Precision Calculations for Coloured Supersymmetric Particle Production at the Large Hadron Collider

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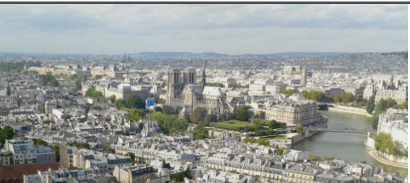
with Wim Beenakker, Michael Krämer, Anna Kulesza, and Eric Laenen



PSR Workshop

Paris, 05 July 2016

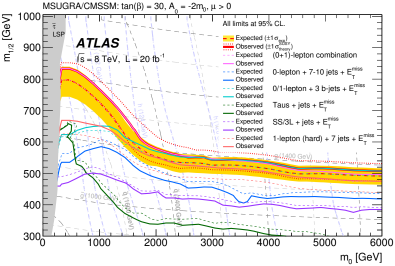
Outline

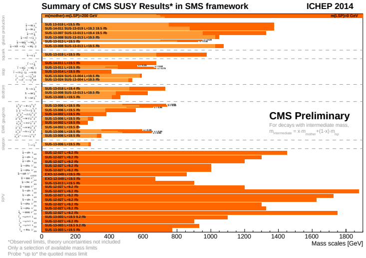
1. Motivation
2. Calculations
    - Soft-gluon resummation at NNLL
    - Coulomb resummation and bound states below threshold
3. Results
    - Impact of resummed PDFs on the predictions: NNPDF3.0 studies
    - Updated predictions for squark and gluino production: NNLL-fast
    - Comparison to SCET
4. Summary

Searches for supersymmetry

Main sparticle production processes:

$$pp \quad \to \quad \tilde{g}\tilde{g} \quad , \quad \tilde{q}\tilde{q}^* \quad , \quad \tilde{q}\tilde{g} \quad , \quad \tilde{q}\tilde{q} \quad , \quad \tilde{t}_1\tilde{t}_1^*$$





Searches for supersymmetry

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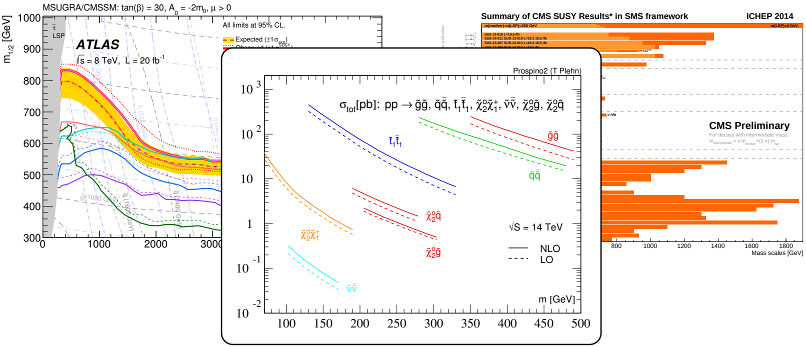

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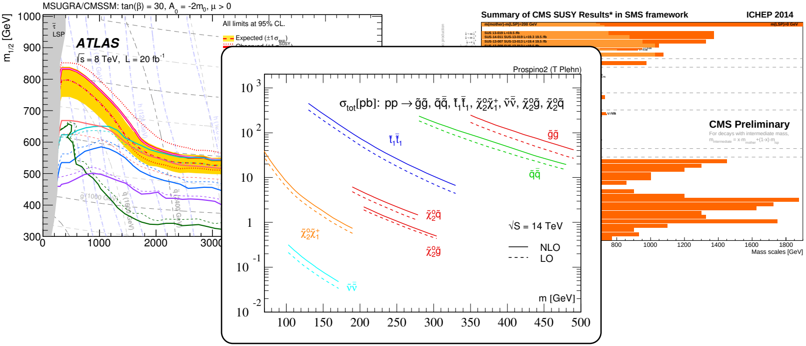


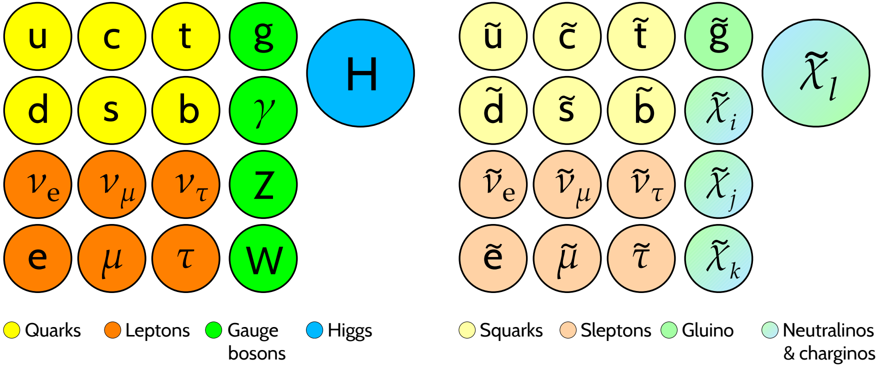
⇒ Cross sections needed at high precision for experimental searches

Supersymmetry?

Supersymmetry (SUSY) connects bosons to fermions and vice versa

→ New particles that "only" differ in their spin quantum number

Standard Model particles **Supersymmetric partners**



("Reality": broken symmetry, heavy SUSY particles, R-parity, ...)

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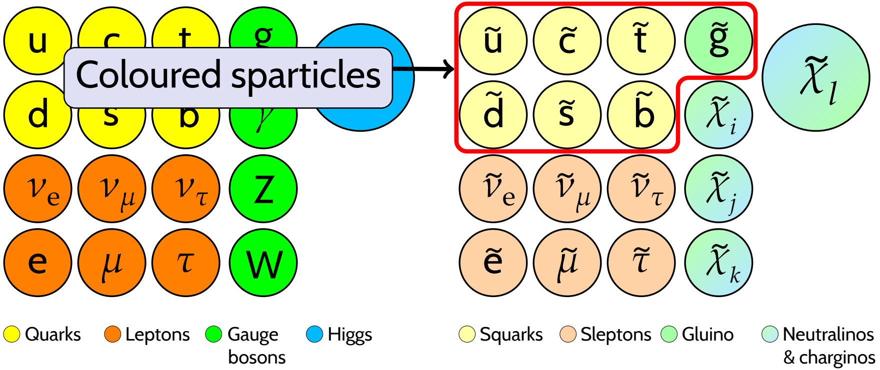
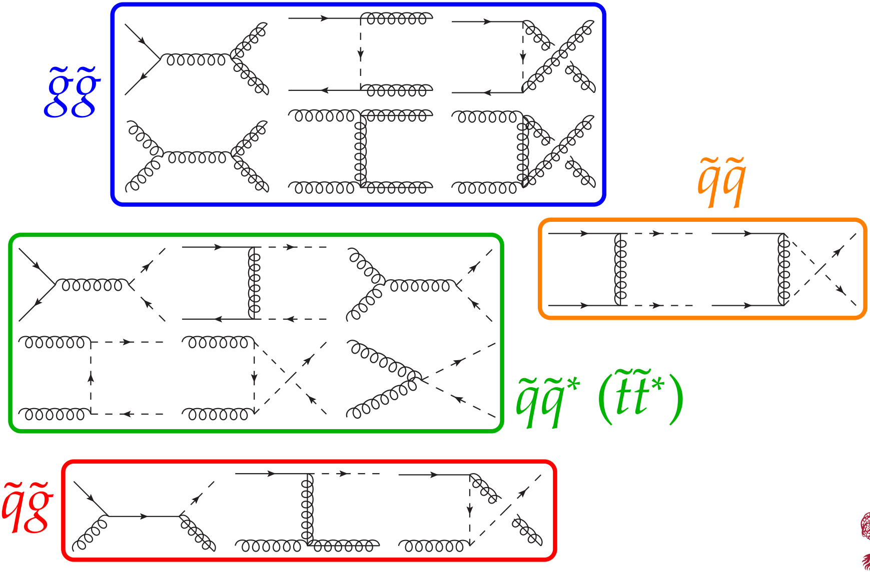


(“Reality”: broken symmetry, heavy SUSY particles, R-parity, ...)

LO production of squarks and gluinos



Particle production close to threshold

Heavy SUSY particles ⇒ production in the **threshold limit** $\sqrt{\hat{s}} \to 2m$:

$$\beta = \sqrt{1-\hat{\rho}} := \sqrt{1 - \frac{4m^2}{\hat{s}}} \to 0$$

with $\sqrt{\hat{s}}$: partonic centre-of-mass energy, m: average mass of final state particles

⇒ Just enough energy to produce the two sparticles

⇒ **Real radiation processes are soft**

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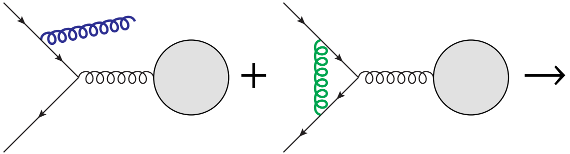
Remainder after cancellation of IR divergencies:



$$\sim \alpha_s^n \ln^m \beta^2, \quad m \leq 2n$$

Soft & collinear gluons

Particle production close to threshold

Heavy SUSY particles $\Rightarrow$ production in the **threshold limit** $\sqrt{\hat{s}} \to 2m$:

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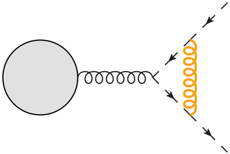
- $\Rightarrow$ Just enough energy to produce the two sparticles
- $\Rightarrow$ **Real radiation processes are soft**

Additionally:



$\longrightarrow$ $\sim \alpha_s^n/\beta^n$

Coulomb gluons

Particle production close to threshold

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⇒ **Real radiation processes are soft**

Enhanced partonic cross sections close to threshold:

- Soft & collinear gluons: $\alpha_s^n \ln^m \beta^2 \sim 1$
- Coulomb gluons: $\alpha_s^n/\beta^n \sim 1$

⇒ Endangering the perturbative series

⇒ Systematic treatment of these terms required

Treating large logarithms

Threshold logarithms in Mellin-moment space (threshold limit: $\beta \to 0 \mathrel{\hat=} N \to \infty$):

$$\ln \beta^2 \xrightarrow{\text{Mellin}} \ln N =: L$$

(neglect subleading terms $\mathcal{O}(1/N)$)

Reordering of the perturbative series in α_s **and** L

- Resummation via renormalisation group equations

Exponential form (g_1, g_2, g_3 known):

$$\tilde{\sigma} \sim \tilde{\sigma}^{(0)} \times C(N, \alpha_s) \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots\right]$$

[Kodaira, Trentadue '82][Sterman '87][Catani, D'Emilio, Trentadue '88][Catani, Trentadue '89][Kidonakis, Sterman '96][Kidonakis, Oderda, Sterman '98][Contopanagos, Laenen, Sterman '96][Catani, de Florian, Grazzini '01][Moch, Vermaseren, Vogt '04][Beneke, Falgari, Schwinn '09][Czakon, Mitov, Sterman '09][Ferroglia, Neubert, Pecjak, Yang '09] ...

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Higher order terms of different origin; **split-up close to threshold**:

[Beneke, Falgari, Schwinn '09-10]

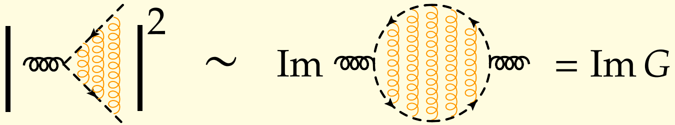
- $\mathcal{C}^{\text{Hard}}(\alpha_s)$: **hard matching coefficients** (independent of N)

  [Beenakker, Janssen, Lepoeter, Krämer, Kulesza, Laenen, Niessen, Thewes, Van Daal '13][Broggio, Ferroglia, Neubert, Vernazza, Yang '13]

- $\mathcal{C}^{\text{Coul}}(N, \alpha_s)$: **Coulomb terms** (final state gluon exchange)

  [Kulesza, Motyka '09][Beneke, Falgari, Schwinn '10][Falgari, Schwinn, Wever '12]
  [Fadin, Khoze '87][Peskin, Strassler '91][Hagiwara, Yokoya '09][Kauth, Kühn, Marquard, Steinhauser '09-11][Kauth, Kress, Kühn '11]

Coulomb Green's function



Calculation of ladder diagrams leads to **non-relativistic Schrödinger equation** *[Peskin, Strassler '91]*:

$$\left\{\left[\frac{(-i\nabla)^2}{2m_{\mathrm{red}}} + V_C(\vec{r})\right] - (E + i\Gamma)\right\} G(\vec{r}, E + i\Gamma) = \delta^{(3)}(\vec{r})$$

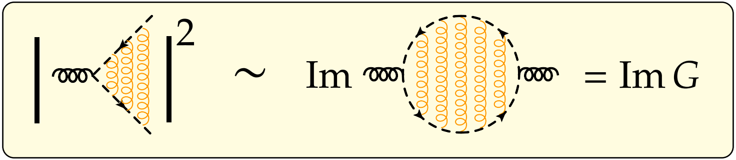
with $E = \sqrt{\hat{s}} - 2m$: energy and Γ: average decay width of the final state particles, m_{red}: reduced mass,

and the **Coulomb potential**:

$$V_C(\vec{r}) = -\mathscr{D}_{R_\alpha} \frac{\alpha_s}{|\vec{r}|} + \mathcal{O}(\alpha_s^2)$$

with $\mathscr{D}_{R_\alpha}$: colour factor related to Casimir invariants

Coulomb Green's function



The solution at origin is *[Beneke, Signer, Smirnov '99][Pineda, Signer '06]*:

$$G(\vec{0}, E + i\Gamma) = i\frac{m_{\text{red}}^2}{\pi} v\,\Delta_{\text{nC}} + \mathscr{D}_{R_\alpha}\frac{\alpha_s m_{\text{red}}^2}{\pi}\left[g_{\text{LO}}\,\Delta_{\text{nC}} + \frac{\alpha_s}{4\pi} g_{\text{NLO}} + \ldots\right]$$

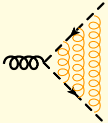
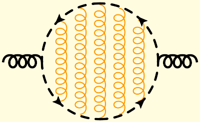
with g_{LO} (g_{NLO}) contributions from LO (NLO) Coulomb potential ($g_{\text{LO}} \sim$ Sommerfeld factor)
and Δ_{nC}: spin-dependent terms of non-Coulombic origin *[Beneke, Czakon, Falgari, Mitov, Schwinn '11]*

Incorporate into resummation framework:

$$\hat{\sigma}^{\text{Coul},res} = \hat{\sigma}^{\text{LO}} \times \frac{\text{Im}\, G(\vec{0}, E + i\Gamma)}{\text{Im}\, G^{\text{free}}(\vec{0}, E + i\Gamma)}$$

with the velocity $v = \sqrt{\frac{E + i\Gamma}{2m_{\text{red}}}} \approx \sqrt{\frac{m}{2m_{\text{red}}}}\beta$

Coulomb Green's function

$\Big|$  $\Big|^2 \sim \mathrm{Im}$  $= \mathrm{Im}\, G$

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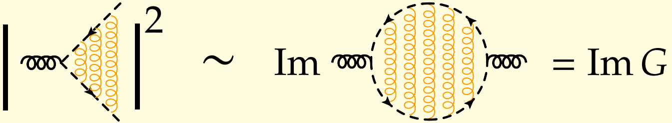
Relevant scale for Coulomb effects *[Beneke, Falgari, Schwinn '10]*:

$$\mu_C = \max\left\{\mu_B, 2\sqrt{2m\, m_{\mathrm{red}}\beta}\right\}$$

with μ_B the Bohr scale, definition see next slide!

Coulomb Green's function: bound states



$G(\vec{0}, E + i\Gamma)$ develops poles **below** threshold → bound states for attractive Coulomb potential ($\mathscr{D}_{R_\alpha} > 0$):

$$\operatorname{Im} G(\vec{0}, E + i\Gamma) = \operatorname{Im} \sum_n \frac{|\psi(0)|^2}{E_n - (E + i\Gamma)} \rightarrow \sum_n |\psi(0)|^2 \pi \delta\left(E - E_n\right)$$

with $\psi(0)$ the wave function for the bound-state system at origin and E_n the bound-state energies

Relevant scale for bound-state effects: **Bohr scale**

$$\mu_B = 2 m_{\text{red}}\, \mathscr{D}_{R_\alpha}\, \alpha_s\left(\mu_B\right)$$

with the equation being solved iteratively

Combining and matching

Fixed-order part at $\text{NNLO}_{\text{Approx}}$:

PROSPINO

$$\sigma^{\text{NNLO}_{\text{Approx}}} = \sigma^{\text{NLO}} + \Delta\sigma^{\text{NNLO}_{\text{Approx}}}$$

($\Delta\text{NNLO}_{\text{Approx}}$: dominant terms in β for $\beta \to 0$ *[Beneke, Czakon, Falgari, Mitov, Schwinn '09]*)

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Combining soft and Coulomb gluon resummation:

$$\tilde{\sigma}^{\text{soft+Coul,res}} \sim \mathcal{C}^{\text{hard}} \times \Delta\Delta\Delta \times \int_0^1 \mathrm{d}x\, x^{N-1}\, \hat{\sigma}^{\text{Coul,res}}$$

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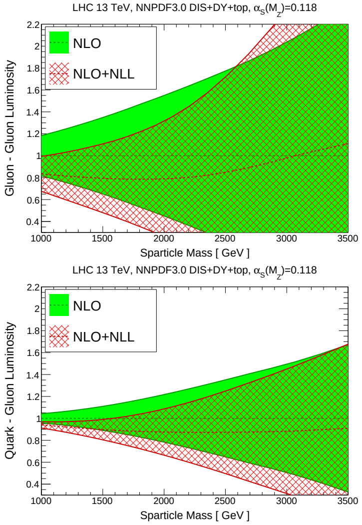
Matching to fixed order:

$$\sigma^{\mathrm{matched,res}} \sim \sigma^{\mathrm{NNLO_{Approx}}} + \sigma^{\mathrm{BS}} + \int_{C_{\mathrm{MP}}} \mathrm{d}N \left[\tilde{\sigma}^{\mathrm{soft+Coul,res}}(N) - \tilde{\sigma}^{\mathrm{soft+Coul,res}}(N)\Big|_{\mathrm{NNLO}} \right]$$

Threshold-improved NNPDFs: luminosities

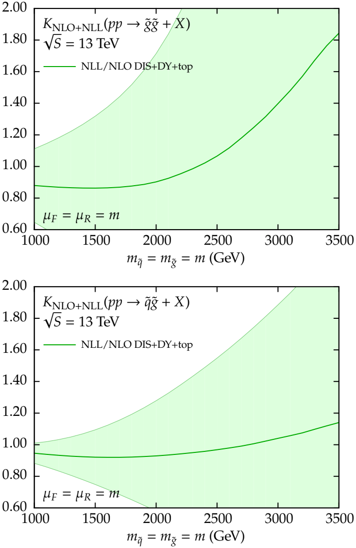
[Bonvini, Marzani, Rojo, Rottoli, Ubiali, Ball, Bertone, Carrazza, Hartland '15]



DIS+DY+top fits (reduced data sets compared to "global fit" NNPDF3.0 analysis)

NLO+NLL with threshold-improved NNPDFs (1)

[Beenakker, CB, Krämer, Kulesza, Laenen, Marzani, Rojo, Ubiali; arXiv: 1510.00375]



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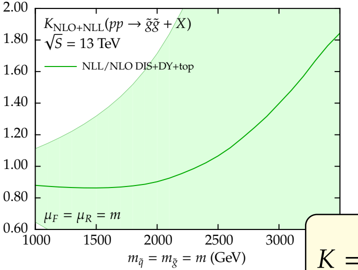
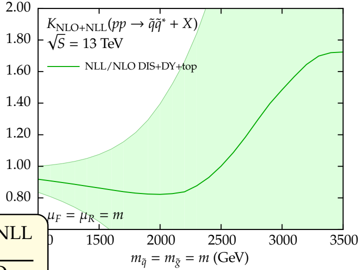
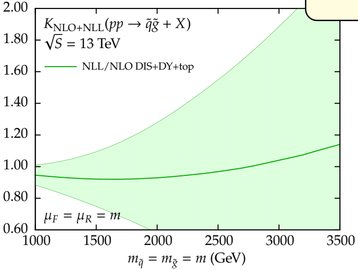
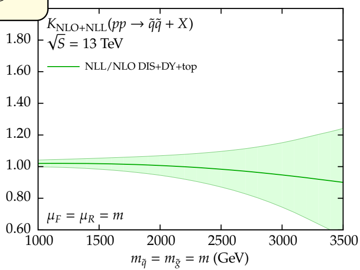




$$K = \frac{\sigma^{\mathrm{NLO+NLL}}}{\sigma^{\mathrm{NLO}}}$$





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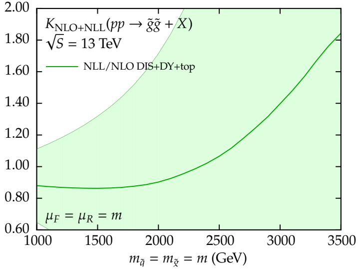

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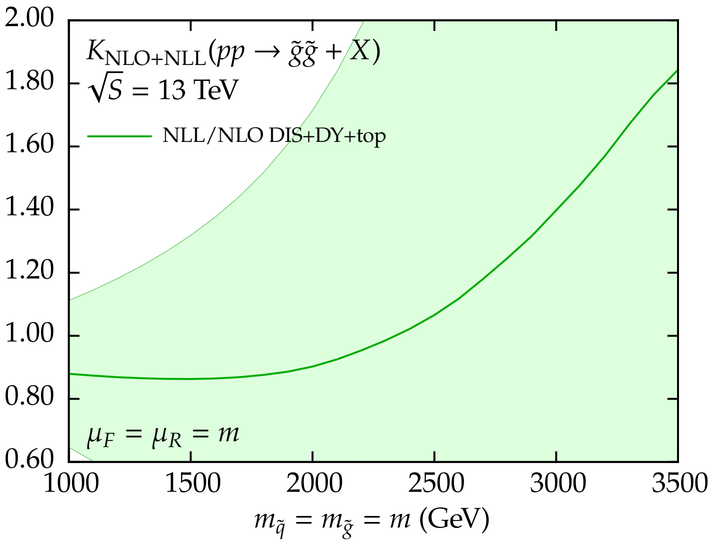

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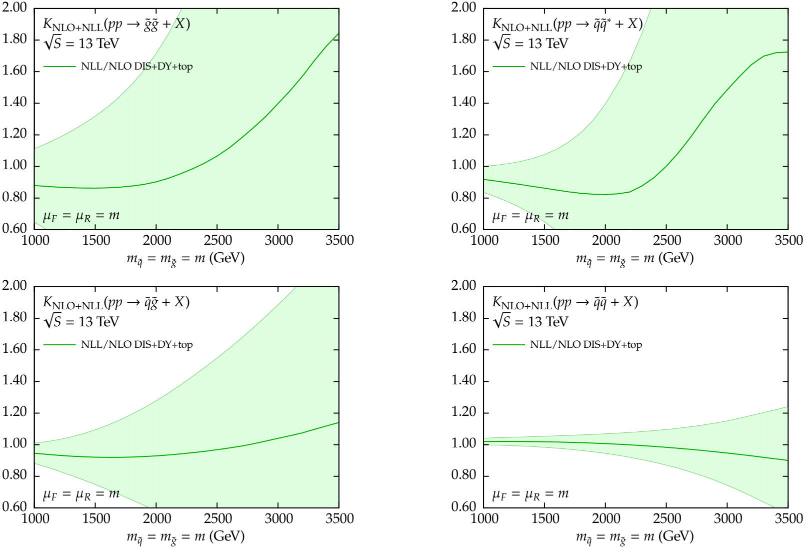
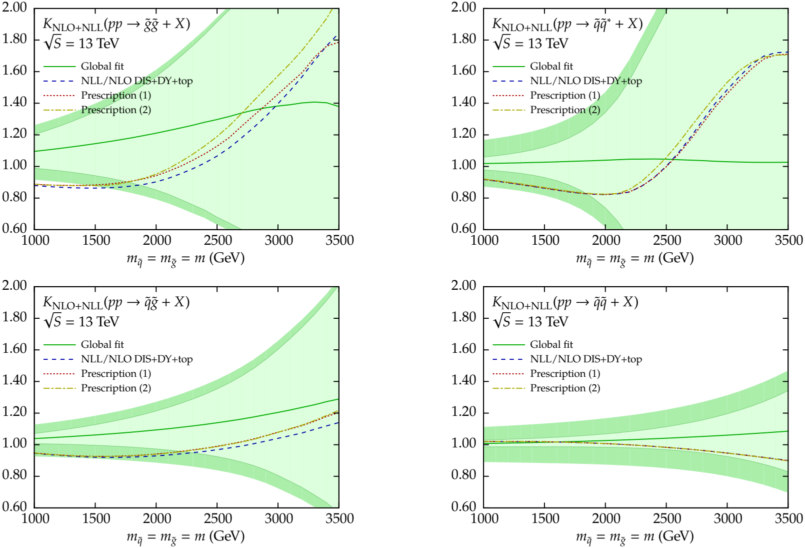


NLO+NLL with threshold-improved NNPDFs (2)

[Beenakker, CB, Krämer, Kulesza, Laenen, Marzani, Rojo, Ubiali; arXiv: 1510.00375]



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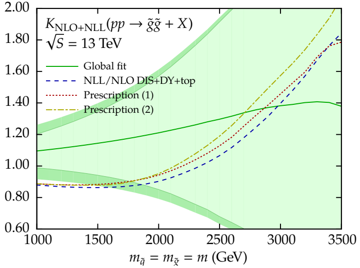

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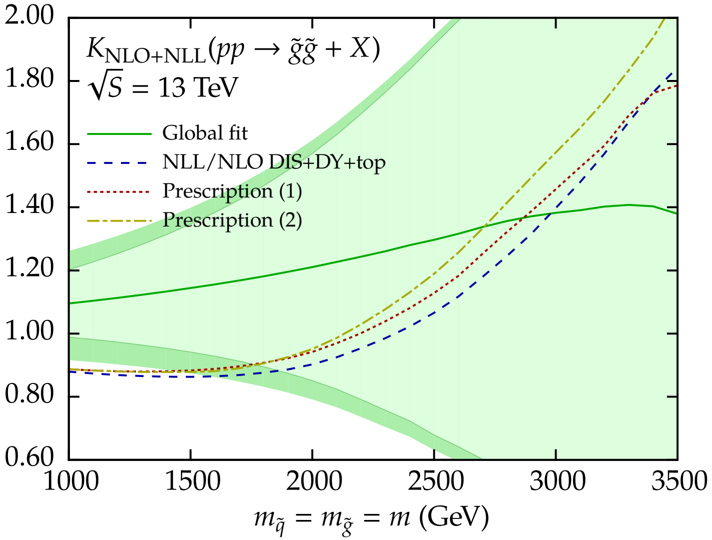

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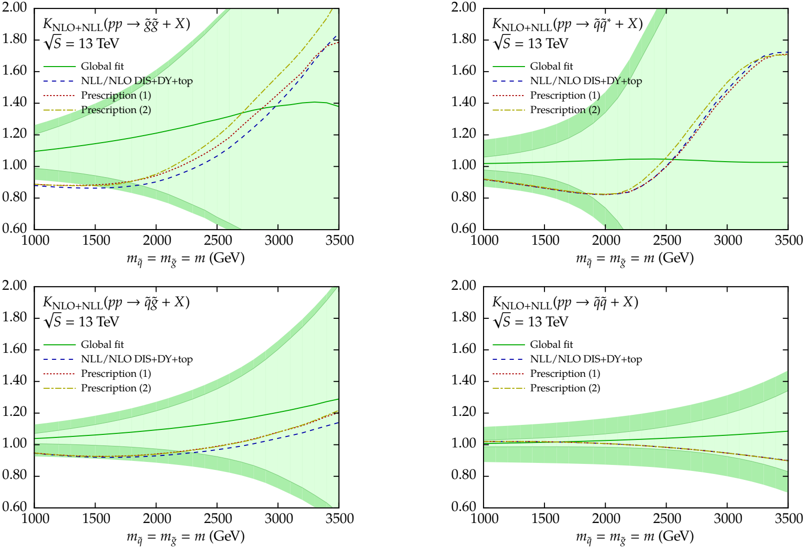
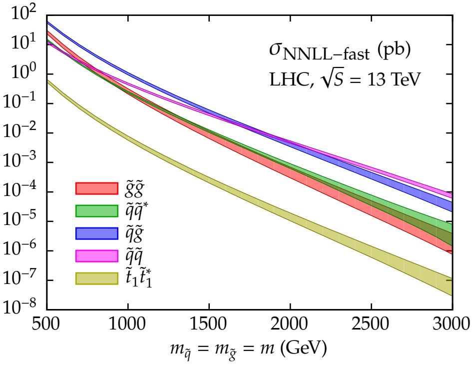


Numerical package: NNLL-fast

Code package to compute **NNLO$_{\text{Approx}}$+NNLL** cross sections including **Coulomb resummation** and **bound states**

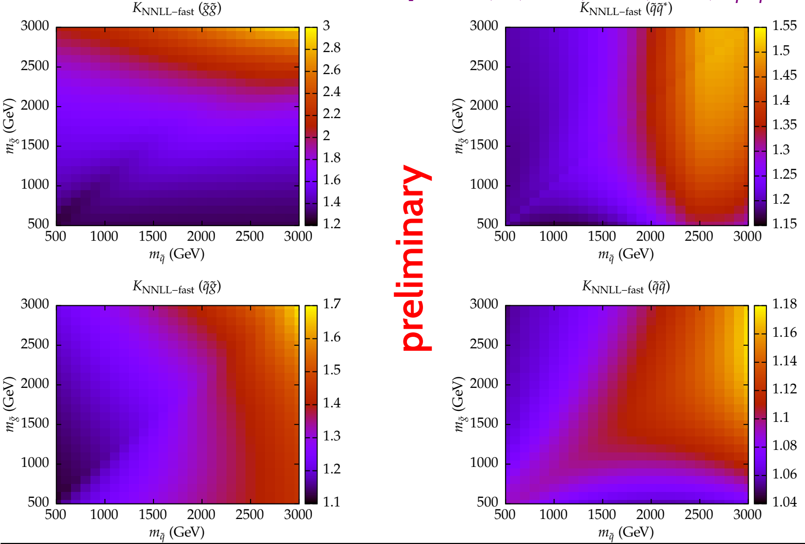
- $\tilde{g}\tilde{g}$, $\tilde{q}\tilde{q}^*$, $\tilde{q}\tilde{g}$, $\tilde{q}\tilde{q}$, $\tilde{t}\tilde{t}^*$ and **decoupling limit for** $\tilde{g}$ **and** $\tilde{q}$
- Including α_s, PDF, and scale variation
- At the current LHC Run II energy: $\sqrt{S} = \mathbf{13\ TeV}$, or upon request
- Notable increase of cross sections compared to NLL-fast



Available soon! *Stay tuned!*

NNLL-fast: 2D squark and gluino mass grids

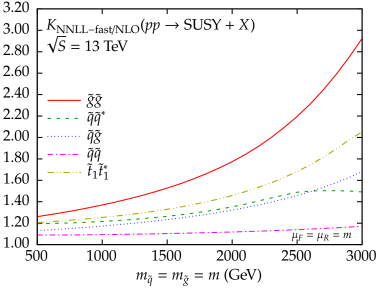
[Beenakker, CB, Krämer, Kulesza, Laenen; in preparation]



PDF4LHC15_MC sets; K-factor wrt. NLO

NNLL-fast: equal-mass case

[Beenakker, CB, Krämer, Kulesza, Laenen; in preparation]



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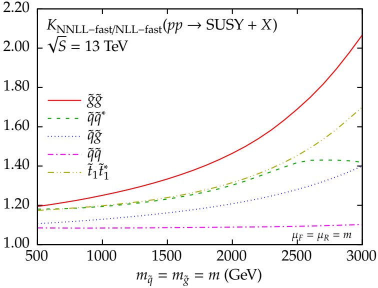


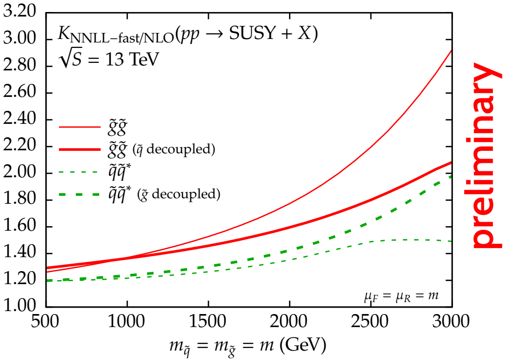
$$K = \frac{\sigma^{\text{NNLL-fast}}}{\sigma^{\text{NLO}}}$$

$$K' = \frac{\sigma^{\text{NNLL-fast}}}{\sigma^{\text{NLL-fast}}}$$

- Strongest relative enhancement for $\tilde{g}\tilde{g}$
- Shape of K' in principle following K; stronger relative enhancement for $\tilde{q}\tilde{q}^*$ than for $\tilde{q}\tilde{g}$
- Kink for high-mass $\tilde{q}\tilde{q}^*$: effect of setting negative PDF replicas to zero

NNLL-fast: decoupling scenarios

[Beenakker, CB, Krämer, Kulesza, Laenen; in preparation]



Decoupling scenarios:

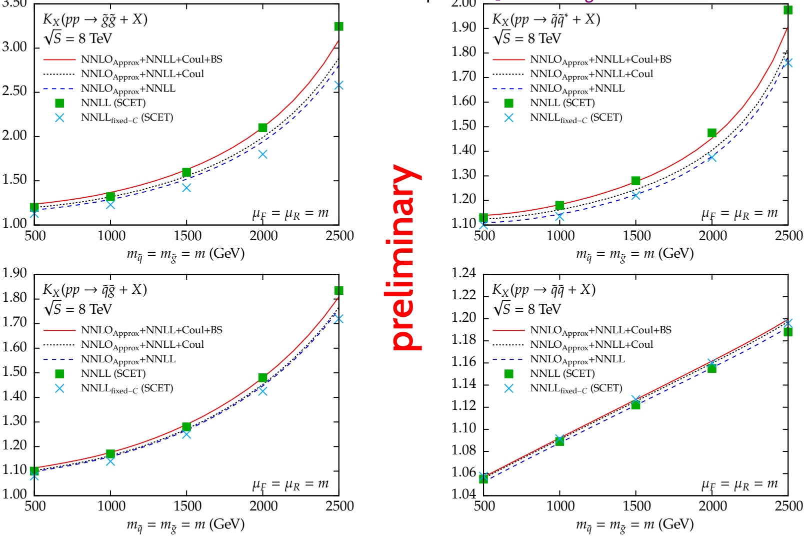
- gluinos with decoupled (= very heavy) squarks
- squarks with decoupled gluinos

At high masses, different behaviour from non-decoupling scenarios

- Squarks with decoupled $\tilde{g}$ very similar to $\tilde{t}_1\tilde{t}_1^*$ K-factor, smaller effect from negative replicas

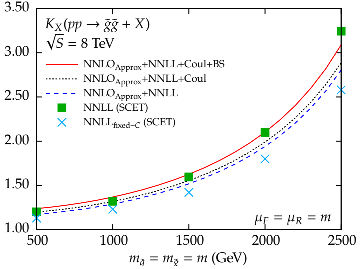
Comparison to SCET: *K*-factor

compared to *[Beneke, Falgari, Piclum, Schwinn, Wever '14]*

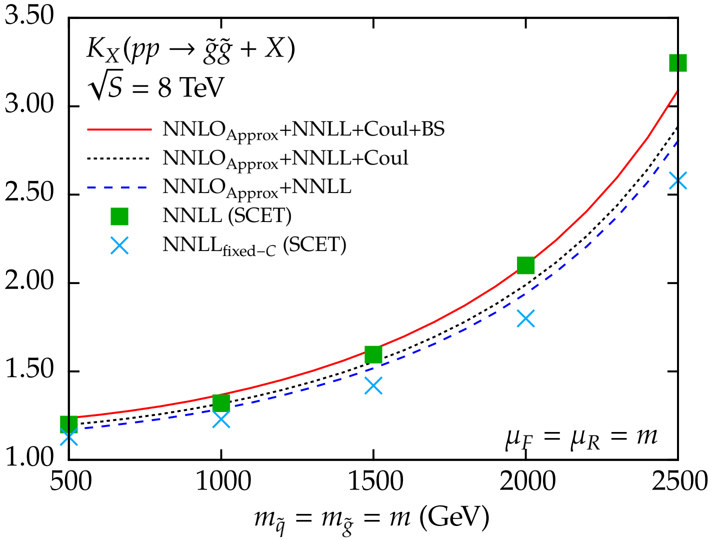


Using MSTW 2008 PDFs; improved agreement due to Coulomb resummation and BS

Comparison to SCET: K-factor



Comparison to SCET: K-factor



Comparison to SCET: *K*-factor

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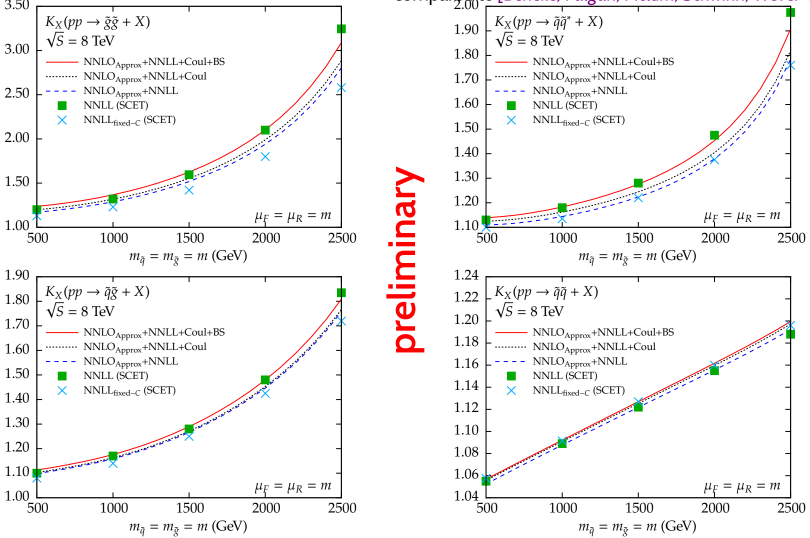
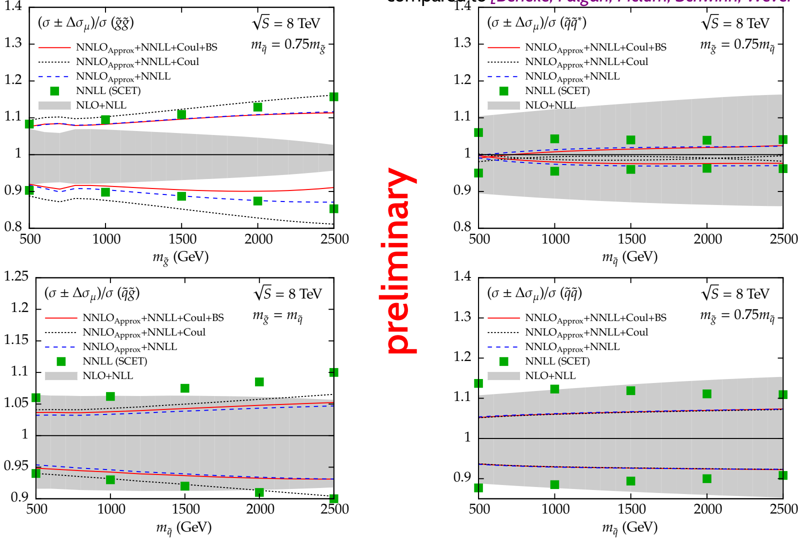


Using MSTW 2008 PDFs; improved agreement due to Coulomb resummation and BS

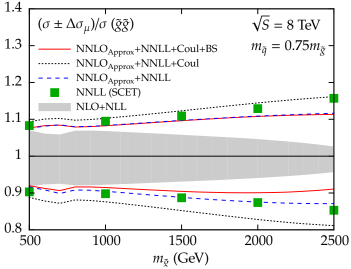
Comparison to SCET: scale uncertainty

compared to *[Beneke, Falgari, Piclum, Schwinn, Wever '14]*

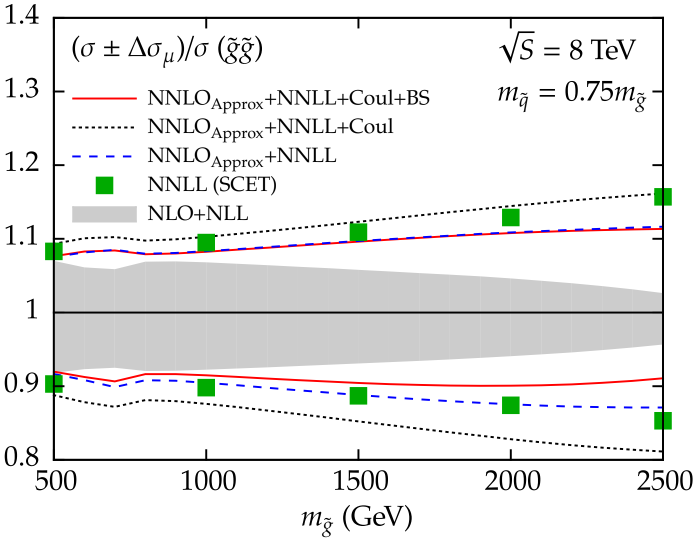


For almost all processes, scale dependence within NLO+NLL band

Comparison to SCET: scale uncertainty

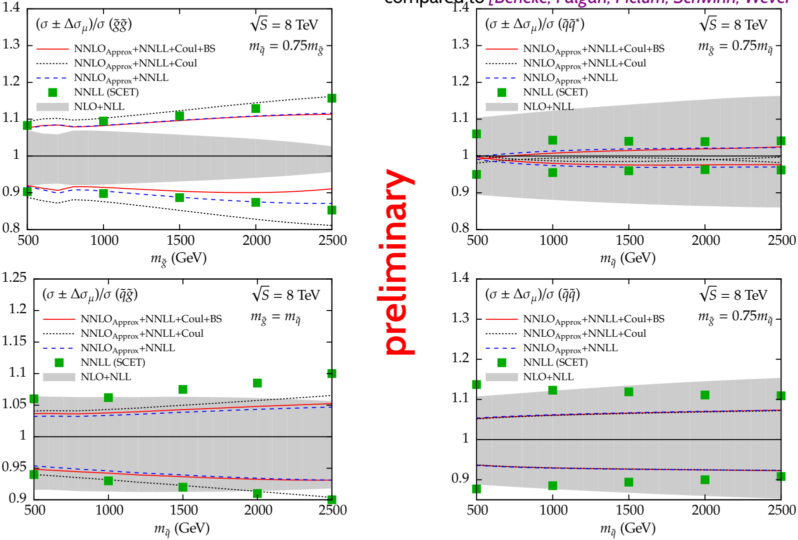


Comparison to SCET: scale uncertainty



Comparison to SCET: scale uncertainty

compared to *[Beneke, Falgari, Piclum, Schwinn, Wever '14]*



For almost all processes, scale dependence within NLO+NLL band

Summary

✓ Threshold-improved PDFs change both the **quantitative** and **qualitative** behaviour of the cross sections

- In the future, relevant for phenomenological studies

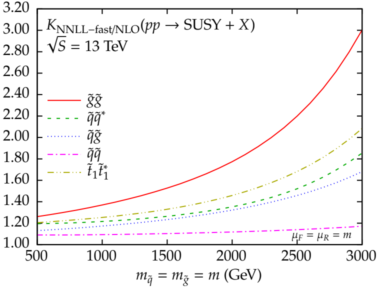
✓ Combined resummation of **soft** and **Coulomb gluons** **in Mellin-moment space at NNLL**, including bound states

- For all processes of squark and gluino production
- Useful for many phenomenological studies:
  - ★ Production of stops treated separately from $\tilde{q}\tilde{q}^*$ due to strong mixing
  - ★ Including special scenarios where only $\tilde{g}$ or $\tilde{q}$ are within experimental reach
- Available for squark and gluino masses from 0.5 to 3 TeV
- Results compatible with SCET

Outlook:

✍ Public code with PDF4LHC15 in preparation: `NNLL-fast`

NNLL-fast: equal-mass case with negative replicas



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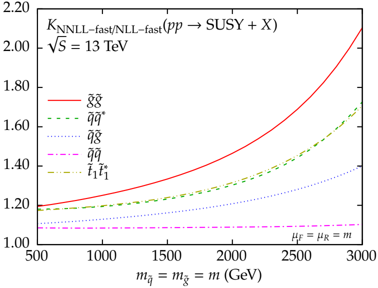
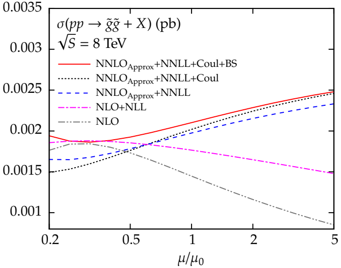


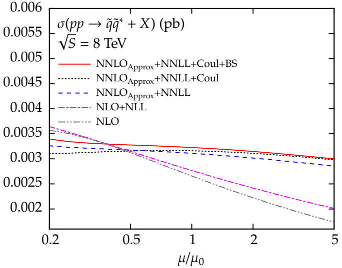
$$K = \frac{\sigma^{\mathrm{NNLL-fast}}}{\sigma^{\mathrm{NLO}}}$$

$$K' = \frac{\sigma^{\mathrm{NNLL-fast}}}{\sigma^{\mathrm{NLL-fast}}}$$

- Process mainly affected by negative replicas: $\tilde{q}\tilde{q}^*$
- Only at high masses where the PDF uncertainties are large
- $\tilde{q}\tilde{g}$ and $\tilde{q}\tilde{q}$ not affected

Scale dependence for squarks and gluinos





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