pi}\right)^2\left\{\mathcal{H}^{F,(2)} + \tilde{D}^{(2)}\log\bar{N}\right\}$$

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E^{\mathrm{NNLL}}_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_Q^2) = &-\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{Q^2}{q^2} + \tilde{B}_N(\alpha_s(q))\right] \\
&+\int_{\mu_F^2}^{Q^2}\frac{dq^2}{q^2}2\gamma(N,\alpha_s(q)) - 2\log\bar{N}\int_{\mu_F^2}^{Q^2/\chi^2}\frac{dk_T^2}{k_T^2}A_a(\alpha_s(k_T)) \\
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&-\int_{Q^2(\bar{b}^2+1)/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(A^{(3)}_{Q_T} - A^{(3)}_{\mathrm{thr}}\right)\left(\frac{\alpha_s(q)}{\pi}\right)^3\log\frac{Q^2}{q^2}
\end{aligned}$$

NLO DGLAP

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Overview NNLL

$$\mathcal{H}^{F,\ \mathrm{NNLL}} = \frac{\alpha_s}{\pi}\left\{\mathcal{H}^{F,(1)} + A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\right\} + \left(\frac{\alpha_s}{\pi}\right)^2\left\{\mathcal{H}^{F,(2)} + \tilde{D}^{(2)}\log\bar{N}\right\}$$

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E^{\mathrm{NNLL}}_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_Q^2) = &-\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{Q^2}{q^2} + \tilde{B}_N(\alpha_s(q))\right] \\
&+\int_{\mu_F^2}^{Q^2}\frac{dq^2}{q^2}2\gamma(N,\alpha_s(q)) - 2\log\bar{N}\int_{\mu_F^2}^{Q^2/\chi^2}\frac{dk_T^2}{k_T^2}A_a(\alpha_s(k_T)) \\
&-\frac{1}{2}\int_{Q^2/\bar{N}^2}^{Q^2}\frac{dq^2}{q^2}\tilde{D}_a(\alpha_s(q)) + 2\beta_0 A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(\frac{\alpha_s(q)}{\pi}\right)^2 \\
&-\int_{Q^2(\bar{b}^2+1)/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(A^{(3)}_{Q_T} - A^{(3)}_{\mathrm{thr}}\right)\left(\frac{\alpha_s(q)}{\pi}\right)^3\log\frac{Q^2}{q^2}
\end{aligned}$$

Only N^{3}LL solution of integral

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Overview NNLL

$$\mathcal{H}^{F,\,\mathrm{NNLL}} = \frac{\alpha_s}{\pi}\left\{\mathcal{H}^{F,(1)} + A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\right\} + \left(\frac{\alpha_s}{\pi}\right)^2\left\{\mathcal{H}^{F,(2)} + \tilde{D}^{(2)}\log\bar{N}\right\}$$

$$\begin{aligned} E^{\mathrm{NNLL}}_{\{I\}}(\alpha_s(\mu_R), b, N, Q^2/\mu_R^2, Q^2/\mu_Q^2) = &-\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{Q^2}{q^2} + \tilde{B}_N(\alpha_s(q))\right] \\ &+\int_{\mu_F^2}^{Q^2}\frac{dq^2}{q^2}2\gamma(N,\alpha_s(q)) - 2\log\bar{N}\int_{\mu_F^2}^{Q^2/\chi^2}\frac{dk_T^2}{k_T^2}A_a(\alpha_s(k_T)) \\ &-\frac{1}{2}\int_{Q^2/\bar{N}^2}^{Q^2}\frac{dq^2}{q^2}\tilde{D}_a(\alpha_s(q)) + 2\beta_0 A^{(1)}\left[\zeta_2 - \mathrm{Li}_2\left(\frac{\bar{b}^2}{\chi^2}\right)\right]\int_{Q^2/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(\frac{\alpha_s(q)}{\pi}\right)^2 \\ &-\int_{Q^2(\bar{b}^2+1)/\chi^2}^{Q^2}\frac{dq^2}{q^2}\left(A^{(3)}_{Q_T} - A^{(3)}_{\mathrm{thr}}\right)\left(\frac{\alpha_s(q)}{\pi}\right)^3\log\frac{Q^2}{q^2} \end{aligned}$$

Collinear anomaly

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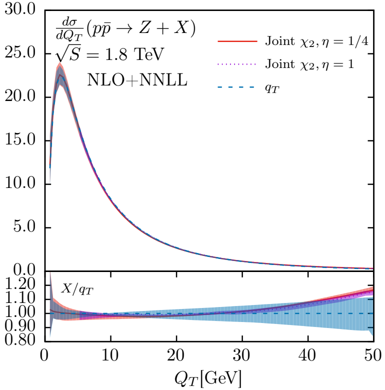
Results

[Ferrera, Marzani, VT, in preperation]

PDFs used: MSTW2008NNLO



preliminary

Conclusions

- Application of joint threshold, q_T extended to NNLL
- Agreement for low q_T
- Lower scale uncertainty mid to high q_T

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Conclusions

- Application of joint threshold, q_T extended to NNLL
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Thank you for your attention

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