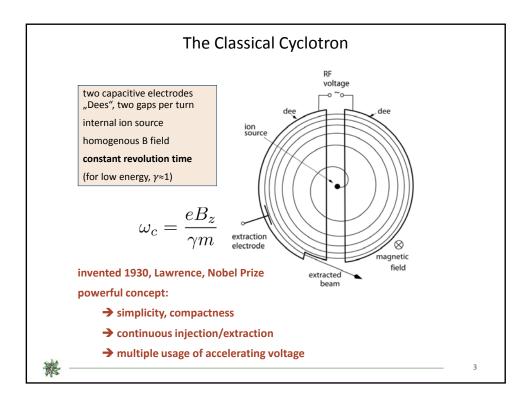


Outline

- Cyclotron Basics scaling and isochronicity, focusing, turn separation, classical cyclotrons and derived types
- Injection for Cyclotrons internal source, electrostatic inflectors, horizontal injection, optics matching, bunching
- Extraction for Cyclotrons
 electrostatic septum, stepwidth calculation, charge
 exchange extraction





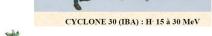
wide spectrum of cyclotrons ...

compact and cost optimized for series production e.g. medical nuclide production → Internal source, extraction or internal target



huge and complex for variable research purposes, e.g. R.I.B. production or high intensity → External source, injection





RIKEN s.c. Ring Cyclotron- "as big as a house"

cyclotron basics: isochronicity and scalings

continuous acceleration \rightarrow revolution time should stay constant, though E_k , R vary

magnetic rigidity:

$$BR = \frac{1}{e} p = \beta \gamma \frac{m_0 c}{e}$$

orbit radius from isochronicity:

$$R = \frac{c}{\omega_c}\beta = R_{\infty}\beta$$

deduced scaling of B:

$$R \propto \beta; BR \propto \beta \gamma \longrightarrow B(R) \propto \gamma(R)$$

thus, to keep the isochronous condition, B must be raised in proportion to $\gamma(R)$; this contradicts the focusing requirements!

field index
$$k$$
: $k = \frac{R}{B} \frac{dB}{dR}$

$$= \frac{\beta}{\gamma} \frac{d\gamma}{d\beta}$$

$$= \gamma^2 - 1$$



5

cyclotron basics: stepwidth (nonrelativistic, B const)

relation between energy and radius

$$qRB_z = \sqrt{2mE_k}$$

$$\frac{dR}{R} = \frac{1}{2}\frac{dE_k}{E_k}$$

use:

 $\Delta E_k = \text{const}; B_z = \text{const}; E_k \propto R^2$



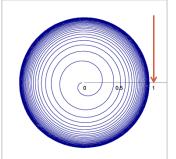
"cyclotron

language"

thus:

$$\Delta R \propto \frac{R}{E_k} \propto \frac{1}{R}$$

radius increment per turn decreases with increasing radius → extraction becomes more and more difficult at higher energies





focusing in a cyclotron

focusing: consider small deviations x from beam orbit R (r = R + x):

$$\ddot{x} + \frac{q}{m}vB_z(R+x) - \frac{v^2}{R+x} = 0, \qquad \text{using: } \omega_c = qB_z/m = v/R$$

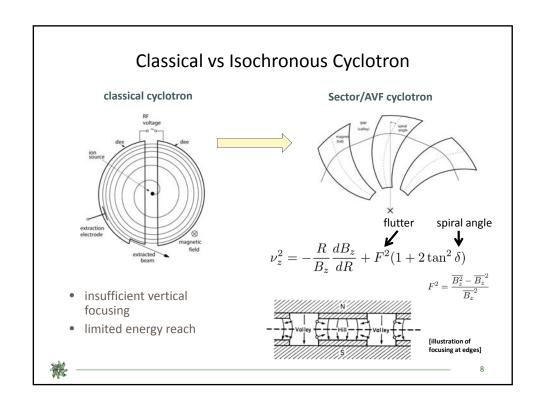
$$\ddot{x} + \omega_c^2(1+k)x = 0. \qquad k = \frac{R}{B}\frac{dB}{dR}$$

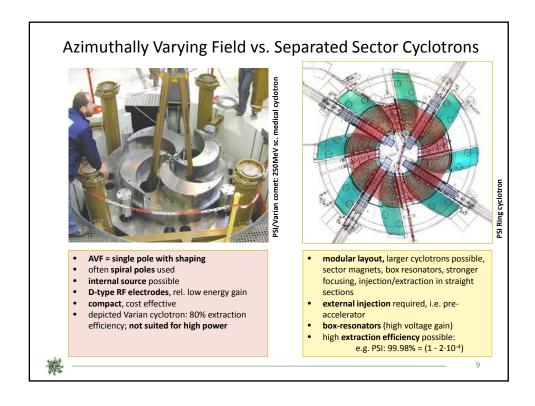
thus in radial $\omega_r = \omega_c \sqrt{1+k} = \omega_c \nu_r$ plane: $\nu_r = \sqrt{1+k} \quad \text{using isochronicity condition}$

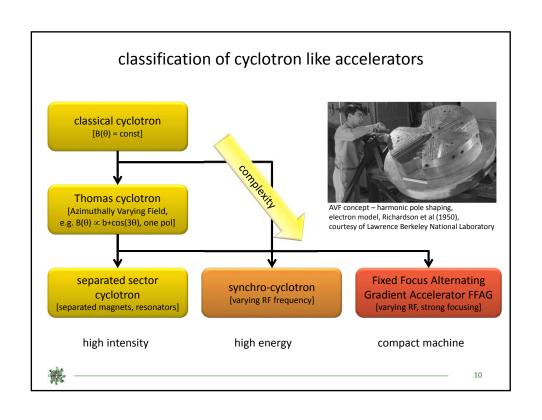
in vertical $\nu_z=\sqrt{-k}$ plane:

k<0 to obtain vertical focus.

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Injection – Overview

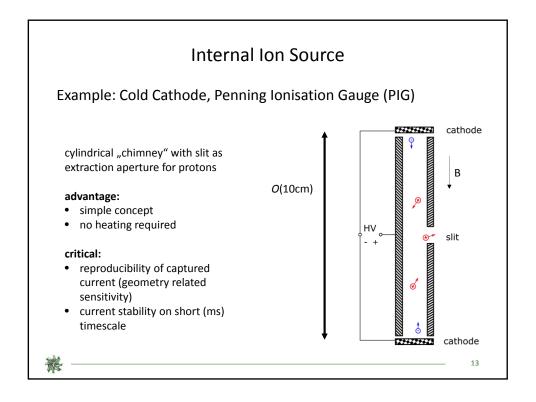
Injection Techniques

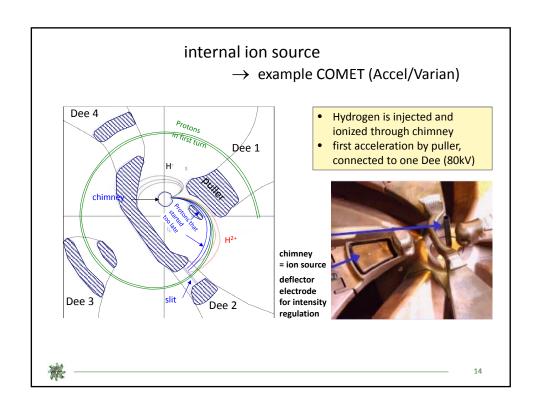
- internal source
- axial injection
 - mirrow inflector
 - spiral inflector
 - hyperbolic inflector
- radial injection
 - electrostatic septum
 - stripping injection

Aspects to be considered

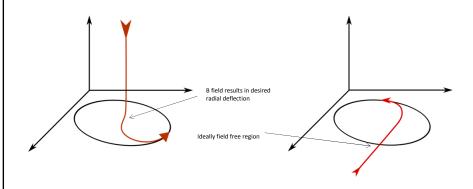
- overall central region design
- radial centering
- matching of beam optics
- vertical centering
- bunching / long. capture
- minimize overall losses for high intensity application







external source: axial vs. horizontal injection



axial: suited for compact cyclotron with field covering entire plane

horizontal: suited for sector cyclotron with gaps between magnets



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Beam Deflection by Electric Field

 $\Delta p_{\perp} = \int F_{\perp} dt = \int \frac{F_{\perp}}{\beta c} ds, F = qE$ momentum change:



resulting angle:

electric rigidity:

$$\theta = \frac{\Delta p_{\perp}}{p} = \frac{qlE}{\gamma \beta^2 E_0}$$

rest energy

field strength deflector

 $\rho = \frac{l}{\theta}$ bending radius:

kinetic energy

bending radius charge

 $E\rho = \frac{\gamma^2 - 1}{\gamma} \frac{E_0}{q} = \frac{\gamma + 1}{\gamma} \frac{E_k}{q}$

length

 $E\rho \approx 2U_{\rm acc}$ low energy at source:

acc.voltage (source)

Bending radius in B and E:

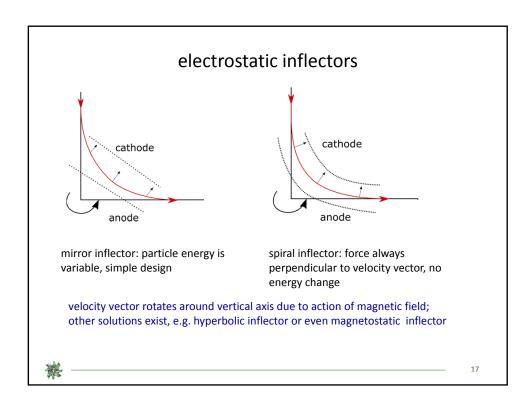
comparison electric and magnetic force on protons

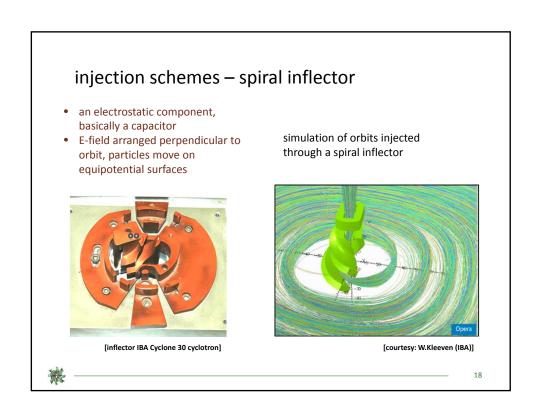
$$\vec{F_E} = e \cdot \vec{E}, \quad \vec{F_B} = e \cdot \vec{v} \times \vec{B}$$

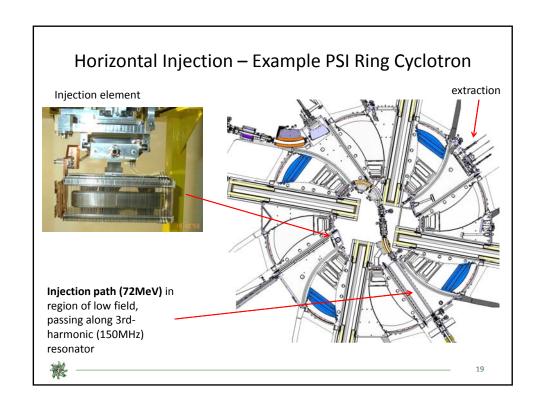
table: bending radius, varying E_k

E _k	B = 1T	E = 10MV/m
60 keV	35 mm	12 mm
1 MeV	140 mm	200 mm
1 GeV	5.6 m	150 m









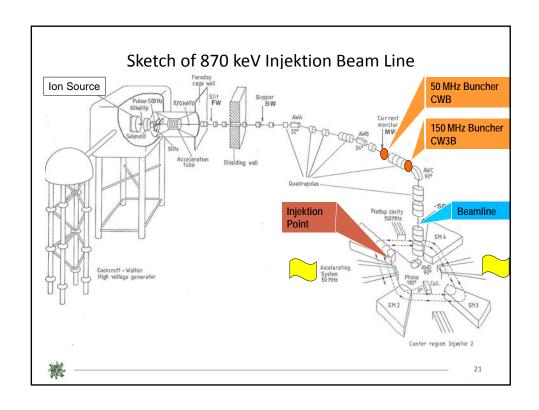
Bunching for Cyclotrons

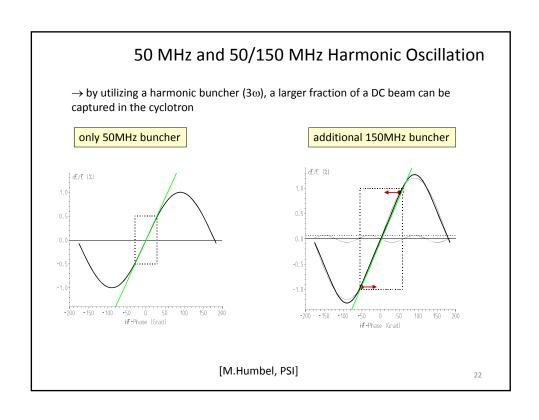
Ion sources deliver DC beam; for acceleration in an RF field the beam must be bunched; unbunched beam should be removed at low energy (\leq 5MeV) to avoid uncontrolled losses and activation

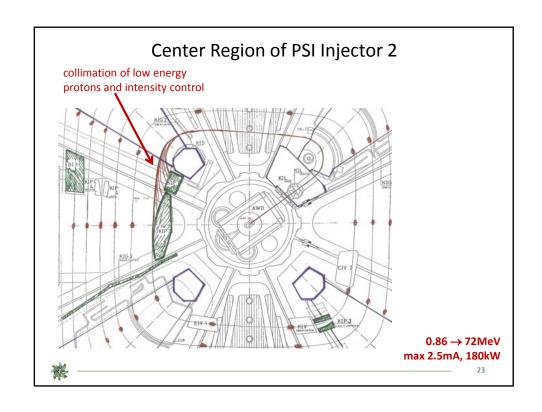
schemes applied in practice:

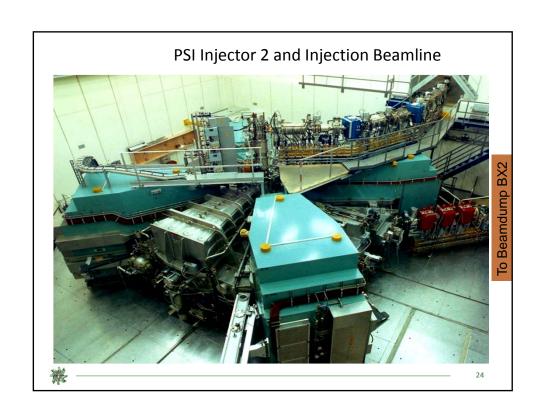
	bunching in cyclotron	external buncher cavities	comment
internal source	x		lowest cost and complication
external source	х	x	higher intensity, variety of ions
DC pre-accelerator Cockcroft-Walton		x	low ΔE , costly
Radio Freq. Quadrupole (RFQ)		х	compact, costly

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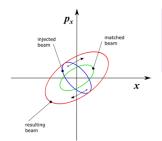






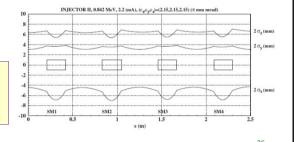
Transverse Matching

 Similar to a synchrotron the envelope function β varies around the circumference; the beam at injection must be matched to avoid blow up and sub-optimal beam distributions



nonetheless of the short «storage» time of a beam in a cyclotron, the distribution starts to filament, if not properly matched

example: beam sizes around the circumference for Inj II cyclotron, PSI [Ch.Baumgarten, [7]]



彩

transverse space charge

especially at low energy space charge effects are critical for the injection of high intensity beams

vertical force from space charge: $F_y = \frac{n_v e^2}{\epsilon_0 \gamma^2} \cdot y, \ n_v = \frac{N}{(2\pi)^{\frac{3}{2}} \sigma_y D_f R \Delta R}$ [constant charge density, D_f = I_{avg}/I_{peak}]

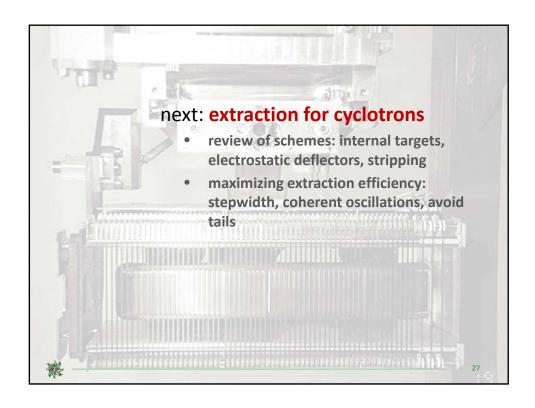
thus, eqn. of motion:

$$\ddot{y} + \left(\omega_c^2 \nu_{y0}^2 - \frac{n_v e^2}{\epsilon_0 m_0 \gamma^3}\right) y = 0$$

 \rightarrow tune shift results in **intensity limit** (see [6])!

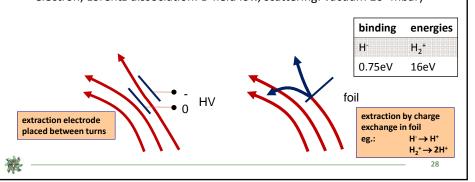
tune shift from forces: $\Delta \nu_y \quad \approx \quad -n_v \frac{2\pi r_p R^2}{\beta^2 \gamma^3 \nu_{y0}}$





electrostatic septum and charge exchange extraction

- simplest solution: use beam without extraction → **internal target**; use some mechanism to exchange target
- **electrostatic deflectors** with thin electrodes, deflecting element should affect just one turn, not neighboured turn → critical, cause of losses
- alternative: charge exchange by stripping foil; accelerate H⁻ or H₂⁺ to extract protons (problem: significant probability for unwanted loss of electron; Lorentz dissociation: B-field low, scattering: vacuum 10⁻⁸mbar)



derivation of relativistic turn separation in a cyclotron

starting point: bending strength

- → compute total log.differential
- \rightarrow use field index $k = R/B \cdot dB/dR$

$$BR = \sqrt{\gamma^2 - 1} \frac{m_0 c}{e}$$

$$\frac{dB}{B} + \frac{dR}{R} = \frac{\gamma d\gamma}{\gamma^2 - 1}$$

$$\frac{dR}{d\gamma} = \frac{\gamma R}{\gamma^2 - 1} \frac{1}{1 + k}$$

radius change per

$$\frac{dR}{dn_t} = \frac{dR}{d\gamma} \frac{d\gamma}{dn_t}$$
 [U_t = energy gain per turn]
$$= \frac{U_t}{m_0 c^2} \frac{\gamma R}{(\gamma^2 - 1)(1 + k)}$$
 isochronicity not conserved (last turns)
$$= \frac{U_t}{m_0 c^2} \frac{R}{(\gamma^2 - 1)\gamma}$$
 isochronicity conserve (general scaling)



discussion: scaling of turn separation

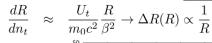
for clean extraction a large stepwidth (turn separation) is of utmost importance; in the PSI Ring most efforts were directed towards maximizing the turn separation

general scaling at extraction:

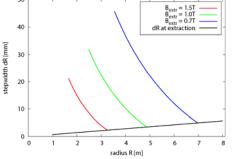
$$\Delta R(R_{\rm extr}) \quad = \quad \frac{U_t}{m_0 c^2} \frac{R_{\rm extr}}{(\gamma^2-1)\gamma} \quad \begin{array}{c} \text{desirable:} \\ \bullet \quad \text{limited energy (< 1GeV)} \\ \bullet \quad \text{large radius } R_{\rm extr} \\ \bullet \quad \text{high energy gain } U_t \end{array}$$

scaling during acceleration:

illustration:



stepwidth vs. radius in cyclotrons of different sizes; 100MeV inj → 800MeV extr



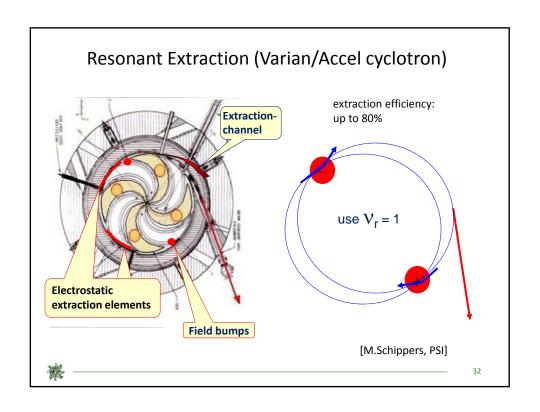
methods to enhance turn separation

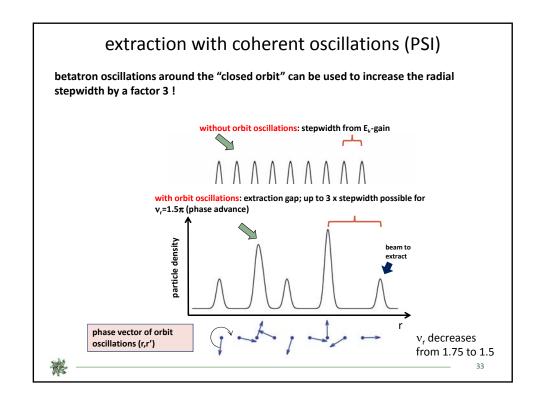
several techniques were invented to "artificially" increase turn separation beyond the magnitude achieved by simple acceleration

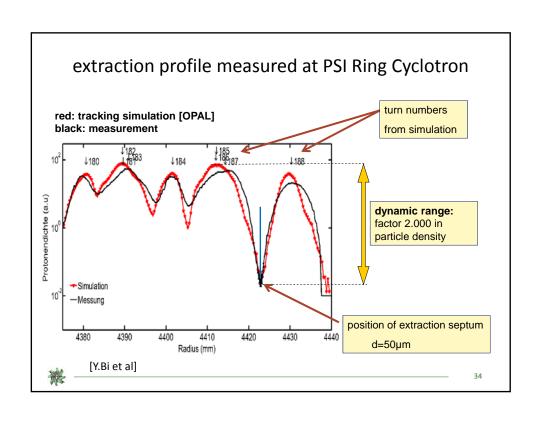
"brute force"	resonant orbit distortion is excited by harmonic coils beyond a certain radius
precessional extraction	resonant excitation at $\nu_{\rm r}\!\!=\!\!1$ plus steep $\nu_{\rm r}$ slope in fringe field
regenerative extraction	using coherent excitation at half integer resonance by gradient bump

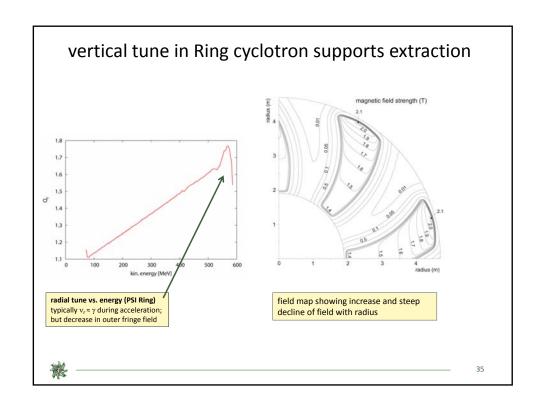
taken from Kleeven [1]

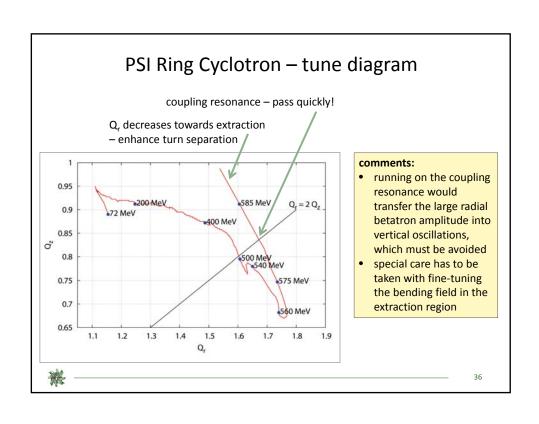


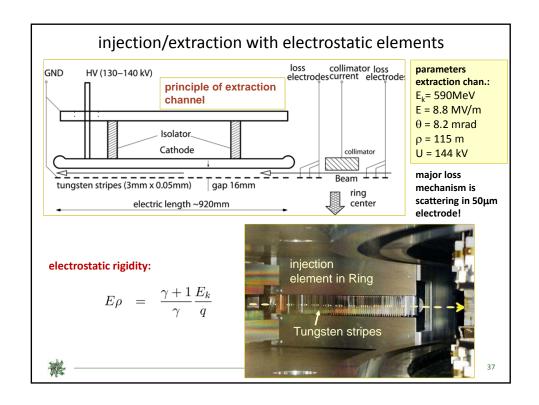


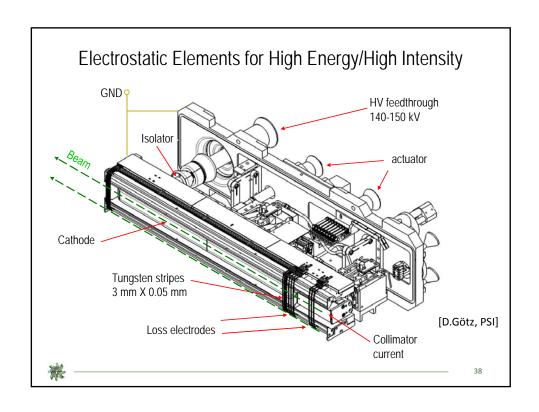












longitudinal space charge (tails at extraction)

sector model:

- → accumulated energy spread transforms into transverse tails
- consider rotating uniform sectors of charge (overlapping turns)
- test particle "sees" only fraction of sector due to shielding of vacuum chamber with gap height 2w

two factors are proportional to the number of turns:

- 1) the charge density in the sector
- 2) the time span the force acts

$$\Delta U_{sc} \approx 2.800\Omega \cdot eI_p \cdot \frac{n_{\max}^2}{\beta_{\max}}$$

derivation see [4]: Joho 1981



3) the inverse of turn separation at extraction: $\frac{1}{\Delta R_{
m extr}} \propto$

 \rightarrow the attainable current at constant losses scales as n_{max}^{-3}

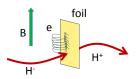


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extraction foil

- thin foil, for example carbon, removes the electron(s) with high probability
- new charge state of ion brings it on a new trajectory → separation from circulating beam
- · lifetime of foil is critical due to heating, fatigue effects, radiation damage
- conversion efficiencies, e.g. generation of neutrals, must be considered carefully

electrons removed from the ions spiral in the magnetic field and may deposit energy in the foil



How much power is carried by the electrons?

ightarrow velocity and thus γ are equal for p and e

$$E_k = (\gamma - 1)E_0$$

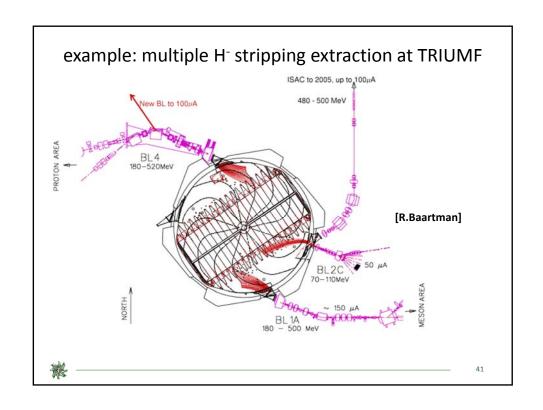
$$\to E_k^e = \frac{E_0^e}{E_0^p} E_k^p = 5.4 \cdot 10^{-4} E_k^p$$

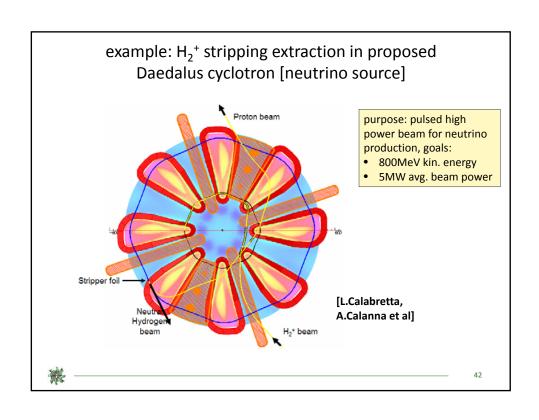
Bending radius of electrons?

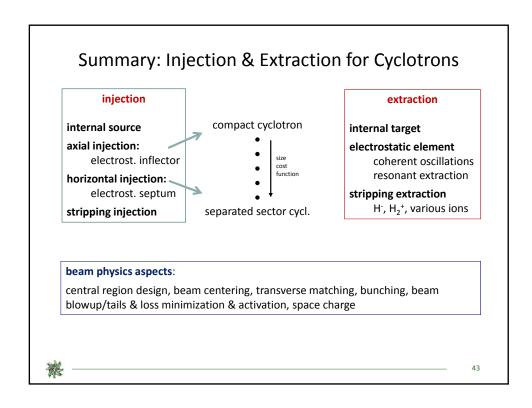
$$\rho^e = \frac{E_0^e}{E_0^p} \rho^p$$

→ typically mm









literature w.r.t. cyclotron injection/extraction comprehensive review of [1] W.Kleeven (IBA), Injection and Extraction for Cyclotrons inj./extr. concepts many examples and [2] P.Heikkinen (Jyväsyla), Injection and Extraction for Cyclotrons calculations for compact machines calculations and matching [3] W.Kleeven & R.Baartman, 2x paper on spiral inflectors, Particle on spiral inflectors Accelerators 41 (1993), pages 41 and 55 extraction for very high [4] W.Joho, High Intensity Problems in Cyclotrons, Proc. 5th intl. intensity Conf. on Cyclotrons and their Applications, Caen, 337-347 (1981) OPAL simulations; [5] Y.Bi, A. Adelmann, et al. Phys. Rev. STAB Vol. 14, 054402 (2011) extraction profile Intensity limitation [6] R.Baartman, Space Charge limit in separate Turn Cyclotrons, Intl. Cycl. Conf. (2013) rn.ch/AccelConf/CYCLOTRONS2013/papers/we2pb01.pdf formation of round Ch.Baumgarten, transverse-longitudinal coupling by space charge in bunches and matching cyclotrons approach ps.org/prab/abstract/10.1103/PhysRevSTAB.14.114201

